Derivative of Exponential Function

Q.1. Differentiate $3^x/(2 + \sin x)$.

Solution : 1

Let $y = 3x/(2 + \sin^{x})$

Then $dy/dx = \{(2 + \sin x).d/dx(3^{x}) – 3^{x}.d/dx(2 + \sin x)\}/(2 + \sin x)^{2}$

$= \{(2 + \sin x).\ 3^{x} \log 3 – 3x.(0 + \cos x)\}/(2 + \sin x)^{2}$

$= 3^{x}\{(2 + \sin x) \log 3 – \cos x\}/(2 + \sin x)^{2}$.

Q.2. If $e^{x + y} = x y$, show that $dy / dx = [y (1 – x)] / [x (y – 1)]$.

Solution : 2

We have, $e^{x + y} = x y$,

Taking log of both the sides,

$(x + y) \text{loge}_{e} = \text{loge } x + \log_{e} y$

Or, $x + y = \text{loge } x + \log_{e} y$ [As, $\text{loge } e = 1$]

Differentiating both sides with respect to x we get,

$1 + dy / dx = 1 / x + (1/y) . dy / dx$

Or, $[1 – (1 / y)] dy / dx = (1 / x) – 1$

Or, $dy / dx = (1 – x) / x \times y / (y – 1)$

$= [y (1 – x)] / [x (y – 1)]$ **[Proved.]**

Q.3. If $y = e^{\wedge}\sin x^{2}$, find dy / dx.

Solution : 3

We have , y = e^sin x^2,

dy / dx = e^sin x^2 × d/dx (sin x^2)

= e^sin x^2 × cos x^2 × d / dx (x^2)

= e^sin x^2× cos x^2 × (2x)

= 2 x cos x^2 e^sin x^2.

Q.4. If y = [e^x + e^{-x}] / [e^x – e^{-x}], find dy / dx.

Solution : 4

y = [e^x + e^{-x}] / [e^x– e^{-x}]

= [e^x + 1 / e x] / [e^x – 1 / e^x]

= [e^{2x} + 1] /[e^{2x} –1]

dy / dx = [(e^{2x} –1) × d / dx (e^{2x} + 1) – (e^{2x} + 1) × d / dx (e^{2x} –1)] / [e^{2x} –1]2

= [(e^{2x} –1) × 2 e^{2x} – (e^{2x} + 1) × 2e^{2x}] / [e^{2x} –1]^2

= [{e^{2x} –1 – e^{2x}–1}× 2e^{2x}] / [e^{2x} –1]^2

= – 4 e^{2x}/[e^{2x} –1]^2.

Q.5. If y = e^ e x, find dy / dx.

Solution : 5

Let y = e ^ e^x = e^u, where u = e^x ,

du / dx = $e^x$and dy / du = e u = e ^ e^x ,

Hence, dy / dx = dy / du × du / dx = e ^ e^x . e^x.

Q.6. If $y = \sqrt{\{(1 + e^{x}) / (1 - e^{x})\}}$, find dy / dx.

Solution : 6

$dy / dx = d / dx [\sqrt{\{(1 + e^{x}) / (1 - e^{x})\}}]$

$= [\sqrt{(1 - e^{x})}\ d / dx \{\sqrt{(1 + e^{x})}\} - \sqrt{(1 + e^{x})}\ d / dx \{\sqrt{(1 - e^{x})}] / \{\sqrt{(1 - e^{x})}\}^{2}$

$= [\sqrt{(1 - e^{x})} \times 1 / 2\sqrt{(1 + e^{x})} \times e^{x} - \sqrt{(1 + e^{x})} \times 1/2\sqrt{(1 - e^{x})} \times (- e^{x})] / (1 - e^{x})$

$= [(e^{x}) / 2] [\{\sqrt{(1 - e^{x})}\}/\{\sqrt{(1 + e^{x})}\} + \{\sqrt{(1 + e^{x})}\}/\{\sqrt{(1 - e^{x})}\}] / (1 - e^{x})$

$= [e^{x}\{(1 - e^{x}) + (1 + e^{x})\}/ 2 \sqrt{(1 - e^{2x})}] / (1 - e^{x})$

$= e^{x}/ [(1 - e^{x}) \sqrt{(1 - e^{2x})}]$.

Q.7. If $y = e^{ax} / \sin (bx + c)$, find dy/dx .

Solution : 7

$dy / dx = [\sin (bx + c) . d / dx (e^{ax}) - e\ ax . d / dx \{\sin (bx + c)\}] / \sin^{2} (bx + c)$

$= [\sin (bx + c) . ae^{ax} - e^{ax} . \cos (bx + c) . b] / \sin2 (bx + c)$

$= e^{ax} [a \sin (bx + c) - b \cos (bx + c)] / \sin2 (bx + c)$.

Q.8. If $y = \{x + \sqrt{(x^{2} - 1)}\}^{m}$, prove that $(x^{2} - 1)(dy/dx)^{2} = m^{2} . y^{2}$.

Solution : 8

We have , $y = \{x + \sqrt{(x^{2} - 1)}\}^{m}$,

Then, $dy/dx = m\{x + \sqrt{(x^{2} - 1)}\}^{(m -1)} \times \{1 + 1/2. 2x/\sqrt{(x^{2} - 1)}\}$

$= m \{x + \sqrt{(x^{2} - 1)}\}^{(m -1)} \times [\{\sqrt{(x^{2} - 1)} + x \}/\sqrt{(x^{2} - 1)}]$

$= m\{x + \sqrt{(x^{2} - 1)}\}^{m} /\sqrt{(x^{2} - 1)}$ Or, $\sqrt{(x^{2} - 1)} . dy/dx$

$= m \{ x + \sqrt{(x^{2} - 1)} \}^{m}$

Squaring both sides , we get

$(x^2 – 1) . (dy/dx)^2 = m^2 [\{x + \sqrt{(x^2 – 1)}\}m]^2$

$= m^2 . y^2$. **[Proved.]**