Session 7

Differentiability in an Interval

(i) A function $f(x)$ defined in an open interval (a, b) is said to be differentiable or derivable in open interval (a, b), if it is differentiable at each point of (a, b).

(ii) A function $f(x)$ defined in a close interval $[a, b]$ is said to be differentiable or derivable at the end points a and b, if it is differentiable from the right at a and from the left at b. In other words, $\lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$ and $\lim_{x \to b^-} \frac{f(x) - f(b)}{x - b}$ both exist.

Example 44 Discuss the differentiability of $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Sol. We have, $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\Rightarrow f'(x) = \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \times \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$= \frac{(1+x^2)}{\sqrt{(1+x^2)^2 - 4x^2}} \times \left[\frac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2}\right]$$

$$= \frac{(1+x^2)}{\sqrt{1 + 2x^2 + x^4 - 4x^2}} \times \frac{(2 + 2x^2 - 4x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)}{\sqrt{1 - 2x^2 + x^4}} \times \frac{(2 - 2x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)}{\sqrt{(1-x^2)^2}} \times \frac{(2-2x^2)}{(1+x^2)^2} = \frac{(1+x^2)}{|1-x^2|} \times \frac{2(1-x^2)}{(1+x^2)^2}$$

[since $1 + x^2 \neq 0$]

$$\Rightarrow f'(x) = \frac{1}{|(1-x^2)|} \times \frac{2(1-x^2)}{(1+x^2)} \quad \text{...(i)}$$

Here, in Eq. (i), $f'(x)$ exists only if, $|1 - x^2| \neq 0$

$\Rightarrow \quad 1 - x^2 \neq 0$

$\Rightarrow \quad x^2 \neq 1 \Rightarrow x \neq \pm 1$

Thus, $f'(x)$ exists only, if $x \in R - \{-1, 1\}$.

$\therefore$ $f(x)$ is differentiable for all $x \in R - \{1, -1\}$.

Remark

The above example, can also be solved as follows

$y = f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, let $x = \tan\theta$

$\therefore \quad y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \Rightarrow y = \sin^{-1}(\sin 2\theta)$

$\therefore \quad y = 2\theta$ or $y = 2\tan^{-1} x$

$\frac{dy}{dx} = \frac{2}{1+x^2}$, which states $f'(x)$ exists for all $x \in R$. "Which is wrong as we have not checked the domain of $f(x)$." So, students are advised to solve these problems carefully, while applying this method.

Example 45 ***Let [ ] denotes the greatest integer function and*** $f(x) = [\tan^2 x]$***, then*** ***[IIT JEE 1993]***

(a) $\lim_{x \to 0} f(x)$ doesn't exist (b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) $f'(0) = 1$

Sol. Here, [ ] denotes the greatest integral function.

Thus, $\quad -45° < x < 45°$

$\Rightarrow \quad \tan(-45°) < \tan x < \tan(45°)$

$\Rightarrow \quad -1 < \tan x < 1 \Rightarrow 0 < \tan^2 x < 1$

Since, $\quad f(x) = [\tan^2 x] = 0$

Therefore, $f(x)$ is zero for all values of x from $(-45°)$ to $(45°)$. Thus, $f(x)$ exists when $x \to 0$ and also it is continuous at $x = 0$, $f(x)$ is differentiable at $x = 0$ and has a value 0. (i.e. $f(0) = 0$).

Hence, (b) is the correct answer.

Theorems of Differentiability

Theorem 1 If $f(x)$ and $g(x)$ are both derivable at $x = a$, $f(x) \pm g(x)$, $f(x) \cdot g(x)$ and $\frac{f(x)}{g(x)}$ will also be derivable at $x = a$ $\left\{\text{only if } g(a) \neq 0 \text{ for } \frac{f(x)}{g(x)}\right\}$.

Theorem 2 If $f(x)$ is derivable at $x = a$ and $g(x)$ is not differentiable at $x = a$, then $f(x) \pm g(x)$ will not be derivable at $x = a$.

e.g. $f(x) = \cos|x|$ is derivable at $x = 0$ and $g(x) = |x|$ is not derivable at $x = 0$.

Then, $\cos|x| + |x|$ is not derivable at $x = 0$.

However, nothing can be said about the product function, as in this case

$$f(x) = x \text{ is derivable at } x = 0$$
$$g(x) = |x| \text{ is not derivable at } x = 0$$

But, $f(x) \cdot g(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

which is derivable at $x = 0$.

Theorem 3 If both $f(x)$ and $g(x)$ are non-derivable, then nothing can be said about the sum/difference/product function.

e.g. $f(x) = \sin|x|$, not derivable at $x = 0$
$g(x) = |x|$, not derivable at $x = 0$

Then, the function

$F(x) = \sin|x| + |x|$, not derivable at $x = 0$
$G(x) = \sin|x| - |x|$, derivable at $x = 0$

Theorem 4 If $f(x)$ is derivable at $x = a$ and $f(a) = 0$ and $g(x)$ is continuous at $x = a$.

Then, the product function $F(x) = f(x) \cdot g(x)$ will be derivable at $x = a$.

Proof $F'(a^+) = \lim_{h \to 0} \dfrac{f(a+h) \cdot g(a+h) - 0}{h} = f'(a) \cdot g(a)$

$$F'(a^-) = \lim_{h \to 0} \frac{f(a-h) \cdot g(a-h) - 0}{-h} = f'(a) \cdot g(a)$$

$\therefore$ Derivable at $x = a$.

Theorem 5 Derivative of a continuous function need not be a continuous function.

e.g. $$f(x) = \begin{cases} x^2 \cdot \sin\left(\dfrac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Here, $f(0^+) = 0$ and $f(0^-) = 0$

$\therefore$ Continuous at $x = 0$.

and $$f'(x) = \begin{cases} 2x \cdot \sin\dfrac{1}{x} - x^2 \cdot \cos\left(\dfrac{1}{x}\right) \cdot \dfrac{1}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\Rightarrow$ $f'(x)$ is not continuous at $x = 0$.

$$\left[\text{as } \lim_{x \to 0} f'(x) \text{ doesn't exist}\right]$$

Remark

One must remember the formula which we can write as

$$\max\{f(x), g(x)\} = \frac{f(x) + g(x)}{2} + \left|\frac{f(x) - g(x)}{2}\right|$$

$$\min\{f(x), g(x)\} = \frac{f(x) + g(x)}{2} - \left|\frac{f(x) - g(x)}{2}\right|$$

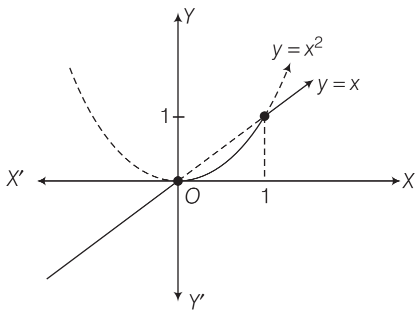
Example 46 ***Let $h(x) = \min\{x, x^2\}$ for every real number of x. Then,*** ***[IIT JEE 1998]***

(a) h is not continuous for all x
(b) h is differentiable for all x
(c) $h'(x) = 1$ for all x
(d) h is not differentiable at two values of x

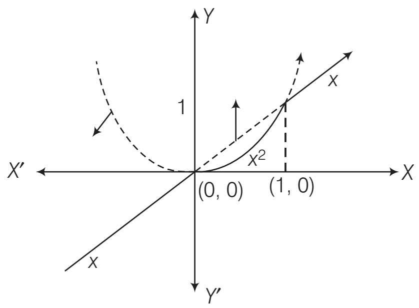
Sol. Here, $h(x) = \min\{x, x^2\}$ can be drawn on graph in two steps.

(a) Draw the graph of $y = x$ and $y = x^2$ also find their point of intersection.

Clearly, $x = x^2 \Rightarrow x = 0, 1$



(b) To find $h(x) = \min\{x, x^2\}$ neglecting the graph above the point of intersection, we get



Thus, from the above graph,

$$h(x) = \begin{cases} x, & x \leq 0 \text{ or } x \geq 1 \\ x^2, & 0 \leq x \leq 1 \end{cases}$$

which shows $h(x)$ is continuous for all x. But not differentiable at $x = \{0, 1\}$.

Thus, $h(x)$ is not differentiable at two values of x.

Hence, (d) is the correct answer.

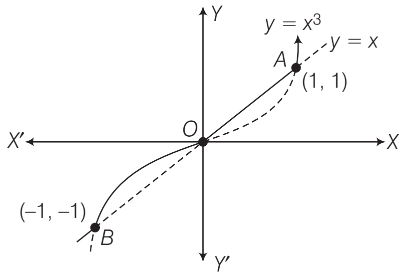
Example 47 Let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of all points where $f(x)$ is not differentiable, is ***[IIT JEE 2001]***

(a) $\{-1, 1\}$
(b) $\{-1, 0\}$
(c) $\{0, 1\}$
(d) $\{-1, 0, 1\}$

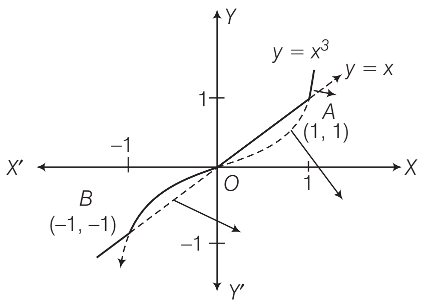
Sol. $f(x) = \max\{x, x^3\}$. Consider the graph separately of $y = x^3$ and $y = x$ and find their point of intersection;

Clearly, $x^3 = x$

$\Rightarrow$ $x = 0, 1, -1$



Now, to find $f(x) = \max\{x, x^3\}$ neglecting the graph below the point of intersection, we get the required graph of $f(x) = \max\{x, x^3\}$.



Thus, from above graph, $f(x) = \begin{cases} x, & \text{if } x \in (-\infty, -1] \cup [0,1] \\ x^3, & \text{if } x \in [-1, 0] \cup [1, \infty) \end{cases}$

which shows $f(x)$ is not differentiable at 3 points, i.e. $x = \{-1, 0, 1\}$. (Due to sharp edges)

Hence, (d) is the correct answer.

Example 48 *Let $f(x)$ be a continuous function, $\forall\ x \in R$, $f(0) = 1$ and $f(x) \neq x$ for any $x \in R$, then show $f(f(x)) > x, \forall\ x \in R^+$.*

Sol. Let $g(x) = f(x) - x$

So, $g(x)$ is continuous and $g(0) = f(0) - 0$.

$\Rightarrow$ $g(0) = 1$

Now, it is given that $g(x) \neq 0$ for any $x \in R$

[as $f(x) \neq x$ for any $x \in R$]

So, $g(x) > 0, \forall\ x \in R^+$

i.e. $f(x) > x, \forall\ x \in R^+$

$\Rightarrow$ $f(f(x)) > f(x) > x, \ \forall\ x \in R^+$

or $f(f(x)) > x, \forall\ x \in R^+$

Example 49 *The total number of points of non-differentiability of $f(x) = \max\left\{\sin^2 x, \cos^2 x, \frac{3}{4}\right\}$ in $[0, 10\pi]$, is*

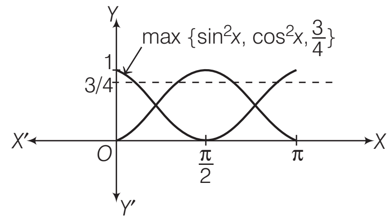
(a) 40

(b) 30

(c) 20

(d) 10

Sol. Here, $f(x) = \max\left\{\sin^2 x, \cos^2 x, \frac{3}{4}\right\}$



Since, $\sin^2 x$ and $\cos^2 x$ are periodic with period π

and in $[0, \pi]$, there are four points of non-differentiability of $f(x)$.

$\therefore$ In $[0, 10\pi]$, there are 40 points of non-differentiability.

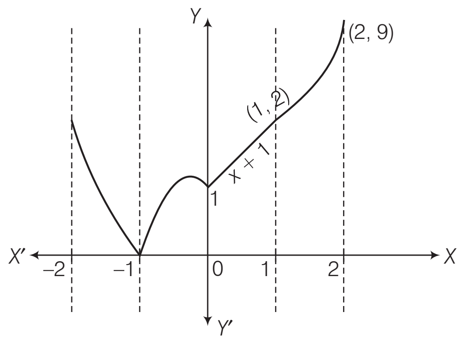
Hence, (a) is the correct answer.

Example 50 If $f(x) = |x+1|\{|x| + |x-1|\}$, then draw the graph of $f(x)$ in the interval $[-2, 2]$ and discuss the continuity and differentiability in $[-2, 2]$.

Sol. Here, $f(x) = |x+1|\{|x| + |x-1|\}$

$$f(x) = \begin{cases} (x+1)(2x-1), & -2 \le x < -1 \\ -(x+1)(2x-1), & -1 \le x < 0 \\ (x+1), & 0 \le x < 1 \\ (x+1)(2x-1), & 1 \le x \le 2 \end{cases}$$

Thus, the graph of $f(x)$ is



Clearly, continuous for $x \in R$ and has differentiability for $x \in R - \{-1, 0, 1\}$

Example 51 If the function

$$f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2),$$

(where [ ] denotes the greatest integer function) is continuous and differentiable in $(4, 6)$*, then*

(a) $a \in [8, 64]$ (b) $a \in (0, 8]$
(c) $a \in [64, \infty)$ (d) None of these

Sol. We have, $x \in (4, 6) \Rightarrow 2 < x - 2 < 4$

$$\Rightarrow \quad \frac{8}{a} < \frac{(x-2)^3}{a} < \frac{64}{a} \qquad [\because a > 0]$$

For $f(x)$ to be continuous and differentiable in $(4, 6)$, $\left[\frac{(x-2)^3}{a}\right]$ must attain a constant value for $x \in (4, 6)$.

Clearly, this is possible only when $a \geq 64$.

In that case, we have

$f(x) = a\cos(x-2)$, which is continuous and differentiable

$\therefore \quad a \in [64, \infty)$

Hence, (c) is the correct answer.

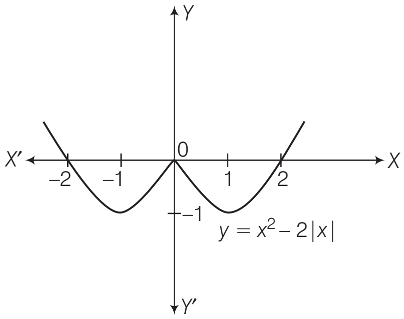
Example 52 If $f(x) = x^2 - 2|x|$ and

$$g(x) = \begin{cases} \min\{f(t) : -2 \leq t \leq x,\ -2 \leq x \leq 0\} \\ \max\{f(t) : 0 \leq t \leq x, \quad 0 \leq x \leq 3\} \end{cases}$$

(i) Draw the graph of $f(x)$ and discuss its continuity and differentiability.

(ii) Find and draw the graph of $g(x)$. Also, discuss the continuity.

Sol. (i) Graph of $f(x) = \begin{cases} x^2 - 2x, & x \geq 0 \\ x^2 + 2x, & x < 0 \end{cases}$ is shown as



which shows $f(x)$ is continuous for all $x \in R$ and differentiable for all $x \in R - \{0\}$.

(ii) We know that,

If $f(x)$ is an increasing function on $[a, b]$, then

$\max\{f(t); a \leq t \leq x,\ a \leq x \leq b\} = f(x)$

$\min\{f(t); a \leq t \leq x,\ a \leq x \leq b\} = f(a)$

If $f(x)$ is decreasing function on $[a, b]$, then

$\max\{f(t); a \leq t \leq x,\ a \leq x \leq b\} = f(a)$

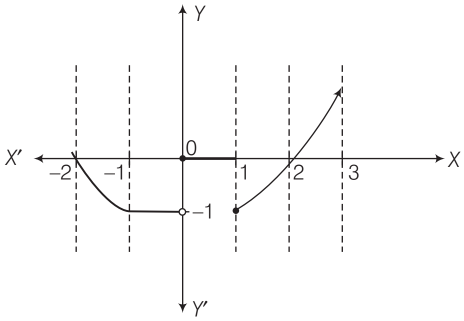
$\min\{f(t); a \leq t \leq x,\ a \leq x \leq b\} = f(x)$

From graph of $f(x)$,

$$g(x) = \begin{cases} f(x), & \text{for } -2 \leq x < -1 \\ -1, & \text{for } -1 \leq x < 0 \\ 0, & \text{for } 0 \leq x < 1 \\ f(x), & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow \quad g(x) = \begin{cases} x^2 + 2x, & \text{for } -2 \leq x < -1 \\ -1, & \text{for } -1 \leq x < 0 \\ 0, & \text{for } 0 \leq x < 1 \\ x^2 - 2x, & \text{for } x \geq 1 \end{cases}$$

Thus, graph of $g(x)$ is



From above figure, it is clear that $g(x)$ is not continuous at $x = 0, 1$.

Example 53 *Let* $f(x) = \phi(x) + \psi(x)$ *and* $\phi'(a), \psi'(a)$ *are finite and definite. Then,*

(a) $f(x)$ is continuous at $x = a$
(b) $f(x)$ is differentiable at $x = a$
(c) $f'(x)$ is continuous at $x = a$
(d) $f'(x)$ is differentiable at $x = a$

Sol. We know that the sum of two continuous (differentiable) functions is continuous (differentiable).

$\therefore$ $f(x)$ is continuous and differentiable at $x = a$.

Hence, (a) and (b) are the correct answers.

Example 54 If $f(x) = x + \tan x$ and $g(x)$ is the inverse of $f(x)$, then $g'(x)$ is equal to

(a) $\dfrac{1}{1 + (g(x) - x)^2}$ (b) $\dfrac{1}{2 + (g(x) + x)^2}$
(c) $\dfrac{1}{2 + (g(x) - x)^2}$ (d) None of these

Sol. We have, $f(x) = x + \tan x$

$$\Rightarrow \quad f(f^{-1}(x)) = f^{-1}(x) + \tan(f^{-1}(x))$$

$$\Rightarrow \quad x = g(x) + \tan(g(x)) \qquad \text{...(i)}$$

$$[\because g(x) = f^{-1}(x)]$$

$$1 = g'(x) + \sec^2(g(x)) \cdot g'(x)$$

$$\Rightarrow \quad g'(x) = \frac{1}{1 + \sec^2(g(x))}$$

$$\Rightarrow \quad g'(x) = \frac{1}{2 + \tan^2(g(x))}$$

$$\Rightarrow \quad g'(x) = \frac{1}{2 + (x - g(x))^2} \qquad \text{[from Eq. (i)]}$$

Hence, (c) is the correct answer.

Example 55 If $f(x)$ is differentiable function and $(f(x) \cdot g(x))$ is differentiable at $x = a$, then

(a) $g(x)$ must be differentiable at $x = a$
(b) $g(x)$ is discontinuous, then $f(a) = 0$
(c) $f(a) \neq 0$, then $g(x)$ must be differentiable
(d) None of the above

Sol. $$\left[\frac{d}{dx}(f(x) \cdot g(x))\right]_{x=a} = f'(a)g(a) + \lim_{h \to 0} \frac{g(a+h) - g(a)}{h} \cdot f(a)$$

If $f(a) \neq 0 \Rightarrow g'(a)$ must exist.

Also, if $g(x)$ is discontinuous, $f(a)$ must be 0 for $f(x) \cdot g(x)$ to be differentiable.

Hence, (b) and (c) are the correct answers.

Example 56 If $f(x) = [x^{-2}[x^2]]$, (where $[\cdot]$ denotes the greatest integer function) $x \neq 0$, then incorrect statement

(a) $f(x)$ is continuous everywhere
(b) $f(x)$ is discontinuous at $x = \sqrt{2}$
(c) $f(x)$ is non-differentiable at $x = 1$
(d) $f(x)$ is discontinuous at infinitely many points

Sol. Here, $$0 \leq [x^2] \leq x^2$$

$$\Rightarrow \quad 0 \leq x^{-2}[x^2] \leq 1 \Rightarrow [x^{-2}[x^2]] = 0 \text{ or } 1$$

$f(x)$ is discontinuous at $x^2 = n, n \in N \Rightarrow x = \sqrt{n}$

$\therefore f(x)$ is neither continuous nor differentiable at $x = \sqrt{n}, n \in N$.

Hence, (b), (c) and (d) are the correct answers.

Example 57 *If* $f(x) = \begin{cases} x^2(\operatorname{sgn}[x]) + \{x\}, & 0 \leq x \leq 2 \\ \sin x + |x - 3|, & 2 \leq x \leq 4 \end{cases}$, ***where*** $[\ ]$ ***and*** $\{\ \}$ ***represents greatest integer and fractional part function respectively, then***

(a) $f(x)$ is differentiable at $x = 1$
(b) $f(x)$ is continuous but non-differentiable at $x = 2$
(c) $f(x)$ is non-differentiable at $x = 2$
(d) $f(x)$ is discontinuous at $x = 2$

Sol. For continuity at $x = 1$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 \operatorname{sgn}[x] + \{x\} = 1 + 0 = 1$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 \operatorname{sgn}[x] + \{x\} = 0 + 1 = 1$$

Also, $f(1) = 1$

$\therefore f(x)$ is Continuous at $x = 1$

$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ x^2 + x - 1, & 1 \leq x < 2 \end{cases}, \text{ non-differentiable at } x = 1$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 \operatorname{sgn}[x] + \{x\} = 4 \times 1 + 1 = 5$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \sin x + |x - 3| = 1 + \sin 2$$

Thus, $f(x)$ is neither continuous nor differentiable at $x = 2$.

Hence, (c) and (d) are the correct answers.

Example 58 ***A real valued function*** $f(x)$ ***is given as***

$$f(x) = \begin{cases} \int_0^x 2\{x\}dx, & x + \{x\} \in I \\ x^2 - x + \frac{1}{2}, & \frac{1}{2} < x < \frac{3}{2} \text{ and } x \neq 1, \\ x^2 - x + \frac{1}{6}, & \text{otherwise} \end{cases}$$

where $[\ ]$ ***denotes greatest integer less than or equals to*** x ***and*** $\{\ \}$ ***denotes fractional part function of*** x***. Then,***

(a) $f(x)$ is continuous and differentiable in $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$
(b) $f(x)$ is continuous and differentiable in $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$
(c) $f(x)$ is continuous and differentiable in $x \in \left[\frac{1}{2}, \frac{3}{2}\right]$
(d) $f(x)$ is continuous but not differentiable in $x \in (0, 1)$

Sol. Here, $$x + \{x\} \in I \Rightarrow x + x - [x] \in I$$

$$\Rightarrow \quad 2x - [x] \in I, \text{ possible for } x = \frac{n}{2}, n \in I$$

$$\therefore \quad f\left(\frac{1}{2}\right) = \int_0^{1/2} 2\{x\}dx = \frac{1}{4}, \ f\left(\frac{3}{2}\right) = \int_0^{3/2} 2\{x\}dx = \frac{5}{4}$$

and $$f\left(\frac{-1}{2}\right) = \int_0^{-1/2} 2\{x\}dx = \frac{-3}{4}, \ f(1) = 1$$

$$\text{Then, } f(x) = \begin{cases} \frac{1}{4}, & x = \frac{1}{2} \\ \frac{5}{4}, & x = \frac{3}{2} \\ \frac{-3}{4}, & x = \frac{-1}{2} \\ 1, & x = 1 \\ x^2 - x + \frac{1}{2}, & \frac{1}{2} < x < \frac{3}{2} \text{ and } x \neq 1 \\ x^2 - x + \frac{1}{6}, & \text{otherwise} \end{cases}$$

Clearly, continuous for $x \in (0, 1)$ but not differentiable.

Hence, (d) is the correct answer.

Exercise for Session 7

1. If $f(x)=\sin(\pi(x-[x])), \forall\, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, where $[\cdot]$ denotes the greatest integer function, then

(a) $f(x)$ is discontinuous at $x=\{-1, 0, 1\}$
(b) $f(x)$ is differentiable for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)-\{0\}$
(c) $f(x)$ is differentiable for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)-\{-1, 0, 1\}$
(d) None of these

2. Let $f(x)=\begin{cases} x-1, & -1 \le x < 0 \\ x^2, & 0 \le x \le 1 \end{cases}$, $g(x)=\sin x$ and $h(x)=f(|g(x)|)+|f(g(x))|$. Then,

(a) $h(x)$ is continuous for $x \in [-1, 1]$
(b) $h(x)$ is differentiable for $x \in [-1, 1]$
(c) $h(x)$ is differentiable for $x \in [-1, 1]-\{0\}$
(d) $h(x)$ is differentiable for $x \in (-1, 1)-\{0\}$

3. If $f(x)=\begin{cases} |1-4x^2|, & 0 \le x < 1 \\ [x^2-2x], & 1 \le x < 2 \end{cases}$, where [ ] denotes the greatest integer function, then

(a) $f(x)$ is continuous for all $x \in [0, 2)$
(b) $f(x)$ is differentiable for all $x \in [0, 2)-\{1\}$
(c) $f(x)$ is differentiable for all $x \in [0, 2)-\left\{\frac{1}{2}, 1\right\}$
(d) None of these

4. Let $f(x)=\int_0^1 |x-t|\, t\, dt$, then

(a) $f(x)$ is continuous but not differentiable for all $x \in R$
(b) $f(x)$ is continuous and differentiable for all $x \in R$
(c) $f(x)$ is continuous for $x \in R-\left\{\frac{1}{2}\right\}$ and $f(x)$ is differentiable for $x \in R-\left\{\frac{1}{4}, \frac{1}{2}\right\}$
(d) None of these

5. Let $f(x)$ be a function such that $f(x+y)=f(x)+f(y)$ for all x and y and $f(x)=(2x^2+3x)\cdot g(x)$ for all x, where $g(x)$ is continuous and $g(0)=3$. Then, $f'(x)$ is equal to

(a) 6
(b) 9
(c) 8
(d) None of these

6. If a function $g(x)$ which has derivatives $g'(x)$ for every real x and which satisfies the following equation $g(x+y)=e^y g(x)+e^x g(y)$ for all x and y and $g'(0)=2$, then the value of $\{g'(x)-g(x)\}$ is equal to

(a) e^x
(b) $\frac{2}{3}e^x$
(c) $\frac{1}{2}e^x$
(d) $2e^x$

7. Let $f: R \to R$ be a function satisfying $f\left(\frac{xy}{2}\right)=\frac{f(x)\cdot f(y)}{2}, \forall\, x, y \in R$ and $f(1)=f'(1) \ne 0$. Then, $f(x)+f(1-x)$ is (for all non-zero real values of x)

(a) constant
(b) can't be discussed
(c) x
(d) $\frac{1}{x}$

8. Let $f(x)$ be a derivable function at $x=0$ and $f\left(\frac{x+y}{K}\right)=\frac{f(x)+f(y)}{K}$ $(K \in R, K \ne 0, 2)$. Then, $f(x)$ is

(a) even function
(b) neither even nor odd function
(c) either zero or odd function
(d) either zero or even function

9. Let $f: R-(-\pi, \pi)$ be a differentiable function such that $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$.
If $f(1)=\frac{\pi}{2}$ and $\lim_{x \to 0} \frac{f(x)}{x}=2$. Then, $f(x)$ is equal to

(a) $2\tan^{-1} x$
(b) $\frac{1}{2}\tan^{-1} x$
(c) $\frac{\pi}{2}\tan^{-1} x$
(d) $2\pi \tan^{-1} x$

10. Let $f(x)=\sin x$ and $g(x)=\begin{cases} \max\{f(t), 0 \le t \le x\}, & \text{for } 0 \le x \le \pi \\ \dfrac{1-\cos x}{2}, & \text{for } x > \pi \end{cases}$. Then, $g(x)$ is

(a) differentiable for all $x \in R$
(b) differentiable for all $x \in R-\{\pi\}$
(c) differentiable for all $x \in (0, \infty)$
(d) differentiable for all $x \in (0, \infty)-\{\pi\}$

Answers

Exercise for Session 7

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (d)
7. (a)	**8.** (c)	**9.** (a)	**10.** (c)		