(MATHEMATICS) PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

DPP – 14

CLASS – 10^{th}

TOPIC – WORD PROBLEMS

Q.1 A person rowing at the rate of 5km/h in still water, takes thrice as much as time in going 40 km upstream as in going 40km downstream. Find the speed of the stream.

Q.2 Ramesh travels 760km to his home partly by train and partly by car. He takes 8 hours if he travels 160km by train and the rest by car. He takes 12 minutes more if he travels 240km by train and the rest by car. Find the speed of the train and car respectively.

Q.3 A man travels 600 km partly by train and partly by car. If he covers 400km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200km by train and the rest by car, he takes half an hour longer. Find the speed of the train and the speed of the car.

Q.4 Places A and B are 80 km apart from each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite direction, they meet in 1hour and 20 minutes. Find the speeds of the cars.

DPP – 14

CLASS – 10th

TOPIC – WORD PROBLEMS

Sol.1 Let's assume x to be the speed of the stream.

So, we know that

Speed of boat in downstream = (5 + x) and,

Speed of boat in upstream = (5 – x)

It is given that,

The distance in one way is 40km.

And,

Time taken during upstream = 3 × time taken during the downstream

Expressing it by equations, we have

$40/(5 - x) = 3 \times 40/(5 + x)$ [∵ time = distance/ speed]

By cross multiplication, we get

$(5+x) = 3(5-x)$

$\Rightarrow \quad 5 + x = 3(5 - x)$

$\Rightarrow \quad x + 3x = 15 - 5$

$\Rightarrow \quad x = 10/4 = 2.5$

Therefore, the speed of the stream is 2.5 km/hr.

Sol.2 Let's assume,

The speed of the train be x km/hr

The speed of the car = y km/hr

From the question, it's understood that there are two parts

Part 1: When Ramesh travels 160 Km by train and the rest by car.

Part 2: When Ramesh travels 240Km by train and the rest by car.

Part 1,

Time taken by Ramesh to travel 160 km by train = 160/x hrs [∵ time = distance/ speed]

Time taken by Ramesh to travel the remaining (760 – 160) km i.e., 600 km by car =600/y hrs

So, the total time taken by Ramesh to cover 760Km = 160/x hrs + 600/y hrs

It's given that,

Total time taken for this journey = 8 hours

So, by equations its

$160/x + 600/y = 8$

$20/x + 75/y = 1$ [on dividing previous equation by 8] …………………… (i)

Part 2,

Time taken by Ramesh to travel 240 km by train = 240/x hrs

Time taken by Ramesh to travel (760 – 240) = 520km by car = 520/y hrs

For this journey, it's given that Ramesh will take a total of is 8 hours and 12 minutes to finish.

$240/x + 520/y = 8\text{hrs } 12\text{mins} = 8 + (12/60) = 41/5$ hr

$240/x + 520/y = 41/5$

$6/x + 13/y = 41/200$ ………. (ii)

Solving (i) and (ii), we get the required solution

Let's take $1/x = u$ and $1/y = v$,

So, (i) and (ii) becomes,

$20u + 75v = 1$ ……….. (iii)

$6u + 13v = 41/200$ ……. (iv)

On multiplying (iii) by 3 and (iv) by 10,

$60u + 225v = 3$

$60u + 130v = 41/20$

Subtracting the above two equations, we get

$(225 – 130)v = 3 – 41/20$

$95v = 19/ 20$

⇒ $v = 19/ (20 \times 95) = 1/100$

⇒ $y = 1/v = 100$

Using $v = 1/100$ in (iii) to find v,

$20u + 75(1/100) = 1$

$20u = 1 – 75/100$

⇒ $20u = 25/100 = 1/4$

⇒ $u = 1/80$

⇒ $x = 1/u = 80$

So, the speed of the train is 80km/hr and the speed of car is100km/hr.

Sol.3 Let's assume,

The speed of the train be x km/hr

The speed of the car = y km/hr

From the question, it's understood that there are two parts

Part 1: When the man travels 400 km by train and the rest by car.

Part 2: When Ramesh travels 200 km by train and the rest by car.

Part 1,

Time taken by the man to travel 400 km by train = $400/x$ hrs [∵ time = distance/ speed]
Time taken by the man to travel $(600 – 400) = 200$ km by car = $200/y$ hrs
Time taken by a man to cover 600 km = $400/x$ hrs + $200/y$ hrs
Total time taken for this journey = 6 hours + 30 mins = $6 + 1/2 = 13/2$
So, by equations its

$400/x + 200/y = 13/2$

$400/x + 200/y = 13/2$

$400/x + 200/y = 13/2$

$200\ (2/x + 1/y) = 13/2$

$2/x + 1/y = 13/400$ ....(i)

Part 2,

Time taken by the man to travel 200 km by train = 200/x hrs. [∵ time = distance/ speed]

Time taken by the man to travel (600 – 200) = 400km by car = 200/y hrs

For the part, the total time of the journey is given as 6hours 30 mins + 30 mins that is 7hrs,

$200/x + 400/y = 7$

$200\ (1/x + 2/y) = 7$

$1/x + 2/y = 7/200$ .....(ii)

Taking $1/x = u$, and $1/y = v$,

So, the equations (i) and (ii) becomes,

$2u + v = 13/400$ ...... (iii)

$u + 2v = 7/200$ .... (iv)

Solving (iii) and (iv), by

(iv) x2 – (iii) ⇒ $3v = 14/200 – 13/400$

$3v = 1/400 \times (28 – 13)$

$3v = 15/400$

$v = 1/80$

⇒ $y = 1/v = 80$

Now, using v in (iii) we find u,

$2u + (1/80) = 13/400$

$2u = 13/400 – 1/80$

$2u = 8/400$

$u = 1/100$

⇒ $x = 1/u = 100$

Hence, the speed of the train is 100 km/hr and the speed of the car is 80 km/hr.

Sol.4 Let's consider the car starting from point A as X and its speed as x km/hr.

And, the car starts from point B as Y and its speed as y km/hr.

It's seen that there are two cases in the question:

Case 1: Car X and Y are moving in the same direction

Case 2: Car X and Y are moving in the opposite direction

Let's assume that the meeting point in case 1 as P and in case 2 as Q.

Now, solving for case 1:

The distance travelled by car X = AP

And, the distance travelled by car Y = BP

As the time taken for both the cars to meet is 8 hours,

The distance travelled by car X in 7 hours = 8x km [∵ distance = speed × time]

⇒ AP = 8x

Similarly,

The distance travelled by car Y in 8 hours = 8y km

⇒ BP = 8Y

As the cars are moving in the same direction (i.e. away from each other), we can write

AP – BP = AB

So, 8x – 8y = 80

⇒ x – y = 10 ……………………. (i) [After taking 8 common out]

Now, solving for case 2:

In this case, as it's clearly seen,

The distance travelled by car X = AQ

And,

The distance travelled by car Y = BQ

As the time taken for both the cars to meet is 1 hour and 20 min, ⇒1 + (20/60) = 4/3 hr

The distance travelled by car x in 4/3 hour = 4x/3 km

⇒ AQ = 4x/3

Similarly,

The distance travelled by car y in 4/3 hour = 4y/3 km

⇒ BQ = 4y/3

Now, since the cars are moving in the opposite direction (i.e., towards each other), we can write

AQ + BQ = AB

⇒ 4x/3 + 4y/3 = 80

⇒ 4x + 4y = 240

⇒ x + y = 60 …………… (ii) [After taking LCM]

Hence, by solving (i) and (ii) we get the required solution

From (i), we have x = 10 + y……. (iii)

Substituting this value of x in (ii).

⇒ (10 + y) + y = 60

⇒ 2y = 50

⇒ y = 25

Now, using y = 30 in (iii), we get

⇒ x = 35

Therefore,

– Speed of car X = 35 km/hr.

– Speed of car Y = 25 km/hr.