RELATIVE MOTION

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JEE (Advanced) Syllabus

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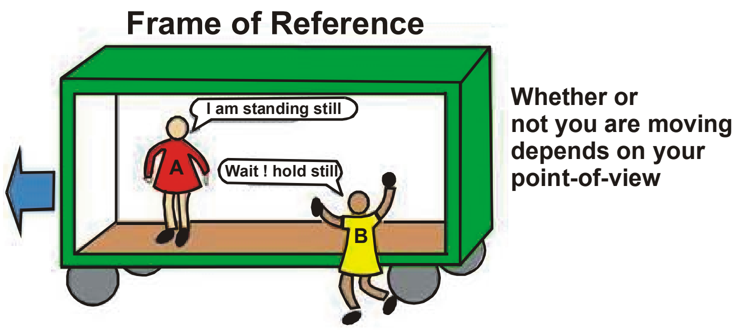
RELATIVE MOTION

1 RELATIVE MOTION

Motion is a combined property of the object under study as well as the observer.

Motion is always relative ; there is no such thing like absolute motion or absolute rest.

motion is a relative concept. To analyze motion of a body say A, therefore we have to fix our reference frame to some other body say B. The result obtained is motion of body A relative to body B.



From inside the box-car, the girl A is stationary (at rest)

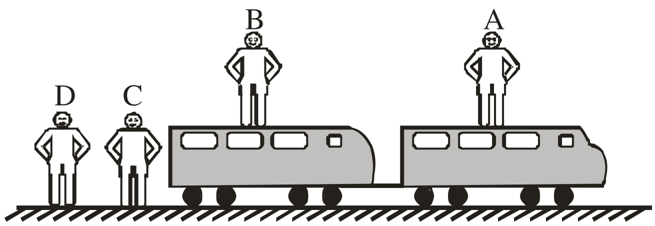
From outside the box-car, the girl A is moving.

So what is it ? Is she moving or stationary ?

Reference frame :

Reference frame is an axis system from which motion is observed along with a clock attached to the axis, to measure time. Reference frame can be stationary or moving.

☞ Suppose there are two persons A and B sitting in a train moving at constant speed. Two stationary persons C and D observe them from the ground.



Here B appears to be moving for C and D, but at rest for A. Similarly C appears to be at rest for D but moving backward for A and B.

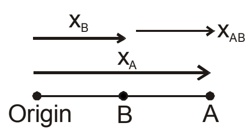
2 RELATIVE MOTION IN ONE DIMENSION :

Relative Position :

Relative position is the position of a particle w.r.t. observer.

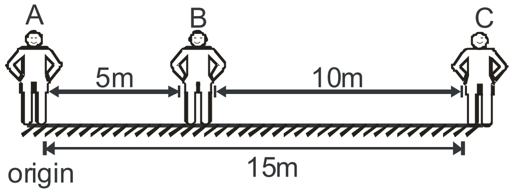
In general if position of A w.r.t. to origin is x_A and that of B w.r.t. origin is x_B then "Position of A w.r.t. B" x_{AB} is

$$\boxed{x_{AB} = x_A - x_B}$$



SOLVED EXAMPLE

Example 1. See the figure (take +ve direction towards right and –ve towards left)
Find x_{BA}, x_{CA}, x_{CB}, x_{AB} and x_{AC}.



Solution : Here, Position of B w.r.t. A is 5 m towards right . (x_{BA} = +5m)
Position of C w.r.t. A is 15 m towards right . (x_{CA} = +15m)
Position of C w.r.t. B is 10 m towards right (x_{CB} = +10m)
Position of A w.r.t. B is 5 m towards left. (x_{AB} = –5 m)
Position of A w.r.t. C is 15 m towards left. (x_{AC} = –15m)

RELATIVE VELOCITY

Definition **:** Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest.

OR

Relative velocity is the velocity with which A appears to move as seen by B considering itself to be at rest.

NOTE 1 : *All velocities are relative & have no significance unless observer is specified. However, when we say "velocity of A", what we mean is , velocity of A w.r.t. ground which is assumed to be at rest.*

Relative velocity in one dimension -

If x_A is the position of A w.r.t. ground, x_B is position of B w.r.t. ground and x_{AB} is position of A w.r.t. B then we can say v_A = velocity of A w.r.t. ground = $\frac{dx_A}{dt}$

v_B = velocity of B w.r.t. ground = $\frac{dx_B}{dt}$

and v_{AB} = velocity of A w.r.t. B = $\frac{dx_{AB}}{dt} = \frac{d}{dt}(x_A - x_B)$

$$= \frac{dx_A}{dt} - \frac{dx_B}{dt}$$

Thus

$$\boxed{\overrightarrow{v_{AB}} = \overrightarrow{v_A} - \overrightarrow{v_B}}$$

NOTE 2. : *Velocity of an object w.r.t. itself is always zero.*

SOLVED EXAMPLE

Example 2. An object A is moving with 5 m/s and B is moving with 20 m/s in the same direction. (Positive x-axis)

(i) Find velocity of B with respect to A.

(ii) Find velocity of A with respect to B

Solution : (i) v_B = +20 m/s v_A = +5 m/s $\mathbf{v_{BA}} = v_B - v_A$ = +15 m/s

(ii) v_B = +20 m/s, v_A = +15 m/s ; $v_{AB} = v_A - v_B$ = – 15 m/s

Note : $\overrightarrow{v_{BA}} = -\overrightarrow{v_{AB}}$

Example 3. Two cars A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.

(i) Find the velocity of A with respect to B.

(ii) Find the velocity of B with respect to A

Solution : $v_A = +10$, $v_B = -12$

(i) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.

(ii) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

RELATIVE ACCELERATION

It is the rate at which relative velocity is changing.

$$a_{AB} = \frac{dv_{AB}}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

Equations of motion when relative acceleration is constant.

$$v_{rel} = u_{rel} + a_{rel}\, t$$

$$s_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2$$

$$v^2_{rel} = u^2_{rel} + 2a_{rel}\, s_{rel}$$

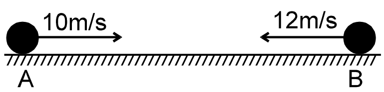
Velocity of Approach / Separation

It is the component of relative velocity of one particle w.r.t. another, along the line joining them.

- If the separation is decreasing, we say it is velocity of approach
- If separation is increasing, then we say it is velocity of separation.
- In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach / separation is simply equal to magnitude of relative velocity of A w.r.t. B.

SOLVED EXAMPLE

Example 4. A particle A is moving with a speed of 10 m/s towards right and another particle B is moving at speed of 12 m/s towards left. Find their velocity of approach.

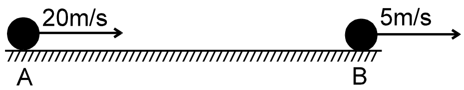


Solution : $V_A = +10$, $V_B = -12$ $\Rightarrow$ $V_{AB} = V_A - V_B$ $\Rightarrow$ $10 - (-12) = 22$ m/s

since separation is decreasing hence $V_{app} = |V_{AB}| = 22$ m/s

Ans. : 22 m/s

Example 5. A particle A is moving with a speed of 20 m/s towards right and another particle B is moving at a speed of 5 m/s towards right. Find their velocity of approach.



Solution : $V_A = +20$, $V_B = +5$

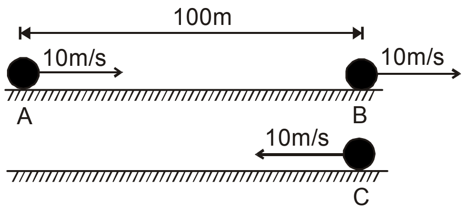
$V_{AB} = V_A - V_B$

$20 - (+5) = 15$ m/s

since separation is decreasing hence $V_{app} = |V_{AB}| = 15$ m/s

Ans. : 15 m/s

Example 6. A particle A is moving with a speed of 10 m/s towards right, particle B is moving at a speed of 10 m/s towards right and another particle C is moving at speed of 10 m/s towards left. The separation between A and B is 100 m. Find the time interval between C meeting B and C meeting A.



Solution : $t = \dfrac{\text{seperation between A and C}}{V_{app} \text{ of A and C}}$

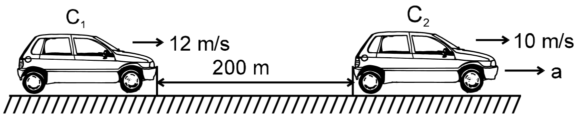
$= \dfrac{100}{10-(-10)} = 5$ sec.

Ans. : 5 sec.

Note : $a_{app} = \left(\dfrac{d}{dt}\right) v_{app}$, $a_{sep} = \dfrac{d}{dt} v_{sep}$

$v_{app} = \int a_{app} dt$, $v_{sep} = \int a_{sep} dt$

Example 7. Two cars C_1 and C_2 moving in the same direction on a straight single lane road with velocities 12 m/s and 10 m/s respectively. When the separation between the two was 200 m C_2 started accelerating to avoid collision. What is the minimum acceleration of car C_2 so that they don't collide.



Solution : Acceleration of car 1 w.r.t. car 2

$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \vec{a}_{C_1} - \vec{a}_{C_2} = 0 - a = (-a)$

$\vec{u}_{12} = \vec{u}_1 - \vec{u}_2 = 12 - 10 = 2$ m/s.

The collision is just avoided if relative velocity becomes zero just at the moment the two cars meet each other.

i.e. $v_{12} = 0$ When $s_{12} = 200$

Now $v_{12} = 0$, $\vec{u}_{12} = 2$, $\vec{a}_{12} = -a$ and $s_{12} = 200$

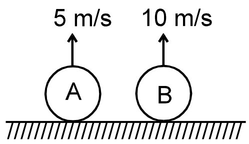
$\therefore \quad v_{12}^2 - u_{12}^2 = 2a_{12}s_{12}$

$\Rightarrow 0 - 2^2 = -2 \times a \times 200 \qquad \Rightarrow a = \frac{1}{100}$ m/s^2 = 0.1 m/s^2 = 1 cm/s^2.

$\therefore$ Minimum acceleration needed by car C_2 = 1 cm/s^2

Example 8. A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively (g = 10 m/s^2. Find separation between them after one second

Solution : $S_A = ut - \frac{1}{2} gt^2$



$= 5t - \frac{1}{2} \times 10 \times t^2$

$= 5 \times 1 - 5 \times 1^2$

$= 5 - 5 = 0$

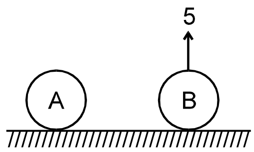
$S_B = ut - \frac{1}{2} gt^2 . = 10 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5$

$\therefore S_B - S_A$ = separation = 5m.

Aliter :

$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$

Also $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s



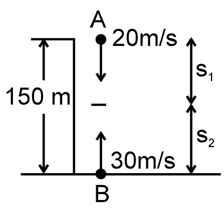
$\therefore \quad \vec{s}_{BA}$ (in 1 sec) $= \vec{v}_{BA} \times t$

$= 5 \times 1$

$= 5$ m

$\therefore$ Distance between A and B after 1 sec = 5 m.

Example 9. A ball is thrown downwards with a speed of 20 m/s from the top of a building 150 m high and simultaneously another ball is thrown vertically upwards with a speed of 30 m/s from the foot of the building. Find the time after which both the balls will meet. (g = 10 m/s^2)



Solution : $S_1 = 20\,t + 5\,t^2$

$S_2 = 30\,t - 5\,t^2$

$S_1 + S_2 = 150$

$\Rightarrow \quad 150 = 50\,t$

$\Rightarrow \quad t = 3$ s

Aliter :

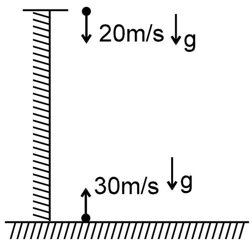
Relative acceleration of both is zero since both have same acceleration in downward direction

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = g - g = 0$

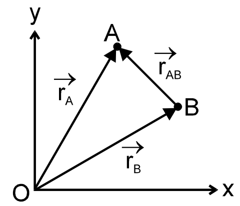
$\vec{v}_{BA} = 30 - (-20) = 50$

$s_{BA} = v_{BA} \times t$

$t = \frac{s_{BA}}{v_{BA}} = \frac{150}{50} = 3$ s



3. RELATIVE MOTION IN TWO DIMENSION



$\vec{r}_A$ = position of A with respect to O

$\vec{r}_B$ = position of B with respect to O

$\vec{r}_{AB}$ = position of A with respect to B.

$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$ (The vector sum $\vec{r}_A - \vec{r}_B$ can be done by Δ law of addition or resolution method)

$$\therefore \quad \frac{d(\vec{r}_{AB})}{dt} = \frac{d(\vec{r}_A)}{dt} - \frac{d(\vec{r}_B)}{dt}.$$

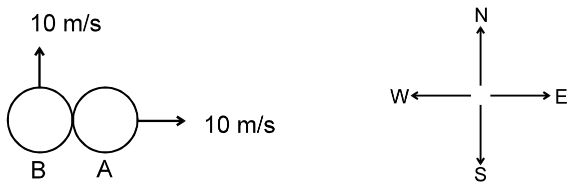
$\Rightarrow \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$$\frac{d(\vec{v}_{AB})}{dt} = \frac{d(\vec{v}_A)}{dt} - \frac{d(\vec{v}_B)}{dt}$$

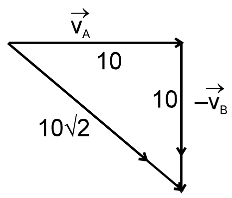
$\Rightarrow \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

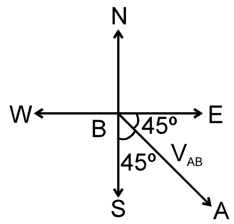
SOLVED EXAMPLE

Example 10. Object A and B both have speed of 10 m/s. A is moving towards East while B is moving towards North starting from the same point as shown. Find velocity of A relative to B ($\vec{v}_{AB}$)



Solution : **Method 1**





$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\therefore |\vec{v}_{AB}| = 10\sqrt{2}$

Method 2

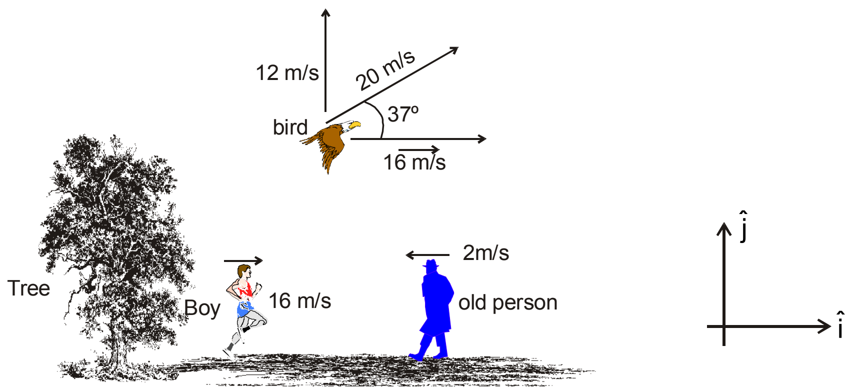
$\vec{v}_A = 10\hat{i}$, $\vec{v}_B = 10\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 10\hat{i} - 10\hat{j}$

$\therefore\ |\vec{v}_{AB}| = 10\sqrt{2}$

Note : $|\vec{v}_A - \vec{v}_B| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$, where θ is angle between $\vec{v}_A$ and $\vec{v}_B$

Example 11. An old man and a boy are walking towards each other and a bird is flying over them as shown in the figure.



(1) Find the velocity of tree, bird and old man as seen by boy.

(2) Find the velocity of tree, bird and boy as seen by old man

(3) Find the velocity of tree, boy and old man as seen by bird.

Solution : (1) With respect to boy :

v_{tree} = 16 m/s $(\leftarrow)$ or $-16\hat{i}$

v_{bird} = 12 m/s $(\uparrow)$ or $12\hat{j}$

$v_{old\ man}$ = 18 m/s $(\leftarrow)$ or $-18\hat{i}$

(2) With respect to old man :

v_{Boy} = 18 m/s $(\rightarrow)$ or $18\hat{i}$

v_{Tree} = 2 m/s $(\rightarrow)$ or $2\hat{i}$

v_{Bird} = 18 m/s $(\rightarrow)$ and 12 m/s $(\uparrow)$ or $18\hat{i} + 12\hat{j}$

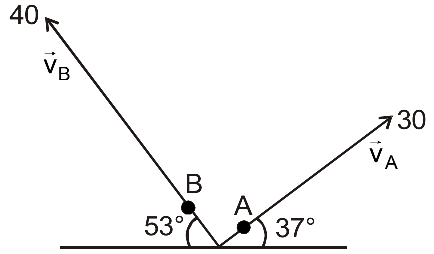
(3) With respect to Bird :

v_{Tree} = 12 m/s $(\downarrow)$ and 16 m/s $(\leftarrow)$ or $-12\hat{j} - 16\hat{i}$

$v_{old\ man}$ = 18 m/s $(\leftarrow)$ and 12 m/s $(\downarrow)$ or $-18\hat{i} - 12\hat{j}$

v_{Boy} = 12 m/s $(\downarrow)$. or $-12\hat{j}$

Example 12. Two particles A and B are projected in air. A is thrown with a speed of 30 m/sec and B with a speed of 40 m/sec as shown in the figure. What is the separation between them after 1 sec.



Solution : $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = \vec{g} - \vec{g} = 0$

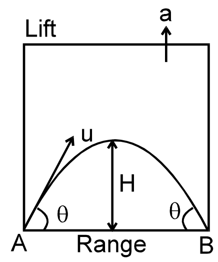
$\therefore \quad \vec{v}_{AB} = \sqrt{30^2 + 40^2} = 50$

$\therefore \quad s_{AB} = v_{AB}t = 50\,t = 50$ m

Relative Motion in Lift

Projectile motion in a lift moving with acceleration a upwards

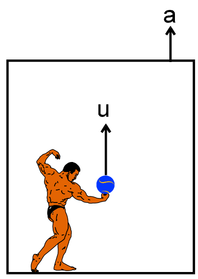
- In the reference frame of lift, acceleration of a freely falling object is g + a
- Velocity at maximum height = u cos θ
- $T = \dfrac{2u\sin\theta}{g+a}$
- Maximum height (H) = $\dfrac{u^2\sin^2\theta}{2(g+a)}$
- Range = $\dfrac{u^2\sin 2\theta}{g+a}$



SOLVED EXAMPLE

Example 13 A lift is moving up with acceleration a. A person inside the lift throws the ball upwards with a velocity u relative to hand.
(a) What is the time of flight of the ball?

(b) What is the maximum height reached by the ball in the lift?



Solution : **(a)** $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = g + a$

$\vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}_{BL}\,t^2$

$0 = uT - \frac{1}{2}(g + a)T^2$

$\therefore \quad T = \dfrac{2u}{(g+a)}$

(b) $v^2 - u^2 = 2\,as$

$0 - u^2 = -2(g + a)\,H$

$H = \dfrac{u^2}{2(g+a)}$

4. RELATIVE MOTION IN RIVER FLOW

If a man can swim relative to water with velocity $\vec{v}_{mR}$ and water is flowing relative to ground with velocity $\vec{v}_R$, velocity of man relative to ground $\vec{v}_m$ will be given by :

$$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$$

or $$\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$$

If $\vec{v}_R = 0$, then $\vec{v}_m = \vec{v}_{mR}$

in words, velocity of man in still water = velocity of man w.r.t. river

River Problem in One Dimension :

☞ Velocity of river is u & velocity of man in still water is v.

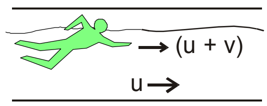
Case - 1

Man swimming downstream (along the direction of river flow)

In this case velocity of river $v_R = +u$

velocity of man w.r.t. river $v_{mR} = +v$

now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = u + v$



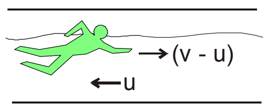
Case - 2

Man swimming upstream (opposite to the direction of river flow)

In this case velocity of river $\vec{v}_R = -u$

velocity of man w.r.t. river $\vec{v}_{mR} = +v$

now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = (v - u)$



SOLVED EXAMPLE

Example 14. A swimmer capable of swimming with velocity 'v' relative to water jumps in a flowing river having velocity 'u'. The man swims a distance d down stream and returns back to the original position. Find out the time taken in complete motion.

Solution : Total time = time of swimming downstream + time of swimming upstream

$$t = t_{down} + t_{up} = \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$ **Ans.**

Motion of Man Swimming in a River

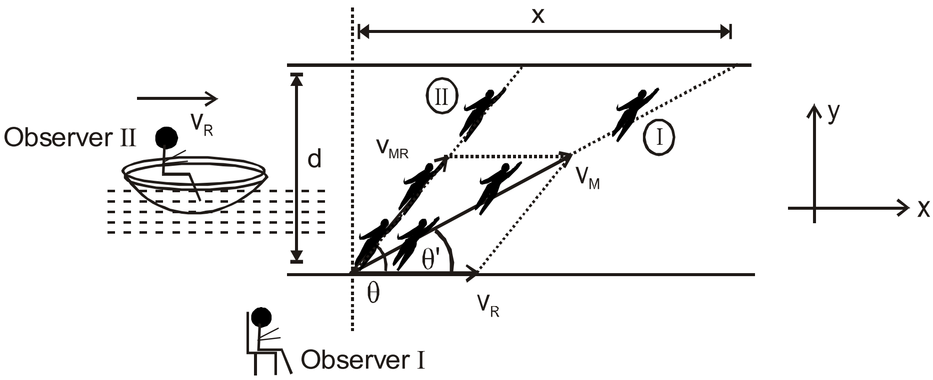
Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow

The velocity of river is $\vec{V}_R$.

Let there be two observers I and II, observer I is on ground and observer II is on a raft floating along with the river and hence moving with the same velocity as that of river. Hence motion w.r.t. observer II is same as motion w.r.t. river. i.e. the man will appear to swim at an angle θ with the river flow for observer II.

For observer I the velocity of swimmer will be $\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$,

Hence the swimmer will appear to move at an angle θ' with the river flow.



I : Motion of swimmer for observer I

II : Motion of swimmer for observer II

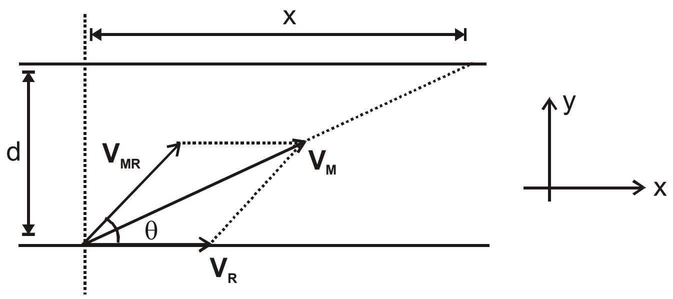
River problem in two dimension (crossing river) :

Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow The velocity of river is V_R and the width of the river is d

$$\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$$

$$\Rightarrow \vec{v}_M = (v_{MR}\cos\theta\,\hat{i} + v_{MR}\sin\theta\,\hat{j}) + v_R\,\hat{i}$$

$$\Rightarrow \vec{v}_M = (v_{MR}\cos\theta + v_R)\,\hat{i} + v_{MR}\sin\theta\,\hat{j}$$



Here $v_{MR}\sin\theta$ is the component of velocity of man in the direction perpendicular to the river flow.
This component of velocity is responsible for the man crossing the river. Hence if the time to cross the river is t , then

$$t = \frac{d}{v_y} = \frac{d}{v_{MR}\sin\theta}$$

Drift

It is defined as the displacement of man in the direction of river flow. (see the figure).
It is simply the displacement along x-axis, during the period the man crosses the river. $(v_{MR}\cos\theta + v_R)$ is the component of velocity of man in the direction of river flow and this component of velocity is responsible for drift along the river flow. If drift is x then,
Drift = $v_x \times t$

$$x = (v_{MR}\cos\theta + v_R) \times \frac{d}{v_{MR}\sin\theta}$$

Crossing the river in shortest time

As we know that $t = \dfrac{d}{v_{MR}\sin\theta}$. Clearly t will be minimum when $\theta = 90°$ i.e. time to cross the river will be minimum if man swims perpendicular to the river flow. Which is equal to $\dfrac{d}{v_{MR}}$.

Crossing the river in shortest path, Minimum Drift

The minimum possible drift is zero. In this case the man swims in the direction perpendicular to the river flow as seen from the ground. This path is known as ***shortest path***

here $x_{min} = 0 \Rightarrow (v_{MR}\cos\theta + v_R) = 0$

or $\cos\theta = -\dfrac{v_R}{v_{MR}}$

☞ since $\cos\theta$ is – ve, $\therefore \theta > 90°$, i.e. for minimum drift the man must swim at some angle ϕ with the perpendicular in backward direction.

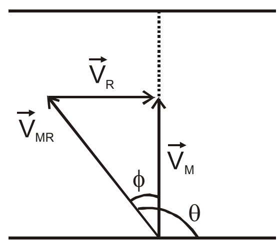
Where $\sin\phi = \dfrac{v_R}{v_{MR}}$

☞ $\theta = \cos^{-1}\left(\dfrac{-v_R}{v_{MR}}\right) \quad \therefore \left|\dfrac{v_R}{v_{MR}}\right| \le 1 \quad$ i.e. $v_R \le v_{MR}$

i.e. minimum drift is zero if and only if velocity of man in still water is greater than or equal to the velocity of river.

☞ Time to cross the river along the shortest path

$$t = \frac{d}{v_{MR}\sin\theta} = \frac{d}{\sqrt{v_{MR}^2 - V_R^2}}$$



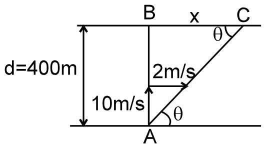
Note :

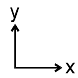
- ***If $v_R > v_{MR}$ then it is not possible to have zero drift. In this case the minimum drift (corresponding to shortest possible path is non zero and the condition for minimum drift can be proved to be $\cos\theta = -\dfrac{v_{MR}}{v_R}$ or $\sin\phi = \dfrac{v_{MR}}{v_R}$ for minimum but non zero drift.***

SOLVED EXAMPLE

Example 15. A 400 m wide river is flowing at a rate of 2.0 m/s. A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.
(a) Find the time taken by the boat to reach the opposite bank.
(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.
(c) In what direction does the boat actually move, with river flow (downstream).

Solution :





(a) time taken to cross the river

$$t = \frac{d}{v_y} = \frac{400\,m}{10\,m/s} = 40\ s \qquad \textbf{Ans.}$$

(b) drift (x) = $(v_x)(t)$ = (2m/s) (40s) = 80 m **Ans.**

(c) Actual direction of boat,

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1} 5, \text{ (downstream) with the river flow.}$$

Example 16. A man can swim at the rate of 5 km/h in still water. A 1 km wide river flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.
(a) Along what direction must the man swim?
(b) What should be his resultant velocity?
(c) How much time will he take to cross the river?

Solution : The velocity of man with respect to river v_{mR}= 5 km/hr, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The angle of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^\circ + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^\circ + \sin^{-1}\left(\frac{3}{5}\right) = 90^\circ + 37^\circ$$

= 127° w.r.t. the river flow or 37° w.r.t. perpendicular in backward direction **Ans.**

(b) Resultant velocity will be $v_m = \sqrt{v_{mR}^2 - v_R^2} = \sqrt{5^2 - 3^2}$ = 4 km/hr
along the direction perpendicular to the river flow.

(c) time taken to cross the

$$t = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}} = \frac{1\,km}{4\,km/hr} = \frac{1}{4}\ h = 15\ min$$

Example 17. A man wishing to cross a river flowing with velocity u jumps at an angle θ with the river flow.
(i) Find the net velocity of the man with respect to ground if he can swim with speed v in still water.
(ii) In what direction does the man actually move.
(iii) Find how far from the point directly opposite to the starting point does the man reach the opposite bank, if the width of the river is d. (i.e. drift)

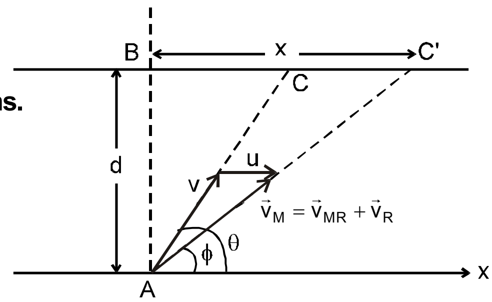
Solution : (i) $v_{MR} = v$, $v_R = u$

$$\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$$

$\therefore$ Velocity of man , $v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$ **Ans.**

(ii) $\tan\phi = \dfrac{v\sin\theta}{u + v\cos\theta}$ **Ans.**

(iii) $(v \sin\theta)\, t = d$



$$\Rightarrow \quad t = \frac{d}{v\sin\theta}$$

$$x = (u + v\cos\theta)\,t$$

$$= (u + v\cos\theta)\frac{d}{v\sin\theta} \quad \textbf{Ans.}$$

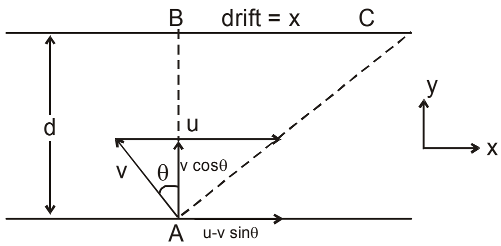
Example 18. A boat moves relative to water with a velocity v which is n times less than the river flow velocity u. At what angle to the stream direction must the boat move to minimize drifting?

Solution : (In this problem, one thing should be carefully noted that the velocity of boat is less than the river flow velocity. Hence boat cannot reach the point directly opposite to its starting point. i.e. drift can never be zero.)

Suppose boat starts at an angle θ from the normal direction up stream as shown.

Component of velocity of boat along the river, $v_x = u - v\sin\theta$

and velocity perpendicular to the river, $v_y = v\cos\theta$.



time taken to cross the river is $t = \frac{d}{v_y} = \frac{d}{v\cos\theta}$.

Drift $x = (v_x)t$

$$= (u - v\sin\theta)\frac{d}{v\cos\theta}$$

$$= \frac{ud}{v}\sec\theta - d\tan\theta$$

The drift x is minimum, when $\frac{dx}{d\theta} = 0$,

or $\quad \left(\frac{ud}{v}\right)(\sec\theta \,.\, \tan\theta) - d\sec^2\theta = 0$

or $\quad \frac{u}{v}\sin\theta = 1$

or $\quad \boxed{\sin\theta = \frac{v}{u}}$

This is the result we stated without proof as a note in section 4.5

so, for minimum drift, the boat must move at an angle $\theta = \sin^{-1}\left(\frac{v}{u}\right) = \sin^{-1}\frac{1}{n}$ from normal direction.

5. WIND AIRPLANE PROBLEMS

This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind.

Thus,

velocity of aeroplane with respect to wind

$$\vec{v}_{aw} = \vec{v}_a - \vec{v}_w$$

or $\quad \vec{v}_a = \vec{v}_{aw} + \vec{v}_w$

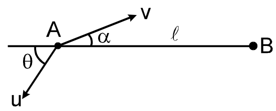
where, $\quad \vec{v}_a$ = velocity of aeroplane w.r.t. ground

and, $\quad \vec{v}_w$ = velocity of wind.

SOLVED EXAMPLE

Example 19. An aeroplane flies along a straight path A to B and returns back again. The distance between A and B is ℓ and the aeroplane maintains the constant speed v w.r.t. wind. There is a steady wind with a speed u at an angle θ with line AB. Determine the expression for the total time of the trip.

Solution : Suppose plane is oriented at an angle α w.r.t. line AB while the plane is moving from A to B :



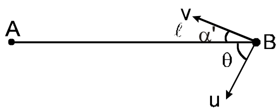
Velocity of plane along AB = v cosα – ucosθ ,

and for no-drift from line AB

$v \sin\alpha = u\sin\theta \quad \Rightarrow \quad \sin\alpha = \dfrac{u\sin\theta}{v}$

time taken from A to B : $t_{AB} = \dfrac{\ell}{v\cos\alpha - u\cos\theta}$

Suppose plane is oriented at an angle α' w.r.t. line AB while the plane is moving from B to A :



velocity of plane along BA = vcosα + u cosθ and for no drift from line AB

$v\sin\alpha = u\sin\theta$

$\Rightarrow \quad \sin\alpha = \dfrac{u\sin\theta}{v}$

$\Rightarrow \quad \alpha = \alpha'$

time taken from B to A : $t_{BA} = \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

total time taken = $t_{AB} + t_{BA} = \dfrac{\ell}{v\cos\alpha - u\cos\theta} + \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

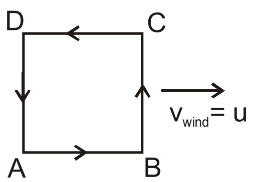
$$= \frac{2v\ell\cos\alpha}{v^2\cos^2\alpha - u^2\cos^2\theta}$$

$$= \frac{2v\ell\sqrt{1-\dfrac{u^2\sin^2\theta}{v^2}}}{v^2 - u^2}.$$

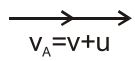
Example 20. Find the time an aeroplane having velocity v, takes to fly around a square with side a if the wind is blowing at a velocity u along one side of the square.

Answer : $\dfrac{2a}{v^2-u^2}\left(v+\sqrt{v^2-u^2}\right)$

Solution :



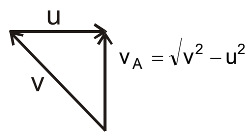
Velocity of aeroplane while flying through AB



$v_A = v + u$

$t_{AB} = \dfrac{a}{v+u}$

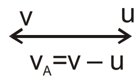
Velocity of aeroplane while flying through BC



$v_A = \sqrt{v^2 - u^2}$

$t_{BC} = \dfrac{a}{\sqrt{v^2 - u^2}}$

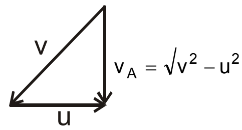
Velocity of aeroplane while flying through CD



$v_A = v - u$

$t_{CD} = \dfrac{a}{v-u}$

Velocity of aeroplane while flying through DA



$v_A = \sqrt{v^2 - u^2}$

$t_{DA} = \dfrac{a}{\sqrt{v^2 - u^2}}$

Total time = $t_{AB} + t_{BC} + t_{CD} + t_{DA}$

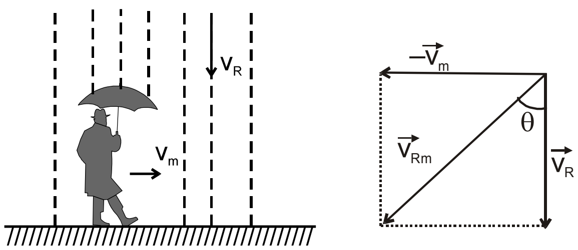
$$= \frac{a}{v+u} + \frac{a}{\sqrt{v^2-u^2}} + \frac{a}{v-u} + \frac{a}{\sqrt{v^2-u^2}} = \frac{2a}{v^2-u^2}\left(v + \sqrt{v^2-u^2}\right)$$

6. RAIN PROBLEM

If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m \qquad \text{or} \qquad v_{Rm} = \sqrt{v_R^2 + v_m^2}$$

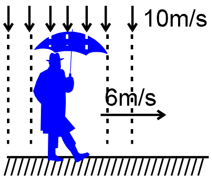
and direction $\theta = \tan^{-1}\left(\dfrac{v_m}{v_R}\right)$ with the vertical as shown in figure.



SOLVED EXAMPLE

Example 21 Rain is falling vertically at speed of 10 m/s and a man is moving with velocity 6 m/s. Find the angle at which the man should hold his umbrella to avoid getting wet.

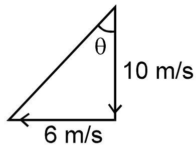
Solution :



$\vec{v}_{rain} = -10\,\hat{j}$ $\Rightarrow$ $\vec{v}_{man} = 6\,\hat{j}$

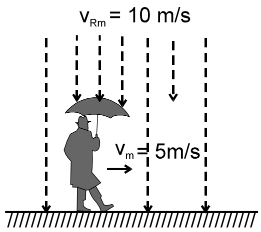
$\vec{v}_{r.w.r.t.\ man} = -10\,\hat{j} - 6\,\hat{i}$

$\tan\theta = \frac{6}{10}$ $\Rightarrow$ $\theta = \tan^{-1}\left(\frac{3}{5}\right)$



Where θ is angle with vertical

Example 22 A man moving with 5m/s observes rain falling vertically at the rate of 10 m/s. Find the speed and direction of the rain with respect to ground.



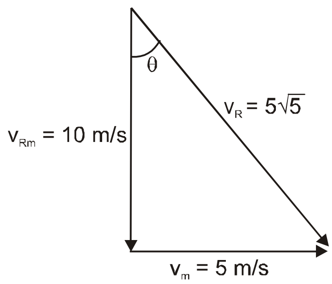
Solution :

$v_{RM} = 10$ m/s, $v_M = 5$ m/s

$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$\Rightarrow$ $\vec{v}_R = \vec{v}_{RM} + \vec{v}_M$ $\Rightarrow$ $\vec{v}_R = 5\sqrt{5}$

$\tan\theta = \frac{1}{2}$, $\theta = \tan^{-1}\frac{1}{2}$.

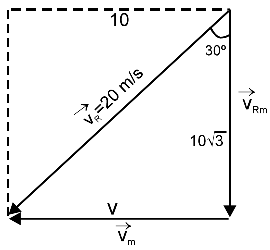


Example 23. A standing man, observes rain falling with velocity of 20 m/s at an angle of 30º with the vertical.

(i) Find the velocity with which the man should move so that rain appears to fall vertically to him.

(ii) Now if he further increases his speed, rain again appears to fall at 30º with the vertical. Find his new velocity.

Solution : (i) $\vec{v}_m = -v\,\hat{i}$ (let)



$\vec{v}_R = -10\,\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_{RM} = -(10 - v)\,\hat{i} - 10\sqrt{3}\,\hat{j}$

$\Rightarrow$ $-(10 - v) = 0$ (for vertical fall, horizontal component must be zero)

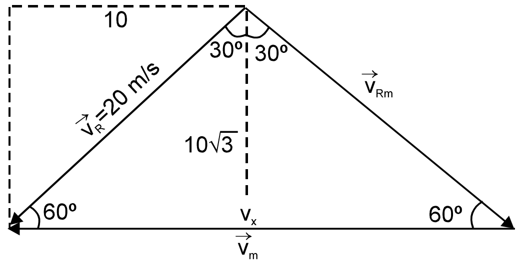
or $v = 10$ m/s **Ans.**

(ii) $\vec{v}_R = -10\hat{i} - 10\sqrt{3}\hat{j}$

$\vec{v}_m = -v_x\hat{i}$

$\vec{v}_{RM} = -(10 - v_x)\hat{i} - 10\sqrt{3}\hat{j}$

Angle with the vertical = 30º



$\Rightarrow \quad \tan 30^\circ = \dfrac{10 - v_x}{-10\sqrt{3}} \quad \Rightarrow \quad v_x = 20$ m/s

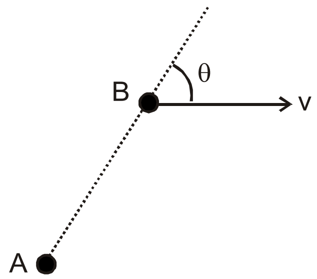
7. VELOCITY OF APPROACH / SEPARATION IN TWO DIMENSION

It is the component of relative velocity of one particle w.r.t. another, along the line joining them.

If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

SOLVED EXAMPLE

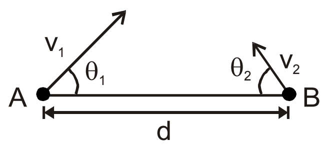
Example 24. Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at t = 0. Find their velocity of separation



Solution : $v_{BA} = v_B - v_A = v$

v_{sep} = component of v_{BA} along line AB = $v\cos\theta$

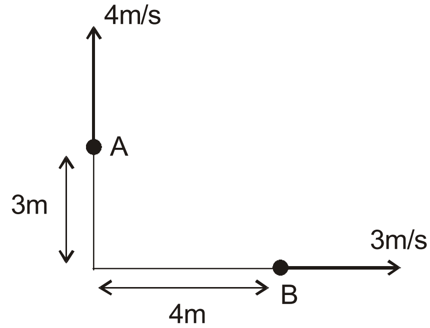
Example 25. Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.



Solution : Velocity of approach is relative velocity along line AB

$v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$

Example 26. Particles A and B are moving as shown in the diagram at t = 0. Find their velocity of separation
(i) at t = 0
(ii) at t = 1 sec.

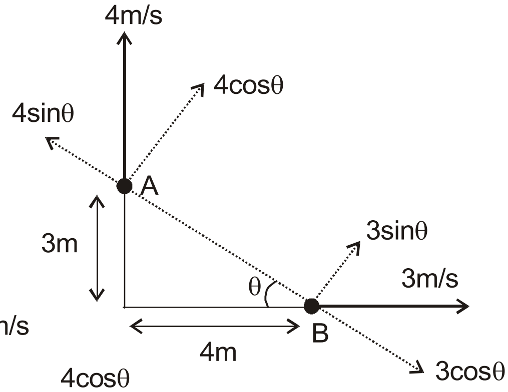


Solution : (i) $\tan\theta = 3/4$

v_{sep} = relative velocity along line AB

$= 3\cos\theta + 4\sin\theta$

$= 3.\dfrac{4}{5} + 4.\dfrac{3}{5} = \dfrac{24}{5} = 4.8$ m/s

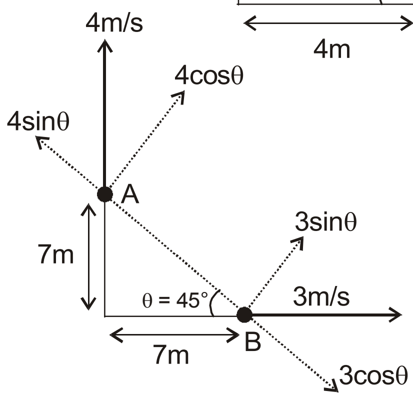


(ii) $\theta = 45°$

v_{sep} = relative velocity along line AB

$= 3\cos\theta + 4\sin\theta$

$= 3.\dfrac{1}{\sqrt{2}} + 4.\dfrac{1}{\sqrt{2}} = \dfrac{7}{\sqrt{2}}$ m/s

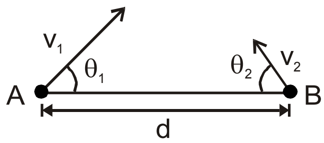


CONDITION FOR UNIFORMLY MOVING PARTICLES TO COLLIDE

If two particles are moving with uniform velocities and the relative velocity of one particle w.r.t. other particle is directed towards each other then they will collide.

SOLVED EXAMPLE

Example 27. Two particles A and B are moving with constant velocities v_1 and v_2 . At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B.
(i) Find the condition for A and B to collide.
(ii) Find the time after which A and B will collide if separation between them is d at t = 0



Solution : (i) For A and B to collide, their relative velocity must be directed along the line joining them. Therefore their relative velocity along the perpendicular to this line must be zero.
Thus $v_1 \sin\theta_1 = v_2 \sin\theta_2$.

(ii) $v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$

$$t = \frac{d}{v_{app}} = \frac{d}{v_1\cos\theta_1 + v_2\cos\theta_2}$$

Minimum / Maximum distance between two particles

If the separation between two particles decreases and after some time it starts increasing then the separation between them will be minimum at the instant, velocity of approach changes to velocity of separation. (at this instant $v_{app} = 0$)

Mathematically S_{AB} is minimum when $\frac{dS_{AB}}{dt} = 0$

Similarly for maximum separation $v_{sep} = 0$.

Note :

- If the initial position of two particles are $\vec{r}_1$ and $\vec{r}_2$ and their velocities are $\vec{v}_1$ and $\vec{v}_2$ then shortest distance between the particles, $d_{shortest} = \frac{|\vec{r}_{12} \times \vec{v}_{12}|}{|\vec{v}_{12}|}$ and time after which this situation will occur, $t = -\frac{\vec{r}_{12} \cdot \vec{v}_{12}}{|\vec{v}_{12}|^2}$

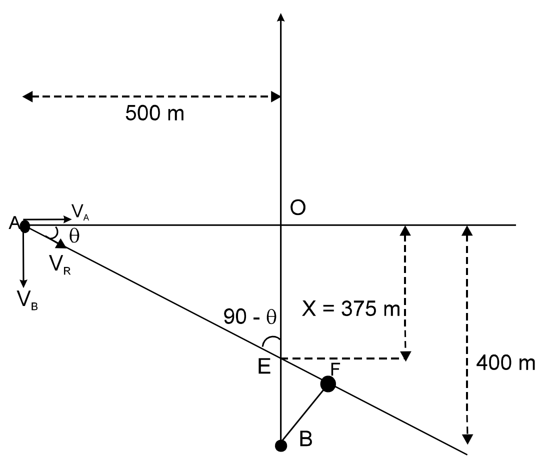
SOLVED EXAMPLE

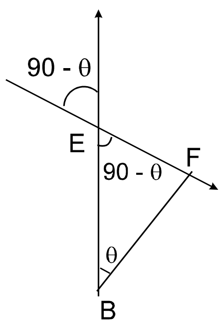
Example 28. Two cars A and B are moving west to east and south to north respectively along crossroads. A moves with a speed of 72 kmh^{-1} and is 500 m away from point of intersection of cross roads and B moves with a speed of 54 kmh^{-1} and is 400 m away from point of intersection of cross roads. Find the shortest distance between them?

Solution : **Method – I (Using the concept of relative velocity)**

In this method we watch the velocity of A w.r.t. B. To do this we plot the resultant velocity V_r. Since the accelerations of both the bodies is zero, so the relative acceleration between them is also zero. Hence the relative velocity will remain constant. So the path of A with respect to B will be straight line and along the direction of relative velocity of A with respect to B. The shortest distance between A & B is when A is at point F (i.e. when we drop a perpendicular from B on the line of motion of A with respect to B).

From figure





$\tan\theta = \frac{V_B}{V_A} = \frac{15}{20} = \frac{3}{4}$ ....(i)

This θ is the angle made by the resultant velocity vector with the x-axis.

Also we know that from figure

$OE = \frac{x}{500} = \frac{3}{4}$ ....(ii)

From equation (i) & (ii) we get

x = 375 m

$\therefore$ EB = OB – OE = 400 – 375 = 25 m

But the shortest distance is BF.

From magnified figure we see that BF = EB $\cos\theta$ = 25 × $\frac{4}{5}$

$\therefore$ BF = 20 m

Method II **(Using the concept of maxima – minima)**

A & B be are the initial positions and A',B' be the final positions after time t.

B is moving with a speed of 15 m/sec so it will travel a distance of BB' = 15t during time t.

A is moving with a speed of 20 m/sec so it will travel a distance of AA' = 20t during time t.

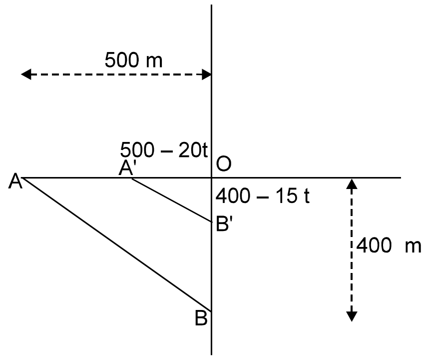
So

OA' = 500 – 20 t

OB' = 400 – 15 t

$\therefore A'B'^2 = OA'^2 + OB'^2 = (500 - 20t)^2 + (400 - 15t)^2$ .....(i)

For A'B' to be minimum $A'B'^2$ should also be minimum



$\therefore \frac{d(A'B'^2)}{dt} = \frac{d(400-15t)^2 + (500-20t)^2}{dt} = 0$

= 2(400 – 15t) (–15) + 2(500 – 20t) (–20) = 0

= – 1200 + 45t = 2000 – 80 t

$\therefore$ 125 t = 3200

$\therefore t = \frac{128}{5}$ s. Hence A and B will be closest after $\frac{128}{5}$ s.

Now $\frac{d^2A'B'}{dt^2}$ comes out to be positive hence it is a minima.

On substituting the value of t in equation (i) we get

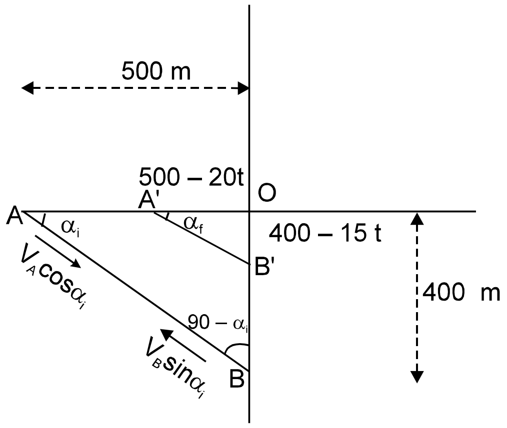
$\therefore A'B'^2 = \left(400 - 15\times\frac{128}{5}\right)^2 + \left(500 - 20.\frac{128}{5}\right)^2 = \sqrt{16^2 + (-12)^2}$ = 20 m

$\therefore$ Minimum distance A'B' = 20 m.

Method III (Using the concept of relative velocity of approach)

After time t let us plot the components of velocity of A and B in the direction along AB. When the distance between the two is minimum, the relative velocity of approach is zero.

$\therefore \quad V_A \cos\alpha_f + V_B \sin\alpha_f = 0$



(where α_f is the angle made by the line A'B' with the x-axis)

$$20\cos\alpha_f = -15\sin\alpha_f$$

$$\therefore \quad \tan\alpha_f = -\frac{20}{15} = -\frac{4}{3}$$

(Here do not confuse this angle with the angle θ in method (I) because that θ is the angle made by the net relative velocity with x-axis, but α_f is the angle made by the line joining the two particles with x-axis when velocity of approach in zero.)

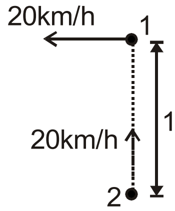
$$\therefore \quad \frac{400-15t}{500-20t} = -\frac{4}{3}$$

$$\therefore \quad t = \frac{128}{5}$$

So, OB' = 16 m and OA' = – 12m

$$A'B' = \sqrt{16^2+(-12)^2} = 20 \text{ m}$$

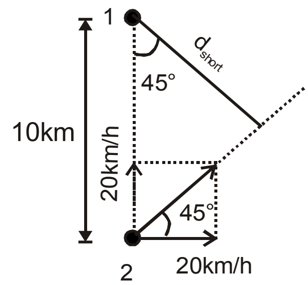
Example 29. Two ships are 10 km apart on a line joining south to north. The one farther north is steaming west at 20 km h^{-1}. The other is steaming north at 20 km h^{-1}. What is their distance of closest approach ? How long do they take to reach it ?



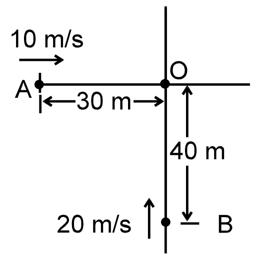
Solution : Solving from the frame of particle -1

we get $d_{short} = 10\cos 45° = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ km

$$t = \frac{10\sin 45}{|\vec{V}_{21}|} = \frac{10\times 1/\sqrt{2}}{20\sqrt{2}} = \frac{1}{4}\text{h} = 15 \text{ min.}$$



Example 30. Two particles A and B are moving with uniform velocity as shown in the figure given below at t = 0.
(i) Will the two particle collide
(ii) Find out shortest distance between two particles



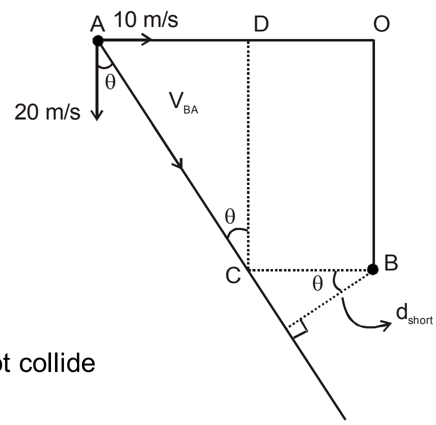
Solution : Solving from the frame of B

we get $\tan\theta = \frac{10}{20} = \frac{1}{2}$

again $\tan\theta = \frac{AD}{CD} = \frac{AD}{40} = \frac{1}{2}$

$\Rightarrow$ AD = 20 $\Rightarrow$ DO = 10 $\Rightarrow$ BC = 10

$$d_{short} = BC\cos\theta = 10\cos\theta = \frac{10\times 2}{\sqrt{5}} = 4\sqrt{5} \text{ m}$$



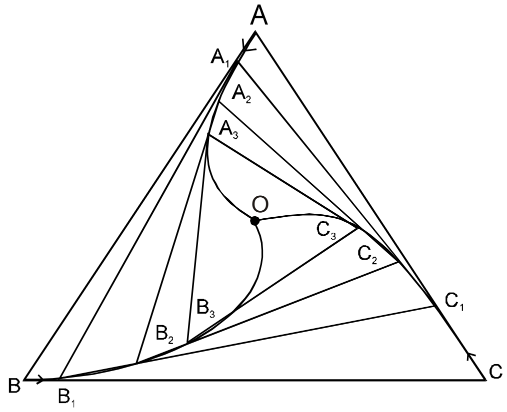
Since closest distance is non zero therefore they will not collide

MISCELLANEOUS PROBLEMS ON COLLISION

SOLVED EXAMPLE

Example 31. There are particles A, B and C are situated at the vertices of an equilateral triangle ABC of side a at t = 0. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particle meet each other?

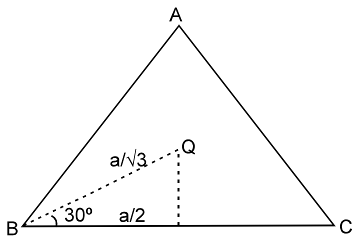
Solution : The motion of the particles is roughly sketched in figure. By symmetry they will meet at the centroid O of the triangle. At any instant the particles will from an equilateral triangle ABC with the same



Centroid O. All the particles will meet at the centre. Concentrate on the motion of any one particle, say B. At any instant its velocity makes angle 30º with BO. The component of this velocity along BO is v cos 30º. This component is the rate of decrease of the distance BO.

Initially.

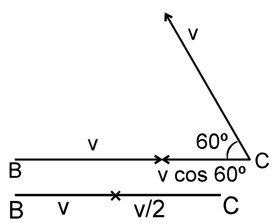
$BO = \dfrac{a/2}{\cos 30^\circ} = \dfrac{a}{\sqrt{3}}$ = displacement of each particle. Therefore, the time taken for BO to become zero



$$= \frac{a/\sqrt{3}}{v\cos 30^\circ} = \frac{2a}{\sqrt{3}v \times \sqrt{3}} = \frac{2a}{3v}.$$

Aliter : Velocity of B is v along BC. The velocity of C is along CA. Its component along BC is v cos 60º = v/2. Thus, the separation BC decreases at the rate of approach velocity.

$\therefore$



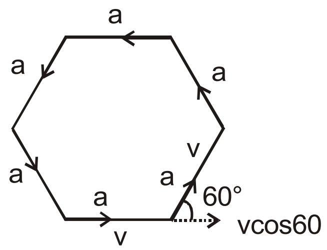
$\therefore$ approach velocity $= v + \dfrac{v}{2} = \dfrac{3v}{2}$

Since, the rate of approach is constant, the time taken in reducing the separation BC from a to zero is

$$t = \frac{a}{\frac{3v}{2}} = \frac{2a}{3v}$$

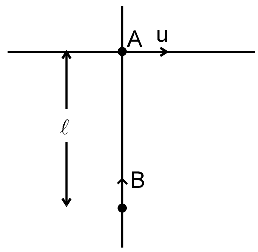
Example 32. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

Solution : V_{app} = V – Vcos 60° = V – V/2 = V/2



$$t = \frac{a}{V_{app}} = \frac{a}{V/2} = \frac{2a}{V}$$

Example 33. 'A' moves with constant velocity u along the 'x' axis. B always has velocity towards A. After how much time will B meet A if B moves with constant speed v. What distance will be travelled by A and B.



Solution : Let at any instant the velocity of B makes an angle α with that of x axis and the time to collide is T.

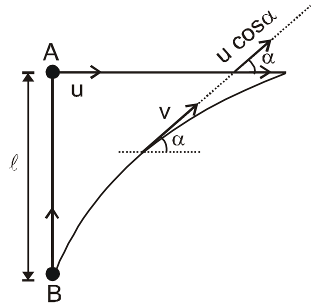
$v_{app} = v - u\cos\alpha$

$$\ell = \int_0^T v_{app}\,dt = \int_0^T (v - u\cos\alpha)\,dt \quad \text{............ (1)}$$

Now equating the displacement of A and B along x direction we get

$$uT = \int v\cos\alpha\,dt \quad \text{....... (2)}$$

Now from (1) and (2) we get



$$\ell = vT - \int_0^T u\cos\alpha\,dt = vT - \frac{u}{v}\int_0^T v\cos\alpha\,dt = vT - \frac{u}{v}.uT$$

$$\Rightarrow T = \frac{\ell v}{v^2 - u^2}$$

Now distance travelled by A and B

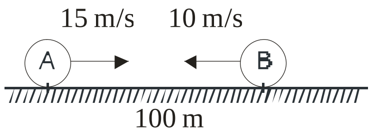
$$= u \times \frac{\ell v}{v^2 - u^2} \text{ and } v \times \frac{\ell v}{v^2 - u^2} = \frac{uv\ell}{v^2 - u^2} \text{ and } \frac{v^2\ell}{v^2 - u^2}$$

Exercise # 1

PART-I : SUBJECTIVE QUESTIONS

Section : (A) Relative motion in one dimension

A-1. An object A is moving with 15 m/s and B is moving with 10 m/s in the opposite direction of as shown. A is 100 m apart from B. Find the time taken by A to meet B.



A-2. Two parallel rail tracks run north-south. Train A moves due north with a speed of 54 km h^{-1} and train B moves due south with a speed of 90 km h^{-1}. A monkey runs on the roof of train A with a velocity of 18 km/h w.r.t. train A in a direction opposite to that of A. Calculate the (a) relative velocity of B with respect to A (b) relative velocity of ground with respect to B (c) velocity of a monkey as observed by a man standing on the ground. (d) velocity of monkey as observed by a passenger of train B.

A-3. A train is moving with a speed of 40 km/h. As soon as another train going in the opposite direction passes by the window, the passenger of the first train starts his stopwatch and notes that other train passes the window in 3 s. Find the speed of the train going in the opposite direction if its length is 75 m.

A-4. The driver of a train A running at 40 m s^{-1} sights a train B moving in the same direction on the same track with 20 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 2 ms^{-2}. what should be the minimum distance between the trains to avoid the accident .

A-5. A ball is thrown vertically upwards with a velocity 'u' w.r.t. ground from the balloon descending with velocity v. The ball will pass by the balloon after time.

Section : (B) Relative motion in two dimensions

B-1. A helicopter is flying due south with constant velocity 80 km/h and a train is running with the same speed due east. Direction of velocity of the helicopter relative to the train as observed by the passengers in the train is

B-2. A ship is steaming due east at 12 ms^{-1}. A woman runs across the deck at 5 ms^{-1} (relative to ship) in a direction towards north. Calculate the velocity of the woman relative to sea.

B-3. As two boats approach the mumbai, the velocity of boat 1 relative to boat 2 is $10\sqrt{3}$ kmhr^{-1} in a direction of 60° north of east. If boat 2 has a velocity of 15 kmhr^{-1} due south. What is the velocity of boat 1 ?

B-4. A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor with respect to ground .

B-5. A motorboat is observed to travel 10 km h^{-1} relative to the earth in the direction 37° north of east. If the velocity of the boat due to the wind only is 2 km h^{-1} westward and that due to the current only is 4 km h^{-1} southward, what is the magnitude and direction of the velocity of the boat due to its own power ?

Section : (C) Relative motion in river flow & Air flow

C-1. A boat speed in the direction of flow of river is 10 km h^{-1}. boat speed against the direction of flow of river is 6 km h^{-1}. Calculate the boat speed in still water and the velocity of flow of the river.

C-2. A man can swim in still water with a speed of 25 m/min. If the speed of the stream is 15 m/min, and its width is 100 m, the time taken to cross the stream by shortest route and the quickest route is

C-3. A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, swims across the shortest path distance. In what direction should he swim ?

Section : (D) Relative motion in Rain and wind

D-1. A boy is running on the plane road with velocity v with a long hollow tube in his hand. The water is falling vertically downwards with velocity u. At what angle to the vertical, he must incline the tube so that the water drops enters in it without touching its side ?

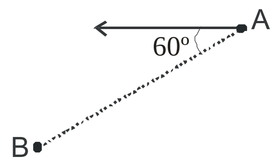
D-2. A car is moving uniformly westwards with 8 m/s, driver in the car can see rain falling vertical at a velocity 16 m/s. Find the speed of rain drops with respect to ground.

D-3. To a man walking at the rate of 2 km/hour with respect to ground, the rain appears to fall vertically. When he increases his speed to 4 km/hour in same direction of his motion, rain appears to meet him at an angle of 45° with horizontal, find the real direction and speed of the rain.

Section : (E) Velocity of separation & approach

E-1. A person standing at origin. Another person start its motion from (10,0) with the velocity $-6\hat{i}+8\hat{j}$. Find their closest distance of approach.

E-2. A particle A is moving with a constant velocity of $20\sqrt{3}$ m/sec. Another particle B is moving with a constant but unknown velocity. At an instant, the line joining A and B makes an angle of 60° with velocity of A. Find the minimum possible magnitude of velocity of B, if they collide after some time. (see figure)



E-3. When two bodies move uniformly towards each other, the distance between them diminishes by 16 m every 10 s. If bodies move with velocities of the same magnitude and in the same direction as before the distance between then will decrease 3 m every 5 s. Calculate the velocity of each body.

PART-II : OBJECTIVE QUESTIONS

Marked Questions are having more than one correct option.

Section : (A) Relative motion in one dimension

A-1. A culprit is running away on a straight road with a speed of 12 m s^{-1}. A police jeep is chasing him at a speed of 15 m s^{-1}. If the instantaneous separation of the jeep from the culprit is 90 m, how long will it take for the police jeep to catch the culprit ?

(A) 1s (B) 5s (C) 60s (D) 30s

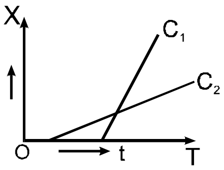
A-2. A stone is thrown upwards from a tower with a velocity 50 ms^{-1}. Another stone is simultaneously thrown downwards from the same location with a velocity 50 ms^{-1}. When the first stone is at the highest point, the relative velocity of the second stone with respect to the first stone is (assume that second stone has not yet reached the ground) :

(A) Zero (B) 50 ms^{-1} (C) 100 ms^{-1} (D) 150 ms^{-1}

A-3. An aeroplane is flying vertically upwards with a uniform speed of 500 m/s. When it is at a height of 1000 m above the ground a shot is fired at it with a speed of 700 m/s from a point directly below it. The minimum uniform acceleration of the aeroplane now so that it may escape from being hit?
($g = 10\ m/s^2$)

(A) 10 m/s^2 (B) 8 m/s^2 (C) 12 m/s^2 (D) None of these

A-4. Shown in the figure are the position time graph for two children going home from the school. Which of the following statements about their relative motion is true after both of them started moving ?
Their relative velocity :



(A) first increases and then decreases (B) first decreases and then increases
(C) is zero (D) is non zero constant.

A-5. Two cars P_1 and P_2 which are 50000 m apart are travelling on two parallel tracks in opposite direction with initial speed of 25 km/hr. Car P_1 accelerates at 10 km/hr^2 & car P_2 retards by 10 km/hr^2.
The distance covered by car P_1 when the cross each other.

(A) 45 km (B) 60 km/ (C) 30 km (D) 25 km

A-6. A jet airplane travelling from east to west at a speed of 500 km h^{-1} eject out gases of combustion at a speed of 1500 km h^{-1} with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground ?

(A) 1000 km h^{-1} in the direction west to east (B) 1000 km h^{-1} in the direction east to west
(C) 2000 km h^{-1} in the direction west to east (D) 2000 km h^{-1} in the direction east to west

A-7. Two cars get closer by 15 m every second while travelling in the opposite directions. They get closer by 3 m every second while travelling in the same directions. What are the speeds of the cars?

(A) 9 ms^{-1} and 6 ms^{-1} (B) 8 ms^{-1} and 7 ms^{-1}
(C) 6 ms^{-1} and 3 ms^{-1} (D) 6 ms^{-1} and 5 ms^{-1}

Section : (B) Relative motion in two dimension

B-1. A monkey is climbing up a tree at a speed of 3 m/s. A dog runs towards the tree with a speed of 4 m/s. What is the relative speed of the dog as seen by the monkey?

(A) > 7 m/s (B) between 5 m/s and 7 m/s
(C) 5 m/s (D) < 5 m/s

B-2 An aeroplane is travelling due west at 20 km/hr. An aeroplane flying 30º west of south is always due south from first aeroplane, speed of second aeroplane.

(A) 40 km/hr (B) 30 km/hr
(C) 50 km/hr (D) 20 km/hr

B-3. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum, when the angle between their velocities is :

(A) zero (B) $\pi/4$ (C) $\pi/2$ (D) π

B-4. Two billiard balls are rolling on a flat table. One has velocity components v_x = 3 m/s, v_y = 3m/s and the other has components v_x = $2\sqrt{3}$ m/s and v_y = 2 m/s. If both the balls start moving from the same point, the angle between their path is -

(A) 60° (B) 45° (C) 22.5° (D) 15°

Section : (C) Relative motion in river flow

C-1. A boat, which has a speed of 8 km/h in still water, crosses a river of width 1 km along the shortest possible path in 30 minutes. The velocity of the river water in km/h is -

(A) 3 (B) $2\sqrt{15}$ (C) $\sqrt{68}$ (D) $\sqrt{41}$

C-2. A boat which can move with a speed of 5 m/s relative to water crosses a river of width 480 m flowing with a constant speed of 4 m/s. What is the time taken by the boat to cross the river along the shortest path.

(A) 80 s (B) 160 s (C) 240 s (D) 320 s

C-3. To cross the river in shortest distance, a swimmer should swim making angle θ with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance. [Assume speed of swimmer in still water is greater than the speed of river flow]

(A) $\cos\theta$ (B) $\sin\theta$ (C) $\tan\theta$ (D) $\cot\theta$

Section (D): Relative motion in Rain and wind

D-1. It is raining vertically downwards with a velocity of 8 km h^{-1} . A man walks in the rain with a velocity of 6 kmh^{-1}. The rain drops will fall on the man with a relative velocity of ;

(A) 10 kmh^{-1} (B) 6 kmh^{-1} (C) 8 kmh^{-1} (D) 5 kmh^{-1}

D-2. To man running at a speed of 5m/s, the rain drops appear to be falling at an angle of 45° from the vertical. If the rain drops are actually falling vertically downwards, then velocity in m/s is

(A) 5 (B) $5\sqrt{3}$ (C) $5\sqrt{2}$ (D) 4

D-3. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is :

(A) 10 m/s (B) $10\sqrt{3}$ m/s (C) 20 m/s (D) $20\sqrt{3}$ m/s

D-4. An aeroplane has to go along straight line from A to B, and back again. The relative speed with respect to wind is 500 km/hr. The wind blows perpendicular to line AB with speed 300 km/hr. The distance between A and B is 1000 km. The total time for the round trip is :

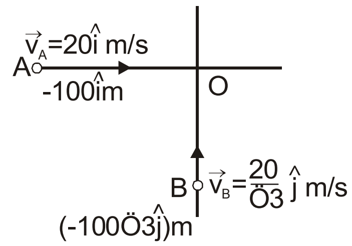
(A) 2.5 hr (B) 5 hr (C) 10 hr (D) 2 hr

Section : (E) Velocity of separation & approach

E-1. Two particles having position vectors $\vec{r}_1 = (3\hat{i} + 5\hat{j})$ metres and $\vec{r}_2 = (-5\hat{i} - 3\hat{j})$ metres are moving with velocities $\vec{v}_1 = (4\hat{i} + 3\hat{j})$ m/s and $\vec{v}_2 = (\alpha\hat{i} + 7\hat{j})$ m/s. If they collide after 2 seconds, the value of 'α' is

(A) 2 (B) 4 (C) 6 (D) 8

E-2. Positions of two vehicles A and B with reference to origin O and their velocities are as shown distance of closest approach is :-



(A) 100 m (B) $100\sqrt{3}$ (C) $100/\sqrt{3}$ (D) 50

PART-III : MATCH THE COLUMN

1. Two particles A and B moving in x-y plane are at origin at t = 0 sec. The initial velocity vectors of A and B are $\vec{u}_A = 8\hat{i}$ m/s and $\vec{u}_B = 8\hat{j}$ m/s. The acceleration of A and B are constant and are $\vec{a}_A = -2\hat{i}$ m/s^2 and $\vec{a}_B = -2\hat{j}$ m/s^2. Column I gives certain statements regarding particle A and B. Column II gives corresponding results. Match the statements in column I with corresponding results in Column II.

Column I	Column II
(A) The time (in seconds) at which velocity of A relative to B is zero	(p) $16\sqrt{2}$
(B) The distance (in metres) between A and B when their relative velocity is zero.	(q) $8\sqrt{2}$
(C) The time (in seconds) after t = 0 sec, at which A and B are at same position	(r) 8
(D) The magnitude of relative velocity of A and B at the instant they are at same position.	(s) 4

2. **Match the following :**

A ball is thrown vertically upward in the air by a passenger (relative to himself) from a train that is moving as given in column I ($v_{ball} << v_{escape}$). Correctly match the situation as described in the column I, with the paths given in column II.

Column I	Column II
(A) Train moving with constant acceleration on a slope then path of the ball as seen by the passenger.	(p) Straight line
(B) Train moving with constant acceleration on a slope then path of the ball as seen by a stationary observer outside.	(q) Parabolic
(C) Train moving with constant acceleration on horizontal ground then path of the ball as seen by the passenger.	(r) Elliptical
(D) Train moving with constant acceleration on horizontal ground then path of the ball as seen by a stationary observer outside.	(s) Hyperbolic
	(t) Circular

Exercise # 2

PART-I : OBJECTIVE QUESTIONS

1. Two cars are moving in the same direction with a speed of 30 km h^{-1}. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. What is the speed of the third car?
(A) 35 km h^{-1} (B) 40 km h^{-1} (C) 45 km h^{-1} (D) 75 km h^{-1}

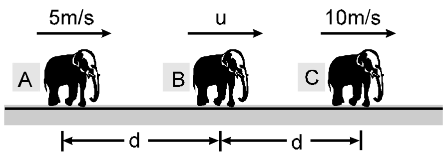
2. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car (as, seen by thief). According to thief in the car ?
(A) 105 m/s (B) 100 m/s (C) 110 m/s (D) 90 m/s

3. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is found to be ' t '. The acceleration with which the lift is moving up is :
(A) $\frac{u-gt}{t}$ (B) $\frac{2u-gt}{t}$ (C) $\frac{u+gt}{t}$ (D) $\frac{2u+gt}{t}$

4. For four particles A, B, C & D, the velocities of one with respect to other are given as $\vec{V}_{DC}$ is 20 m/s towards north, $\vec{V}_{BC}$ is 20 m/s towards east and $\vec{V}_{BA}$ is 20 m/s towards south. Then $\vec{V}_{DA}$ is
(A) 20 m/s towards north (B) 20 m/s towards south
(C) 20 m/s towards east (D) 20 m/s towards west

5. A particle is thrown up inside a stationary lift of sufficient height. The time of flight is T. Now it is thrown again with same initial speed v_0 with respect to lift. At the time of second throw, lift is moving up with speed v_0 and uniform acceleration g upward (the acceleration due to gravity). The new time of flight is–
(A) $\frac{T}{4}$ (B) $\frac{T}{2}$ (C) T (D) 2T

6. Three elephants A, B and C are moving along a straight line with constant speed in same direction as shown in figure. Speed of A is 5 m/s and speed of C is 10 m/s. Initially separation between A & B is 'd' and between B & C is also d. When 'B' catches 'C' separation between A & C becomes 3d. Then the speed of B will be -



(A) 7.5 m/s (B) 15 m/s (C) 20 m/s (D) 5 m/s

7. A train is standing on a platform , a man inside a compartment of a train drops a stone . At the same instant train starts to move with constant acceleration . The path of the particle as seen by the person who drops the stone is :
(A) parabola
(B) straight line for sometime & parabola for the remaining time
(C) straight line
(D) variable path that cannot be defined

8. A battalion of soldiers is ordered to swim across a river 500 m wide . At what minimum rate should they swim perpendicular to river flow in order to avoid being washed away by the waterfall 300 m downstream. The speed of current being 3 m/sec :

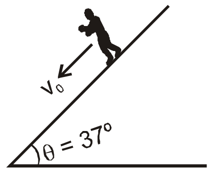
(A) 6 m/sec. (B) 5 m/sec. (C) 4 m/sec (D) 2 m/sec

9. A bucket is placed in the open where the rain is falling vertically. If a wind begins to blow horizontally at double the velocity of the rain, how will be rate of filling of the bucket change?

(A) Remain unchanged (B) Doubled

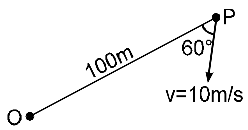
(C) Halved (D) Become four times

10. A man is moving downward on an inclined plane ($\theta = 37^\circ$) with constant velocity v_0 and rain drops appear to him moving in horizontal direction with velocity $2v_0$ towards him. If man increases his velocity to $2v_0$. Velocity of rain drops as observed by man is :



(A) $\sqrt{\frac{41}{5}}\ v_0$ (B) $\sqrt{39}\ v_0$ (C) $7\ v_0$ (D) $6\ v_0$

11. P is a point moving with constant speed 10 m/s such that its velocity vector always maintains an angle 60° with line OP as shown in figure (O is a fixed point in space). The initial distance between O and P is 100 m. After what time shall P reach O.



(A) 10 sec. (B) 15 sec. (C) 20 sec. (D) $20\sqrt{3}$ sec

PART-II : SUBJECTIVE QUESTIONS

1. The driver of a car travelling at a speed of 20 m/s, wishes to overtake a truck that is moving with a constant speed of 20 m s^{-1} in the same lane. The car's maximum acceleration is 0.5 m s^{-2}. Initially the vehicles are separated by 40 m, and the car returns back into its lane after it is 40 m ahead of the truck. The car is 3 m long and the truck 17 m. Then find the minimum time (in sec) required for the car to pass the truck and return back to its lane?

2. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclists are riding in the same direction at 25 km/h at equal intervals of 30 m. At what speed (km/h) an observer travel along the road in opposite direction so that whenever he meets a runner he also meets a cyclist? (neglect the size of cycle)

3. A boat moves relative to water with a velocity half of the river flow velocity. To minimize the drif, at the angle with the stream direction of the boat must be $\frac{2\pi}{n}$. Then find the vlaue of n.

4. A motorboat going down stream overcome a float at a point M which is fixed with respect to ground. 60 minutes later it turned back and after some time passed, the float at a distance of 6 km from the point M. Find the velocity of the stream (in km/h) assuming a constant velocity for the motorboat in still water.

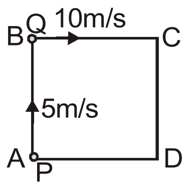
5. An aeroplane has to go from a point A to another point B, 1000 km away due 30° west of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. If the angle at which the pilot should head the plane to reach the point B is $\sin^{-1}\left(\frac{1}{n}\right)$ west of line AB. Then find the value of n.

6. Rain appears to be falling at an angle of 37° with vertical to the driver of a car moving with a velocity of 7 m/sec. When he increases the velocity of the car to 25 m/sec, the rain again appears to fall at an angle 37°. with vertical. the angle of actual velocity of rain with respect to ground is $\frac{n\pi}{180}$ with vertical. Then find the value of n.

7. During a rainy day, rain is falling vertically with a velocity 2m/s. A boy at rest starts his motion with a constant acceleration of $2 m/s^2$ along a straight road the rate at which the angle of the axis of umbrella with vertical should be changed so that the rain always falls parallel to the axis of the umbrella is $\frac{1}{n}$ at t = 2 sec then find the value of n.

8. Two straight tracks AOB and COD meet each other at right angles at point O. A person walking at a speed of 5 km/h along AOB is at the crossing O at 12 o'clock noon. Another person walking at the same speed along COD reaches the crossing O at 1:30 PM. If the time at which the distance between them is least is 12 : n pm then find the value of n ?

9. Two men P & Q are standing at corners A & B of square ABCD of side 10 m. They start moving along the track with constant speed 5 m/s and 10 m/s respectively. The time (in sec) when they will meet for the first time is.



PART - III : ONE OR MORE THAN ONE CORRECT QUESTION

1. A man standing on the edge of the terrace of a high rise building throws a stone vertically up with a speed of 20 m/s. Two seconds later an identical stone is thrown vertically downwards with the same speed of 20 m/s. Then :

(A) the relative velocity between the two stones remain constant till one hits the ground

(B) both will have the same kinetic energy when they hit the ground

(C) the time interval between their hitting the ground is 2 seconds

(D) if the collisions on the ground are perfectly elastic both will rise to the same height above the ground.

2. A man in a lift which is ascending with an upward acceleration 'a' throws a ball vertically upwards with a velocity 'v' with respect to himself and catches it after 't_1' seconds. Afterwards when the lift is descending with the same acceleration 'a' acting downwards the man again throws the ball vertically upwards with the same velocity with respect to him and catches it after 't_2' seconds

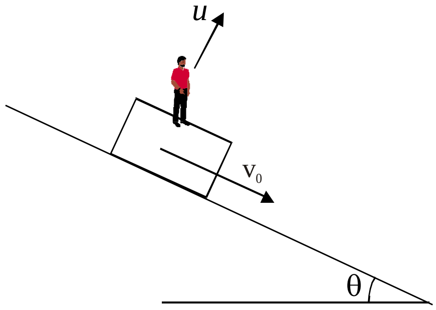
(A) the acceleration of the ball with respect to ground is g when it is in air

(B) the velocity v of the ball relative to the lift is $\frac{g(t_1+t_2)}{t_1 t_2}$

(C) the acceleration 'a' of the lift is $\frac{g(t_2-t_1)}{t_1+t_2}$

(D) the velocity 'v' of the ball relative to the man is $\frac{g t_1 t_2}{(t_1+t_2)}$

3. When a rectangular box is going down a smooth inclined plane with initial velocity v_o, a person standing on the rectangular box throws up a ball perpendicular to the inclined plane with velocity u with respect to himself as shown in the figure. Which of the following is/are true about the ball, coming back to incline?



(A) It will fall behind the person

(B) It will fall ahead of the person

(C) It will come back into the hands of person

(D) Neglecting the height of point of projection above box total time taken by the ball to return on the box is $\frac{2u}{g\cos\theta}$

4. A ball is thrown vertically upward (relative to the train) in a compartment of a moving train. (train is moving horizontally)

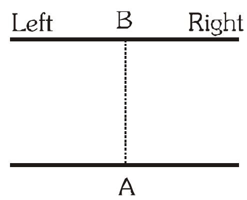
(A) The ball will maintain the same horizontal velocity as that of the person (or the compartment) at the time of throwing.

(B) If the train is accelerating then the horizontal velocity of the ball will be different from that of the train velocity, at the time of throwing.

(C) If the ball appears to be moving backward to the person sitting in the compartment it means that speed of the train is increasing.

(D) If the ball appears to be moving ahead of the person sitting in the compartment it means the train's motion is retarding.

5. Velocity of swimmer in still water is v_S and velocity of river is v_R. If $v_S > v_R$. Then



(A) swimmer can reach right of B

(B) swimmer can reach left of B

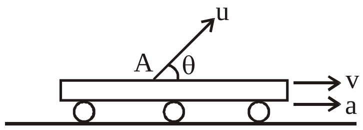
(C) minimum time to cross the river is independent of velocity of river

(D) time taken by swimmer to reach the point B is more than minimum time of crossing river

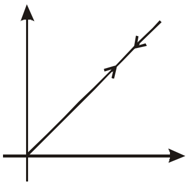
PART - IV : COMPREHENSION

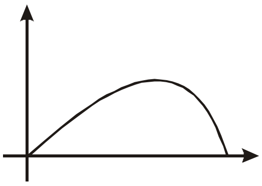
Comprehension # 1

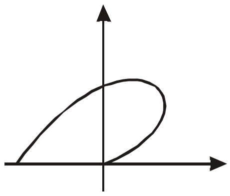
A trolley is moving with speed 'v' and acceleration 'a' horizontally. A ball is projected from trolley with speed u and angle θ w.r.t. trolley as shown in the figure.

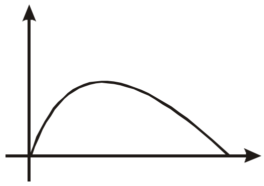


1. Which of the following can not be the trajectory of ball as seen from the trolley.

(A) 

(B) 

(C) 

(D) 

2. If acceleration of trolley is zero which of the following will be affected due to the motion of trolley.

(A) Maximum height (B) Time of fight

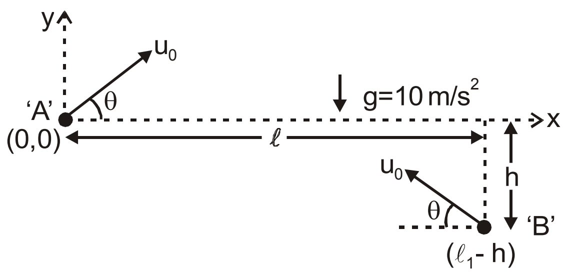
(C) Range (D) Vertical component of velocity

3. When ball is projected, velocity of trolley is zero and acceleration is a. Ball is projected with speed u at an angle θ w.r.t. trolley from point A. What should be the acceleration of trolley so that ball again strike the trolley at A.

(A) $g \cot \theta$ (B) $g \tan \theta$ (C) $g \cos \theta$ (D) $g \sin \theta$

Comprehension # 2

Two particles 'A' and 'B' are projected in the vertical plane with same initial speed u_0 from part (0, 0) and (ℓ , – h) towards each other as shown in figure at t = 0.



4. The path of particle 'A' with respect to particle 'B' will be

(A) parabola (B) straight line parallel to x-axis.

(C) straight line parallel to y-axis (D) none of these.

5. Minimum distance between particle A and B during motion will be :

(A) ℓ (B) h (C) $\sqrt{\ell^2 + b^2}$ (D) ℓ + h

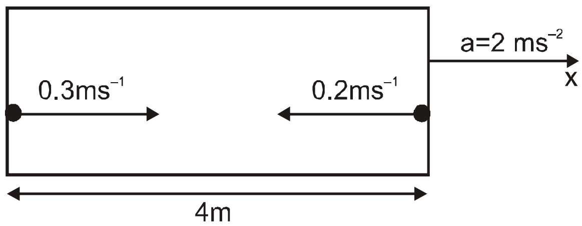
6. The time when separation between A and B is minimum is :

(A) $\frac{x}{u_0 \cos\theta}$ (B) $\sqrt{\frac{2h}{g}}$ (C) $\frac{\ell}{2u_0 \cos\theta}$ (D) $\frac{2\ell}{u_0 \cos\theta}$

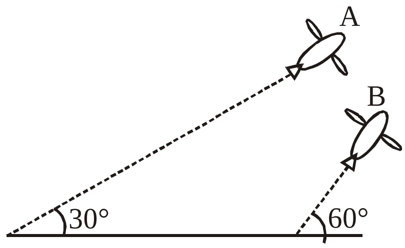
Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A rocket is moving in a gravity free space with a constant acceleration of $2ms^{-2}$ along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of $0.3ms^{-1}$ relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of $0.2ms^{-1}$ from its right end relative to the rocket. The time in seconds when the two balls hit each other is: **[JEE (Advanced)-2014,P-1, 3/60]**



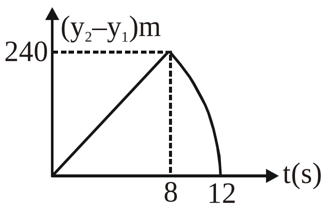
2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is **[JEE (Advanced)-2014,P-1, 3/60]**

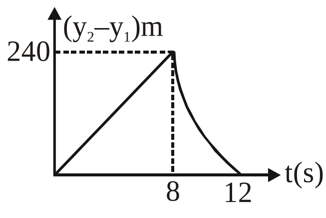


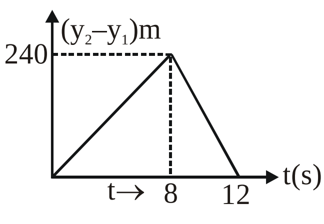
PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

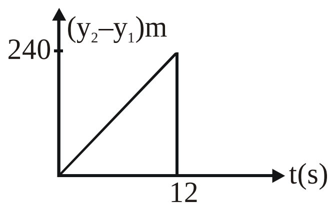
1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s²)
(The figure are schematic and not drawn to scale) **[JEE Main-2015; 4/120, –1]**

(1)


(2)


(3)


(4)


2. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[JEE (Main)-2019,April; 4/120, –1]**

(1) 4.2 hrs. (2) 2.2 hrs. (3) 3.2 hrs. (4) 2.6 hrs.

3. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

[JEE (Main)-2019,April; 4/120, –1]

(1) 60° (2) 150° (3) 90° (4) 120°

4. A particle is moving along the x-axis with its coordinate with the time 't' given be $x(t) = 10 + 8t - 3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is __________.

[JEE (Main)-2020, January; 4/100]

Answers

Exercise - 1

PART - I

Section (A):

A-1. 4 sec.

A-2. (a) 144 km/h due south ; (b) 90 km/hdue north ; (c) 36 km/h due north; (d) 126 km/h due north

A-3. 50 km/h **A-4.** 100 m **A-5.** $\frac{2(u+v)}{g}$

Section (B):

B-1. South-west **B-2.** 13 m/s, $\tan^{-1}\left(\frac{5}{12}\right)$ = 22°37′ north of east

B-3. $5\sqrt{3}$ kmhr^{-1} due east **B-4.** $\hat{i}+\sqrt{2}\,\hat{j}+\hat{k}$, $\hat{i}\rightarrow$ east , $\hat{j}\rightarrow$ north $\hat{k}\rightarrow$ verticle upward

B-5. $10\sqrt{2}$ km/h, 45° N of E

Section (C):

C-1. 8 km/h , 2km/h **C-2.** 5min, 4 min. **C-3.** At an angle 30° west of north

Section (D):

D-1. $\tan^{-1}\frac{v}{u}$ **D-2.** $8\sqrt{5}$ m/s **D-3.** $2\sqrt{2}$ m/s , 45° with vertically away from the man.

Section (E):

E-1. 8 m **E-2.** 30 m/sec. **E-3.** $v_1 = \frac{11}{10}$ m/s and $v_2 = \frac{1}{2}$ m/s.

PART - II

Section (A):

A-1. (D) **A-2.** (C) **A-3.** (A) **A-4.** (D) **A-5.** (C)
A-.6. (A)

Section (B):

B-1. (C) **B-2** (A) **B-3.** (D) **B-4.** (D)

Section (C):

C-1. (B) **C-2.** (B) **C-3.** (B)

Section (D):

D-1. (A) **D-2.** (A) **D-3.** (B) **D-4.** (B)

Section : (E)

E-1. (D) **E-2.** (A)

PART - III

1. (A) $\rightarrow$ s, (B) $\rightarrow$ p, (C) $\rightarrow$ r, (D) $\rightarrow$ q **2.** (A) $\rightarrow$q, (B) $\rightarrow$ q , (C) $\rightarrow$ q, (D) $\rightarrow$ q

Exercise - 2

PART-I

1.	(C)	**2.**	(A)	**3.**	(B)	**4.**	(D)	**5.**	(B)
6.	(B)	**7.**	(C)	**8.**	(B)	**9.**	(A)	**10.**	(A)
11.	(C)								

PART-II

1.	20	**2.**	5	**3.**	(3)	**4.**	(3)	**5.**	15
6.	53	**7.**	5	**8.**	45	**9.**	6		

PART-III

1.	(A,B,C,D)	**2.**	(A,C,D)	**3.**	(C,D)	**4.**	(A,C,D)	**5.**	(A,B,C,D)
6.	(C,D)								

PART-IV

1.	(D)	**2.**	(C)	**3.**	(A)	**4.**	(B)	**5.**	(B)

Exercise - 3

PART - I

1.	2	**2.**	5

PART - II

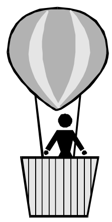
1.	(1)	**2.**	(4)	**3.**	(4)	**4.**	580.00

Ranker Problems

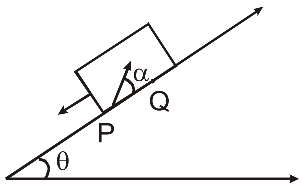
SUBJECTIVE QUESTIONS

1. Two cars A and B are racing along straight line. Car A is leading, such that their relative velocity is directly proportional to the distance between the two cars. When the lead of car A is ℓ_1 = 10 m, its running 10 m/s faster than car B. Find the time car A will take to increase its lead to ℓ_2 = 20 m from car B is :

2. A man in a balloon, throws a stone downwards with a speed of 5 m/s with respect to balloon . The balloon is moving upwards with a constant acceleration of 5 m/s^2. Then velocity of the stone relative to the man after 2 second is :



3. A large heavy box is sliding without friction down a smooth plane of inclination θ From a point P on the bottom of the box, a particle is thrown inside box. The initial speed of the particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown in the figure :

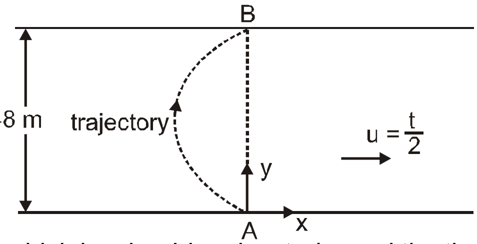


(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected. **[JEE-1998, 5 + 3/120]**

4. Find the time an aeroplane having velocity v (relative to air), takes to fly around a square with side a and the wind blowing at a velocity u, in the two cases,
(a) if the direction of wind is along one side of the square,
(b) If the direction of wind is along one of the diagonals of the square

5. A man starts swimming at time t = 0 from point A on the ground and he wants to reach the point B directly opposite the point A. His velocity in still water is 5 $\frac{m}{sec}$ and width of river is 48 m. River flow velocity 'u' varies with time t (in seconds) as u = $\frac{t}{2}$ $\frac{metre}{sec}$. He always tries to swim in particular fixed direction with river flow. Find the (Given $\sin^{-1}\left(\frac{24}{25}\right)$ = 74º)

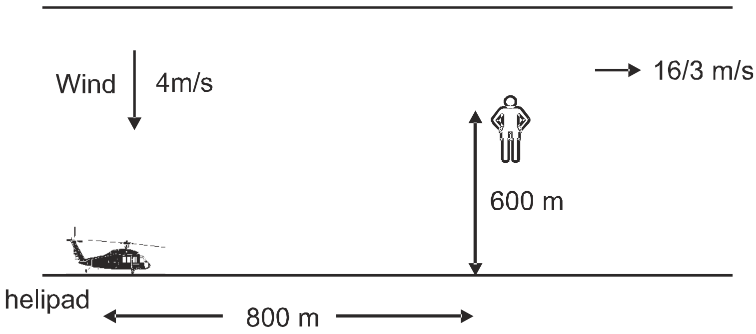


(a) direction (with line AB) in which he should make stroke and the time taken by man to cross the river.
(b) trajectory of path.

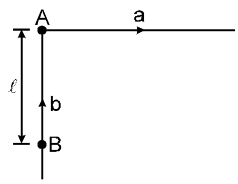
6. A child in danger of drowing in a river is being carried downstream by a current that flows uniformly at a speed of 16/3 m/sec. The child is 600m from shore and 800 m downstream of a helipad when the rescue helicopter sets out and the helicopter follows the same direction throughout the motion. If the helicopter proceeds at its maximum speed of 80/3 m/sec. with respect to air and air is blowing with velocity of 4m/sec perpendicular to river flow velocity as shown, then

(i) Heading at what angle with the shore should the helicopter take off?

(ii) calculate the time taken by helicopter to reach the child.



7. In the figure shown A and B are two particles which start from rest. A has constant acceleration 'a' in the direction shown. B also increases its speed at a coznstant rate 'b', but the direction of velocity is always towards A. Find the time after which B meets A. Also find the total distance travelled by B. (b > a)



8. Two guns situated on the top of a hill of height 10m fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontaly and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at point P. Find :

(a) The time interval between the firings and

(b) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane

9. Two swimmers start from point A on one bank of a river to reach point B on the other bank, lying directly opposite to point A. One of them crosses the river along the straight line AB, while the other swims at right angles to the stream and then walks the distance which he has been carried away by the stream to get to point B. What was the velocity (assumed uniform) of his walking if both the swimmers reached point B simultaneously. Velocity of each swimmer in still water is 2.5 km h^{-1} and the stream velocity is 2 km h^{-1}.

10. A cat runs along a straight line with constant velocity of magnitude v. A dog chases the cat such that the velocity of dog is always directed towards the cat. The speed of dog is 'u' and always constant. At the instant both are separated by distance x and their velocities are mutually perpendicular, the magnitude of acceleration of dog is.

Answers

Ranker Problems

1. $t = (\log_e 2)$ sec

2. 35 m/s

3. (a) PQ = $(u^2 \sin 2\alpha)/ g \cos\theta$ (b) $v = \frac{u\cos(\alpha+\theta)}{\cos\theta}$

4. (a) $\frac{2a}{v^2-u^2}\left(v+\sqrt{v^2-u^2}\right)$ (b) $2\sqrt{2}\,a\left(\frac{\sqrt{2v^2-u^2}}{v^2-u^2}\right)$

5. (a) 37° and 53°, 12 sec. and 16 sec. (b) $x = \frac{3y}{4} + \frac{y^2}{64}$

6. (i) 37° (ii) 50 sec

7. $t = \sqrt{\frac{2\ell b}{b^2-a^2}}, \frac{b^2\ell}{b^2-a^2}$

8. (a) 1s (b) $5\sqrt{3}$ m. + 5 m

9. 3 km/h towards B

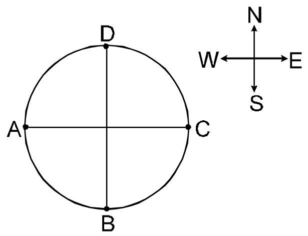
10. $\frac{uv}{x}$

Self Assessment Test

JEE (ADVANCED) PAPER-1

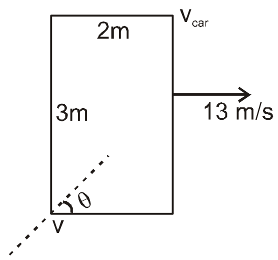
SECTION-1 : ONE OPTION CORRECT (Maximum Marks - 12)

1. Two aeroplanes fly from their respective positions 'A' and 'B' starting at the same time and reach the point 'C' simultaneously when wind was not blowing. On a windy day they head towards 'C' but both reach the point 'D' simultaneously in the same time which they took to reach 'C'. Then the wind is blowing in



(A) North-East direction
(B) North-West direction
(C) Direction making an angle 0 < θ < 90 with North towards West.
(D) North direction

2. A car 2m long and 3m wide is moving at 13 m/sec when a bullet hits it in a direction making an angle $\theta = \tan^{-1} 3/4$ with the car as seen from the ground. The bullet enters one edge of the corner and passes out at the diagonally opposite corner. Neglecting any interaction between bullet and car find the time for the bullet to cross the car :



(A) $\frac{1}{4}$ sec (B) $\frac{13}{10}$ sec (C) $\frac{2}{13}$ sec (D) $\frac{3}{5}$ sec

3. A swimmer crosses the river along the line making an angle of 45º with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be:

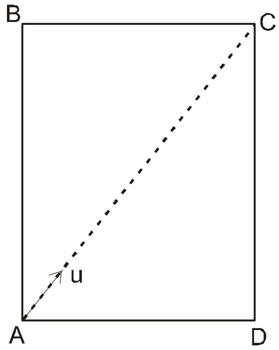
(A) 10 m/s (B) 12 m/s (C) $5\sqrt{5}$ m/s (D) $10\sqrt{2}$ m/s

4. A man who is wearing a hat of extended length of 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops do not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be:

(A) $\frac{15}{2}$ m/s (B) $\frac{40}{3}$ m/s (C) 10 m/s (D) zero

SECTION-2 : ONE OR MORE THAN ONE CORRECT (Maximum Marks - 32)

5. Figure shows an elevator cabin, which is moving downwards with constant acceleration a. A particle is projected from corner A, directly towards diagonally opposite corner C. Then :



(A) particle will hit C only when a = g.

(B) particle may hit the wall CD if a < g

(C) particle will hit the roof BC if a > g

(D) Nothing can be said because numerical data is not given.

6. Consider a boy on a trolley who throws a ball with speed 20 m/s at an angle 37^0 with respect to trolley which moves horizontally with speed 10 m/s. If ball is projected in direction of motion of trolley then :

(A) The horizontal and vertical components of initial velocity of ball are 26 m/s, 12 m/s .

(B) The horizontal range with respect to boy is $\frac{190}{5}$ m

(C) The time of flight & maximum height are $\frac{12}{5}$ s , $\frac{36}{5}$ m

(D) The range with respect to ground is $\frac{312}{5}$ m

7. For previous question If ball is projected opposite to direction of motion of trolley then :

(A) The horizontal and vertical components of initial velocity of ball are –6 m/s, 12 m/s

(B) The horizontal range with respect to boy is $\frac{192}{5}$ m

(C) The range with respect to ground is $\frac{72}{5}$ m

(D) The time of flight & maximum height are $\frac{12}{5}$ s , $\frac{36}{5}$ m

8. A person is standing on a truck moving with a constant velocity of 15 m/s on a horizontal road. The man throws a ball in such a way that it returns to his hand after the truck has moved 60 m. (g = 10 m/s^2)

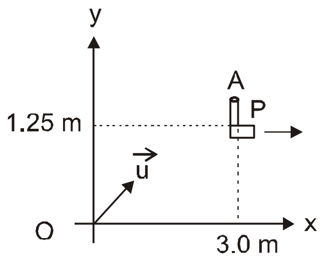
(A) Initially the speed of the ball as seen from the truck is 20 m/s

(B) Initially the direction of initial velocity of ball is upward as seen from the truck

(C) The speed of the ball as seen from the ground is 25 m/s

(D) None of these

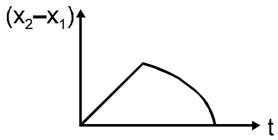
9. An object A is kept fixed at the point x = 3 m and y = 1.25 m on a plank P raised above the ground. At time t = 0 the plank starts moving along the + x direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45^0 to the horizontal. All the motions are in X–Y plane. Find speed and the time after which the stone hits the object. Take g = 10 m/s^2.

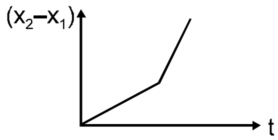


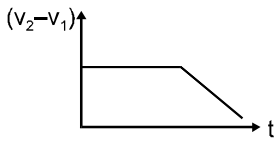
(A) Initial velocity of stone u is 8 m/sec
(B) Initial velocity of stone u is 7.28 m/sec
(C) the time after which the stone hits the object is 1 second.
(D) the time after which the stone hits the object is 2 second.

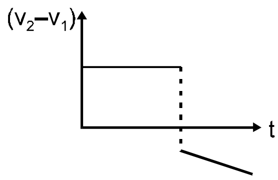
10. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself
(A) The ball will always return to him.
(B) The ball will never return to him.
(C) The ball will return to him if the cart moves with constant velocity.
(D) The ball will fall behind him if the cart moves with some positive acceleration.

11. Two stones are thrown vertically upwards simultaneously from the same point on the ground with initial speed u_1 = 30 m/sec and u_2 = 50 m/sec. Which of the curve represent correct variation (for the time interval in which both reach the ground) of
$(x_2 - x_1)$ = the relative position of second stone with respect to first with time (t).
$(v_2 - v_1)$ = , the relative velocity of second stone with respect to first with time (t).
Assume that stones do not rebound after hitting.

(A) 

(B) 

(C) 

(D) 

12. A man is standing on a road and observes that rain is falling at angle 45° with the vertical. The man starts running on the road with constant acceleration 0.5 m/s^2. After a certain time from the start of the motion, it appears to him that rain is still falling at angle 45° with the vertical, with speed $2\sqrt{2}$ m/s . Motion of the man is in the same vertical plane in which the rain is falling. Then which of the following statement(s) are true.
(A) It is not possible
(B) Speed of the rain relative to the ground is 2 m/s.
(C) Speed of the man when he finds rain to be falling at angle 45° with the vertical, is 4m/s.
(D) The man has travelled a distance 16m on the road by the time he again finds rain to be falling at angle 45°.

SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. A train of length ℓ = 350 m starts moving rectilinearly with constant acceleration $\omega = 3.0 \times 10^{-2}$ m/s^2; t = 30 s after the start the locomotive headlight is switched on (event 1), and τ = 60 sec after that event the train signal light is switched on (event 2). At what constant velocity V (in m/s) relative to the Earth must a certain reference frame K move for the two events to occur in it at the same point?

14. A boy sitting at the rear end of a railway compartment of a train, running at a constant acceleration on horizontal rails, throws a ball towards the fore end of the compartment with a muzzle velocity of 20 m/sec at an angle 37° above the horizontal, when the train is running at a speed of 10 m/sec. If the same boy catches the ball without moving from his seat and at the same height of projection, find the speed of the train at the instant of his catching the ball. [g = 10 m/sec^2; sin 37° = 3/5]

15. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is :

16. Two identical trains take 3 sec to pass one another when going in the opposite direction but only 2.5 sec if the speed of one is increased by 50 %. The time (in sec) one would take to pass the other when going in the same direction at their original speed is :

17. Taxies leaves the station X for station Y every 10 minutes. Simultaneously, taxies also leaves the station Y for station X every 10 minutes. The taxies move at the same constant speed and go form X to Y or vice versa in 2 hours. How many taxies coming from the other side will meet each taxi moving from Y to X in the path.

18. Two particles P and Q are moving with constant velocities of $(\hat{i}+\hat{j})$ m/s and $\left(-\hat{i}+2\hat{j}\right)$ m/s respectively. At time t = 0, P is at origin and Q is at a point with position vector $\left(2\hat{i}+\hat{j}\right)$ m. Equation of the trajectory of Q as observed by P is x + ay = b, then find the value of a + b :

Answers

SAT (Self Assessment Test)

1.	(B)	**2.**	(C)	**3.**	(C)	**4.**	(A)	**5.**	(A,B,C)
6.	(A,C,D)	**7.**	(A,B,C,D)	**8.**	(A,B,C)	**9.**	(B,C)	**10.**	(C,D)
11.	(A,D)	**12.**	(C,D)	**13.**	04.03	**14.**	42.00	**15.**	03.00
16.	15.00	**17.**	23.00	**18.**	06.00				