EAMCET
Engineering Entrance Exam
Solved Paper 2012

Physics

1. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a rigid support. The time taken by the transverse wave formed in the rope to travel through the full length of the rope is

   (Assume $g = 9.8 \text{ m/s}^2$)

   a) 0.5 s b) 1.6 s

   c) 1.2 s d) 1.0 s

2. When a vibrating tuning fork is placed on a sound box of a sonometer, 8 beats per second are heard when the length of the sonometer wire is kept at 101 cm or 100 cm. Then the frequency of the tuning fork is (Consider that the tension in the wire is kept constant)

   a) 1616 Hz b) 1608 Hz

   c) 1632 Hz d) 1600 Hz

3. The objective and eyepiece of an astronomical telescope are double convex lenses with refractive index 1.5. When the telescope is adjusted to infinity, the separation between the two lenses is 16 cm. If the space between the lenses is now filled with water and again telescope is adjusted for infinity, then the present separation between the lenses is

   a) 8 cm b) 16 cm

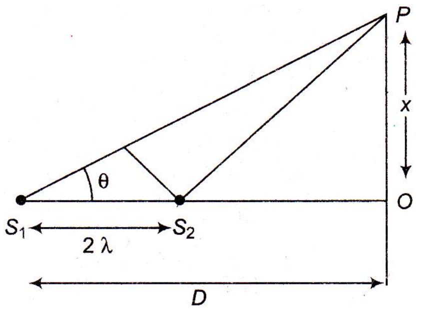
   c) 24 cm d) 32 cm

4. The dispersive powers of the materials of two lenses forming an achromatic combination are in the ratio of 4 : 3. Effective focal length of the two lenses is + 60 cm then the focal lengths of the lenses should be

   a) – 20 cm, 25 cm b) 20 cm, – 25 cm

   c) –15 cm, 20 cm d) 15 cm, – 20 cm

5. Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between them is 2λ as shown in figure. The first bright fringe is formed at P due to interference on a screen placed at a distance D from S_1 ($D >> \lambda$), then OP is



   a) $\sqrt{2}D$ b) 1.5 D

   c) $\sqrt{3}D$ d) 2 D

6. At a certain place a magnet makes 30 oscillations per minute. At another place where the magnetic field is double, its time period will be

   a) 4 s b) 2 s

   c) $\frac{1}{2}$S d) $\sqrt{2}$ s

7. A magnetic needle lying parallel to a magnetic field is turned through 60^o. The work done on it is W. The torque required to maintain the magnetic needle in the position mentioned above is

   a) $\sqrt{3}$ W b) $\frac{\sqrt{3}}{2}$W

   c) $\frac{W}{2}$ d) 2 W

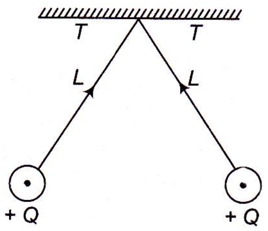
8. A parallel plate capacitor has a capacity 80×10^{-6} F, when air is present between the plates. The volume between the plates is then completely filled with a dielectric slab of dielectric constant 20. The capacitor is now connected to a battery of 30 V by wires. The dielectric slab is then removed. Then, the charge that passes now through the wire is

   a) 45.6×10^{-3}C b) 25.3×10^{-3}C

   c) 120×10^{-3}C d) 125×10^{-3}C

9. Two small spheres each having equal positive charge Q (Coulomb) on each are suspended by two insulating strings of equal length L (metre) from

a rigid hook (shown in figure). The whole set up is taken into satellite where there is no gravity. The two balls are now held by electrostatic forces in horizontal position, the tension in each string is then

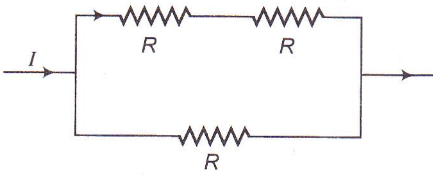


a) $\dfrac{Q^2}{16\pi\varepsilon_0 L^2}$ b) $\dfrac{Q^2}{8\pi\varepsilon_0 L^2}$

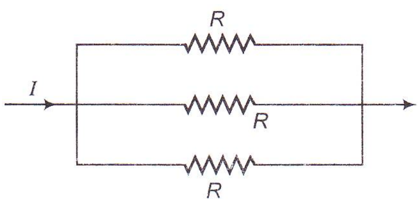
c) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}$ d) $\dfrac{Q^2}{2\pi\varepsilon_0 L^2}$

10. Three resistances of equal values are arranged in four different configurations as shown below. Power dissipation in the increasing order is

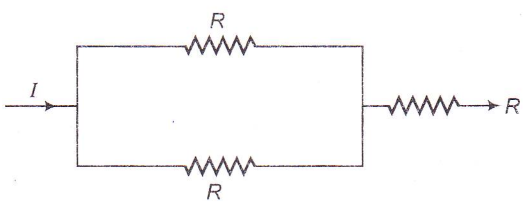
(I)



(II)



(III)



(IV)

a) (III) < (II) < (IV) < (I)

b) (II) < (III) < (IV) < (I)

c) (I) < (III) < (III) < (II)

d) (I) < (III) < (II) < (IV)

11. Four resistors A, B, C and D form a Wheatstones bridge. The bridge is balanced, when C = 100 Ω. If A and B are interchanged, the bridge balances for C = 121 Ω. The value of D is

a) 10 Ω b) 100 Ω

c) 110 Ω d) 120 Ω

12. Total emf produced in a thermocouple does not depend on

a) the metals in the thermocouple

b) thomson coefficients of the metals in the thermocouple

c) temperature of the junctions

d) the duration of time for which the currrent is passed through thermocouple

13. A long curved conductor carries a current I (I is a vector). A small current element of length dl, on the wire induces a magnetic field at a point, away from the current elemnt. If the position vector between the current element and the point is r, making an angle with current element then, the induced magnetic field density; dB (vector) at the point is (μ_0 = permeability of free space)

a) $\dfrac{\mu_0 I dl \times r}{4\pi r}$ (perpendicular to the current element d l)

b) $\dfrac{\mu_0 I \times r \times dl}{4\pi r^2}$ (perpendicular to the current element d l)

c) $\dfrac{\mu_0 I \times dl}{r}$ (perpendicular to the plane containing the current element and position vector r)

d) $\dfrac{\mu_0 I \times dl}{4\pi r^2}$ (perpendicular to the plane containing current element and position vector r)

14. A primary coil and secondary coil are placed close to each other. A current, which changes at the rate of 25 A in a millisecond, is present in the primary

coil. If the mutual inductance is 92×10^{-6} H, then the value of induced emf in the secondary coil is

a) 4.6 V b) 2.3 V

c) 0.368 mV d) 0.23 mV

15. The de-Broglie wavelength of an electron moving with a velocity of 1.5×10^{8} m/s is equal to that of a photon. The ration of kinetic energy of the electron to that of the photon ($c = 3 \times 10^{8}$ m/s)

a) 2 b) 4

c) $\frac{1}{2}$ d) $\frac{1}{4}$

16. A proton when accelerated through a potential difference of V, has a de-Broglie wavelength λ associated with it. If an α -particle is to have the same de-Broglie wavelength λ, it must be accelerated through a potential difference of

a) $\frac{V}{8}$ b) $\frac{V}{4}$

c) 4 V d) 8 V

17. The half-life of Ra^{226} is 1620 years. Then the number of atoms decay in one second in 1 g of radium (Avogadro number $= 6.023 \times 10^{23}$)

a) 4.23×10^{9} b) 3.16×10^{10}

c) 3.61×10^{10} d) 3.16×10^{10}

18. The half-life of a radioactive element is 10 h. The fraction of initial radioactivity of the element that will remain after 40 h is

a) $\frac{1}{2}$ b) $\frac{1}{16}$ c) $\frac{1}{8}$ d) $\frac{1}{4}$

19. In a transistor if $\frac{I_C}{I_E} = \alpha$ and $\frac{I_C}{I_B} = \beta$. If α varies between $\frac{20}{21}$ and $\frac{100}{101}$, then the value of β lies between

a) 1–10 b) 0.95 - 0.99

c) 20 – 100 d) 200 – 300

20. Match Column A (layers in the ionosphere for skywave propagation) with Column B (their height range)

	Column A		Column B
A.	D-layer	I.	250-400 km
B.	E-layer	II.	170-190 km
C.	F_1-layer	III.	95-120 km
D.	F_2-layer	IV.	65-75 km

	A	B	C	D
a)	I	II	III	IV
b)	IV	III	I	II
c)	IV	III	II	I
d)	II	IV	I	II

21. The gravitational field in a region is given by equation E = (5i + 12j) N/kg. If a particle of mass 2 kg is moved from the origin to the point (12 m, 5 m) in this region, the change in gravitational potential energy is

a) – 225 J b) –240 J

c) – 245 J d) – 250 J

22. The time period of a particle in simple harmonic motion is 8 s. At t = 0, it is at the mean position. The ratio of the distances travelled by it in the first and second seconds is

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$

c) $\frac{1}{\sqrt{2}-1}$ d) $\frac{1}{\sqrt{3}}$

23. A tension of 22 N is applied to a cooper wire of cross-sectional area 0.02 cm^2 Young's modulus of copper is 1.1×10^{11} N/m^2 and Poisson's ration 0.32. The decrease in cross-sectional area will be

a) 1.28×10^{-6} cm^2 b) 1.6×10^{-6} cm^2

c) 2.56×10^{-6} cm^2 d) 0.64×10^{-6} cm^2

24. Drops of liquid of density d are floating half immersed in a liquid of density p. If the surface tension of the liquid is T, then the radius of the drop is

a) $\sqrt{\frac{3T}{g(3d-\rho)}}$ b) $\sqrt{\frac{6T}{g(2d-\rho)}}$

c) $\sqrt{\frac{3T}{g(2d-\rho)}}$ d) $\sqrt{\frac{3T}{g(4d-3\rho)}}$

25. A pipe having an internal diameter D is connected to another pipe of same size. Water flows into the second pipe through '*n*' holes, each of diameter *d*. If the water in the first pipe has speed *v*, the speed of water leaving the second pipe is

a) $\frac{D^2v}{nd^2}$ b) $\frac{nD^2v}{d^2}$

c) $\frac{nd^2v}{D^2}$ d) $\frac{d^2v}{nd^2}$

26. When a liquid is heated in copper vessel its coefficient of apparent expansion is $6\times10^{-6}/°C$. When the same liquid is heated in a steel vessel its coefficient of apparent expansion is $24\times10^{-6}/°C$. If coefficient of linear expansion for copper is $18\times10^{-6}/°C$, the coefficient of linear expansion for steel is

a) $20\times10^{-6}/°C$ b) $24\times10^{-6}/°C$

c) $36\times10^{-6}/°C$ d) $12\times10^{-6}/°C$

27. When the temperature of a body increases from T to T + ΔT, its moment of inertia increases from I to I + ΔI. If α is the coefficient of linear expansion of the material of the body, then $\frac{\Delta I}{I}$ is (neglect higher orders of α)

a) $\alpha\Delta T$ b) $2\alpha\Delta T$

c) $\frac{\Delta T}{\alpha}$ d) $\frac{2\alpha}{\Delta T}$

28. A sound wave passing through an ideal gas at NTP produces a pressure change of 0.001 dyne/cm^2 during adiabatic compression. The corresponding change in temperature (γ =1.5 for the gas and atmospheric pressure is 1.013×10^6 dyne/cm^2) is

a) 8.97×10^{-4} K b) 8.97×10^{-6} K

c) 8.97×10^{-8} K d) 8.97×10^{-9} K

29. Work done to increase the temperature of one mole of an ideal gas by 30°C, if it is expanding under the condition $V\propto T^{2/3}$ is, (R = 8.314 J/ mol/K)

a) 116.2 J b) 136.2 J

c) 166.2 J d) 186.2 J

30. Power radiated by a black body at temperature T1 is P and it radiates maximum energy at a wavelength λ_1. If the temperature of the black body is changed from T_1 to T_2, it radiates maximum energy at a wavelength $\frac{\lambda_1}{2}$. The power radiated at T_2 is

a) 2 P b) 4 P c) 8 P d) 16 P

31. Two solid spheres A and B each of radius R are made of materials of densities ρ_A and ρ_B respectively. Their moments of inertia about a diameter are I_A and 1_B respectively. The value of $\frac{I_A}{I_B}$ is

a) $\sqrt{\frac{\rho_A}{\rho_B}}$ b) $\sqrt{\frac{\rho_B}{\rho_A}}$

c) $\frac{\rho_A}{\rho_B}$ d) $\frac{\rho_B}{\rho_A}$

32. Assertion (A) The moment of inertia of a steel sphere is larger than the moment of inertia of a wooden sphere of same radius.

Reason (R) Moment of inertia is independent of mass of the body.

The correct one is

a) Both (A) and (R) are true, and (R) is the correct explanation of (A)

b) Both (A) and (R) are true, and (R) is not the correct explanation of (A)

c) (A) is true but (R) is wrong

d) (A) is wrong but (R) is true

33. When the engine is switched off a vehicle of mass M is moving on a rough horizontal road with momentum p. If the coefficient of friction between the road and tyres of the vehicle is μ_k, the distance travelled by the vehicle before it comes to rest is

a) $\frac{p2}{2\mu_k M^2 g}$ b) $\frac{2\mu_k M^2 g}{p2}$

c) $\frac{p2}{2\mu kg}$ d) $\frac{p^2 M^2}{2\mu kg}$

34. Choose correct statement

(A) The position of centre of mass of a system is dependent on the choice of coordinate system.

(B) Newton's second law of motion is applicable to the centre of mass of the system.

(C) Internal force can not change the state of centre of mass.

(D) Internal force can change the state of centre of mass.

a) Both (A) and (B) are correct

b) Both (B) and (C) are wrong

c) Both (A) and (C) are wrong

d) Both (A) and (C) are wrong

35. A ball A of mass *m* moving along positive *x*-direction with kinetic energy K and momentum p undergoes elastic head on collision with a stationary ball B of mass M after collision the ball A moves along negative x-direction with kinetic energy $\frac{K}{9}$, final momentum of B is

a) p b) $\frac{p}{3}$ c) $\frac{4p}{3}$ d) 4p

36. A straight conductor fo length 4 m moves at a speed of 10 m/s. When the conductor makes on angle of 30^o with the direction of magnetic field of induction of 0.1 Wb- m^2, then induced emf is

a) 8 V b) 4 V c) 1 V d) 2 V

37. The velocity *v* reached by a car of mass m at certain distance from the starting point driven with constant power P is such that

a) $v \propto \frac{3P}{m}$ b) $v^2 \propto \frac{3P}{m}$

c) $v^3 \propto \frac{3P}{m}$ d) $v \propto \left(\frac{3P}{m}\right)^2$

38. It is possible to project a particle with a given velocity in two possible ways so as to make them pass through a point p at a horizontal distance r from the point of projection. If t_1 and t_2 are times taken to reach this point in two possible ways, then the product $t_1 t_2$ is proportional to

a) $\frac{1}{r}$ b) r c) r^2 d) $\frac{1}{r^2}$

39. Sum of magnitudes of two forces acting at a point is 16 N. If their resultant is normal to smaller force, and has a magnitude 8 N, then forces are

a) 6 N, 10 N b) 8 N, 8 N

c) 4 N, 12 N d) 2 N, 14 N

40. The length of a pendulum is measured as 1.01 m and time for 30 oscillations is measured as one minute 3 s. Error length is 0.01 m and error in time is 3 s. The percentage error in the measurement of acceleration due to gravity is

a) 1 b) 5 c) 10 d) 15

Chemistry

1. Benzene 4-hydroxy acetanilide belongs which of the following?

a) Antipyretic b) Antacid

c) Antiseptic d) Antihistamine

2. The site of action of insulin is

a) mitochondria b) nucleus

c) plasma membrane d) DNA

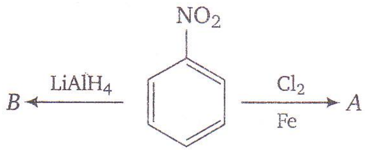
3. The monomer of neoprene is

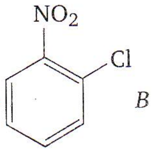
a) 1,3-butadiene b) 2-chloro-1,3-butadiene

c) 2-methyl-1,3-butadiene

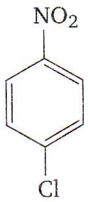
d) vinyl chloride

4. Identify A and B in the following reactions

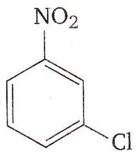


(a) $A =$ 

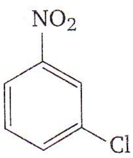
$B = C_6H_5—NH—NH—C_6H_5$

(b) $A =$ 

$B = C_6H_5—N{=}N—C_6H_5$

(c) $A =$ 

$B = C_6H_5—N{=}N—C_6H_5$

(d) $A =$ 

$B = C_6H_5—NH_2$

5. What is the product obtained in the reaction of acetaldehyde with semicarbazide ?

a) $H_3C-CH=N-\overset{O}{\overset{\|}{C}}-NH_2$

b) $H_3C-CH=N-NH_2$

c) $H_3C-CH=N-OH$

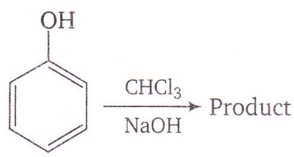
d) $H_3C-\underset{CH_3}{\underset{|}{C}}=N-\overset{O}{\overset{\|}{C}}-NH_2$

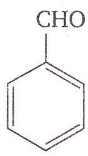
6. Compound A (C_3H_6O) undergoes following reactions to form B and C. Identify A, B and C.

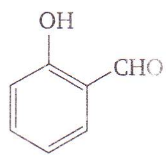
$$C \xleftarrow{Zn-Hg/HCl} \underset{A}{C_3H_6O} \xrightarrow{I_2/NaOH} B$$

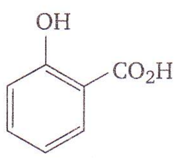
	A	B	C
a)	$H_3C-\overset{O}{\overset{\|}{C}}-CH_3$	CHI_3	$CH_3-CH_2-CH_3$
b)	$H_2C=\underset{H}{\underset{|}{C}}-CH_2OH$	CH_3I	$H_3C-CH_2-CH_2-OH$
c)	H_3C-CH_2-CHO	CHI_3	$H_3C-\underset{OH}{\underset{|}{CH}}-CH_3$
d)	$H_3C-\overset{O}{\overset{\|}{C}}-CH_3$	CHI_3	$H_3C-\underset{OH}{\underset{|}{CH}}-CH_3$

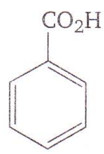
7. Identify the product in the following reaction.



(a) 

(b) 

(c) 

(d) 

8. With respect to chlorobenzene, which of the following statements is not correct ?

a) Cl is ortho / para directing

b) Cl exhibits + M effect

c) Cl is ring deactivating

d) Cl is meta directing

9. Match the following.

	Column I		Column II
A.	Acetaldehyde, vinyl alcohol	1.	Enantiomers
B.	Eclipsed and staggered ethane	2.	Tautomers
C.	(+)2-butanol, (–)2-butanol	3.	Chain isomers
D.	Methyl-*n*-propyl-amine and diethylamine	4.	Conformational isomers
		5.	Metamers

	A	B	C	D
(a)	1	4	3	5
(b)	2	4	1	5
(c)	5	1	4	2
(d)	5	1	3	2

10. Which of the following statements is not correct ?

a) The six carbons in benzene are sp^2 hybridised

b) Benzene has $(4n+2)$ π electrons

c) Benzene undergoes substitution reactions

d) Benzene has two carbon-carbon bond lengths, 1.54 $\overset{o}{A}$ and 1.34 $\overset{o}{A}$

11. Different conformations of the same molecule are called

a) isomers b) epimers

c) enantiomers d) rotarners

12. The chlorination of ethane is an example for which type of the following reactions ?

a) Nucleophilic substitution

b) Electrophilic substitution

c) Free radical substitution

d) Rearrangement

13. The pair of gases responsible for acid rain are

a) H_2, O_3 b) CH_4, O_3

c) NO_2, SO_2 d) CO, CH_4

14. In photoelectric effect, if the energy required to overcome the attractive forces on the electron, (work functions) of Li, Na and Rb are 2.41 eV, 2.30 eV and 2.09 eV respectively, the work function of ‘K’ could approximately be in eV

a) 2.52 b) 2.20

c) 2.35 d) 2.01

15. The quantum number which explains the line spectra observed as doublets in case of hydrogen and alkali metals and doublets and triplets in case of alkaline earth metals is

a) spin b) azimuthal

c) magnetic d) principal

16. Which one of the following cannot form an amph oteric oxide?

a) Al b) Sn c) Sb d) P

17. The formal charges of C and O atoms in CO_2 (:Ö=C=Ö:) are, respectively

CO_2(0 =C =0 :) are, respectively

a) 1, – 1 b) - 1, 1

c) 2, – 2 d) 0, 0

18. According to molecular orbital theory, the total number of bonding electron pairs in O_2 is

a) 2 b) 3 c) 5 d) 4

19. One mole of N_2H_4 loses 10 moles of electrons to form a new compound Z. Assuming that all the nitrogens appear in the new compound, what is the oxidation state of nitrogen in Z? (There is no change in the oxidation state of hydrogen.)

a) – 1 b) - 3 c) + 3 d) + 5

20. Which one of the following equations represents the variation of viscosity coefficient (η) with temperature (T) ?

a) $\eta = Ae^{-E/RT}$ b) $\eta = Ae^{E/RT}$

c) $\eta = Ae^{-E/kT}$ d) $\eta = Ae^{-E/T}$

21. The weight in grams of a non-volatile solute (mol. wt. 60) to be dissolved in 90 g of water to produce a relative lowering of vapour pressure of 0.02 is

a) 4 b) 8 c) 6 d) 10

22. The experimentally determined molar mass of a non-volatile solute, $BaCl_2$ in water by Cottrell’s method, is

a) equal to the calculated molar mass

b) more than the calculated molar mass

c) less than the calculated molar mass

d) double of the calculated molar mass

23. The number of moles of electrons required to deposit 36 g of Al from an aqueous solution of $Al(NO_3)_3$ is (At. wt. of Al = 27)

a) 4 b) 2 c) 3 d) 1

24. The emf (in V) of a Daniell cell containing 0.1 M $ZnSO_4$ and 0.01 M $CuSO_4$ solutions at their respective electrodes is

$$(E^o_{Cu^{2+}/Cu} = +0.34V; E^o_{Zn^{2+}/Zn} = -0.76V)$$

a) 1.10 b) 1.16 c) 1.13 d) 1.07

25. Which one of the following elements, when present as an impurity in silicon makes it a p-type semiconductor?

a) As b) P c) In d) Sb

26. Which one of the following statements is correct for the reaction?

$CH_3COOC_2H_5(aq) + NaOH(aq) \longrightarrow$

$CH_3COONa(aq) + C_2H_5OH(aq)$

a) Order is two but molecularity is one

b) Order is one but molecularity is two

c) Order is one but molecularity is one

d) Order is two but molecularity is two

27. The catalyst and promoter respectively used in the Haber's process of industrial synthesis of ammonia are

a) Mo, V_2O_5 b) V_2O_5, Fe

c) Fe, Mo d) Mo, Fe

28. Which one of the following statements is not correct ?

a) The pH of 1.0×10^{-8} M HCl is less than 7

b) The ionic product of water at 25°C is 1.0×10^{-14} mol^2L^{-2}

c) Cl^- is a Lewis acid

d) Bronsted Lowry theory cannot explain the acidic character of $AlCl_3$

29. Molar heat capacity (Cp) of water at constant pressure is 75 $JK^{-1}mol^{-1}$. The increase in temperature (in K) of 100 g of water when 1 kJ of heat is supplied to it is

a) 2.4 b) 0.24 c) 1.3 d) 0.13

30. Gelly is a colloidal solution of

a) solid in liquid b) liquid in solid

c) liquid in liquid d) solid in solid

31. The product(s) formed when H_2O_2 reacts with disodium hydrogen phosphate is

a) $P_2O_5.Na_3PO_4$ b) $Na_2HPO_4.H_2O_2$

b) NaH_2PO_4, H_2O c) Na_2HPO_4, H_2O

32. Which of the following is not correct ?

a) LiOH is a weaker base than NaOH

b) Salts of Be undergo hydrolysis

c) $Ca(HCO_3)_2$ is soluble in water

d) Hydrolysis of beryllium carbide gives acetylene

33. What is Z in the following reactions ?

$$BCl_3 + H_2 \xrightarrow[450^oC]{Cu\text{-}Al} X + HCl$$

$$X \xrightarrow{methylation} Z$$

a) $(CH_3)BH_2$ b) $(CH_3)_4B_2H_2$

c) $(CH_3)_3B_2H_3$ d) $(CH_3)_6B_2$

34. Which one of the following elements reduces NaOh to Na ?

a) Si b) Pb

c) C d) Sn

35. Which one of the following is used in the preparation of cellulose nitrate ?

a) KNO_3 b) HNO_3

c) KNO_2 d) HNO_2

36. The oxoacid of sulphur which contains two sulphur atoms in different oxidation states is

a) Pyrosulphurous acid

b) hyposulphurous acid

c) pyrosulphuric acid

d) persulphuric acid

37. Bond energy of Cl_2, Br_2 and I_2 follow the order

a) $Cl_2 > Br_2 > I_2$ b) $Br_2 > Cl_2 > I_2$

c) $I_2 > Br_2 > Cl_2$ d) $I_2 > Cl_2 > Br_2$

38. Assertion (A) The boiling points of noble gases increases from He to Xe.

Reason (R) The interatomic van der Walls' attractive forces increases from He to Xe.

The correct answer is

a) Both (A) and (R) are true, and (R) is the correct explanation of (A)

b) Both (A) and (R) are true, and (R) is not the correct explanation of (A)

c) (A) is true but (R) is not true

d) (A) is not true but (R) is true

39. A coordinate complex contains Co^{3+}, Cl^- and NH_3. When dissolved in water, one mole of this complex gave a total of 3 moles of ions. The complex is

a) $[Co(NH_3)_6]Cl_3$

b) $[Co(NH_3)_5Cl]Cl_2$

c) $[Co(NH_3)_4Cl_2]Cl$

d) $[Co(NH_3)_3Cl_3]$

40. Ni anode is used in the electrolytic extraction of

a) Al b) Mg

c) Na by Down's process

d) Na by Castner's process

Mathematics

1. The equation of a straight line passing through the point (1, 2) and inclined at 45o to the line y = 2x + 1 is

a) $5x + y = 7$ b) $3x + y = 5$

c) $x + y = 3$ d) $x - y + 1 = 0$

2. A point moves in the xy-plane such that the sum of its distance from two mutually perpendicular lines is always equal to 5 units. The area (in sq units) enclosed by the locus of the point, is

a) $\frac{25}{4}$ b) 25 c) 50 d) 100

3. The distance between the parallel lines given by $(x+7y)^2 + 4\sqrt{2}(x+7y) - 42 = 0$ is

a) $\frac{4}{5}$ b) $4\sqrt{2}$

c) 2 d) $10\sqrt{2}$

4. If the area of the triangle formed by the pair of lines $8x^2 - 6xy + y^2 = 0$ and the line $2x + 3y = a$ is equal to

a) 14 b) $14\sqrt{2}$

c) $28\sqrt{2}$ d) 28

5. If the pair of lines given by $(x^2 + y^2)\cos^2\theta = (x\cos\theta + y\sin\theta)^2$ are perpendicular to each other, then θ is equal to

a) 0 b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$ d) $\frac{3\pi}{4}$

6. Given the circle C with the equation $x^2 + y^2 - 2x + 10y - 38 = 0$. Match the List I with the List II given below concerning C

	List I		List II
A.	The equation of the polar of (4, 3) with respect to C	I.	$y + 5 = 0$
B.	The equation of the tangent at (9, – 5) on C	II.	$x = 1$
C.	The equation of the normal at (– 7, – 5) on C	III.	$3x + 8y = 27$

	List I		List II
D.	The equation of the diameter of C passing through (1, 3)	IV.	$x + y = 3$
		V.	$x = 9$

The correct answer is

7. Consider the circle $x^2 + y^2 - 4x - 2y + c = 0$ whose centre is A(2, 1). If the point P(10, 7) is such that the line segment PA meets the circle in Q with PQ = 5, then c is equal to

a) – 15 b) 20

c) 30 d) – 20

8. If the line $x + 3y = 0$ is the tangent at (0, 0) to the circle of radius 1, then the centre of one such circle is

a) (3, 0) b) $\left(\frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

c) $\left(\frac{3}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right)$ d) $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

9. A circle passes through the point (3, 4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally; the locus of its centre is a straight line. If the distance of this straight line from the orgin is 25, then a2 is 4equal to

a) 250 b) 225

c) 100 d) 25

10. The equation to the line joining the centres of the circles belonging to the coaxial system of circles $4x^2 + 4y^2 - 12x + 6y - 3 + \lambda(x + 2y - 6) = 0$ is

a) $8x - 4y - 15 = 0$

b) $8x - 4y + 15 = 0$

c) $3x - 4y - 15 = 0$

d) $3x - 4y + 15 = 0$

11. Let x + y = k be a normal to the parabola y2 = 12x. If p is lemgth of the perpendicular from the focus of the parabola onto this normal, then $4k - 2p^2$ is equal to

a) 1 b) 0

c) –1 d) 2

12. If the line $2x + 5y = 12$ intersects the ellipse $4x^2 + 5y^2 = 20$ in two distinct points A and B, then mid-point of AB is

a) (0,1) b) (1, 2)

c) (1, 0) d) (2, 1)

13. Equation of one of the tangents passing through (2, 8) to the hyperbola $5x^2 - y^2 = 5$ is

a) $3x + y - 14 = 0$ b) $3x - y + 2 = 0$

c) $x + y + 3 = 0$ d) $x - y + 6 = 0$

14. The area (in sq units) of the equilateral tringle formed by the tangent at ($\sqrt{3}$, 0) to the hyperbola $x^2 - 3y^2 = 3$ with the pair of asymptotes ofthe hyperbola is

a) $\sqrt{2}$ b) $\sqrt{3}$

c) $\frac{1}{\sqrt{3}}$ d) $2\sqrt{3}$

15. The radius of the circle $r = 12\cos\theta + 5\sin\theta$ is

a) $\frac{5}{12}$ b) $\frac{17}{2}$

c) $\frac{15}{2}$ d) $\frac{13}{2}$

16. If x– coordinate of a point P on the line joining the points Q (2, 2, 1) and R (5, 1, – 2) is 4, then the z-coordinate of P is

a) –2 b) –1

c) 1 d) 2

17. A straight line is equally inclined to all the three coordinate axes. Then, an angle made by the line with the y-axis is

a) $\cos^{-1}\left(\frac{1}{3}\right)$ b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

c) cos–1 $\left(\frac{2}{\sqrt{3}}\right)$ d) $\frac{\pi}{4}$

18. If the foot of the perpendicular from (0, 0, 0) to a plane is (1, 2, 3), then the equation of the plane is
a) $2x + y + 3z = 14$
b) $x + 2y + 3z = 14$
c) $x + 2y + 3z + 14 = 0$
d) $x + 2y - 3z = 14$

19. The equation of the sphere through the points (1, 0, 0), (0, 1, 0) and (1, 1, 1) and having the smallest radius
a) $3(x^2 + y^2 + z^2) - 4x - 4y - 2z + 1 = 0$
b) $2(x^2 + y^2 + z^2) - 3x - 3y - z + 1 = 0$
c) $x^2 + y^2 + z^2 - x - y + z + 1 = 0$
d) $x^2 + y^2 + z^2 - 2x - 2y - 4z + 1 = 0$

20. $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$ is equal to
a) e^4 b) e^6 c) e^5 d) e

21. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} \alpha + \frac{\sin[x]}{x} & , \text{ if } x>0 \\ 2 & , \text{ if } x=0 \\ \beta + \left[\frac{\sin x - x}{x3}\right] & , \text{ if } x<0 \end{cases}$$

Where, [x] denotes the integral part of x. If f continuous at x = 0, then $\beta - \alpha$ is equal to
a) –1 b) 1
c) 0 d) 2

22. If $f(x) = \log\left(e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right)$, then $f\,'(0)$ is equal to
a) $\frac{1}{4}$ b) 4
c) $\frac{-3}{4}$ d) 1

23. If $xy \neq 0$, $x + y \neq 0$ and $x^m y^n = (x + y)^{m+n}$, where $m, n \notin$ N, then $\frac{dy}{dx}$ is equal to
a) $\frac{y}{x}$ b) $\frac{x+y}{xy}$
c) xy d) $\frac{x}{y}$

24. If $x^2 + y^2 = t + \frac{1}{t}$, x4 + y4 = t2 + $\frac{1}{t^2}$, then x^3 $y\frac{dy}{x}$ is equal to
a) –1 b) 1
c) 0 d) t

25. If $f(x) = (x^2 - 1)^7$, then $f^{(14)}(x)$ is equal to
a) 0 b) 2!
c) 7! d) 14!

26. The coordinates of the point P on the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, where the tangent is inclined at an angle $\frac{\pi}{4}$ to x-axis, are
a) $\left[a\left(\frac{\pi}{4}-1\right), a\right]$ b) $\left[a\left(\frac{\pi}{4}+1\right), a\right]$
c) $\left(a\frac{\pi}{4}, a\right)$ d) (a, a)

27. If Δ is the area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, then Δ is equal to
a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{3}$
c) $2\sqrt{3}$ d) 6

28. If the volume of a sphere increases at the rate of 2π cm^3/s, then the rate of increase of its radius (in cm/s), when the volume is 288π cm^3, is
a) $\frac{1}{36}$ b) $\frac{1}{72}$

c) $\frac{1}{18}$ d) $\frac{1}{9}$

29. If $u = f$ (r), where $r^2 = x^2 + y^2$, then $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ is equal to

a) f'' (r) b) f'' (r) + f' (r)

c) $f''(r) + \frac{1}{r} f'(r)$ d) $f''(r) + f'(r)$

30. $\int \frac{dx}{x^2\sqrt{4+x^2}}$ is equal to

a) $\frac{1}{4}\sqrt{4+x^2} + C$ b) $\frac{-1}{4}\sqrt{4+x^2} + C$

c) $\frac{-1}{4x}\sqrt{4+x^2} + C$ d) $\frac{9}{4x}\sqrt{4+x^2} + C$

31. If $\int \sec^2 x \operatorname{cosec}^4 x \, dx = -\frac{1}{3}\cot^3 x + k \tan x - 2 \cot x + C$, then k is equal to

a) 4 b) 3

c) 2 d) 1

32. $\int \frac{dx}{\sqrt{x - x^2}}$ is equal to

a) 2 $\sin^{-1}\sqrt{x}$ + C b) 2 $\sin^{-1} x$ + C

c) 2x $\sin^{-1} x$ + C d) $\sin^{-1}\sqrt{x}$ + C

33. If a > 0, then $\int_{-\pi}^{\pi} \frac{\sin^2 x}{1+d^x} dx$ is equal to

a) $\frac{\pi}{2}$ b) π

c) $\frac{2\pi}{2}$ d) $a\pi$

34. The area (in sq units) bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is

a) $\frac{64}{3}$ b) $\frac{16}{3}$

c) $\frac{8}{3}$ d) $\frac{2}{3}$

35. The value of the integral $\int_0^4 \frac{dx}{1+x^2}$ obtained by using Trapezoidal rule with h = 1 is

a) $\frac{63}{85}$ b) $\tan^{-1}$ (4)

c) $\frac{108}{85}$ d) $\frac{113}{85}$

36. If $\frac{dy}{dx} + 2x \tan(x-y) = 1$, then $\sin(x-y)$ is equal to

a) Ae^{-x^2} b) Ae^{2x}

c) Ae^{x^2} d) Ae^{-2x}

37. An integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} + xy = \frac{x^4}{(1+x^5)}(\sqrt{1-x^2})^3$ is

a) $\sqrt{1-x^2}$ b) $\frac{x}{\sqrt{1-x^2}}$

c) $\frac{x^2}{\sqrt{1-x^2}}$ d) $\frac{x}{\sqrt{1-x^2}}$

38. If $f : R \to R^2$ and $g : R^+ \to R$ are such that $g\{f(x)\} = |\sin x|$ and $f\{g(x)\} = (\sin\sqrt{x})^2$, then a possible choice for f and g is

a) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$

b) $f(x) = \sin x$, $g(x) = |x|$

c) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

d) $f(x) = x^2$, $g(x) = \sqrt{x}$

39. If $f : Z \to Z$ is defined by $f(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even} \\ 0, & \text{if } x \text{ is odd} \end{cases}$, then f is

a) onto but not one - to - one

b) one - to - one but not onto

c) one - to - one and onto

d) neither one - to - one nor onto

40. If $\frac{1}{2\times4}+\frac{1}{4\times6}\frac{1}{6\times8}+\dots(n \text{ terms})=\frac{km}{n\times1}$, then k is equal to

a) $\frac{1}{4}$ b) $\frac{1}{2}$

c) 1 d) $\frac{1}{8}$

41. A regular polygon of n sides has 170 diagonals, then n is equal to

a) 12 b) 17

c) 20 d) 25

42. A committee of 12 members is to be formed from 9 women and 8 men. The number of committees in which the women are in majority is

a) 2720 b) 2702

c) 2270 f) 2278

43. A student has to answer 10 out of 13 questions in an examination choosing atleast 5 questions from the first 6 questions. The number of choice availablae to the student is

a) 63 b) 91

c) 161 d) 196

44. $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}({}^kC_r)$ is equal to

a) $\frac{1}{3}$ b) $\frac{2}{3}$

c) 1 d) 2

45. If $ab\neq0$ and the sum of the coefficients of x7 and x4 in the expansion of $\left(\frac{x^2}{a}-\frac{b}{x}\right)^{11}$ is 0, then

a) a = b b) a + b = 0

c) ab = –1 d) ab = 1

46.

$$\frac{1}{x(x+1)(x+2)\dots(x+n)}=\frac{A_0}{x}+\frac{A_1}{x+1}+\dots\frac{A_n}{x+n},$$

$0\leq i\leq r\Rightarrow A_r$ is equal to

a) $(-1)^r\frac{r!}{(n-r)!}$ b) $(-1)^r\frac{1}{r!(n-r)!}$

c) $\frac{1}{r!(n-r)!}$ d) $\frac{r!}{(n-r)!}$

47. $1+\frac{1}{3.2^2}+\frac{1}{5.2^4}+\frac{1}{7.2^6}+\dots$ is equal to

a) $\log_e 2$ b) $\log_e 3$

c) $\log_e 4$ d) $\log_e 5$

48. In ΔPQR, $<R=\frac{\pi}{4}$, $\tan\left(\frac{P}{3}\right)$, $\tan\left(\frac{Q}{3}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then

a) a + b = c b) b + c = 0

c) a + c = 0 d) b = c

49. The product of real of the equation $|x|^{6/5}-26|x|^{3/5}-27=0$

a) -3^{10} b) -3^{12}

c) $-3^{12/5}$ d) $-3^{21/5}$

50. If α,β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then the coefficient of x in the cubic equation whose roots are $\alpha(\beta+\gamma)$, $\beta(\gamma+\alpha)$ and $\gamma(\alpha+\beta)$ is

a) 2q b) q^2 + pr

c) p^2 – qr d) r(pq – r)

51. Let $A=\begin{vmatrix}2 & ei\pi\\ -1 & i^{2012}\end{vmatrix}, C=\frac{d}{dx}\left(\frac{1}{x}\right)_{x=1}$, $D=\int_{e^2}^{1}\frac{dx}{x}$.

If the sum of two roots of the equation $Ax^3 + Bx^2 + Cx - D = 0$ is equal to zero, then B is equal to

a) –1 b) 0

c) 1 d) 2–

52. $A = \begin{bmatrix} i & i \\ -i & i \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow A^8$

a) 4B b) 8B

c) 64B d) 128B

53. If $f(x)$

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x-1)x(x+1) \end{vmatrix}$$ then f (2012) is equal to

a) 0 b) 1

c) – 500 d) 500

54. Let $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$ and

$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, if a, b and c respectively, denote the ranks of A, B and C, then the correct order of these number is

a) a < b < c b) c < b < a

c) $b < a <$ c d) $a < c < b$

55. Given that, $a\alpha^2 + 2b\alpha + c \neq 0$ and that the system of equations

$(a\alpha^2 + b)x + \alpha y + bz = 0;$

$(b\alpha + c)x + by + cz = 0;$

$(a\alpha + b)y + (b\alpha + c)z = 0;$

has a non-trivial solution, then a, b and c lie in

a) Arithmetic progression

b) Geometric progression

c) Harmonic progression

d) Arithemetico-geometric progression

56. If a, b c and d $\notin$ R such that $a^2 + b^2 = 4$ and $c^2 + d^2 = 2$ and if $(a + ib)^2 = (c + id)^2 (x + iy)$, then $x^2 + y^2$ is equal to

a) 4 b) 3

c) 2 d) 1

57. If z is complex number such that $\left|z - \frac{4}{z}\right| = 2$, then the greatest value of $|z|$ is

a) $1+\sqrt{2}$ b) $\sqrt{2}$

c) $\sqrt{3}+1$ d) $1+\sqrt{5}$

58. If α is a non-real root of the equation $x^6 - 1 = 0$, then $\frac{\alpha^2+\alpha^3+\alpha^4+\alpha^5}{\alpha+1}$ is equal to

a) α b) 1

c) 0 d) –1

59. The minimum value of $27\tan^2\theta + 3\cot^2\theta$ is

a) 15 b) 18

c) 24 d) 30

60. $\cos 36^o - \cos 72^o$ is equal to

a) 1 b) $\frac{1}{2}$

c) $\frac{1}{4}$ d) $\frac{1}{8}$

61. If $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$, then $\tan 3x$ is equal to

a) 3 b) 2

c) 1 d) 0

62. If $3\sin x + 4\cos x = 5$, then $6\tan\frac{x}{2} - 9\tan^2\frac{x}{2}$ is equal to

a) 0 b) 1

c) 3 d) 4

63. If $\frac{1}{2}\frac{1}{2} \le x \le 1$, then

$\cos^{-1}x + \cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right)$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$

c) π d) 0

64. If a, b and c form *a* geometric progression with common ratio r, then the sum of the ordinates of the points of intersection of the line ax + by + c = 0 and the curve $x + 2y^2 = 0$ is

a) $-\frac{r^2}{2}$ b) $-\frac{r}{2}$

c) $\frac{r}{2}$ d) r

65. The point (3, 2) undergoes the following three transformations in the order given

i) Reflection about the line y = x.

ii) Translation by the distance 1 unit in the positive direction of x-axis.

iii) Rotation by an angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

Then, the final position of the point is

a) $(-\sqrt{18}, \sqrt{18})$ b) (–2, 3)

c) $(0, \sqrt{18})$ d) (0, 3)

66. If X is a poisson variate such that $\alpha = P(X = 1) = P(X = 2)$, then $P(X = 4)$ is equal to

a) 2α b) $\frac{\alpha}{3}$

c) αe^{-2} d) αe^2

67. Suppose X follows a binomial distribution with parameters n and p, where 0 < p < 1. If $\frac{P(X = r)}{P(X = n-r)}$ is independent of n for every r, then p is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{8}$

68. In an entrance test there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student know the answer to a question is 9/10. If he gets the correct answer to a question, then the probability that he was guessing is

a) $\frac{37}{40}$ b) $\frac{1}{37}$

c) $\frac{36}{37}$ d) $\frac{1}{9}$

69. There are four machines and it is known that exactly two of them are faulty. They are tested one by one, in a random order till both the faulty machines are identified. The, the probability that only two tests are need is

a) $\frac{1}{3}$ b) $\frac{1}{6}$

c) $\frac{1}{2}$ d) $\frac{1}{4}$

70. A fair coin is tossed 100 times. The probability of getting tails an odd number of times is

a) $\frac{1}{2}$ b) $\frac{1}{4}$

c) $\frac{1}{8}$ d) $\frac{3}{8}$

71. $a = i + j - 2k \Rightarrow \sum \{a \times i) \times j\}^2$ is equal to

a) $\sqrt{6}$ b) 6

c) 36 d) $6\sqrt{6}$

72. Let a, b and c be three non-coplanar vectors and let p, q and r be the vectors defined by $p = \frac{b \times c}{[a\,b\,c]}, q = \frac{c \times a}{[a\,b\,c]}, r = \frac{a \times b}{[a\,b\,c]}$. Then, (a + b). p + (b + c). q + (c + a) . r is equal to

a) 0 b) 1

c) 2 d) 3

73. Let a = i + 2j + k, b = i – j + k, c = i + j – k. $\frac{1}{\sqrt{3}}$ on c. Then, one such vector is

a) 4i + j – 4k b) 3i + j – 3k

c) 4i – j + 4k d) 2i + j + 2k

74. The point if intersection of the lines

l_1: r(t) = (i – 6j + 2k) + t (i + 2j + k)

l_2: R(u) = (4j + k) + u (2i + j + 2k) is

a) (4, 4, 5) b) (6, 4, 7)

c) (8, 8, 9) d) (10, 12, 11)

75. The vectors AB = 3i – 2j + 2k and BC = i – 2k are the adjacent sides of a parallelogram. The angle between its diagonals is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

c) $\frac{3\pi}{4}$ or $\frac{\pi}{4}$ d) None of these

76. If pth, qth, rth terms of a gemetric progression are the positive numbers a, b and c respectively, then the angel between the vectors (log a^2) i + (log b^2) j + (log c^2)k and (q – r)i + (r – p)j + (p – q)k is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$

c) $\sin^{-1} \frac{1}{\sqrt{a^2 + b^2 + c^2}}$

d) $\frac{\pi}{4}$

77. A vertical pole subtends an angle $\tan^{-1}\left(\frac{1}{2}\right)$ at a point P on the ground. If the angles substanded by the upper half and the lower half of the pole at P are respectively α and β, then (tan α , tan β) is equal to

a) $\left(\frac{1}{4}, \frac{1}{5}\right)$ b) $\left(\frac{1}{5}, \frac{2}{9}\right)$

c) $\left(\frac{2}{9}, \frac{1}{4}\right)$ d) $\left(\frac{1}{4}, \frac{2}{9}\right)$

78. If α , β and γ are length of the altitudes of Δ ABC with area Δ, then $\frac{\Delta^2}{R^2}\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)$ is equal to

a) $\sin^2 A + \sin^2 B + \sin^2 C$

b) $\cos^2 A + \cos^2 B + \cos^2 C$

c) $\tan^2 A + \tan^2 B + \tan^2 C$

d) $\cot^2 A + \cot^2 B + \cot^2 C$

79. In an acute angled triangle, cot B cot C + cot A cot C + cot A cot B is equal to

a) –1 b) 0

c) 1 d) 2

80. $x = \log\left(\frac{1}{y} + \sqrt{1 + \frac{1}{y^2}}\right) \Rightarrow$ y is equal to

a) tanh x b) coth x

c) sech x d) cosech x

Answers

Physics

1. (d)	2. (b)	3. (d)	4. (d)	5. (c)	6. (d)	7. (a)	8. (a)	9. (a)
10. (a)	11. (c)	12. (d)	13. (b)	14. (b)	15. (d)	16. (a)	17. (c)	18. (b)
19. (c)	20. (c)	21. (b)	22. (c)	23. (d)	24. (c)	25. (a)	26. (d)	27. (b)
28. (c)	29. (c)	30. (d)	31. (c)	32. (c)	33. (a)	34. (d)	35. (c)	36. (d)
37. (*)	38. (b)	39. (a)	40. (c)					

Chemistry

1. (a)	2. (c)	3. (b)	4. (c)	5. (a)	6. (a)	7. (b)	8. (d)	9. (b)
10. (d)	11. (d)	12. (c)	13. (c)	14. (b)	15. (a)	16. (d)	17. (d)	18. (c)
19. (c)	20. (b)	21. (c)	22. (c)	23. (a)	24. (d)	25. (c)	26. (d)	27. (c)
28. (c)	29. (a)	30. (b)	31. (b)	32. (d)	33. (b)	34. (c)	35. (b)	36. (a)
37. (a)	38. (a)	39. (b)	40. (d)					

Mathematics

1. (b)	2. (c)	3. (c)	4. (d)	5. (b,d)	6. (c)	7. (d)	8. (d)	9. (b)
10. (a)	11. (b)	12. (*)	13. (b)	14. (b)	15. (d)	16. (b)	17. (b)	18. (b)
19. (a)	20. (c)	21. (b)	22. (a)	23. (a)	24. (a)	25. (d)	26. (b)	27. (c)
28. (b),	29. (c)	30. (c)	31. (d)	32. (a)	33. (a)	34. (b)	35. (d)	36. (c)
37. (d)	38. (c)	39. (a)	40. (a)	41. (c)	42. (b)	43. (c)	44. (d)	45. (d)
46. (b)	47. (b)	48. (a)	49. (a)	50. (b)	51. (d)	52. (d)	53. (a)	54. (c)
55. (b)	56. (a)	57. (d)	58. (d)	59. (b)	60. (b)	61. (c)	62. (b)	63. (b)
64. (c)	65. (c)	66. (b)	67. (a)	68. (b)	69. (a)	70. (a)	71. (b)	72. (d)
73. (d)	74. (c)	75. (d)	76. (b)	77. (c)	78. (a)	79. (c)	80. (d)	

Note : None of the given option is correct.

Physics

1. The time taken by the transverse wave

$$t = 2\sqrt{\frac{l}{g}} = 2\sqrt{\frac{2.45}{9.8}} = 1s$$

2. We have, nl = constant

$$n_1 l_1 = n_2 l_2$$

$(n + 8)\ 100 = (n - 8)\ 101$

or $\frac{n+8}{n-8} = \frac{101}{100}$

$$\frac{n}{8} = \frac{201}{1}$$

$$n = 1608$$

3. From the lens marks formula

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

$$\frac{4}{3f'_0} - \frac{1}{\infty} = \frac{1.5-1}{R} + \frac{\frac{4}{3} - 1.5}{-R}$$

$$\Rightarrow f'_0 = R = 2f$$

Now, length = $f'_0 + f'_e$

$= 2f_0 + 2f_e = 2(f_o + f_e)$

$= 2(L) = 2(16) = 32$ cm

4. $P = P_1 + P_2 = \frac{q}{f_1} + \frac{q}{f_2}$

$$\frac{1}{60} = \frac{q}{f_1} + \frac{q}{f_2}$$

$$\frac{1}{60} = \frac{f_1 + f_2}{f_1 f_2}$$

According to question,

$$\frac{\omega_1}{\omega_2} = \frac{f_1}{f_2} \Rightarrow \frac{f_1}{f_2} = \frac{-4}{3}$$

or $f_1 = -\frac{4}{3} f_2$

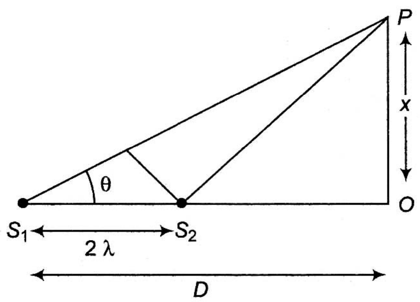
From Eq. (i)

$$\frac{1}{60} = \frac{\frac{4}{3}f_2 + f_2}{(f_2)^2 \frac{4}{3}} \Rightarrow \frac{-\frac{1}{3}}{-\frac{4}{3}f_2}$$

$f_2 = 15$ cm

$\therefore f_1 = -\frac{4}{3} \times 15 = -20$ cm

5. From the figure,



$(S_1P)^2 = D^2 + x^2$

and $(S_2P)^2 = (D - 2\lambda)^2 + x^2$

So, $(S_1P)^2 - (S_2P)^2 = D^2 + x^2$

$- [D^2 + 4\lambda^2 - 4D\lambda + x^2]$

$= 4D\lambda - 4\lambda^2$

$(S_1P + S_2P)\ (S_1P - S_2P) = 4D\lambda - 4\lambda^2$

$[D >> \lambda)$

Since, $[D >> \lambda]$

$\therefore S_1P = S_2P$

and neglecting higher power of λ

$\therefore 2S_1P(S_1P - S_2P) = 4D\lambda$

or $2(2S_1P)\ (\lambda) = 4D\lambda$

or $\sqrt{D^2 + x^2} = 2D$

or $x^2 = 3D^2$

or $x = \sqrt{3}\,D$

Alternative

$2\lambda \cos\theta = \lambda$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°, \tan\theta = \frac{x}{D}$$

$\sqrt{3} = \frac{x}{D} \Rightarrow x = \sqrt{3}D$

6. $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$\therefore \frac{T_1}{T_2} = \sqrt{\frac{(B_H)_2}{(B_H)_1}}$

$T_2 = T_1\sqrt{\frac{(B_H)_1}{(B_H)_2}}$

$n_1 = 30$ oscillation/min

$n_1 = \frac{1}{2}$ oscillation/s

$T_1 = 2s$

$T_2 = 2\sqrt{\frac{B_H}{2B_H}}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}s$

7. Work done W = MB (1 – cos 60°)

$= \frac{MB}{2}$

The torque required to maintain the magnetic needle

$\tau = MB\sin\theta$

$= MB \sin 60°$

$= MB\frac{\sqrt{3}}{2}$

$\tau = \sqrt{3}W$

8. Charge that through the wire

$\Delta q = \Delta CV$

$= (C' - C)V$

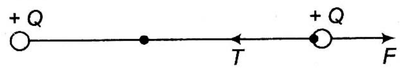
$= (k - 1)\, \sigma V$

$= (20 - 1)\, (80 \times 10^{-6})\, (30)$

$= 4.56 \times 10^{-2}$

$= 45.6 \times 10^{-3}C$

9. In this case, tension will be equal to force



The tension in each string

$T = F$

$= \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2L)^2}$

$= \frac{Q^2}{16\pi\varepsilon_0 L^2}$

10. For 10. For figure I,

Total resistance = 3R

I power = I^2 (3R)

For figure II,

Resistance = $\frac{2R}{3}\Omega$

II power = $I^2\left(\frac{2R}{3}\right)$

For figure III,

Total resistance = $\frac{R}{3}\Omega$

For figure IV,

Total resistance $\frac{3}{2}\Omega$

IV power = $I^2\left(\frac{3}{2}R\right)$

Hence, option (a) is correct.

11. Balancing condition of bridge balance

$\frac{A}{B} = \frac{100}{x}$

$\frac{B}{A} = \frac{121}{x}$

$\therefore \frac{100}{x} = \frac{x}{121}$

x^2 12100

$x = 110\Omega$

12. Total emf produced in a thermocouple does not depend on the duration of time for which the current is passed through thermocouple.

13. The magnetic field

$$dB = \frac{\mu_0 I \times r \times dl}{4\pi r^2}$$

14. The induced emf in secondary coil.

$$e = M\frac{di}{dt} = 92 \times 10^{-6} \times \frac{25}{1 \times 10^{-3}} = 2.3V$$

15. The ratio of kinetic energy of the electron to that of the photon.

$$= \frac{v}{2c}$$

$$= \frac{1.5 \times 10^8}{2 \times 3 \times 10^8} = \frac{1}{4}$$

16. $\lambda_a = \lambda_\alpha$

$(mqV)_p = (mqV)_\alpha$

Potential difference

$$V_\alpha = \frac{V}{8} \quad \begin{bmatrix} \because m_\alpha = 4m_p \\ q_\alpha = 2q_p \end{bmatrix}$$

17. The number of atoms decay in one second

$$\frac{dN}{dt} = \lambda N$$

$$= \frac{0.693}{1620 \times 365 \times 86 \times 400} \times \frac{6.023 \times 10^{23}}{226}$$

$= 3.61 \times 10^{10}$

18. After 4 half-lifes

$$\frac{N_0}{2^4} = \frac{N_0}{16}$$

19. $\beta = \frac{\alpha}{1-\alpha}$

$$\beta_1 = \frac{\frac{20}{21}}{1-\frac{20}{21}} = 20,\ \beta_2 = \frac{\frac{100}{101}}{1-\frac{100}{101}} = 100$$

The value of β lies between 20 – 100.

20. D-layer – 65-75 km

E-layer – 95-120 km

F_1-layer – 170-190 km

F_2-layer – 250-400 km

21. We have,

$dV = -E \,.\, dr = (5i + 12j) \,.\, (12i + 15j)$

$= (60+ 60) = -120$

The change in gravitational potential energy

$U = mdV = 2 \times (-120) = -240$ J

22. Distance $y_1 = A \sin \frac{2\pi}{8} \times 1 = \frac{A}{\sqrt{2}}$

and $y_2 = A - \frac{A}{\sqrt{2}}$

The ratio of the distances travelled by it in the first and seconds.

$$\frac{y_1}{y_2} = \frac{1}{\sqrt{2}-1}$$

23. Young's modulus of materials

$$Y = \frac{F \times l}{A\,\Delta l}, \frac{\Delta l}{l} = \frac{F}{AY}$$

$$= \frac{22}{0.02 \times 10^{-4} \times 1.1 \times 10^{11}} = 10^{-4}$$

Poisson's ratio, $\sigma = \frac{\frac{\Delta l}{l}}{\frac{\Delta r}{r}}$

$$\frac{\Delta l}{l} = \sigma \frac{\Delta r}{r} = 0.32 \times \frac{\Delta r}{r}$$

$= 0.32 \times 10^{-4} = 32 \times 10^{-6}$

The decrease in cross-sectional area,

$$\frac{\Delta A}{A} = \frac{\Delta A}{0.02} = 0.32 \times \frac{\Delta l}{l}$$

$\Delta A = 0.64 \times 10^{-6}$ cm^2

24. According to the question,

$$\frac{4}{3}\pi r^3 dg = \frac{2}{3}\pi r^3 \rho g + T \times 2\pi r$$

or $r = \sqrt{\dfrac{3T}{g(2d-\rho)}}$

25. $\pi\left(\frac{D}{2}\right)^2 X\nu = n\left(\frac{d}{2}\right)^2 X\nu',\ \nu' = \frac{D^2\nu}{nd^2}$

26. We have,

$$\gamma a_1 + 3\alpha_1 = \gamma a_2 + 3\alpha_2$$

$$6 \times 10^{-6} + 3\left(18 \times 10^{-6}\right) = 24 \times 10^{-6} + 3\alpha_2$$

$$\alpha_2 = 12 \times 10^{-6} / °C$$

27. $\frac{\Delta I}{I} = 2\frac{\Delta k}{k} = 2 \propto \Delta T$

$$\frac{\Delta I}{I} = 2 \propto \Delta T$$

28. $T^\gamma p^{1-\gamma}$ = constant

or $\quad T^\gamma = p^{1-\gamma}$

$$T - p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$\therefore \quad \frac{\Delta T}{T} = \frac{\gamma - 1}{\gamma} \times \frac{\Delta p}{p}$

$$\frac{\Delta T}{T} = \left(\frac{1.5-1}{1.5}\right) \times \frac{0.001}{1.013 \times 10^6}$$

$$\Delta T = 8.908 \times 10^{-8} k$$

29. We have, $V \propto T^{2/3}$

and $\quad p \propto \sqrt{V}$

$\therefore \quad p = k\sqrt{V} \Rightarrow p = kV^{1/2}$

Workdone, $W = \int pdV$

$$W = \int kV^{1/2}$$

$$W = \frac{kV^{3/2}}{3/2}$$

$$W = \frac{2}{3}\frac{p}{\sqrt{V}}V^{3/2} = \frac{2}{3}pV \quad \left[\because V = \frac{p}{\sqrt{V}}\right]$$

$$= = \frac{2}{3}nRT = \frac{2}{3} \times 1 \times 8.3 \times 30 = 166.2J$$

30. We have, $P \propto T^4 \propto \frac{1}{\lambda^4} \Rightarrow \frac{P_1}{P_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$

$$\frac{P_1}{P_2} = \frac{1}{16}$$

$\Rightarrow \quad P_2 = 16P_1 = 16P$

31. Moment of inertia

$$I = \frac{2}{3}MR^2$$

$$\therefore \frac{I_A}{I_B} = \frac{\frac{2}{5}M_A R^2}{\frac{2}{5}M_B R^2} = \frac{\frac{4}{3}\pi R^3 \rho_A}{\frac{4}{3}\pi R^3 \rho_B} = \frac{\rho_A}{\rho_B}$$

32. Moment of inertia

$$I = MR^2$$

i.e., the moment of inertia depends on the mass of the body.

33. We have, $\mu_k mgs = \frac{p^2}{2M}$

or $\quad s = \frac{p^2}{2M^2 \mu kg}$

35. Kinetic energy

$$K = \frac{1}{2}mu_1^2$$

$$\mu_1 = \sqrt{\frac{2K}{m}}$$

$$\frac{1}{2}mv_1^2 = \frac{K}{9}$$

$$v_1^2 = \frac{2K}{9m}$$

Velocity, $v_1 = \sqrt{\frac{2K}{9m}}$

Kinetic energy $K = \frac{p^2}{2m}$

From conservation of the momentum

$mu_1 = -mv_1 + p_B$

$m(u_1 + v_1) = p_B$

Final momentum of B

$$p_B = m\left[\sqrt{\frac{2K}{m}} + \sqrt{\frac{2K}{9m}}\right]$$

$$= \sqrt{2mK} + \sqrt{\frac{2mK}{3}} = p + \frac{p}{3} = \frac{4p}{3}$$

36. Induced emf is given by

$$e = Bvl\sin\theta$$

$$= 0.1 \times 10 < 4 \sin 30 = 2V$$

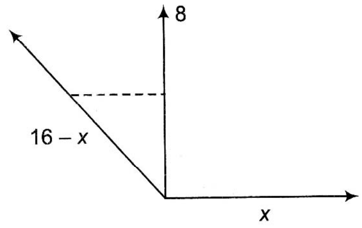
37. $P = \frac{\frac{1}{2}mv^2}{t} \Rightarrow v^2 = \frac{2Pt}{m} \Rightarrow v^2 \propto \frac{2P}{m}$

38. $T_1 = \frac{2u\sin\theta}{g}$

and $T_2 = \frac{2u\cos\theta}{g}$

$$T_1T_2 = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2r}{g} \quad \left[\because R = \frac{v^2\sin 2\theta}{g}\right]$$

39. $(16 - x)^2 = 8^2 + x^2$



$256 + x^2 - 32x = 64 + x^2$

$32x = 192$

$x = \frac{192}{32} = 6$

$\therefore$ Force 6N, 10N.

40. $\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

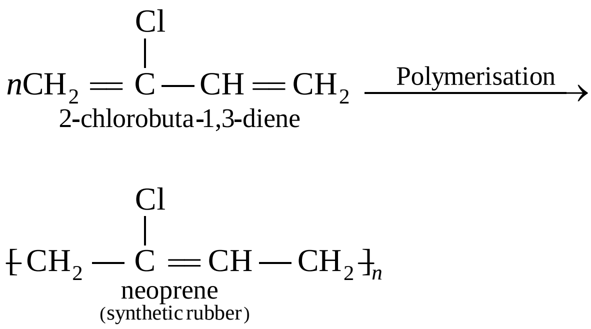
We get, $\frac{\Delta g}{g}\% = 10\%$

Chemistry

1. Benzene-4-hydroxy acentanilide is also called paracetamol. It is an antipyretic (i.e., used for lowering body temperature) as well as analgesics (i.e., used to releive pain).

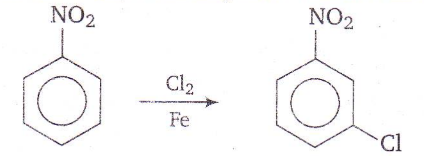
2. Insulin is an hormone, secreted by pancreas. It is transported to different body parts by the blood stream. It maintains glucose level in blood and glucose metabolism. thus, its site of action is plasma membrane.

3. Chloroprene (or 2-chlorobuta-1, 3-diene) is the monomer of synthetic rubber, neoprene.



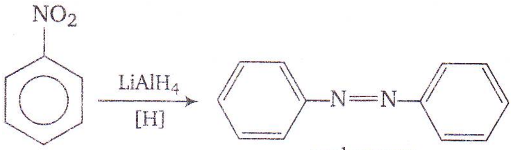
4. NO_2 being meta directing gives m-chloronitrobenzene, when treated with Cl_2, Fe.

m-chloronitrobenzene, when treated with Cl_2,



m-chloronitrobenzene
'*A*'

On reduction with $LiAlH_4$, nitrobenzene gives azobenzene.



azobenzene
'*B*'

4. When acetaldehyde is treated with semicarbazide addition elimination reaction takes place and acetaldehyde semicarbazone is obtained as product.

$$CH_3CHO + H_2N.NHCONH_2 \xrightarrow{-H_2O}$$

$$\underset{\text{acetaldehydesemicarbazone}}{CH_3CH = N\cdot NHCONH_2}$$

6. The compound reacts with I_2/NaOH, so it must be a methyl ketone and it should give iodoform (CHI_3) in this reaction.

$$\therefore \text{ A is } CH_3\overset{O}{\overset{\|}{C}}CH_3 \text{ (acetone)}$$

$$CH_3 - \underset{'A'}{\overset{O}{\overset{\|}{C}}} - CH_3 \xrightarrow{I_2/NaOH} \underset{\substack{'B'\\ \textit{yellow ppt.}}}{CHI_3}$$

$$+ CH_3COONa + NaI + H_2O$$

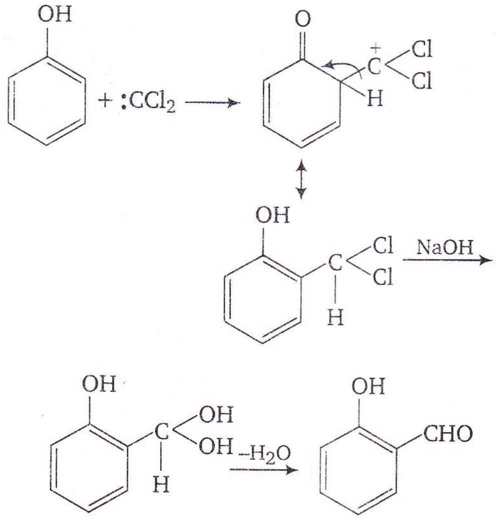
$$CH_3\underset{'A'}{\overset{O}{\overset{\|}{C}}}CH_3 \xrightarrow[\text{(Clemmensen's reduction)}]{\text{Zn-Hg/HCl}} \underset{\substack{\text{propane}\\ 'C'}}{CH_3CH_2CH_3}$$

7. $$CHCl_3 + NaOH \longrightarrow \underset{\text{unstable}}{:CCl_3^-} + Na^+ + H_2O$$

$$\downarrow$$

$$:CCl_2 \quad +Cl^-$$

dichlorocarbene
(electrophile)



salicylaldehyde
(*o*-hydroxy benzaldehyde)
product

This reaction is known as Reimer-Tiemann reaction.

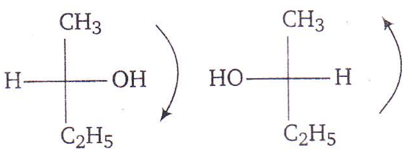
8. Cl exhibits – I and + M effect. Due to which it is ortho/para directing but ring deactivating.

9. $$\underset{\text{acetaldehyde}}{CH_3CHO} \xrightarrow{\text{Tautomerisation}} \underset{\text{vinyl alcohol}}{CH_2 = CH - OH}$$

Thus, these two are tautomers.

Eclipsed and staggered ethane are two conformations of ethane.

(+)2-butanol and (–)2-butanol are enantiomers as these are non-superimposable mirror images.



(+)-2-butanol (–)-2-butanol

$$\underset{\text{(methyl-n-propylamine)}}{CH_3NHC_3H_7}$$

and $$\underset{\text{(diethylamine)}}{C_2H_5NHC_2H_5}$$

Due to difference in the nature of alkyl groups attached to the same functional group, these are called metamers.

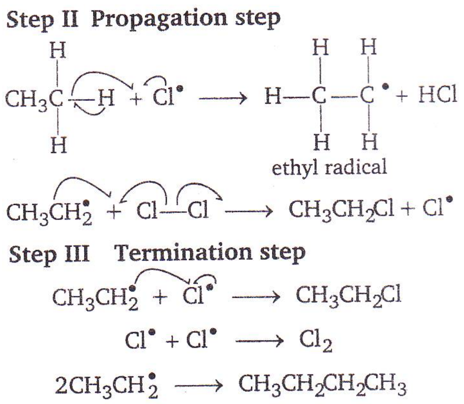
10. In benzene all the C – C bond lengths are equal to 1.39 $\mathring{A}$ due to resonance.

11. Conformational isomers are also known as rotamers and the isomerism as rotamerism.

12. Chlorination of ethane takes place by free radical substitution and involves the following steps

Step I Initiation step

$$Cl_2 \xrightarrow{hv} 2Cl^{\bullet}$$



13. Oxides of nitrogen and sulphur are responsible for acid rain.

$NO_2 + H_2O \longrightarrow HNO_3$

$SO_2 \xrightarrow{[O]} SO_3 + H_2O \rightarrow H_2SO_4$

Due to the presence of HNO_3 and H_2SO_4 (strong acids) rain water becomes acidic and rain is called the acid rain.

14. As the size of atom (element) increases, energy required to overcome the actractive force on outermost electron decreases.

Li, Na, K and Rb are the members of same group and have the following order of size

$$Li < Na < K < Rb$$

Thus, the energy in case of K is intermediate of Rb and Na, i.e.,

$$2.30 < K > 2.09$$

or the energy for K is 2.20 eV.

15. Spin quantum number explains the line spectra observed as doublets in case of H and alkli metals and doublets and triplets in case of alkaline earth metals.

16. Phosphorus being a non-metal always forms acidic oxides like P_2O_3, P_5O_{10}, etc.

Al_2O_3, SnO and Sb_2O_3 are amphoteric oxides i.e., react with acids as well as bases.

17. Formal charge $(FC) = [V - lp - \frac{1}{2}bp]$

[where, V = valence electrons, *l*p = lone pair and dp = bond pair]

For $:\ddot{O}=C=\ddot{O}:$

FC of O = $6 - 4 - \frac{1}{2} \times 4 = 0$

FC of C = $4 - 0 - \frac{1}{2} \times 8 = 0$

18. The molecular orbital electronic configuration of O_2 is

$$O_2 (8+8=16e^-) = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2,$$

$$\sigma 2P_z^2, \pi 2P_x^2, \approx \pi 2P_y^2, \overset{*}{\pi} 2P_x^1 \approx \overset{*}{\pi} 2P_y^1$$

(Unstarred σ and π represent bonding orbitals.)

$\therefore$ number of bonding electrons = 10

and number of bonding electron pairs = 5

19. $N_2H_4 - 10e^- \longrightarrow X^{10+}$ (with two N atoms)

Total oxidation number of two N atoms in N_2H_4,

$\Rightarrow$ $2x + 4 = 0$

$2x = -4$

N_2H_4 loses 10 electrons, so total oxidation number of two N-atoms increases by 10, *i.e.*, the total oxidatio number of two N-atoms in

T = – 4 + 10 = + 6

$\therefore$ Oxidation numbr of each N atom in

X^{10+} = + 3.

20. The variation of viscosity coefficient (η) with temperature (T) is given by the following expression

$$\eta = Ae^{E/RT}$$

21. Relative lowering of vapour pressure,

$$\frac{P^o - Ps}{P^o} = x_A$$

$$= \frac{\frac{w_A}{m_A}}{\frac{w_A}{m_A} + \frac{w_B}{m_B}}$$

(where, w_A and m_A are the mass and molar mass of solute and w_B and m_B are the mass and molar mass of water.)

$$\therefore \qquad 0.02 = \frac{\frac{x}{60}}{\frac{x}{60} + \frac{90}{18}}$$

$$0.02 = \frac{\frac{x}{60}}{\frac{x}{60} + 5}$$

$$\frac{1}{0.02} = \frac{\frac{x}{60} + 5}{\frac{x}{60}}$$

$$50 = 1 + \frac{5 \times 60}{x}$$

$$49 = \frac{300}{x}$$

$$x = \frac{300}{49} = 6\,g$$

22. BaCl2 being ionic in nature, ionises completely to give Ba^{2+} and $2Cl^-$ (1+2 = 3) ions, i.e., number of particles increases in the solution.

$$\therefore \qquad i = \frac{3}{1} = 3 = \frac{Mc}{Mo}$$

(where, Mc = calculated molar mass and M_o = observed molar mass)

$$3 = \frac{Mc}{Mo}$$

or $$Mo = \frac{Mc}{3}$$

i.e., $M_o < M_c$

or observed molecular mass of $BaCl_2$ is less than the calculated molecular mass.

23. $$\underset{}{Al^{3+}} + \underset{3\,mol}{3e^-} \longrightarrow \underset{1\,mol = 27\,g}{Al}$$

$\because$ 27 g of Al is deposited by 3 moles of electrons

$\therefore$ 36 g Al will be deposited by electrons

$$= \frac{3}{27} \times 36 = 4\,mol$$

24. $$E^o_{cell} = E^o_{Cu^{2+}/Cu^-} - E^o_{Zn^{2+}/Zn}$$

$= + 0.34 - (-0.76)$V

$= 1.1$ V

Further $$E_{cell} = E^o_{cell} - \frac{0.059}{n} \log \frac{[products]}{[reactants]}$$

For the reaction,

$$CuSO_4 + Zn \longrightarrow ZnSO_4 + Cu$$

or $$Cu^{2+} + Zn \longrightarrow Zn^{2+} + Cu$$

$$E_{cell} = E^o_{cell} - \frac{0.059}{2} \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

$$= 1.1 - \frac{0.059}{2} \log \frac{0.1}{0.01}$$

$$= 1.1 - \frac{0.059}{2} \log 10$$

$= 1.1 - 0.0295 \times 1$ $\qquad [\therefore \log 10 = 1]$

$= 1.07$ V

25. When silicon is doped with such an impurity valency of which is less than 4, p-type semiconductor is obtained.

    valency of AAs, P and Sb is + 5 and of In is + 3. Thus, In is the impurity which when added to Si gives p-type semiconductor.

26. For the reaction,

    $CH_3COOC\ _2H_5(aq) + NaOH(aq) \longrightarrow CH_3COONa(aq) + C_2H_5OH(aq)$

    the molecularity is two because two molecules are taking part.

    Moreover, rate law for the reaction is,

    rate (r) = k $[CH_3COOC\ _2H_5]$ [NaOH]

    Order of reaction is 1 + 1 = 2

27. In the Haber's process for the synthesis of ammonia from N_2 and H_2, Fe is used as catalyst and Mo works as activator.

    $$N_2 + 3H_2 \xrightleftharpoons{\text{Fe, Mo}} 2NH_3 + 22.4 \text{ kcal}$$

28. Cl^- is a Lewis base, not a Lewis acid as its octet is complete and it have a tendency to give electrons.

29. Molar heat capacity is the amount of heat required to raise the temperature of 1 mole of a substance by 1°C.

    $$\therefore \quad \Delta H = nCp\ \Delta T$$

    $$1000 = \frac{100}{18} \times 75 \times \Delta T$$

    $$\Delta T = \frac{180}{75} = 2.4^o$$

30. In gelly, dispersed phase is liquid and dispersion medium is solid. Thus, it is a colloidal solution of liquid in solid.

31. H_2O_2 forms addition product with disodium hydrogen phosphate.

    $$H_2O_2 + Na_2HPO_4 \longrightarrow \underset{\text{addition product}}{Na_2HPO_4H_2O_2}$$

32. $$Be_2C + 4H_2O \longrightarrow \underset{\substack{\text{methane} \\ 'X'}}{CH_4} + 2Be(OH)_2$$

33. $$BC1_3 + H2 \xrightarrow[450^o C]{Cu-Al} \underset{\text{diborane}}{B_2H_6} + HC1$$

    $$\underset{'X'}{B_2H_6} + 4CH_3C1 \xrightarrow{\text{Methylation}} \underset{'Z'}{(CH_3\ )_4\ B_2H_2}$$

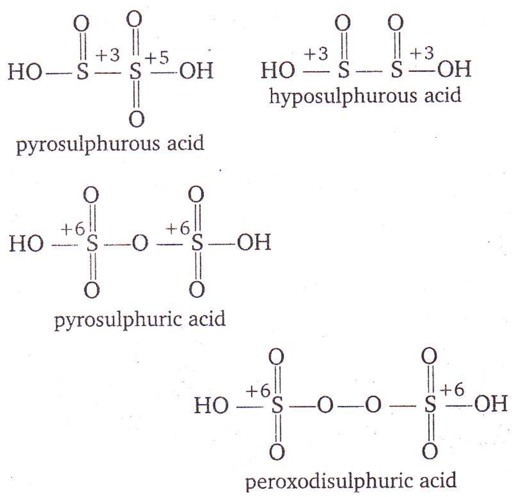
34. NaOH when heitted with coke, gets reduced to Na.

    $$6NaOH + 2C \xrightarrow{\Delta} 2Na + 2Na_2CO_3 + 3H_2 \uparrow$$

    Here, C behaves as reducing agent.

35. Treatment of cellulose with HNO_3 gives cellulose nitrate, which is a very important substance for the manufacture of explosives (gun powder), medicines, paints, lacquers, etc.

36. The structure of the given oxoacids of sulphur are as



    Thus, the oxidation states of two S atoms in pyrosulphurous acid are different.

37. As the size of atom increases, A – A bond length increases and hence, A – A bond energy decreases. (where A = Cl, Br, I)

    $\therefore$ The order of bond energy is

    $C1_2 > Br_2 > I_2$

38. As the size of noble gases increases from He to Xe, van der Waals' forces increases and due to which boiling point increases.

39. When the complex dissolved in water, it gives three ions, which shows that two Cl^- ions are present outside the coordination sphere.

    $\therefore$ The formula of the complex is

$[Co(NH_3)_5C1]C1_2$

$[Co(NH_3)_5C1]C1_2 \xrightarrow{Water}$

$$\underbrace{[Co(NH_3)_5C1]^{2+} + 2C1^-}_{3 \text{ ion}}$$

40. In Castner's process for the extraction of sodium, Ni anode is used.

Mathematics

\

1. Let required equation of line is

$$y = mx + c$$

Since, angle between $y = mx + c$

and $y = 2x + 1$ is $\tan 45°$

$\because \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan 45° = \left|\frac{m-2}{1+2m}\right|$

$\Rightarrow \quad 1 = \pm\frac{m-2}{1+2m}$

$\Rightarrow \quad 1 + 2m = m - 2$

$\Rightarrow \quad 1 + 2m = -m + 2$

$\Rightarrow \quad m = -3 \text{ or } 3m = 1$

$\Rightarrow \quad m = -3 \text{ or } \frac{1}{3}$

On putting $m = -3$ in Eq. (1), we get

$$y = -3x + c$$

Since, it passes through (1, 2).

$\therefore \quad 2 = -3 + c$

$\Rightarrow \quad c = 5$

$\therefore \quad y = -3x + 5$

$\Rightarrow \quad 3x + y = 5$

Again, putting $m = \frac{1}{3}$ in Eq. (i), we get

$$y = -\frac{x}{3} + c$$

Since, it passes through (1, 2).

$\therefore \quad 2 = -3 + c$

$\Rightarrow \quad c = 5$

$\therefore \quad y = -3x + 5$

$\Rightarrow \quad 3x + y = 5$

Again, putting $m = \frac{1}{3}$ in Eq. (i) we get

$$y = -\frac{x}{3} + c$$

Since, it passes through (1, 2).

$\therefore \quad 2 = -\frac{1}{3} + c \Rightarrow c = \frac{7}{6}$

$\therefore$ Equation of line is $y = -\frac{x}{3} + \frac{7}{6}$

$\Rightarrow \quad 6y + 2x = 7$

Hence, option (b) is correct.

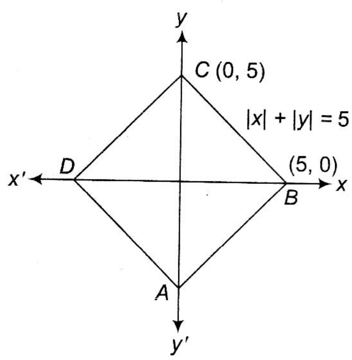
2. In a figure, xx' and yy' are two perpendicular axes.

$\therefore |x| + |y| = 5$

$\therefore$ Coordinates of B and C are (5,0) and (0,5).

$\therefore$ Length of BC = $\sqrt{5^2 + 5^2} = 5\sqrt{2}$

$\therefore$ Area of square ABCD = $(5\sqrt{2})^2 = 50$



3. Given equation is

$(x + 7y)^2 + 4\sqrt{2}\,(x + 7y) - 42 = 0$

Put $\quad x + 7y = t$

$\Rightarrow \quad t^2 + 4\sqrt{2}\,(t) - 42 = 0$

$\Rightarrow \quad t = \dfrac{-4\sqrt{2} \pm \sqrt{32+168}}{2}$

$= \dfrac{-4\sqrt{2} \pm 10\sqrt{2}}{2}$

$= 3\sqrt{2}, -7\sqrt{2}$

$\therefore \quad x + 7y = 3\sqrt{2}$

and $\quad x + 7y = -7\sqrt{2}$

$\Rightarrow \quad x + 7y - 3\sqrt{2} = 0$

and $\quad x + 7y + 7\sqrt{2} = 0$

$\therefore$ Distance between parallel lines

$$= \frac{|7\sqrt{2} + 3\sqrt{2}|}{\sqrt{1^2 + 7^2}} = \frac{10\sqrt{2}}{5\sqrt{2}} = 2$$

4. Given pair of lines is $8x^2 - 6xy + y^2 = 0$.

$\Rightarrow \quad 8\left(\dfrac{x}{y}\right)^2 - 6\left(\dfrac{x}{y}\right) + 1 = 0$

$\Rightarrow \quad \left[4\left(\dfrac{x}{y}\right) - 1\right]\left[2\left(\dfrac{x}{y}\right) - 1\right] = 0$

$\Rightarrow \quad \dfrac{4x}{y} = 1$ and $\dfrac{2x}{y} = 1$

$\Rightarrow \quad 4x = y$ and $2x - y \quad$ ... (i)

Also, other line is $2x + 3y = a \quad$ ... (ii)

On solving, Eqs. (i) and (ii), we get

$O(0, 0)$, $A\left(\dfrac{a}{14}, \dfrac{2a}{7}\right)$ and $B\left(\dfrac{a}{8}, \dfrac{a}{4}\right)$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ \frac{a}{14} & \frac{2a}{7} & 1 \\ \frac{a}{8} & \frac{a}{4} & 1 \end{matrix}\right\|$$

$\Rightarrow \quad 7 = \dfrac{1}{2}\left|\left(\dfrac{a^2}{56} - \dfrac{2a^2}{56}\right)\right|$

$\Rightarrow \quad 7 = \dfrac{1}{2}\left|\dfrac{-a^2}{56}\right|$

$\Rightarrow \quad 14 \times 56 = a^2$

$\Rightarrow \quad a = 28$

5. Given pair of lines is

$(x^2 + y^2)\cos^2\theta = (x\cos\theta + y\sin\theta)^2$

$\Rightarrow (x^2 + y^2)\cos^2\theta = x^2\cos^2\theta + y^2\sin^2\theta$

$\qquad + 2xy\sin\theta\cos\theta$

$\Rightarrow y^2(\cos^2\theta - \sin^2\theta) - 2xy\sin\theta\cos\theta = 0$

$\therefore$ Pair of lines are perpendicular to each other.

$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$\therefore 0 + \cos^2\theta - \sin^2\theta = 0$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

6. Given equation of circle is

$x^2 + y^2 - 2x + 19y - 38 = 0$.

A) Polar equation at point (4, 3) is $S_1 = 0$.

$\Rightarrow x \times 4 + y \times 3 - (x + 4)$

$\qquad + 5(y + 3) - 38 = 0$

$\Rightarrow 3x + 8y = 27$

B) Equation of tangent at point (9, –5) is

$x \times 9 + y \times (-5) - (x + 9) + 5(y - 5) - 38 = 0$

$\Rightarrow \quad 8x - 72 = 0$

$\Rightarrow \quad x = 9$

C) On differentiating given equation

w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} - 2 + 10\frac{dy}{dx} - 0 = 0$$

$$\Rightarrow \frac{dy}{dx}(2y+10) = 2-2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(-7,-5)} = \frac{2-2\times(-7)}{2\times(-5)+10} = \frac{16}{0} = \frac{1}{0}$$

∴ Equation of normal at point (–7, –5) is

$y + 5 = -0\ (x + 7)$

∴ $y + 5 = 0$

D) Centre of circle C is (1, –5).

∴ Equation of diameter passing through

(1, –5) and (1, 3) is

$$y + 5 = \frac{3+5}{1-1}(x-1)$$

$$\Rightarrow \quad y + 5 = \frac{8}{0}(x-1)$$

$\Rightarrow \quad x - 1 = 0$

$\Rightarrow \quad x = 1$

7. Given equation of circle is

$x^2 + y^2 - 4x - 2y + c = 0$

whose centre is A(2, 1).

$$\text{Now, AP} = \sqrt{(2-10)^2+(1-7)^2}$$

$$= \sqrt{(-8)^2+(-6)^2} = \sqrt{64+36} = \sqrt{100}$$

= 10

∴ AQ = AP – PQ = 10 – 5 = 5

So, Q is the mid-point of AP

$$= \left(\frac{10+2}{2}, \frac{7+1}{2}\right) = (6, 4)$$

Since, Q lies on a circle.

∴ $6^2 + 4^2 - 4(6) - 2(4) + c = 0$

$\Rightarrow 36 + 16 - 24 - 8 + c = 0$

$\Rightarrow \quad 20 + c = 0$

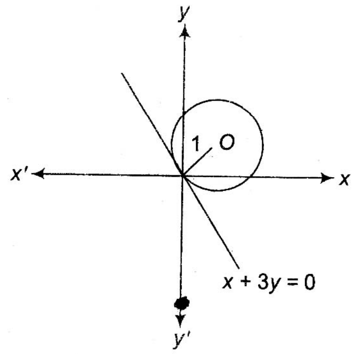
$c = -20$

8. Given line is x + 3y = 0.

∴ Slope of a line $= -\frac{1}{3}$

Let the centres of circle be (+g, +f).

We know that, the perpendicular drawn from the centre to the tangent is equal to radius.



Since perpendicular distance from (g, f) to the line is 1.

$$\therefore \quad \frac{+g+3f}{\sqrt{1^2+3^2}} = 1$$

$$\Rightarrow \quad \frac{+g+3f}{\sqrt{10}} = 1$$

Taking option (d) i.e., centre $= \left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

$$\therefore \frac{+\frac{1}{\sqrt{10}} + 3\times\frac{3}{\sqrt{10}}}{\sqrt{10}} = \frac{10}{10} = 1 \text{ (true)}$$

9. Since, the circle passes through (3, 4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally.

∴ $(x-3)^2 + (y-4)^2 = 0$ ... (i)

Also, $x^2 + y^2 - a^2 = 0$ ... (ii)

∴ Equation of radical axis,

$(x-3)^2 + (y-4)^2 - (x^2 + y^2 - a^2) = 0$

$\Rightarrow x^2 - 6x + 9 + y^2 + 16 - 8y - x^2 - y^2 + a^2 = 0$

$\Rightarrow -6x - 8y + 25 + a^2 = 0$

$\Rightarrow 6x - 8y - 25 - a^2 = 0$ ... (iii)

Now, the distance from (0, 0) to Eq. (iii), we get

$$\left|\frac{6(0)-8(0)-25-a^2}{\sqrt{36+64}}\right|=25$$

$\Rightarrow \quad \dfrac{25+a^2}{10}=25$

$\Rightarrow \quad 25 + a^2 = 250$

$\Rightarrow \quad a^2 = 225$

10. Given coaxial system of circle is

$4x^2 + 4y^2 - 12x + 6y - 3 + \lambda\,(x + 2y - 6) = 0$

or $x^2 + y^2 - 3x + \dfrac{3y}{2} - \dfrac{3}{4} + \dfrac{\lambda}{4}\,(x + 2y - 6) = 0$

Here, radical axis is $x + 2y - 6 = 0$

i.e., line of centre is

$2x - y + k = 0$ ... (ii)

Here, centre of circle is $\left(\dfrac{3}{2}, -\dfrac{3}{4}\right)$ which lies on Eq. (i).

$\therefore \quad \left(\dfrac{3}{2}\right)+\dfrac{3}{4}+k=0$

$\Rightarrow \quad \dfrac{12+3}{4}+k=0$

$\Rightarrow \quad k=-\dfrac{15}{4}$

$\therefore$ Required equation is]

$$2x - y - \frac{15}{4} = 0$$

$\Rightarrow \quad 8x - 4y - 15 = 0$

11. Given equation of parabola is

$$y^2 = 12x$$

Here, $\quad a = 3$

$\therefore$ Equation of normal is

$y = mx - 2am - am^3$ ($\because$ m = –1)

$y = -x - 2(3)(-1) - 3(-1)^3$

$\quad = -x + 6 + 3$

$\Rightarrow \quad x + y = 9$ ... (i)

But given normal is

$$x + y = k$$

$\therefore \quad k = 9$

Focus of a given parabola is S(3, 0).

Now, perpendicular distance from S(3, 0) to the line (1) is

$$p=\frac{|3(1)+0-9|}{\sqrt{1^2+1^2}}=\frac{6}{\sqrt{2}}$$

$$\therefore\ \therefore 4k-2p^2=4(9)-2\left(\frac{6}{\sqrt{2}}\right)^2$$

$$=36-2\times\frac{36}{2}=0$$

12. Since, the line $2x + 5y = 12$ intersect the ellipse $4x^2 + 5y^2 = 20$

$$\therefore\ 4\left(\frac{12-5y}{2}\right)^2 + 5y^2 = 20$$

$\Rightarrow \quad 144 + 25y^2 - 120y + 5y^2 = 20$

$\Rightarrow \quad 30y^2 - 120y + 124 = 0$

$\Rightarrow \quad 15y^2 - 60y + 62 = 0$

$$\therefore\ y=\frac{60\pm\sqrt{(60)^2-4\times15\times62}}{2(15)}$$

$$=\frac{60\pm\sqrt{3600-3720}}{30}=\frac{60\pm\sqrt{-120}}{30}$$

Hence, no real value of y exist.

Hence, no intersection is possible.

13. Given hyperbola is $5x^2 - y^2 = 5$

or it can be rewritten as $\dfrac{x^2}{1}-\dfrac{y^2}{5}=1$

Here, $\quad a^2 = 1, b^2 = 5$

$\therefore$ Equation of tangent is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$\Rightarrow \quad y = mx \pm \sqrt{1m^2 - 5}$ .... (i)

But point (2, 8) lies on it.

$\therefore\ 8 = 2m \pm \sqrt{m^2 - 5}$

$\Rightarrow (8 - 2m) = \pm\sqrt{m^2 - 5}$

On squaring both sides, we get

$64 + 4m^2 - 32m = m^2 - 5$

$\Rightarrow 3m^2 - 32m + 69 = 0$

$\Rightarrow (3m - 23)(m - 3) = 0$

$\Rightarrow \quad m = 3, \frac{23}{3}$

On putting m = 3, Eq. (i), we get

$y = 3x \pm \sqrt{3^2 - 5} = 3x \pm 2$

$\Rightarrow y = 3x + 2$ and $y = 3x - 2$

14. Given equation of hyperbola is $x^2 - 3y^2 = 3$

$\therefore$ Equation of tangent at point $(\sqrt{3}, 0)$ is

$S_1 = 0$

$\Rightarrow x\sqrt{3} - 3y \times 0 = 3$

$\Rightarrow x\sqrt{3} = 3$

$\Rightarrow x = \sqrt{3}$ ..... (i)

The asympotes of given hyperbolas are

$x + \sqrt{3}\,y = 0$ ..... (ii)

and $x - \sqrt{3}\,y = 0$ ..... (iii)

On solving Eqs. (i), (ii) and (iii), we get

(0, 0), $(\sqrt{3}, -1)$ and $(\sqrt{3}, 1)$

$\therefore$ Area of triangle formed by joining the above points

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \sqrt{3} & -1 & 0 \\ \sqrt{3} & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[1\left(\sqrt{3} + \sqrt{3}\right)\right] = \frac{1}{2} \times 2\sqrt{3} = \sqrt{3}$$

15. Given equation of circle is

$r = 12\cos\theta + 5\sin\theta$

Put $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$, we get

$r = 12 \times \frac{x}{r} + 5 \times \frac{y}{r}$

$\Rightarrow \quad r^2 = 12x + 5y$

$\Rightarrow \quad x^2 + y^2 = 12x + 5y$

$\Rightarrow x^2 + y^2 - 12x - 5y = 0$

$\therefore$ Centre is $\left(6, \frac{5}{2}\right)$

$\therefore$ Radius of circle $= \sqrt{(6)^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{144 + 25}{4}}$

$= \sqrt{\frac{169}{4}} = \frac{13}{2}$

16. Let P divide Q (2, 2, 1) and R (5, 1, – 2) in the ratio of m : 1.

$\therefore P\left(\frac{5m+2}{m+1}, \frac{m+2}{m+1}, \frac{-2m+1}{m+1}\right)$

But it is given $\frac{5m+2}{m+1} = 4$

$\Rightarrow 5m + 2 = 4m + 4$

$\Rightarrow m = 2$

$\therefore$ Coordinate of z $= \frac{-2 \times 2 + 1}{2 + 1} = \frac{-3}{3} = -1$

17. Since, a line is equally inclined to the coordinate axes.

$\therefore \quad l = m = n$

$\because \quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 3l^2 = 1$

$\Rightarrow \quad l = \pm\frac{1}{\sqrt{3}}$

$\therefore$ Angle made by line with y-axis = $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

18. The equation of a plane passing through (1, 2, 3) is

$a(x-1) + b(y-2) + c(z-3) = 0$

Also, $(a, b, c) = (1-0, 2-0, 3-0)$

$= (1, 2, 3)$

$\therefore 1(x-1) + 2(y-2) + 3(z-3) = 0$

$\Rightarrow x + 2y + 3z - 1 - 4 - 9 = 0$

$\Rightarrow x + 2y + 3z = 14$

19. Given points are A (1, 0, 0), B (0, 1, 0) and C (1, 1, 1).

$AB = \sqrt{(0-1)^2 + (1-0)^2 + 0^2} = \sqrt{2}$

$BC = \sqrt{(0-1)^2 + 0^2 + 1^2} = \sqrt{2}$

$CA = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$

$\therefore \Delta ABC$ is an equilateral triangle.

$\therefore$ Centre of sphere = Centroid of ΔABC

$= C'\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

$\therefore$ Radius of sphere AC'

$$= \sqrt{\left(\frac{2}{3}-1\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$= \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{1}{9}} = \frac{1}{3}\sqrt{6}$$

$\therefore$ Equation of sphere is

$$\left(x - \frac{2}{3}\right)^2 + \left(y - \frac{2}{3}\right)^2 + \left(z - \frac{1}{3}\right)^2 = \left(\frac{\sqrt{6}}{3}\right)^2$$

$$\Rightarrow x^2 + y^2 + z^2 - \frac{4x}{3} - \frac{4y}{3} - \frac{2z}{3} + \frac{4}{9} + \frac{4}{9} + \frac{1}{9}$$

$$= \frac{6}{9} = \frac{2}{3}$$

$\Rightarrow 3(x^2 + y^2 + z^2) - 4x - 4y - 2z + 1 = 0$

20. $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left(1 + \frac{5}{x+1}\right)^{x+4}$

$$= e^{5\lim_{x\to\infty}\left(\frac{x+4}{x+1}\right)} = e^{5\lim_{x\to\infty}\left(\frac{1+\frac{4}{x}}{1+\frac{1}{x}}\right)} = e^5$$

21. Given, $f(x) = \begin{cases} \alpha + \frac{\sin[x]}{x}, & \text{if } x > 0 \\ 2, & \text{if } x = 0 \\ \beta + \left[\frac{\sin x - x}{x^3}\right], & \text{if } x < 0 \end{cases}$

Since, f is continuous at x = 0

$\therefore$ LHS = f (0) = RHL ..... (i)

Now, LHL = $\lim_{x\to 0^-} f(x)$

$$= \lim_{x\to 0^-}\left[\beta + \left(\frac{\sin x - x}{x^3}\right)\right]$$

$$= \lim_{h\to 0}\left[\beta + \left(\frac{\sin h + h}{-h^3}\right)\right]$$

$$= \beta + 0$$

$$\text{RHL} = \lim_{x\to 0^+}\left[\alpha + \frac{\sin[x]}{x}\right]$$

$$= \lim_{h\to 0}\left[\alpha + \frac{\sin[h]}{h}\right]$$

$$= \alpha + 1$$

and f(0) = 2

$\therefore$ From Eq. (i), we get

$\beta + 0 = 2 = \alpha + 1$

$\Rightarrow \beta - \alpha = 1$

22. Given, $f(x) = \log\left[e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right]$

$\Rightarrow f(x) = x + \frac{3}{4}[\log(x-2) - \log(x+2)]$

On differentiating w.r.t. x, we get

$$f'(x) = 1 + \frac{3}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$$

$$= 1 + \frac{3}{4}\left(\frac{4}{x^2-4}\right) = 1 + \frac{3}{x^2-4}$$

$$\Rightarrow f'(0) = 1 + \frac{3}{0-4} = \frac{1}{4}$$

23. Given, $x^m y^n = (x+y)^{m+n}$

On taking log on both sides, we get

$m \log x + n \log y = (m+n) \log (x+y)$

On differentiating w.r.t. x, we get

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{m}{x} - \frac{m+n}{x+y} = \frac{dy}{dx}\left(\frac{m+n}{x+y} - \frac{n}{y}\right)$$

$$\Rightarrow \frac{my-nx}{x(x+y)} = \frac{dy}{dx}\left(\frac{my-nx}{(x+y)y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

24. Given, $x^4 + y^4 = t^2 + \frac{1}{t^2}$

$$= \left(t + \frac{1}{t}\right)^2 - 2$$

$$= (x^2+y^2)^2 - 2 \quad \left(\because x^2 + y^2 = t + \frac{1}{t}, \text{ given}\right)$$

$\Rightarrow x^4 + y^4 = x^4 + y^4 + 2x^2y^2 - 2$

$\Rightarrow x^2 + y^2 = 1$

$$\Rightarrow y^2 = \frac{1}{x^2}$$

On differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = -\frac{2}{x^3}$$

$$\Rightarrow x^2y\frac{dy}{dx} = -1$$

25. Given, $f(x) = (x^2 - 1)^7$

When we expand above expansion it will give one term of x^{14} whose coefficient is 1.

$\therefore f^{14}(x) = 14!$

26. Given coordinate is

$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$

On differentiating w.r.t. x, we get

$$\frac{dx}{d\theta} = a(1 + \cos\theta),\ \frac{dy}{d\theta} = a(0 + \sin\theta)$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\tan\frac{\pi}{4} = \tan\frac{\theta}{2} \left(\because \frac{dy}{dx} = \tan x\right)$$

$$\Rightarrow \quad \frac{\pi}{4} = \frac{\theta}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

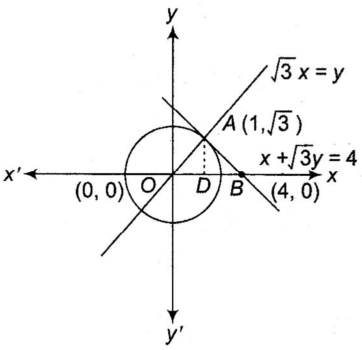
$\therefore$ Coordinate of

$$P\left[a\left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right),\ a\left(1 - \cos\frac{\pi}{2}\right)\right]$$

$$= P\left[a\left(\frac{\pi}{2} + 1\right),\ a\right]$$

27. Given equation of circle is

$$x^2 + y^2 = 4$$



On differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1, \sqrt{3})} = -\frac{1}{\sqrt{3}}$$

$\therefore$ Equation of tangent at $(1, \sqrt{3})$ is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x-1)$$

$$\Rightarrow \sqrt{3}y - 3 = -x + 1$$

$$\Rightarrow x + \sqrt{3}y = 4 \qquad \text{............. (i)}$$

and equation of normal at $(1, \sqrt{3})$ is

$$y - \sqrt{3} = \sqrt{3}(x-1)$$

$$\Rightarrow \sqrt{3}x - y = 0 \qquad \text{.......... (ii)}$$

$\therefore$ Intersection point of Eqs. (i) and (ii) is $(1, \sqrt{3})$.

$\therefore$ Area of ΔOAB

$$= \frac{1}{2} \times OB \times AD = \frac{1}{2} \times 4 \times \sqrt{3}$$

$$= 2\sqrt{3}$$

28. Given, $\frac{dV}{dt} = 2\pi \text{ cm}^3/\text{s}$

$\because$ Volume of sphere, $V = \frac{4}{3}\pi r^3$

On differentiating w.r.t., we get

$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$$

$$\Rightarrow 2\pi = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{2r^2} = \frac{1}{2 \times 6^2} = \frac{1}{72} \text{cm/s}$$

$$\left[\because V = 288\pi = \frac{4}{3}\pi r^3 \Rightarrow 216 = r^3 \Rightarrow r = 6\right]$$

29. Given, $u = f(r)$ and $r^2 = x^2 + y^2$

$$\therefore u_x = f'(r)\frac{\delta r}{\delta x} = f'(r)\frac{x}{r}$$

$$\Rightarrow u_{xx} = f''(r) \times \frac{x^2}{r^2} + f'(r)\frac{r^2 - x \times x}{r^2}$$

$$= f''(r) \times \frac{x^2}{r^2} + f'(r)\frac{r^2 - x^2}{r^3}$$

$$u_y = f'(r)\frac{\delta r}{\delta y} = f'(r)\frac{(2y)}{2\sqrt{x^2 + y^2}}$$

$$= f'(r)\left(\frac{y}{r}\right)$$

$$\text{and } u_{yy} = f''(r)\frac{y^2}{r^2} + f'(r)\frac{r^2 - y^2}{r^3}$$

$$\therefore u_{xx} + u_{yy} = f''(r)\left(\frac{x^2}{r^2} + \frac{y^2}{r^2}\right)$$

$$+ f'(r)\left(\frac{r^2 - x^2}{r^3} + \frac{r^2 - y^2}{r^3}\right)$$

$$= f''(r)\left(\frac{r^2}{r^2}\right) + f'(r)\frac{2r^2 - (x^2 + y^2)}{r^3}$$

$$= f''(r) + f'(r)\frac{r^2}{r^3}$$

$= f''(r) + \frac{f'(r)}{r}$

30. Let $I = \int \frac{dx}{x^2\sqrt{4+x^2}}$

$= \int \frac{dx}{x^3\sqrt{\frac{4}{x^2}+1}}$

Put $\frac{4}{x^2}+1 = t$

$\Rightarrow -\frac{8}{x^3}\,dx = dt$

$\therefore\ I = \int \frac{dt}{-8\sqrt{t}}$

$= -\frac{1}{8}\times\frac{\sqrt{t}}{1/2}+C$

$= -\frac{1}{4}\sqrt{\frac{4}{x^2}+1}+C$

$= -\frac{1}{4x}\sqrt{4+x^2}+C$

31. Let $I = \int \sec^2 x\ \operatorname{cosec}^4 x\ dx$

$= \int \frac{1}{\sin^4 x\cos^2 x}\ dx\ = \int \frac{\sin^2 x+\cos^2 x}{\sin^4 x\cos^2 x}\ dx$

$(\because\ 1 = \sin^2 x + \cos^2 x)$

$= \int \frac{dx}{\sin^2 x\cos^2 x} + \int \frac{dx}{\sin^4 x}$

$= \int \frac{(\sin^2 x+\cos^2 x)dx}{\sin^2 x\cos^2 x} + \int \operatorname{cosec}^4 x\ dx$

$(\because\ 1 = \sin^2 x + \cos^2 x)$

$= \int (\sec^2 x + \operatorname{cosec}^2 x)\ dx$

$+ \int \operatorname{cosec}^2 x(1+\cot^2 x)\ dx$

$= \tan x - \cot x + \int \operatorname{cosec}^2 x\ dx$

$+ \int \operatorname{cosec}^2 x\cot^2 x\ dx$

$= \tan x - \cot x - \cot x - \frac{\cot^3 x}{3} + C$

$= -\frac{1}{3}\cot^3 x + \tan x - 2\cot x + C$

But it given that,

$I = -\frac{1}{3}\cot^3 x + k\tan x - 2\cot x + C$

$\therefore\ k = 1$

32. Let $I = \int \frac{dx}{\sqrt{x-x^2}}$

$= \int \frac{1}{\sqrt{x}}\times\frac{dx}{\sqrt{1-x}}$

Put $\sqrt{x} = \sin\theta$

$\Rightarrow \frac{1}{2\sqrt{x}}\,dx = \cos\theta\ d\theta$

$\therefore\ I = \int \frac{2\cos\theta\ d\theta}{\sqrt{1-\sin^2\theta}}$

$= \int \frac{2\cos\theta}{\cos\theta}d\theta$

$= \int 2\ d\theta\ = 2\theta + C$

$= 2\sin^{-1}\sqrt{x} + C$

33. Let $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}dx$ .... (i)

Put x = – x, we get

$I = -\int_{\pi}^{-\pi} \frac{\sin^2(-x)}{1+a^{-x}}dx$

$\Rightarrow I = \int_{-\pi}^{\pi} a^x\frac{\sin^2 x}{1+a^x}dx$ .... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-\pi}^{\pi} \frac{(1+a^x)\sin^2 x}{(1+a^x)} dx$$

$$= \int_{-\pi}^{\pi} \sin^2 x \, dx$$

$$= 2\int_0^{\pi} \sin^2 x \, dx = \int_0^{\pi} (1-\cos 2x)\, dx$$

$$= \left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \left(\pi + \frac{\sin 2\pi}{2} - 0 - 0\right)$$

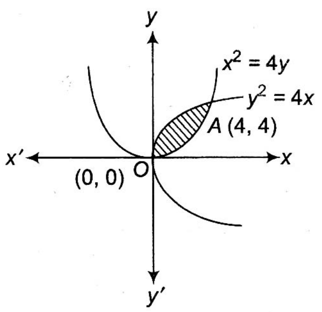
$\Rightarrow 2I = \pi$

$\Rightarrow I = \frac{\pi}{2}$

34. Given curves are $y^2 = 4x$ and $x^2 = 4y$

The intersection point is

$$x^4 = 16y^2 = 16(4x)$$



$\Rightarrow \quad x(x^3 - 64) = 0$

$\Rightarrow \quad x = 0, 4$

$\Rightarrow \quad y = 0, 4$

Hence, intersection point is O (0, 0) and A (4, 4).

$\therefore$ Required area

= Shaded region of the curve

$$= \int (y_2 - y_1) dx$$

$$= \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$$

$$= \left[\frac{2x^{3/2}}{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \left[\frac{4}{3}(4)^{3/2} - \frac{(4)^3}{12} - (0-0)\right]$$

$$= \left(\frac{32}{3} - \frac{64}{12}\right) = \frac{128-64}{12}$$

$$= \frac{64}{12} = \frac{16}{3} \text{ sq. units}$$

35. Given integration is $\int_0^4 \frac{dx}{1+x^2}$ and $h = 1$

x	0	1	2	3	4
$f(x)$	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$

By using Trapezoidal rule,

$$\int_0^4 f(x)\, dx = \frac{h}{2}\left[(y_0 + y_1) + 2(y_1 + y_2 + y_3)\right]$$

$$= \frac{1}{2}\left[\left(1 + \frac{1}{17}\right) + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)\right]$$

$$= \frac{1}{2}\left[\frac{18}{17} + 2\left(\frac{5+2+1}{10}\right)\right]$$

$$= \frac{1}{2}\left(\frac{18}{17} + \frac{8}{5}\right) = \frac{1}{2}\left(\frac{90+136}{85}\right)$$

$$= \frac{1}{2}\left(\frac{226}{85}\right) = \frac{113}{85}$$

36. Given differential equation is

$$\frac{dy}{dx} + 2x \tan(x-y) = 1$$

Put $x - y = t$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$$

$$\therefore 1 - \frac{dt}{dx} + 2x \tan t = 1$$

$\Rightarrow \quad \frac{dt}{\tan t} = 2x\ dx$

$\Rightarrow \quad \cot t\ dt = 2x\ dx$

On integrating both sides, we get

$$\log \sin t = x^2 + \log A$$

$\Rightarrow \log \frac{\sin(x-y)}{A} = x^2$

$\Rightarrow \sin(x-y) = Ae^{x^2}$

37. Given differential equation can be rewritten as

$$\frac{dy}{dx} + \frac{xy}{1-x^2} = \frac{x^4\left(\sqrt{1-x^2}\right)^3}{(1+x^5)(1-x^2)}$$

or $\frac{dy}{dx} + \frac{xy}{1-x^2} = \frac{x^4\sqrt{1-x^2}}{(1+x^5)}$

On comparing by linear differential equation is

$$\frac{dy}{dx} + Py = Q$$

$\therefore P = \frac{x}{1-x^2}$

$\therefore$ IF $= e^{\int P\ dx} = e^{\int \frac{x}{1-x^2} dx}$

$= e^{-\frac{1}{2}\int \frac{-2x}{1-x^2} dx} = e^{-\frac{1}{2}\log(1-x^2)}$

$= e^{\log(1-x^2)^{-1/2}} = \frac{1}{\sqrt{1-x^2}}$

38. Given, g{f(x)} = |sin x|

and, f}g(x)} = $\left(\sin\sqrt{x}\right)^2$

Let us consider f(x) = $\sin^2 x$ and g(x) = $\sqrt{x}$

$\therefore$ f{g(x)} = $f(\sqrt{x}) = (\sin^2\sqrt{x}) = (\sin\sqrt{x})^2$

and g{f(x)} = g($\sin^2 x$) = $\sqrt{\sin^2 x}$ = |sin x|

39. Given, $f: Z \to Z$, $f(x) = \begin{cases} \frac{x}{2}, \text{ if } x \text{ is even} \\ 0, \text{ if } x \text{ is odd} \end{cases}$

when x is odd, f(1) = f(3) = 0

$\Rightarrow$ f(x) is not one-one function.

when x is even i.e., x = 0, ± 2, ± 4, ± 6, ... ∞

$\Rightarrow \frac{x}{2} = 0, \pm1, \pm 2, \pm3, \ldots\ldots\ldots \infty$

Hence, range of f is Z. So, it is onto.

Hence, f is not one-one but it is onto.

40. Given series can be rewritten as

$\frac{1}{2}\left[\frac{1}{2} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + \ldots + \frac{1}{2n} - \frac{1}{2n+2}\right]$

$= \frac{1}{4}\left(1 - \frac{1}{n+1}\right)$

$= \frac{1}{4}\left(\frac{n}{n+1}\right)$

On comparing given equation, we get

$$k = \frac{1}{4}$$

41. We know that,

Number of diagonals = $\frac{n(n-3)}{2}$

$\therefore 170 = \frac{n(n-3)}{2}$

$\Rightarrow$ 340 = n(n – 3)

$\Rightarrow$ 20 × 17 = n (n – 3)

$\Rightarrow$ n = 20

42. A committee of 12 members is to be formed when women are in majority.

Case I 9 women and 3 men

$\therefore$ Number of ways

$= {}^9C_3 \times {}^8C_3 = 1 \times \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

Case II 8 women and 4 men

$\therefore$ Number of ways $= {}^9C_8 \times {}^8C_4$

$= 9 \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 630$

Case III 7 women and 5 men

$\therefore$ Number of ways $= {}^9C_7 \times {}^8C_5$

$= \frac{9 \times 8}{2 \times 1} \times \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$

$= 36 \times 56 = 2016$

$\therefore$ Required number of ways

$= 56 + 630 + 2016 = 2702$

43. There are two cases arise.

Case I When 5 questions are selected from first 6 questions and next 5 questions are selected from 7 questions.

$\therefore$ Number of ways $= {}^6C_5 \times {}^7C_5$

$= 6 \times \frac{7 \times 6}{2 \times 1}$

$= 126$

Case II When 6 questions are selected from first 6 questions and next 4 questions are selected from 7 questions.

$\therefore$ Number of ways $= {}^6C_6 \times {}^7C_4$

$= 1 \times \frac{7 \times 6 \times 5}{3 \times 2} = 35$

$\therefore$ Required number of ways = 126 + 35 =161

44. $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}\left({}^kC_r\right)$

$= \sum_{k=1}^{\infty}\frac{1}{3^k}\left({}^kC_0 + {}^kC_1 + {}^kC_2 + \ldots + {}^kC_k\right)$

$= \sum_{k=1}^{\infty}\frac{2^k}{3^k} = \sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k$

$= \frac{2}{3} + \left(\frac{2}{3}\right)^1 + \ldots\ldots\infty$

$= \frac{2/3}{1-\frac{2}{3}} \quad (\because \text{Common ratio} = \frac{2}{3})$

$= 2$

45. Given expansion is $\left(\frac{x^2}{a} - \frac{b}{x}\right)^{11}$.

$\therefore$ The general term is

$T_{r+1} = {}^{11}C_r \left(\frac{x^2}{a}\right)^{11-r}\left(-\frac{b}{x}\right)^r$

$= {}^{11}C_r (x)^{22-3r}(-b)^r\left(\frac{1}{a}\right)^{11-r}$

$\therefore$ For coefficient x^7, put $22 - 3r = 7$

$\Rightarrow 3r = 15$

$\Rightarrow r = 5$

and for coefficient of x^4, put $22 - 3r = 4$

$\Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore T_6 = {}^{11}C_5\left(\frac{1}{a}\right)^6(-b)^5$

and $T_7 = {}^{11}C_5\left(\frac{1}{a}\right)^6(-b)^6$

According to the given condition,

$T_6 + T_7 = 0$

$\therefore {}^{11}C_5\left(\frac{1}{a}\right)^6(-b)^5 + {}^{11}C_6\left(\frac{1}{a}\right)^5(-b)^6 = 0$

$\Rightarrow {}^{11}C_5\left(\frac{1}{a}\right)^5(-b)^5\left(\frac{1}{a} - b\right) = 0$

$\Rightarrow \frac{1}{a} - b = 0$

$\Rightarrow ab = 1$

46. Given, $\frac{1}{x(x+1)....(x+n)}$

$$= \frac{A_0}{x} + \frac{A_1}{x+1} + ... + \frac{A_r}{x+r} + ... + \frac{A_n}{x+n}$$

$$\Rightarrow \frac{1}{x(x+1)...(x+r-1).(x+r+1)..(x+n)}$$

$$= \frac{x+r}{x}A_0 + ... + A_r + ... + \frac{x+r}{x+n}A_n$$

$$\lim_{x \to -r} \frac{1}{x(x+1)(x+2)..(x+n)}$$

$$= 0 + ... + 0 + A_r + + ... + 0$$

$$A_r = \frac{1}{-r(-r+1)....(-1).1 \,.\, 2 \,....(-r+n)}$$

$$= \frac{(-1)^r}{r(r-1)...(1)[1.2.....(n-r)]}$$

$$\Rightarrow A_r = \frac{(-1)^r}{r!(n-r)!}$$

47. Given series is $1 + \frac{1}{3.2^2} + \frac{1}{5.2^4} + \frac{1}{7.2^6} + ...$

$$= 2\left[\frac{1/2}{1} + \frac{(1/2)^3}{3} + \frac{(1/2)^5}{5} + ...\right]$$

$$= \log_e \left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \log_e \left(\frac{3/2}{1/2}\right)$$

$$\left[\because \log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{3} + ...\right)\right]$$

$$= \log_e 3$$

48. Given, $R = \frac{\pi}{4}$

Also, P + Q + R = π

$$\Rightarrow P + Q + \frac{\pi}{4} = \pi$$

$$\Rightarrow P + Q = \frac{3\pi}{4}$$

$$\Rightarrow \frac{P}{3} + \frac{Q}{3} = \frac{\pi}{4}$$

$$\Rightarrow \tan\left(\frac{P}{3} + \frac{Q}{3}\right) = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{\tan\frac{P}{3} + \tan\frac{Q}{3}}{1 - \tan\frac{P}{3}\tan\frac{Q}{3}} = 1 \quad .... \text{(i)}$$

Since, $\tan\frac{P}{3}$ and $\tan\frac{Q}{3}$ are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \tan\frac{P}{3} + \tan\frac{Q}{3} = -\frac{b}{a}$$

$$\text{and } \tan\frac{P}{3}.\tan\frac{Q}{3} = \frac{c}{a}$$

∴ From Eq. (i),

$$\frac{-\frac{b}{a}}{1-\frac{c}{a}} = 1$$

$$\Rightarrow \frac{-b}{a-c} = 1$$

⇒ a + b = c

49. Given equation is

$|x|^{6/5} - 26\,|x|^{3/5} - 27 = 0$

Put $|x|^{3/5} = t$

$\therefore t^2 - 26t - 27 = 0$

$\Rightarrow t^2 - 27t + t - 27 = 0$

⇒ t (t – 27) + 1 (t – 27) = 0

⇒ (t + 1) (t – 27) = 0

$\Rightarrow t = 27$ or -1

$\Rightarrow |x|^{3/5} = 27$

$(\because |x|^{3/5}$ cannot be negative)

$\Rightarrow |x|^3 = (3^3)^5$

$\Rightarrow |x| = 3^5$

$\Rightarrow x = 3^5$ or -3^5

$\therefore$ Product of $x = 3^5 \times (-3)^5 = -3^{10}$

50. Since, α, β and γ are the roots of

$f(x) = x^3 + px^2 + qx + r = 0$

$\therefore \alpha + \beta + \gamma = -p,\ \alpha\beta + \beta\gamma + \gamma\alpha = q$

and $\alpha\beta\gamma = -r$

Let $x = \alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma + \beta\gamma - \dfrac{\alpha\beta\gamma}{\alpha}$

$\Rightarrow x = q + \dfrac{r}{\alpha} \Rightarrow \alpha = \dfrac{r}{x-q}$

$\therefore \alpha$ satisfies the given equation.

$\therefore f(\alpha) = \alpha^3 + p\alpha^2 + q\alpha + r = 0$

$$\frac{r^3}{(x-q)^3} + \frac{pr^2}{(x-q)^2} + \frac{qr}{(x-q)} + r = 0$$

$\Rightarrow (x-q)^3 + q(x-q)^2 + pr(x-q) + r^2 = 0$

$\therefore$ Coefficient of $x = 3q^2 - 2q^2 + rp$

$= q^2 + pr$

51. Given, $A = \begin{vmatrix} 2 & e^{i\pi} \\ -1 & i^{2012} \end{vmatrix}$

$$= \begin{vmatrix} 2 & \cos\pi + i\sin\pi \\ -1 & (1^4)^{503} \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 2 - 1 = 1$$

$C = \dfrac{d}{dx}\left(\dfrac{1}{x}\right)_{x=1} = \left(-\dfrac{1}{x^2}\right)_{x=1} = -1$

and $D = \displaystyle\int_{e^2}^{1} \frac{dx}{x}$

$= [\log x]_{e^2}^{1}$

$= \log 1 - \log e^2 = 0 - 2$ .... (i)

$= -2$

Let α, β and γ are the roots of the equation

$Ax^3 + Bx^2 + Cx + D = 0$

$\therefore x^3 + Bx^2 - x + 2 = 0$ .... (ii)

$\therefore \alpha + \beta + \gamma = -B$

$\Rightarrow \gamma = -B$ ($\because \alpha + \beta = 0$ given)

$\therefore -B$ satisfies the Eq. (ii),

$\therefore (-B)^3 + B^2 + B + 2 = 0$

$\Rightarrow B = 2$

52. Given, $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$= \begin{bmatrix} i^2 + i^2 & -i^2 - i^2 \\ -i^2 - i^2 & i^2 + i^2 \end{bmatrix}$

$= \begin{bmatrix} -1-1 & 1+1 \\ 1+1 & -1-1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = -2B$

$B^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$= 2B$ .... (i)

Now, $A^8 = (A^2)^4 = (-2B)^4$

$= 16B^4 = 16(B^2)^2\ 16 \cdot (2B)^2$

$= 16 \times 4 \times B^2$

$= 16 \times 4 \times 2B$ [from Eq.(i)]

$= 128B$

53. Given, f(x)

$$= \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x-1)(x)(x+1) \end{vmatrix}$$

Taking x and x(x – 1) common from R_2 and R_3 and (x + 1) common from C_3.

$$= x \times x(x-1) \times (x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & (x-1) & 1 \\ 3 & (x-2) & 1 \end{vmatrix}$$

(applying $C_2 \to C_1 + C_2$)

$$= x^2(x^2-1) \begin{vmatrix} 1 & x+1 & 1 \\ 2 & x+1 & 1 \\ 3 & x+1 & 1 \end{vmatrix}$$

$$= x^2(x^2-1)(x+1) \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow f(x) = 0$

$\therefore f(2012) = 0$

54. Given, $A = \begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

$\therefore |A| = -1(24-25) + 2(18-20) - 3(15-16)$

$= 1 - 4 + 3 = 0$

Now, $\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1 \neq 0$

$\therefore$ Rank of A = a = 2

$$B = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$$

$\therefore |B| = 2 - 2 = 0$

$\therefore$ Rank of B = b = 1

and $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$|C| = 2(4-0) = 8 \neq 0$

$\therefore$ Rank of C = 3, c = 3

$b < a < c \ (\because 1 < 2 < 3)$

55. Given system of equations is

$(a\alpha + b)x + ay + bz = 0,$

$(b\alpha + c)x + by + cz = 0$

and $(a\alpha + b)y + (b\alpha + c)z = 0$

For non-trivial solution,

$$\begin{vmatrix} a\alpha+b & a & b \\ b\alpha+c & b & c \\ 0 & a\alpha+b & b\alpha+c \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - aR_1 - R_2$

$$\Rightarrow \begin{vmatrix} a\alpha+b & a & b \\ b\alpha+c & b & c \\ -(a\alpha^2+2b\alpha+c) & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow -(a\alpha^2 + 2b\alpha + c)(ac - b^2) = 0$

$\Rightarrow ac - b^2 = b \quad (\because a\alpha^2 + 2b\alpha + c \neq 0)$

$\Rightarrow ac = b^2$

Hence, a, b and c are in GP.

56. Given, $(a + ib)^2 = (c + id)^2 (x + iy)$

$\Rightarrow |a + ib|^2 = |c + id|^2 |x + iy|$

$\Rightarrow a^2 + b^2 = (c^2 + d^2)\left(\sqrt{x^2 + y^2}\right)$

$\Rightarrow 4 = 2\left(\sqrt{x^2 + y^2}\right)$

$(\because a^2 + b^2 = 4$ and $c^2 + d^2 = 2$ given)

$\Rightarrow \sqrt{x^2 + y^2} = 2$

$\Rightarrow x^2 + y^2 = 4$

57. Given, $\left|z - \frac{4}{z}\right| = 2$

$$\therefore |z| = \left|z - \frac{4}{z} + \frac{4}{z}\right|$$

$$\le \left|z-\frac{4}{z}\right|+\left|\frac{4}{z}\right|$$

$$\Rightarrow |z| \le 2+\frac{4}{|z|}$$

$$\Rightarrow |z|^2 - 2|z| \le 4$$

$$\Rightarrow \left(|z|^2 - 1\right)^2 \le 5$$

$$\Rightarrow (|z|-1) \le \sqrt{5}$$

$$\Rightarrow |z| \le \sqrt{5}+1$$

58. Since, α is a non-real of the equation

$x^6 - 1 = 0$

$\therefore\ \alpha^6 - 1 = 0$

$\Rightarrow \alpha^6 - 1 = 0 \times \alpha - 1$

$$\Rightarrow \frac{\alpha^6 - 1}{\alpha - 1} = 0$$

$\Rightarrow 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 = 0$

$\Rightarrow (1 + \alpha) + \alpha^2 (1 + \alpha) + \alpha^4 (1 + \alpha) = 0$

$\Rightarrow (1 + \alpha)(1 + \alpha^2 + \alpha^4) = 0$ .... (i)

$$\therefore \frac{\alpha^2+\alpha^3+\alpha^4+\alpha^5}{\alpha+1} = \frac{\alpha^2(\alpha+1)+\alpha^4(\alpha+1)}{\alpha+1}$$

$$= \frac{(\alpha+1)(\alpha^2+\alpha^4)}{\alpha+1} = \alpha^2 + \alpha^4 = -1 \text{ [from Eq. (i)]}$$

59. Given trigonometrical equation is

$27 \tan^2\theta + 3\cot^2\theta$

(using AM $\ge$ GM)

$$\therefore \frac{27\tan^2\theta + 3\cot^2\theta}{2} \ge \sqrt{27\tan^2\theta \, . \, 3\cot^2\theta}$$

$$\Rightarrow \frac{27\tan^2\theta + 3\cot^2\theta}{2} \ge \sqrt{81}$$

$\Rightarrow 27\tan^2\theta + 3\cot^2\theta \ge 18$

Hence, minimum value of given equation is 18.

60. $\cos 36° - \cos 72° = -2\sin\left(\frac{36°+72°}{2}\right)$

$$\sin\left(\frac{72°-36°}{2}\right)$$

$= -2\sin 54° \sin(-18°)$

$= 2\sin 54° \sin 18°$

$$= 2 \times \frac{\sqrt{5}+1}{4} \times \frac{\sqrt{5}-1}{4} = 2 \times \frac{5-1}{4\times 4} = \frac{4}{8}$$

$$= \frac{1}{2}$$

61. LHS $= \tan x + \tan\left(x+\frac{\pi}{3}\right) + \tan\left(x+\frac{2\pi}{3}\right)$

$$= \tan x + \frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1+\sqrt{3}\tan x}$$

$$= \tan x + \frac{\left[\begin{array}{l}\left(\tan x+\sqrt{3}\right)\left(1+\sqrt{3}\tan x\right) \\ +\left(\tan x-\sqrt{3}\right)\left(1-\sqrt{3}\tan x\right)\end{array}\right]}{1-3\tan^2 x}$$

$$= \frac{\left[\begin{array}{l}\tan x\left(1-3\tan^2 x\right)+\tan x+\sqrt{3} \\ +\sqrt{3}\tan^2 x+3\tan x+\tan x-\sqrt{3} \\ \qquad -\sqrt{3}\tan^2 x+3\tan x\end{array}\right]}{1-3\tan^2 x}$$

$$\Rightarrow \frac{\tan x\left(1-3\tan^2 x\right)+8\tan x}{1-3\tan^2 x} = 3 \text{ (given)}$$

$\Rightarrow \tan x\,(1 - 3\tan^2 x) + 8\tan x$

$= 3(1 - 3\tan^2 x)$

$\Rightarrow \tan x\,(9 - 3\tan^2 x) = 3(1 - 3\tan^2 x)$

$\Rightarrow \tan x\,(3 - \tan^2 x) = 1 - 3\tan^2 x$

$$\Rightarrow \frac{3\tan x - \tan^3 x}{1-3\tan^2 x} = 1$$

$\Rightarrow \tan 3x = 1$

62. Given, $3\sin x + 4\cos x = 5$

$$\Rightarrow 3\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)=5$$

$$\Rightarrow 6\tan\frac{x}{2}+4-4\tan^2\frac{x}{2}=5\left(1+\tan^2\frac{x}{2}\right)$$

$$\Rightarrow 6\tan\frac{x}{2}-9\tan^2\frac{x}{2}=1$$

63. $\therefore \cos^{-1}x+\cos^{-1}\left(\frac{x}{2}+\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$

$$=\cos^{-1}x+\cos^{-1}\left(x\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\sqrt{1-x^2}\right)$$

$$=\cos^{-1}x+\cos^{-1}\left(\frac{1}{2}\right)-\cos^{-1}x$$

$$=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

64. Since, a, b and c form a geometric progression

$\therefore$ a = a, b = ar, c = ar^2

Therefore, given line becomes

$$ax+ary+ar^2=0$$

$\Rightarrow x+ry+r^2=0$

$\Rightarrow x=-ry-r^2$ .... (i)

On putting $x=-ry-r^2$ in given curve

$x+2y^2=0$, we get

$$-ry-r^2+2y^2=0$$

$\Rightarrow 2y^2-ry-r^2=0$

$\therefore$ Sum of ordinates $=\frac{r}{2}$

65. Given point is (3, 2).

(i) Reflection of point (3, 2) about the line y = x is 2, 3)

(ii) Translation of a point through 1 unit distance in the positive direction of x-axis is (3, 3).

(iii) $X=-x\cos\theta+y\sin\theta$

$$=\left(-\frac{3}{\sqrt{2}}+\frac{3}{\sqrt{2}}\right)=0$$

and $Y=x\sin\theta+y\cos\theta$

$$=\left(\frac{3}{\sqrt{2}}+\frac{3}{\sqrt{2}}\right)=3\sqrt{2}=\sqrt{18}$$

Hence, final position is $\left(0,\sqrt{18}\right)$

66. Given X is a position variate such that

$\alpha=P(X=1)=P(X=2)$

$\therefore \frac{e^{-\lambda}\lambda}{1!}=\frac{e^{-\lambda}\lambda^2}{2!}\Rightarrow\lambda=2$

$\therefore \alpha=P(X=1)$

$$=e^{-2}\times 2=\frac{2}{e^2} \qquad ....(i)$$

$\therefore P(X=4)=\frac{e^{-\lambda}\lambda^4}{4!}=\frac{e^{-2}(2)^4}{24}$

$=\frac{e^{-2}\times 16}{24}=\frac{2}{3}e^{-2}=\frac{1}{3}\alpha$ [from Eq.(i)]

67. Given $P(X=r)={}^nC_rp^rq^{n-r}$

$$\therefore \frac{P(X=r)}{P(X=n-r)}=\frac{{}^nC_rp^rq^{n-r}}{{}^nC_{n-r}p^{n-r}q^r}$$

$$=\left(\frac{q}{p}\right)^{n-2r}$$

For independent of n, $\frac{q}{p}=1$

$\Rightarrow q=p$

$\because p+q=1$

$\therefore 2p=1\Rightarrow p=\frac{1}{2}$

68. Let E_1 : The event that the students knows the nswer and

E_2 : The event that the student guesses the answer.

$\therefore$ P(E_1) = $\frac{9}{10}$ and P(E_2) = 1 – $\frac{9}{10}$ = $\frac{1}{10}$

Let E : The answer is correct.

The probability that the student answered correctly, given that he knows the answer

i.e., $P\left(\frac{E}{E_1}\right) = 1$

The probability that the students answered correctly, given that he guessed is $\frac{1}{4}$.

i.e., $P\left(\frac{E}{E_2}\right) = \frac{1}{4}$

By using Baye's theorem,

$$P\left(\frac{E_2}{E}\right) = \frac{P\left(\frac{E}{E_2}\right)P(E_2)}{P\left(\frac{E}{E_1}\right)P(E_1) + P\left(\frac{E}{E_2}\right)P(E_2)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{10}}{1\times\frac{9}{10}+\frac{1}{4}\times\frac{1}{10}}$$

$$= \frac{\frac{1}{40}}{\frac{9}{10}+\frac{1}{40}} = \frac{\frac{1}{40}}{\frac{36+1}{40}} = \frac{1}{37}$$

69. $\therefore$ Required probability = P(In first two test either oth are faulty or both are not faulty)

= P(First two are faulty)

+ P(First two are not faulty)

$= \frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}$

$= \frac{4}{12} = \frac{1}{3}$

70. Probability of getting a tail in a single toss

p = $\frac{1}{2}$ and not getting tail, q = $\frac{1}{2}$

Using Binomial distribution,

$\therefore$ Required probability

= P(X=1)+P(X=3)+P(X=5)+....+P(X=99)

$= {}^{100}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{99} + {}^{100}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{97}$

$+ {}^{100}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{95} + ... + {}^{100}C_9\left(\frac{1}{2}\right)^{99}\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^{100}\left({}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + ... + {}^{100}C_{99}\right)$

$= \frac{1}{2^{100}}\left(2^{99}\right) = \frac{1}{2}$

71. Given, a = i + j – 2k

$\therefore$ $\Sigma[(a \times i) \times j]^2 = \Sigma[(a \, . \, j)i]^2$

$= a^2 = |i + j - 2k|^2$

$= \left(\sqrt{1+1+4}\right)^2 = 6$

72. Given, p = $\frac{b \times c}{[a\ b\ c]}$, q = $\frac{c \times a}{[a\ b\ c]}$

and r = $\frac{a \times b}{[a\ b\ c]}$

Now, (a + b) . p = (a + b) . $\frac{(b \times c)}{[a\ b\ c]}$

$= \frac{[a\ b\ c]}{[a\ b\ c]} + \frac{[b\ b\ c]}{[a\ b\ c]}$

= 1 + 0 = 1

Similarly, (b + c) . q = 1

and (c + a) . r = 1

$\therefore$ (a + b) . p + (b + c) . q + (c + a) . r

= 1 + 1 + 1 = 3

73. Since, vectors a and b are in a same plane.

$\therefore$ r = a + tb

= (i + 2j + k) + t(i – j + k)

= (1 + t) i + (2 – t) j + (1 + t) k ..... (i)

$\therefore$ Projection of r on c = $\frac{r \cdot c}{|c|}$

$$\frac{1}{\sqrt{3}} = \frac{\left[\begin{array}{r}\{(1+t)i+(2-t)j+(1+t)k\} \\ (i+j-k)\end{array}\right]}{\sqrt{1^2+1^2+(-1)^2}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1+t+2-t-(1+t)}{\sqrt{3}}$$

$\Rightarrow$ 1 = 2 – t

$\Rightarrow$ t = 1

On putting t = 1 in Eq. (i), we get

r = 2i + j + 2k

74. Put r = xi + yj + zk

$\therefore$ xi + yj + zk = (i – 6j + 2k) + t(i + 2j + k)

$\therefore$ Any point on line is P(1+t, – 6 + 2t, 2+t) is atisfied the second equation of line.

$\therefore$ (1 + t) i + (– 6 + 2t) j + (2 + t) k

= 2ui + (4 + u)j + (1 + 2u)k

On equating the coefficients of i, j and k, we get

1 + t = 2u

$\Rightarrow$ t – 2u + 1 = 0 .... (i)

– 6 + 2t = 4 + u

$\Rightarrow$ 2t – u – 10 = 0 .... (ii)

and 2 + t = 1 + 2u

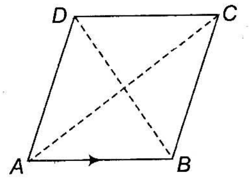
$\Rightarrow$ t – 2u + 1 = 0 .... (iii)

On solving Eqs. (i) and (ii), we get

t = 7, u = 4

$\therefore$ P(1 + 7, – 6 + 2 × 7, 2 + 7) = P(8, 8, 9)

75. Given, AB = 3i – 2j + 2k



and BC = i – 2k

Diagonals AC = AB + BC

= 3i – 2j + 2k + i – 2k

= 4i – 2j

and BD = BC – AB

= i – 2k – (3i – 2j + 2k)

= – 2i + 2j – 4k

$$\therefore \cos\theta = \frac{AC \cdot BD}{|AC||BD|}$$

$$= \frac{(4i-2j)\cdot(-2i+2j-4k)}{\sqrt{4^2+(-2)^2}\sqrt{(-2)^2+(2)^2(-4)^2}}$$

$$= \frac{-8-4}{\sqrt{20}\sqrt{24}} = -\frac{12}{4\sqrt{30}} = -\frac{3}{\sqrt{30}} = -\sqrt{\frac{3}{10}}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\sqrt{\frac{3}{10}}\right)$$

or $\pi - \cos^{-1}\left(-\sqrt{\frac{3}{10}}\right)$

76. Let first term of a GP be u and common ratio z.

$\therefore$ $T_p = uz^{p-1} = a$

$\Rightarrow$ log u + (p – 1) log z = log a .... (i)

$T_q = uz^{q-1} = b$

$\Rightarrow$ log u + (q – 1) log z = log b .... (ii)

and $T_r = uz^{r-1} = c$

$\Rightarrow$ log u + (r – 1) log z = log c .... (iii)

Let θ be the angle between

(log a^2) i + (log b^2) j + (log c^2)k

and (q – r) i + (r – p) j + (p – q) k is

$$\cos\theta = \frac{\left[\begin{array}{c}(\log a^2)(q-r)+(\log b^2)(r-p)\\ +(\log c^2)(p-q)\end{array}\right]}{\left[\begin{array}{c}\sqrt{(\log a^2)^2+(\log b^2)^2+(\log c^2)^2}\\ \sqrt{(q-r)^2+(r-p)^2+(p-q)^2}\end{array}\right]}$$

.... (iv)

From Eqs. (i), (ii) and (iii)

q – r = log b – log c, r – p = log c – log a

p – q = log a – log b

$\therefore$ From Eq. (iv), taking numerator term

= 2 log a (log b – log c) + 2 log b (log c–log a)

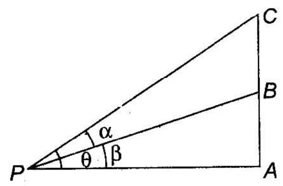
+ 2log c(log a– log b)

= 0

$\therefore$ From Eq. (i), we get

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

77. Let AC be a pole and point P be the position on of the ground.



Given, $\theta = \tan^{-1}\frac{1}{2}$

$\Rightarrow \tan\theta = \frac{1}{2}$

Also, $\theta = \alpha + \beta$

$\Rightarrow \tan\theta = \tan(\alpha+\beta)$

$$\Rightarrow \frac{1}{2} = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

(a) When $(\tan\alpha, \tan\beta) = \left(\frac{1}{4}, \frac{1}{5}\right)$

$$\therefore \text{RHS} = \frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{1}{4}\times\frac{1}{5}} = \frac{\frac{9}{20}}{\frac{19}{20}}$$

$$= \frac{9}{19} \neq \frac{1}{2}, \text{ not true}$$

(b) When $(\tan\alpha, \tan\beta) = \left(\frac{1}{5}, \frac{2}{9}\right)$

$$\text{RHS} = \frac{\frac{1}{5}+\frac{2}{9}}{1-\frac{1}{5}\times\frac{2}{9}} = \frac{\frac{19}{45}}{\frac{43}{45}}$$

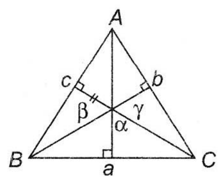
$$= \frac{19}{43} \neq \frac{1}{2}, \text{ not true}$$

(c) When $(\tan\alpha, \tan\beta) = \left(\frac{2}{9}, \frac{1}{4}\right)$

$$\text{RHS} = \frac{\frac{2}{9}+\frac{1}{4}}{1-\frac{2}{9}\times\frac{1}{4}} = \frac{\frac{17}{36}}{\frac{34}{36}} = \frac{1}{2}, \text{ true}$$

78. $\because$ Area of triangle = $\frac{1}{2}$ base $\times$ altitude

$$\therefore \Delta = \frac{1}{2}a\alpha = \frac{1}{2}b\beta = \frac{1}{2}c\gamma$$



$$\Rightarrow \alpha = \frac{2\Delta}{a}, \beta = \frac{2\Delta}{b} \text{ and } \gamma = \frac{2\Delta}{c}$$

$$\therefore \frac{\Delta^2}{R^2}\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)$$

$$= \frac{\Delta^2}{R^2}\left(\frac{a^2}{4\Delta^2}+\frac{b^2}{4\Delta^2}+\frac{c^2}{4\Delta^2}\right)$$

$= \frac{1}{4R^2}(4R^2 \sin^2 A + 4R^2 \sin^2 B + 4R^2 \sin^2 C)$

$$\left[\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}\right]$$

$= \sin^2 A + \sin^2 B + \sin^2 C$

79. $\because$ In ΔABC,

$A + B + C = 180°$

$\Rightarrow A + B = 180° - C$

$\Rightarrow \cot(A + B) = \cot(180° - C)$

$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = -\cot C$

$\Rightarrow \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

80. Given, $x = \log\left(\frac{1}{y} + \sqrt{1 + \frac{1}{y^2}}\right)$

$\therefore x = \text{cosech}^{-1} y$

$\Rightarrow y = \text{cosec h } x$