Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The HCF of the smallest 2-digit number and the smallest composite number is **[1]**

a) 4 b) 10

c) 20 d) 2

2. If **p** and **q** are natural numbers and **p** is the multiple of **q**, then what is the HCF of **p** and **q**? **[1]**

a) p b) q

c) pq d) p + q

3. The sum of reciprocals of Sharma's age 3 years ago and 5 years from now is $\frac{1}{3}$, then his present age is **[1]**

a) 7 years b) 10 years

c) 6 years d) 8 years

4. The graphs of the equations 6x - 2y + 9 = 0 and 3x - y + 12 = 0 are two lines which are **[1]**

a) perpendicular to each other b) parallel

c) coincident d) intersecting exactly at one point

5. $9x^2 + 12x + 4 = 0$ have **[1]**

a) Real and Distinct roots b) No real roots

c) Distinct roots d) Real and Equal roots

6. Two vertices of △ABC are A (-1, 4) and B(5, 2) and its centroid is G(0, -3). Then, the coordinates of C are **[1]**

a) (4, 3) b) (4, 15)

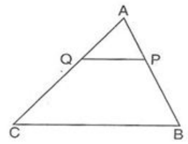
c) (-4, -15) d) (-15, -4)

7. $\triangle$ABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. If $\triangle$DEF ~ $\triangle$ABC and EF = 4 cm, then perimeter of $\triangle$DEF is **[1]**

a) 30 cm b) 7.5 cm

c) 22.5 cm d) 15 cm

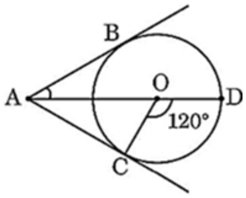
8. In the adjoining figure P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7cm, AQ = 3cm, QC = 6cm and PQ = 4.5cm. The measure of BC is equal to **[1]**



a) 9 cm b) 15 cm

c) 12.5 cm d) 13.5 cm

9. In the given figure, AC and AB are tangents to a circle centered at O. If $\angle$COD = 120^o, then $\angle$BAO is equal to: **[1]**



a) 30^o b) 45^o

c) 90^o d) 60^o

10. $\left(\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}\right)$ is equal to: **[1]**

a) sin 60^o b) tan 60^o

c) cos 30^o d) cos 60^o

11. A man is standing on the deck of a ship, which is 10 m above water level. He observes the angle of elevation of the top of a hill as 45^o and the angle of depression of the base of the hill as 30^o. Calculate the distance of the hill from the ship and the height of the hill. (use $\sqrt{3}$ = 1.732) **[1]**

a) 17.89 m, 28.32 m b) 17.32 m, 27.32 m

c) 18.32 m, 28.32 m d) 8.32 m, 29.22 m

12. 2 $\cos^2\theta(1 + \tan^2\theta)$ is equal to: **[1]**

a) 3 b) 2

c) 1 d) 0

13. The perimeter of a certain sector of a circle of radius 6.5 cm is 31 cm. The area of the sector will be **[1]**

a) 58.5 cm^2 b) 45 cm^2

c) 49 cm^2 d) 48.5 cm^2

14. If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20π cm^2, then find it's radius **[1]**

a) 10 cm b) 16 cm

c) 12 cm d) 8 cm

15. A bag contains 3 white, 4 red and 5 black balls. One ball is drawn at random. What is the probability that the ball drawn is neither black nor white? **[1]**

a) $\frac{1}{3}$ b) $\frac{1}{4}$

c) $\frac{1}{2}$ d) $\frac{4}{3}$

16. Median = ? **[1]**

a) $l+\left\{h\times\frac{\left(cf-\frac{N}{2}\right)}{f}\right\}$ b) $l-\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

c) $l+\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$ d) $l+\left\{h\times\frac{\left(\frac{N}{2}+cf\right)}{f}\right\}$

17. A cylindrical vessel 32 cm high and 18 cm as the radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, the radius of its base is **[1]**

a) 36 cm b) 24 cm

c) 12 cm d) 48 cm

18. $\frac{\text{Upper class limit} + \text{Lower class limit}}{2} =$ **[1]**

a) frequency b) class mark

c) Class interval d) class size

19. **Assertion (A):** Point P(0, 2) is the point of intersection of y-axis with the line 3x + 2y = 4. **[1]**

Reason (R): The distance of point P(0, 2) from x-axis is 2 units.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\sqrt{2}$(5 - $\sqrt{2}$) is an irrational number. **[1]**

Reason (R): Product of two irrational numbers is always irrational.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

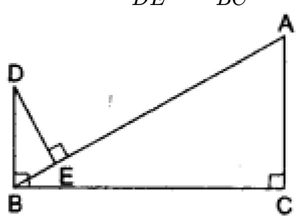
d) A is false but R is true.

Section B

21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the pair of linear equations are consistent, or inconsistent: $\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12. **[2]**

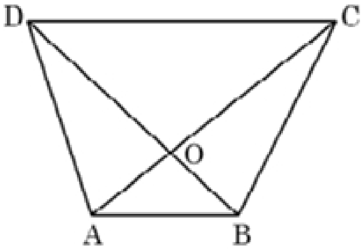
22. In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. **[2]**

Prove that $\frac{BE}{DE} = \frac{AC}{BC}$



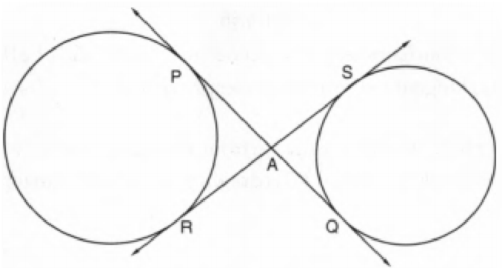
OR

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{OD}$



Show that quadrilateral ABCD is a trapezium.

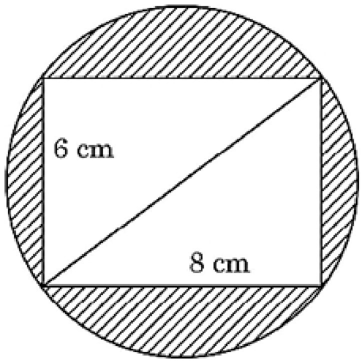
23. In fig common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS. **[2]**



24. Prove the trigonometric identity: **[2]**

$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

25. Reeti prepares a Rakhi for her brother Ronit. The Rakhi consists of a rectangle of length 8 cm and breadth 6 cm inscribed in a circle as shown in the figure. Find the area of the shaded region. (Use π = 3·14) **[2]**



OR

The minute hand of a clock is 7.5 cm long. Find the area of the face of the clock described by the minute hand in 56 minutes.

Section C

26. Show that $5-\sqrt{3}$ is irrational. **[3]**

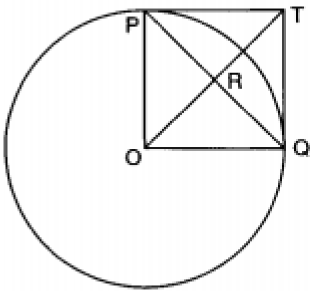
27. Find the zeroes of the polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial. **[3]**

28. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu? Solve the pair of the linear equation obtained by the elimination method. **[3]**

OR

A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs 3,000 as hostel charges whereas Mansi who takes food for 25 days pays Rs 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

29. In figure $PO \perp QO$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other. **[3]**



30. Prove the identity: **[3]**

$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A- cosec^3 A} = \sin^2 A \cos^2 A$

OR

If $\sin(A-B) = \frac{1}{2}, \cos(A+B) = \frac{1}{2}, 0^o < A+B \leqslant 90^o, A > B$, find the values of A and B.

31. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25? **[3]**

Section D

32. If x = - 2 is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal. **[5]**

OR

A and B jointly finish a piece of work in 15 days. When they work separately, A takes 16 days less than the number of days taken by B to finish the same piece of work. Find the number of days taken by B to finish the work.

33. AD and PM are medians of triangles ABC and PQR respectively where $\triangle$ABC $\sim$ $\triangle$PQR. Prove that $\frac{AB}{PQ}$ = $\frac{AD}{PM}$. **[5]**

34. A student was asked to make a model shaped like a cylinder with two cones attached to its ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its total length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model. **[5]**

OR

A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. Also, find the volume of the vessel.

35. If the median of the distribution given below is 28.5, then find the values of x and y. **[5]**

Class Interval	**frequency**
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

i. Find the production during first year. (1)

ii. Find the production during 8th year. (1)

iii. Find the production during first 3 years. (2)

OR

In which year, the production is ₹ 29,200. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

To raise social awareness about the hazards of smoking, a school decided to start a 'No smoking' campaign. 10 students are asked to prepare campaign banners in the shape of a triangle. The vertices of one of the triangles are P(-3, 4), Q(3, 4) and R(-2, -1).

i. What are the coordinates of the centroid of △PQR? (1)

ii. If T be the mid-point of the line joining R and Q, then what are the coordinates of T? (1)

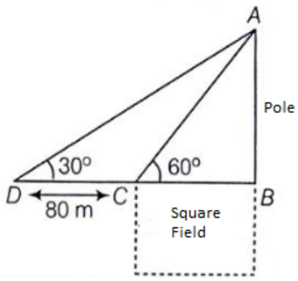
iii. If U be the mid-point of line joining R and P, then what are the coordinates of U? (2)

OR

What are the coordinates of centroid of △STU? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^{o}.



i. Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole. (1)

ii. Find the length of his square field so that he can buy material to do the fencing work accordingly. (1)

iii. Find the Distance from Farmer at position C and top of the pole? (2)

OR

Find the Distance from Farmer at position D and top of the pole? (2)

Solution

Section A

1.

(d) 2

Explanation: Smallest two digit number is 10 and smallest composite number is 4 .

Clearly, 2 is the greatest factor of 4 and 10, so their H.C.F. is 2.

2.

(b) q

Explanation: q is a factor of p. In this case, the highest common factor (HCF) of p and q is q itself because it is the largest number that can evenly divide both p and q. Therefore, if p is a multiple of q, the HCF of p and q is q.

3. **(a)** 7 years

Explanation: Let Sharma's present age be x years

then, his age 3 years ago is (x - 3) years and 5 years from now is (x + 5) years. According to question,

$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2+5x-3x-15}=\frac{1}{3}$

$\Rightarrow$ 6x + 6 = x^2 + 2x - 15

$\Rightarrow$ x^2 - 4x - 21 = 0

$\Rightarrow$ x^2 - 7x + 3x - 21 = 0

$\Rightarrow$ x(x - 7) + 3(x - 7) = 0

$\Rightarrow$ (x + 3)(x - 7) = 0

$\Rightarrow$ x + 3 = 0 and x - 7 = 0

$\Rightarrow$ x = -3 and x = 7

But x = -3 does not satisfy the given condition.

Therefore, Sharma's present age is 7 years.

4.

(b) parallel

Explanation: We have,

6x - 2y + 9 = 0

And, 3x - y + 12 = 0

Here, a_1 = 6, b_1 = - 2 and c_1 = 9

a_2 = 3, b_2 = - 1 and c_2 = 12

$\frac{a1}{a2}=\frac{6}{3}=\frac{2}{1}, \frac{b1}{b2}=\frac{-2}{-1}=\frac{2}{1}$ and $\frac{c_1}{c_2}=\frac{9}{12}=\frac{3}{4}$

Clearly, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Hence, the given system has no solution and the lines are parallel.

5.

(d) Real and Equal roots

Explanation: Comparing the given equation to the below equation

ax^2 + bx + c = 0

a = 9, b = 12, c = 4

D = b^2 - 4ac

D = 12^2 - 4 $\times$ 9 $\times$ 4

D = 144 - 144

D = 0

If $b^2 - 4ac = 0$ then equation have equal and real roots.

6.

(c) (-4, -15)

Explanation: Let the vertex C be C (x,y). Then

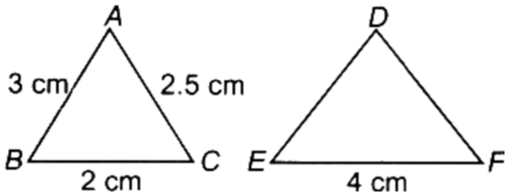
$\frac{-1+5+x}{1} = 0$ and $\frac{4+2+y}{3} = -3 \Rightarrow x + 4 = 0$ and $6 + y = -9$

$\therefore \quad x = -4$ and $y = -15$

so, the coordinates of C are (-4, -15).

7.

(d) 15 cm

Explanation: 

Since, $\triangle$DEF ~ $\triangle$ABC [Given]

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{3}{DE} = \frac{2}{4} \Rightarrow$ DE = 6 cm

Also, $\frac{AC}{DF} = \frac{BC}{EF} \Rightarrow \frac{2.5}{DF} = \frac{2}{4} \Rightarrow$ DF = 5 cm

$\therefore$ Perimeter of $\triangle$DEF = DE + EF + FD

= 6 cm + 4 cm + 5 cm = 15 cm

8.

(d) 13.5 cm

Explanation: In $\Delta \mathrm{ABC}$,

$\Rightarrow \frac{\mathrm{AQ}}{\mathrm{QC}} = \frac{\mathrm{AP}}{\mathrm{PB}} \Rightarrow \frac{3}{9} = \frac{3.5}{10.5} \Rightarrow \frac{1}{3} = \frac{1}{3}$

Since $\frac{\mathrm{AQ}}{\mathrm{QC}} = \frac{\mathrm{AP}}{\mathrm{PB}}$,

therefore, $\mathrm{QP} \parallel \mathrm{BC}$

$\therefore \frac{\mathrm{AQ}}{\mathrm{AC}} = \frac{\mathrm{QP}}{\mathrm{BC}}$

$\Rightarrow \frac{3}{9} = \frac{4.5}{\mathrm{BC}}$

$\Rightarrow$ BC = 13.5 cm

9. **(a)** 30^o

Explanation: $\angle$ACO = 90^o (angle between radius and tangent)

$\angle$OAC + $\angle$ACO = 120^o. {Sum of two interior opposite angle is equal to exterior angle}

$\angle$OAC + 90^o = 120^o

$\angle$OAC = 30^o

Now, In $\triangle$ACO and $\triangle$ABO

OC = OB (radius)

$\angle$ACO = $\angle$ABO = 90^o

AC = AB (Length of tangent from external Point)

$\therefore$ $\triangle$ACO $\cong$ $\triangle$ABO (by SAS)

hence $\angle$BAO = $\angle$CAO = 30^o

$\therefore \angle$ BAO = 30^o

10.

(d) cos 60^o

Explanation: $\frac{1-\tan^2 30^o}{1+\tan^2 30^o} = \frac{1-(\frac{1}{\sqrt{3}})^2}{1+(\frac{1}{\sqrt{3}})^2}$

$= \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$

$= \frac{\frac{2}{3}}{\frac{4}{3}}$

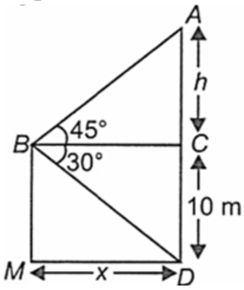
$= \frac{1}{2}$

$= \cos 60^o$

11.

(b) 17.32 m, 27.32 m

Explanation: Let x be the distance of hill from the ship and h + 10 be the total height of hill.



In $\triangle$ACB, $\tan 45^o = \frac{AC}{BC} = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} = x$

In $\triangle$BCD,

$\tan 30^o = \frac{CD}{BC} = \frac{10}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x} \Rightarrow x = 10\sqrt{3}$ m

$\therefore$ Height of hill = h + 10

$= 10\sqrt{3} + 10 = 10 \times 1.732 + 10 = 27.32$ m

Distance of ship from hill = x = $10\sqrt{3}$ m = 17.32 m

12.

(b) 2

Explanation: 2

13. **(a)** 58.5 cm^2

Explanation: Perimeter of a sector of circle = 31 cm

Radius = 6.5 cm

Arc length = 31 - (6.5 + 6.5) = 18 cm

Now, Area of sector $= \frac{1}{2} \times$ Arc length $\times$ radius $= \frac{1}{2} \times 18 \times 6.5$ = 58.5 cm^2

14.

(d) 8 cm

Explanation: We have given length of the arc and area of the sector bounded by that arc and we are asked to find the radius of the circle.

We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$.

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

Now we will substitute the values.

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

$20\pi = \frac{\theta}{360} \times \pi r^2$ ....(1)

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

$5\pi = \frac{\theta}{360} \times 2\pi r$ ....(2)

$\frac{20\pi}{5\pi} = \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r}$

$\frac{20}{5} = \frac{r^2}{2r}$

$\therefore 4 = \frac{r}{2}$

$\therefore$ r = 8

Therefore, radius of the circle is 8 cm.

15. **(a)** $\frac{1}{3}$

Explanation: Total number of balls in the bag = 3 + 4 + 5 = 12.

Number of non-black and non-white balls = 4.

$\therefore$ P (getting a ball which is neither black nor white) $= \frac{4}{12} = \frac{1}{3}$

16.

(c) $l+\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

Explanation: $l+\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

17. **(a)** 36 cm

Explanation: Radius of a cylindrical vessel (r_1) = 18 cm

and height (h_1) = 32 cm

$\therefore$ Volume of sand filled in it = $\pi r_1^2 h_1$

= π $(18)^2$ $\times$ 32 = π $\times$ 324 $\times$ 32 cm^3

= 10368π cm^3

Now height of the conical heap (h_2) = 24 cm

Let r_2 be the its radius, then

$\frac{1}{3}\pi r_2^2 h_2$ = 10368π

$\Rightarrow \frac{1}{3}\pi r_2^2\times 24=10368\pi$

$\Rightarrow 8\pi r_2^2=10368\pi$

$r_2^2=\frac{10368\pi}{8\pi}$ = 1296

$\therefore r_2=\sqrt{1296}$ = 36

Hence radius of the base of the heap = 36 cm

18.

(b) class mark

Explanation: In each class interval of grouped data, there are two limits or boundaries (upper limit and lower limit) while the mid-value is equal to $\frac{\text{Upper class limit}+\text{Lower class limit}}{2}$. These mid-values are also known as Classmark.

19.

(c) A is true but R is false.

Explanation: Put (0, 2) in 3x + 2y = 4

We get LHS = RHS

Assertion is true.

Reason is also true. But it is not the correct explanation of Assertion (A).

Hence option B is the answer.

20.

(c) A is true but R is false.

Explanation: A is true but R is false.

Section B

21. Given equations are:

$\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12

Compare equation $\frac{4}{3}x+2y=8$ with $a_1x + b_1y + c_1 = 0$ and 2x + 3y = 12

with $a_2x + b_2y + c_2 = 0$, We get, $a_1=\frac{4}{3}$, $a_1=\frac{4}{3}$, $b_1 = 2$, $c_1 = -8$, $a_2 = 2$, $b_2 = 3$, $c_2 = -12$

$\frac{a_1}{a_2}=\frac{\frac{4}{3}}{2}=\frac{2}{3}, \frac{b_1}{b_2}=\frac{2}{3}$ and $\frac{c_1}{c_2}=\frac{8}{12}=\frac{2}{3}$

Here $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, the lines have infinitely many solutions.

Hence, they are consistent.

22. In $\triangle BED$ and $\triangle ACB$, we have

$\angle BED=\angle ACB=90^{\circ}$

$\because \angle B+\angle C=180^0$

$\therefore BD \| AC$

$\angle EBD=\angle CAB$ (Alternate angles)

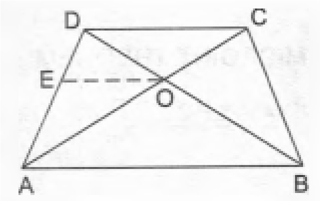
Therefore, by AA similarity theorem, we get

$\triangle BED \sim \triangle ACB$

$\Longrightarrow \frac{BE}{AC} = \frac{DE}{BC}$

$\Longrightarrow \frac{BE}{DE} = \frac{AC}{BC}$

OR



Given A quadrilateral ABCD whose diagonals AC and BD intersect at a point O such that $\frac{AO}{OC} = \frac{BO}{OD}$

To prove ABCD is a trapezium, i.e., AB || DC.

Construction Draw EO || DC, meeting AD at E.

Proof In $\triangle ACD, EO \| DC$

$\therefore \frac{AO}{OC} = \frac{AE}{ED}$ [by Thales' theorem].

But, $\frac{AO}{OC} = \frac{BO}{OD}$ (given)

$\therefore \frac{BO}{OD} = \frac{AE}{ED} \Rightarrow \frac{DO}{OB} = \frac{DE}{EA}$ in ΔDAB

So, EO || AB [by the converse of Thales' theorem].

But, EO || DC.

Hence, AB || DC, i.e., ABCD is a trapezium.

23. We have in given figure common tangents PQ and RS to two circles intersect at A. Since tangents drawn from an external points to a circle are equal.

$\therefore$ AP = AR

and AQ = AS

$\therefore$ AP + AQ = AR + AS [Adding]

$\Rightarrow$ PQ = RS

Hence proved.

24. We have,

$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$\Rightarrow \quad \text{LHS} = \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$\Rightarrow \quad \text{LHS} = \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

$\Rightarrow \quad \text{LHS} = \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$\Rightarrow \quad \text{LHS} = \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow \quad \text{LHS} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta = \text{RHS}$

25. Diagonal of rectangle = $\sqrt{6^2 + 8^2}$ = 10

$\therefore$ Radius of circle r = $\frac{10}{2}$ = 5

Area of circle = 3.14 $\times$ 5 $\times$ 5

= 78.5

Area of rectangle = 6 $\times$ 8 = 48

Area of shaded region = 78.5 - 48

= 30.5 cm^2

$\therefore$ Area of shaded region is 30.5 cm^2

OR

Angle described by the minute hand in 60 minutes = 360^0

$\therefore$Angle described by the minute hand in 56 minutes = $\left(\frac{360}{60} \times 56\right)^0 = 336^\circ$

$\therefore \theta = 336^\circ$ and r = 7.5 cm

$\therefore$Area swept by the minute hand in 56 minutes $= \left(\frac{\pi r^2 \theta}{360}\right)$

$= \left(3.14 \times 7.5 \times 7.5 \times \frac{336}{360}\right) \text{cm}^2$

= 165 cm^2

Section C

26. Let us assume, to the contrary, that $5-\sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b ≠ 0) such that $5-\sqrt{3}=\frac{a}{b}$

Therefore, $5-\frac{a}{b}=\sqrt{3}$

Rearranging this equation, we get $\sqrt{3}=5-\frac{a}{b}=\frac{5b-a}{b}$

Since a and b are integers, we get $5-\frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5-\sqrt{3}$ is rational.

So, we conclude that $5-\sqrt{3}$ is irrational.

27. $7y^2 - \frac{11}{3}y-\frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}(3y - 2)(7y + 1)$

$\Rightarrow y=\frac{2}{3},\frac{-1}{7}$ are zeroes of the polynomial.

If Given polynimoal is $7y^2 - \frac{11}{3}y-\frac{2}{3}$

Then a = 7 , b = $-\frac{11}{3}$ and c = $-\frac{2}{3}$

Sum of zeroes = $\frac{2}{3}+\frac{-1}{7}=\frac{14-3}{21}=\frac{11}{21}$ .... (i)

Also, $\frac{-b}{a}=\frac{-\left(\frac{-11}{3}\right)}{7}=\frac{11}{21}$ .... (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Now, product of zeroes = $\frac{2}{3}\times\frac{-1}{7}=\frac{-2}{21}$ .... (iii)

Also, $\frac{c}{a}=\frac{\frac{-2}{3}}{7}=\frac{-2}{21}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

28. Let the present age of Nuri and Sonu be x years and y years respectively.

Then, according to the question,

x – 5 = 3(y – 5)

⇒ x – 5 = 3y – 15

⇒x – 3y = –10 .... (1)

x + 10 = 2(y + 10)

x + 10 = 2y + 20

⇒ x – 2y = 10 ------------ (2)

Subtracting equation (2) from equation (1), we get

– y= – 20

⇒ y = 20

Subtracting equation (2) from equation (1), we get

x – 2(20) = 10

⇒ x – 40 = 10

⇒ x = 40 + 10

⇒ x = 50

Hence, Nuri and Sonu are 50 years and 20 years old respectively at present.

Verification.Subtracting the value of x = 50 and y = 20,we find that both the eqations (1) and (2) are satisfied as shown below:

x – 3y = 50 – 3(20) = 50 – 60 = – 10

x – 2y = 50 – 2(20) = 50 – 40=10

Hence,the solution is correct.

OR

Let fixed charge be Rs x and charge taken per day for food be Rs y

x + 20y = 3000 ....(i)

x + 25y = 3500 ......(ii)

Subtracting (i) from (ii)

$$\begin{array}{r} x + 25y = 3500 \\ x + 20y = 3000 \\ - \quad - \qquad - \\ \hline 5y = 500 \\ \hline y = 100 \end{array}$$

Substituting this value of y in (i)

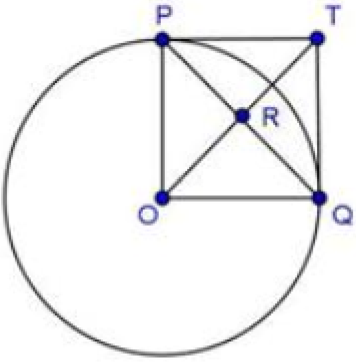
x + 20(100) = 3000

x = 1000

x = 1000 and y = 100

Fixed charge and cost of food per day are Rs 1000 and Rs100

29. Given, PO⊥QO and The tangents to the circle at P and Q intersect at a point T.



Consider, $\Delta TPO \text{ and } \Delta TQO$

PT = TQ [∵ Tangents from external point are equal in length]

OT = OT [Common]

$\angle TPO = \angle TQO = 90^\circ$

So, by RHS rule, we have

$\Delta TPO \cong \Delta TQO$

$\Rightarrow \angle PTO = \angle QTO$ ...(i) [C.P.C.T.]

Now, In ΔPTR and ΔQTR

PT = TQ [∵ Tangents from external point are equal in length]

$\angle PTO = \angle QTO$ [By equation (i)]

TR = TR [Common]

So, by SAS rule, we have

$\Delta PTR \cong \Delta QTR$

∴ PR = RQ ...(ii)

And, $\angle TRP = \angle TRQ$

But, $\angle TRP + \angle TRQ = 180^\circ$

$\Rightarrow 2\angle TRP = 180^\circ$

$\Rightarrow \angle TRP = 90^\circ$ ...(iii)

Therefore, PQ and OT are right bisectors of each other.

30. We have,

$$\text{LHS} = \frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{\left(\frac{1}{\cos^3 A}-\frac{1}{\sin^3 A}\right)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)}{\left(\frac{\sin^3 A-\cos^3 A}{\sin^3 A\cos^3 A}\right)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{1}{\sin A\cos A}\right)(\sin A-\cos A)\sin^3 A\cos^3 A}{(\sin^3 A-\cos^3 A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{(\sin A\cos A+1)(\sin A-\cos A)\sin^2 A\cos^2 A}{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}$$ [∵ $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

$$\Rightarrow \quad \text{LHS} = \frac{(\sin A\cos A+1)\sin^2 A\cos^2 A}{(1+\sin A\cos A)}$$ = $\sin^2 A \cos^2 A$ = RHS

OR

It is given that $\sin(A-B) = \frac{1}{2}, \cos(A+B) = \frac{1}{2}, 0^o < A+B \leqslant 90^o, A > B$, we have to find the values of A and B.

Now, $\sin(A-B)=\frac{1}{2}$
$\Rightarrow \sin(A-B)=\sin 30^{o}$ $[\because \sin 30^{o}=\frac{1}{2}]$
On equating both sides , we get
$A-B=30^{o}$ ...(i)
Also, $\cos(A+B)=\frac{1}{2}$
$\Rightarrow \cos(A+B)=\cos 60^{o}$ $[\because \cos 60^{o}=\frac{1}{2}]$
On equating both sides, we get
$A+B=60^{o}$ ...(ii)
On Adding Eq(i) and Eq(ii), we get
$2A=90^{o}$
$\Rightarrow A=45^{o}$
From Eq(i) , we get $45^{o}-B=30^{o}$
$\Rightarrow B=45^{o}-30^{o}$
$\therefore B=15^{o}$
Hence, $A=45^{o}$ *and* $B=15^{o}$

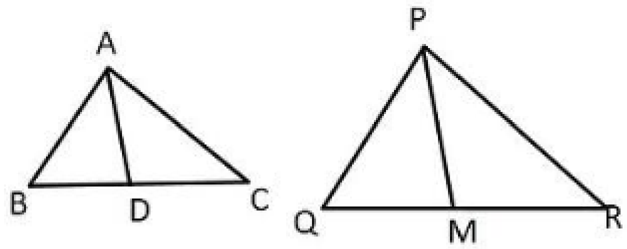
31. The person having higher probability of getting the number 25 has the better chance.
When a pair of dice is thrown, there are 36 elementary events which are as follows:
(1, 1) , (1, 2), (1,3), (1,4), (1,5), (1, 6)
(2, 1) , (2, 2), (2,3), (2,4),(2,5), (2, 6)
(3,1) , (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1) , (4,2), (4,3),(4,4), (4,5), (4,6)
(5,1) , (5,2), (5,3), (5,4), (5,5), (5,6)
(6, 1), (6, 2),(6, 3), (6, 4), (6, 5), (6, 6)
Therefore, the product of numbers on two dice can take values 1, 2, 3, ..., 36.
We observe that the product of two numbers on two dice will be 25 if both the dice show number 5. Therefore, there is only one elementary event, viz., (5, 5), which is favourable for getting 25.
p_1 = Probability that Peter throws $25=\frac{1}{36}$
Rina throws a die on which she can get any one of the six numbers 1, 2, 3, 4, 5, 6 as an outcome. If she gets number 5 on the upper face of the die thrown,then the square of the number is 25.
p_2 = Probability that the square of number obtained is $25=\frac{1}{6}$
Therefore, $p_2 > p_1$. Therefore, Rina has better chance to get the number 25.

Section D

32. Here x = - 2 is the root of the equation $3x^2+7x+p=0$
then, $3(-2)^2+7(-2)+p=0$
or, p=2
Roots of the equation $x^2+4kx+k^2-k+2=0$ are equal,then,
$16k^2-4\left(k^2-k+2\right)=0$
or, $16k^2-4k^2+4k-8=0$
or, $12k^2+4k-8=0$
or, $3k^2+k-2=0$
or, (3k-2)(k + 1) = 0
or, $k=\frac{2}{3},-1$
Hence, roots = $\frac{2}{3},-1$

OR

Let the number of days taken by B be x days.
$\therefore$ number of days taken by A = (x - 16) days
$\frac{1}{\mathrm{x}}+\frac{1}{\mathrm{x}-16}=\frac{1}{15}$
$\therefore x^2 - 46x + 240 = 0$
(x - 40) (x - 6) = 0
x = 40, 6 Rejected ($\because$ 6 - 16 is -ve)
$\therefore$ Number of days taken by B = 40 days

33. 

Given: In $\triangle$ABC and $\triangle$PQR, AD is the median of $\triangle$ABC, PM is the median of $\triangle$PQR and $\triangle ABC \sim \triangle PQR$.

To Prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof:

Since AD is the median

BD = CD = $\frac{1}{2}$BC

Similarly, PM is the median

QM = RM = $\frac{1}{2}$QR

Now,

$\triangle ABC \sim \triangle PQR$. ($\because$ given)

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ($\because$ Corresponding sides of similar triangle are proportional)

So,

$\frac{AB}{PQ} = \frac{BC}{QR}$

$\frac{AB}{PQ} = \frac{2BD}{2QM}$ (Since AD & PM are medians)

$\frac{AB}{PQ} = \frac{BD}{QM}$ ....(1)

Also, since $\triangle ABC \sim \triangle PQR$.

$\angle B = \angle Q$ ($\because$ Corresponding angles of similar triangles are equal).............(2)

Now,

In $\triangle$ABD & $\triangle$PQM

$\angle B = \angle Q$ [$\because$ from (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [$\because$ from (1)]

Hence by SAS similarly,

$\triangle ABD \sim \triangle PQM$

Since corresponding sides of similar triangles are proportional,

$\frac{AB}{PQ} = \frac{AD}{PM}$

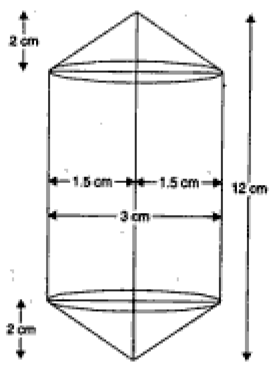
Hence proved.

34. For upper conical portion

Radius of the base(r) = 1.5 cm

Height (h_1) = 2 cm

$\therefore$ Volume = $\frac{1}{3}$ $\pi r^2 h_1 = \frac{1}{3}\pi(1.5)^2(2) = 1.5\pi$ cm^3



For lower conical portion

Volume = 1.5π cm^3

For central cylindrical portion

Radius of the base(r) = 1.5 cm

Height (h_2) = 12 - (2 + 2) = 12 - 4 = 8 cm

$\therefore$ Volume = $\pi r^2 h_2 = \frac{1}{3}\pi(1.5)^2(8) = 18\pi$ cm^3

Therefore, volume of the model = $1.5\pi + 1.5\pi + 18\pi = 21\pi = 21 \times \frac{22}{7} = 66$ cm^3

Hence, the volume of the air contained in the model that Rechel made is 66 cm^3.

OR

Radius of hemispherical bowl = radius of cylinder = 7 cm
Height of cylinder = 13 - 7 = 6 cm

Inner surface area of the vessel = $2\pi rh + 2\pi r^2$
$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 7(6 + 7)$

$= 44 \times 13 = 572$ cm^2

Volume of the vessel = $\pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 (h + \frac{2}{3}r)$
$= \frac{22}{7} \times 7 \times 7 (6 + \frac{14}{3})$
$= \frac{4928}{3}$ cm^3 or 1642.67 cm^3

35.

Monthly Consumption	**Number of consumers** (f_i)	**Cumulative Frequency**
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
Total	$\sum f_i = n = 60$	

Here, $\sum f_i = n = 60$, then $\frac{n}{2} = \frac{60}{2} = 30$, also, median of the distribution is 28.5, which lies in interval 20 – 30.
$\therefore$ Median class = 20 – 30
So, l = 20, n = 60, f = 20, cf = 5 + x and h = 10
$\because 45 + x + y = 60$
$\Rightarrow x + y = 15$ ………..........(i)
Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$
$\Rightarrow 28.5 = 20 + \left[\frac{30-(5+x)}{20}\right] \times 10$
$\Rightarrow 28.5 = 20 + \frac{30-5-x}{2}$
$\Rightarrow 28.5 = \frac{40+25-x}{2}$
$\Rightarrow 57.0 = 65 - x$
$\Rightarrow x = 65 - 57 = 8$
$\Rightarrow$ x = 8
Putting the value of x in eq. (i), we get,
8 + y = 15
$\Rightarrow$ y = 7
Hence the value of x and y are 8 and 7 respectively.

Section E

36. i. Let 1st year production of TV = x
Production in 6th year = 16000
$t_6 = 16000$
$t_9 = 22,600$
$t_6 = a + 5d$
$t_9 = a + 8d$

$$16000 = x + 5d \ ...(i)$$
$$22600 = x + 8d \ ...(ii)$$
$$- \quad - \quad -$$
$$-6600 = -3d$$

d = 2200

Putting d = 2200 in equation ...(i)

16000 = x + 5 $\times$ (2200)

16000 = x + 11000

x = 16000 - 11000

x = 5000

$\therefore$ Production during 1^{st} year = 5000

ii. Production during 8th year is (a + 7d) = 5000 + 7(2200) = 20400

iii. Production during first 3 year = Production in (1^{st} + 2^{nd} + 3^{rd}) year

Production in 1^{st} year = 5000

Production in 2^{nd} year = 5000 + 2200

= 7200

Production in 3^{rd} year = 7200 + 2200

= 9400

$\therefore$ Production in first 3 year = 5000 + 7200 + 9400

= 21,600

OR

Let in n^{th} year production was = 29,200

$t_n = a + (n - 1)d$

29,200 = 5000 + (n - 1) 2200

29,200 = 5000 + 2200n - 2200

29200 - 2800 = 2200n

26,400 = 2200n

$\therefore n = \frac{26400}{2200}$

n = 12

i.e., in 12^{th} year, the production is 29,200

37. i. We have, P(-3, 4), Q(3, 4) and R(-2, -1).

$\therefore$ Coordinates of centroid of $\triangle$PQR

= $\left(\frac{-3+3-2}{3}, \frac{4+4-1}{3}\right) = \left(\frac{-2}{3}, \frac{7}{3}\right)$

ii. Coordinates of T = $\left(\frac{-2+3}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

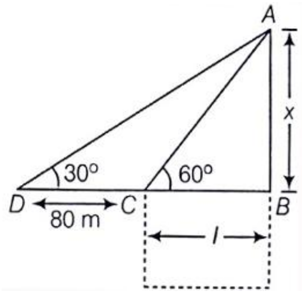
iii. Coordinates of U = $\left(\frac{-2-3}{2}, \frac{-1+4}{2}\right) = \left(\frac{-5}{2}, \frac{3}{2}\right)$

OR

The centroid of the triangle formed by joining the mid-points of sides of a given triangle is the same as that of the given triangle.

So, centroid of $\triangle$STU = $\left(\frac{-2}{3}, \frac{7}{3}\right)$

38. i. The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

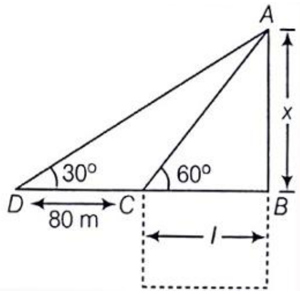
Now, l = 40 metres

We get,

x = $\sqrt{3}$ l = 40$\sqrt{3}$ = 69.28

Thus, height of the pole is 69.28 metres.

ii. The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

In △ABC,

tan 60° = $\frac{x}{l}$

$\sqrt{3} = \frac{x}{l}$

x = $\sqrt{3}$ l ...(i)

Now, in △ABD,

tan 30° = $\frac{x}{80+l}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(i))

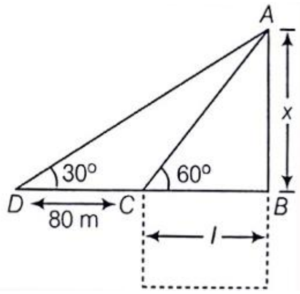
80 + l = 3l

2l = 80

l = 40

Thus, length of the field is 40 metres.

iii. The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position C and top of the pole is AC.

In △ABC

cos 60° = $\frac{CB}{AC}$

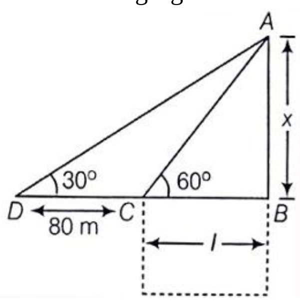
$\Rightarrow AC = \frac{CB}{\cos 60^0}$

$\Rightarrow AC = \frac{40}{\frac{1}{2}}$

⇒ AC = 80 m

OR

The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position D and top of the pole is AD

In △ABC

cos 30^o = $\frac{DB}{AD}$

$\Rightarrow AD = \frac{DB}{\cos 30^0}$

$\Rightarrow AD = \frac{120}{\frac{\sqrt{2}}{2}} = \frac{240}{\sqrt{3}}$

$\Rightarrow$ AC = 138.56 m