4

Linear Equations in Two Variables

4.01. Introduction

In previous classes you have studied about linear equations in one variable. The equations in which degree of variables is one are called linear equation. Some of the examples of the linear equations are

(i) $x+3=8$ (ii) $2y+10=28$

(iii) $4x-7=2x+3$ (iv) $5m=40$

The equations satified by a value of the variable, which when substuituted in place of variable in the equation i.e. the left hand side and right hand side of the equation is equal is called the solution of the equation.

We know the fact about the equations that the solution of a linear equation is not affected when :

(i) the same number is added or subtracted from both sides of an equation.

(ii) the same number is multiplied or same non zero number divide both the sides of the equation.

In general, a linear equation in one variable can be expressed in the form of $ax+b=0$, where a and b are real number. Here $a \neq 0$ and x is the variable. The solution of equation $ax+b=0$ is $x=\frac{-b}{a}$. The equaltion in one variable has a unique (one and only one) solution, that is called the root of equation .

Linear Equations in Two Variables

The equation in which there are two variables of degree one is called linear equation in two variables.

Let us understand the co-ordinate system before study of the equations in two variables.

4.02. Rectangular Co-ordinate System

We shall clear the concept of rectangular co-ordinate system before solving a linear equation graphically.

(a) Rectangular Co-ordinate System :

We have already learnt that how to represent a real number on the number line. There are many situations, in which to find a point we are need to describe its position with reference to more than one line. Sometimes a point does not fall on the number line instead it is located somewhere in the plane. So we expand the principle related to number line.

So a dot in the plane can be represented by two perpendicular lines one of them is horizontal and other one is vertical. The horizontal line is called x-axis as well as vertical line is called y-axis and they are represented as XOX' and YOY' respectively.

The intersecting point of both lines known as origin and is denoted by symbol O. Positive integers lie to the right side of origin on x-axis (towards OX) and negative integers lie to the left side (towards OX'). Similarly positive integers lie above the origin of y-axis (towards OY) and negative integers lie below the origin (towards OY')

(b) Quadrants

Two axes XX' and YY' divide the plane into four parts which are called quadrants (means one fourth part). The expansion of these quadrants is infinite. XOY, YOX', $X'OY'$ and $Y'OX$ are the first, second, third and fourth quadrants respectively.

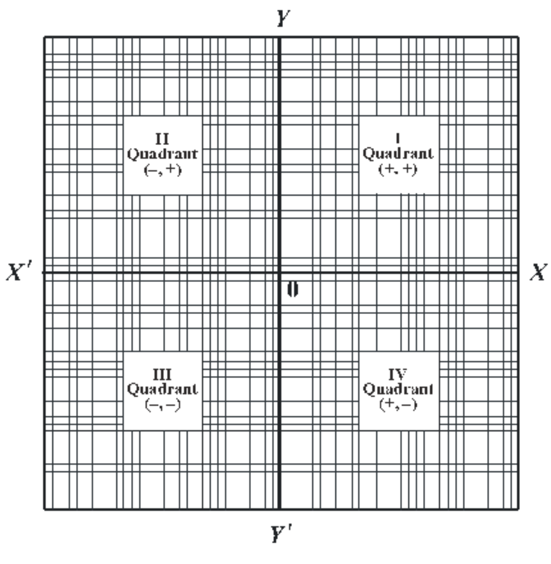

Fig. 4.01

(c) Plotting of Points

Let P be a point in the first quadrant. To reach this point we have to move 3 units towards OX and then 4 units towards OY, then this point can be expressed as $P(3,4)$. 3 is the x-coordinate of P on x-axis and 4 is the y-coordinate of P on y-axis. x-coordinate is called **abscissa** and y-coordinate is called **ordinate.** In this way, there is an abscissa x and an ordinate y for every point in the plane. These are represented by a ordered pair (x,y). Ordered pair (x,y) is called the coordinate of that point.

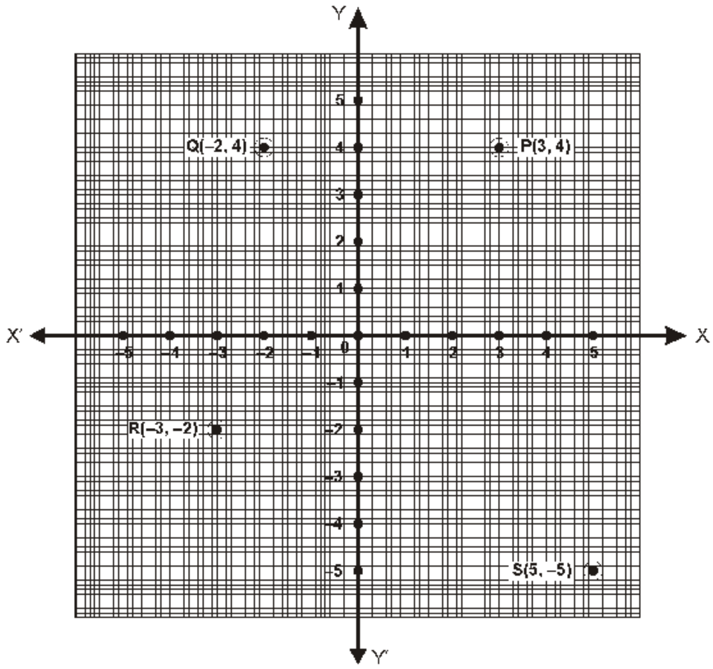

Fig. 4.02

Now consider the point $P(3,4)$. We observe that this point is above the x-axis and right to the y-axis, so its both abscissa and ordinate are positive. Therefore, the point P (3,4) lie in the first quadrant.

Similary we see that the coordinate in ordered pair form of points Q, R and S are resectively $(-2,4)$, $(-3,-2)$ and $(5,-5)$ as shown in the figure 4.02.

Again if we plote the point P (3, 4) then we move 3 unit in the direction towards OX right to O and then we move upward 4 unit parallel to OY from that point. This is the actual position of point $P(3, 4)$ in the plane.

So, to locate the point $Q(-2,4)$ we move 2 units in the direction of OX' form O and then we move upward 4 units parallel to OY. The point Q lie in the IInd quadrant.

In the same way we can plot the points $\text{R}(-3,-2)$ and $\text{S}(5,-5)$ in the plane.

Note :

1. The ordinate of each point on x-axis is zero.
2. The abscissa of each point on y-axis is zero.
3. Coordinates of origin are (0,0).

4.03. Graph of Linear equation in two variables

Take an example $x + y = 9$

The solutions of the equation are the values of variables x and y which satisfy the given equation. Let us see what values of x and y satisfy the above equation. See the following table.

x	0	1	2	3	4	5	6	7
y	9	8	7	6	5	4	3	2

These are some solutions that satisfy the equation but we can say that infinitely many values of variables x and y satisfy the solution.

We should plot the value of x variable on the X-axis and the value of variable y on Y-axis, and cordinates of x and y are written as (x,y). When all these points are joined, we get a straight line that is called the graph of the equation. Solutions of the equation are pointed on the obtained line and according to the graph every point on the line is the solution of the equation.

Construction of a line is a series of infinitely many points. So we can say that an equation in two variables has infinitely many solutions.

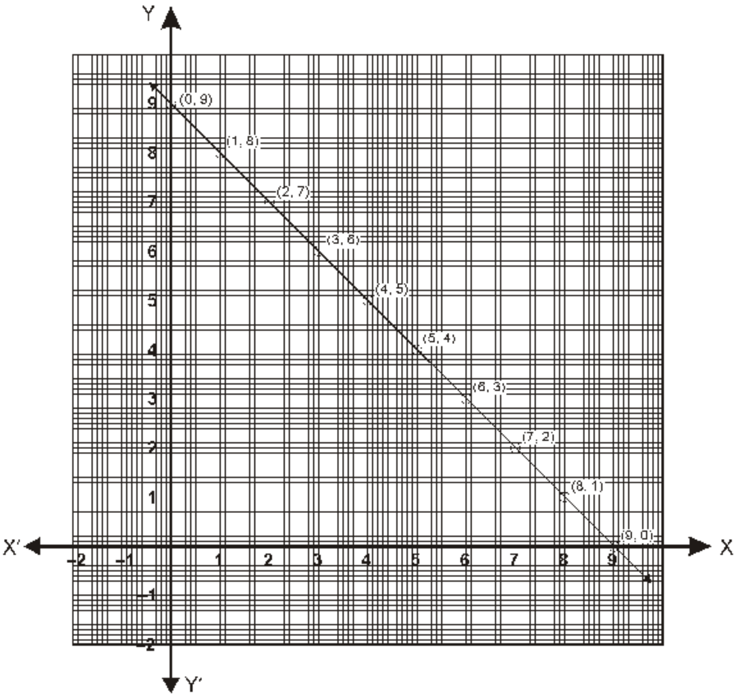

Fig. 4.03

Note :

1. The graphical representation of an equation in two variables is always a straight line.

2. Every point on the graph line gives the solution of the equation.

3. Any point, which does not lie on graph line, is not a solution of the equation. To obtain the graph of a linear equation in two variables, it is enough to plot two points corresponsing to two solutions and join them by a line. However, it is advisable to plot more than two such points so that we can immediately check the corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that we can immediately check the correction of the graph.

Example 1. Draw the graph of equation $3x + y = 2$

Solution : Given equation is

$$3x + y = 2 \quad \Rightarrow \quad y = 2 - 3x$$

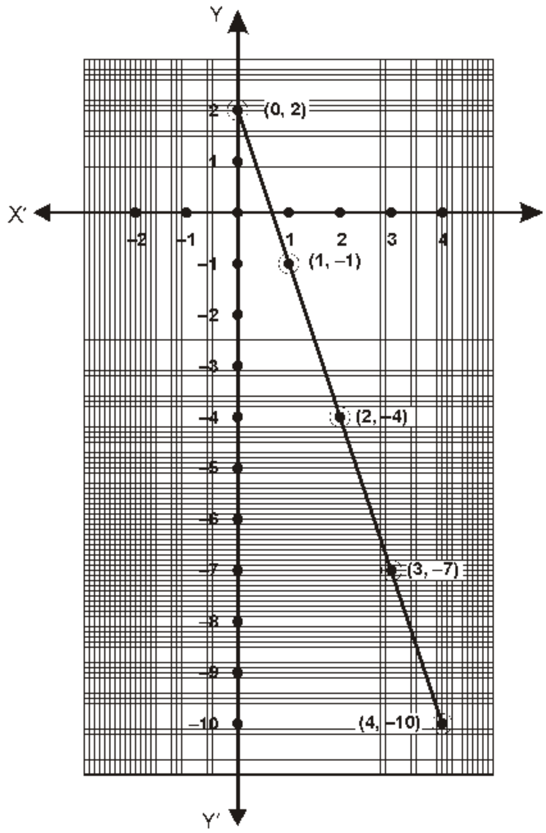

Fig. 4.04

We prepare a table as follows by writing the values of y below the corresponding values of x

x	0	1	2	3	4
y	2	-1	-4	-7	-10

From the graph of the equation, it is clear that every point on the graph line is a solution of the equation. If more than one equations in two variables are to be graphed on the same graph paper, then following conditions may be obtained :

(i) The graph line of two equations may intersect each other on a point.

(ii) Graph lines of two equations may be parallel and they never intersects each other.

(iii) Both lines may be coincident. In the first condition, the intersecting point of two lines shows the solution of both the lines. So,the coordinates of that point satisfy the two equations.

To obtain the unique solution of a linear equation in two variables, two linear equations will be required. Such linear equations in two variables are called the simultaneous equations.

The solution of the pair of linear equation by graph.

Illustrative Examples

Example 2. Solve the following equations by graphical method.

$$x + y = 3 \ ; \ 3x - 2y = 4$$

Solution : By the given equations we make the separate tables from their possible solutions.

Given, $x + y = 3$ . . . (i)

$\Rightarrow$ $x = 3 - y$

x	1	2	3
y	2	1	0

Similarly, $3x - 2y = 4$ or $x = \dfrac{4 + 2y}{3}$ . . . (ii)

x	2	4	6
y	1	4	7

Draw the graph by plotting the above points given in the tables and then by joining the line.

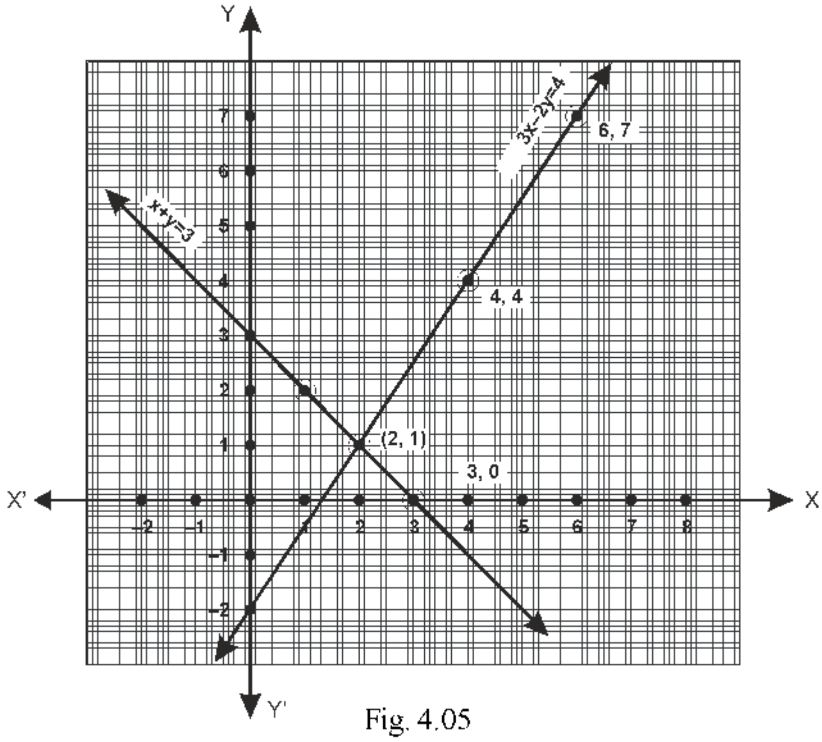

Fig. 4.05

Both the lines intersect at a point. The coordinates of point are (2, 1). Thus, the solution of the given equations is $x = 2$ and $y = 1$.

Example 3. **Solve the following pair of equations for x and y by graphical Method : $2x + 3y = 13$; $5x - 2y = 4$**

Solution : Given system of equations is :

$$2x + 3y = 13 \quad \text{...(1)}$$

$$5x - 2y = 4 \quad \text{...(2)}$$

Graph of the equation $2x + 3y = 13$ or $y = \dfrac{13 - 2x}{3}$

We have the following table for some possible solutions of the equation :

x	–1	2	5
y	5	3	1

Similarly euation $5x - 2y = 4$ or $y = \dfrac{5x - 4}{2}$.

x	–2	4	2
y	–7	8	3

Plotting the points $(-1, 5)$, $(2, 3)$ and $(5, 1)$ and drawing a line joining them, we get the graph of the equation $2x+3y=13$ as shown in Fig. 4.06. We have the following table for values of (x, y).

Plotting the points $(-2,-7),(4,8)$ on the same graph paper and drawing a line joining them, we obtain the graph of the equation $5x-2y=4$.

Clearly, the two lines intersect at point P(2, 3). Thus, $x = 2,\ y = 3$ is the solution of the given system.

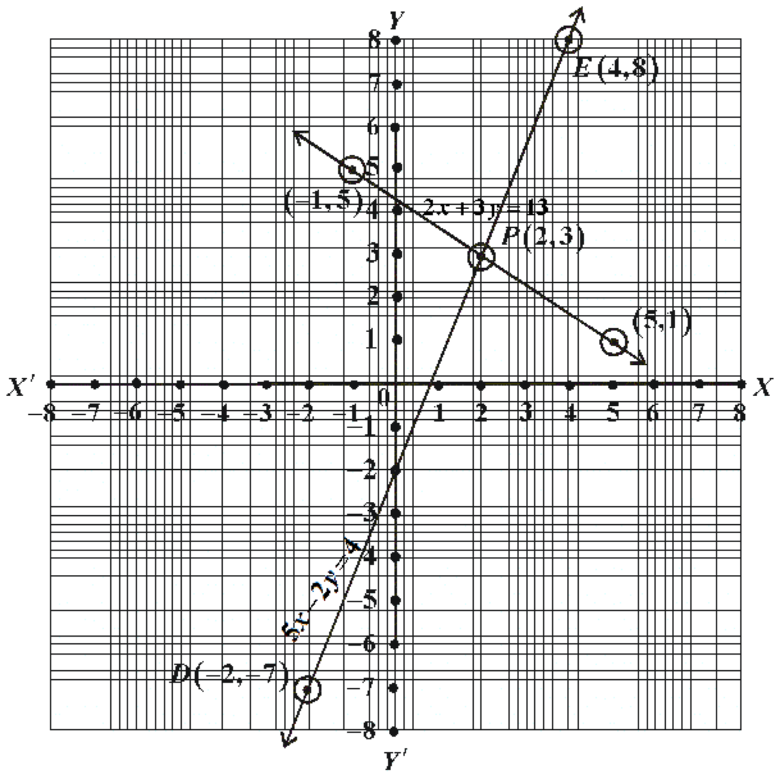

Fig. 4.06

Example 4. Solve the following equations by graphical method.

$$\mathbf{2x-6y+10=0\ ;\ 3x-9y+15=0}$$

Solution : Given system of equation is :

$2x-6y+10=0 \qquad \ldots(1)$

$3x-9y+15=0 \qquad \ldots(2)$

Table for equation $2x-6y+10=0$			
x	1	–5	7
y	2	0	4

Table for equation $3x-9y+15=0$			
x	4	–2	–8
y	3	1	–1

Now, we plot the points $(1,2),(-5,0)$ and $(7,4)$ on a graph and join them. We get a straight line AB in the form of the graph of equation $2x-6y+10=0$. Again, we plot the points $(4,3),(-2,-1)$ and $(-8,-1)$. We see that all three points lie on the line AB. So, both the lines are coincide. So, the equations have infinitely many solutions. And the solution of the equation $2x-6y+10=0$, will be the solution of the system.

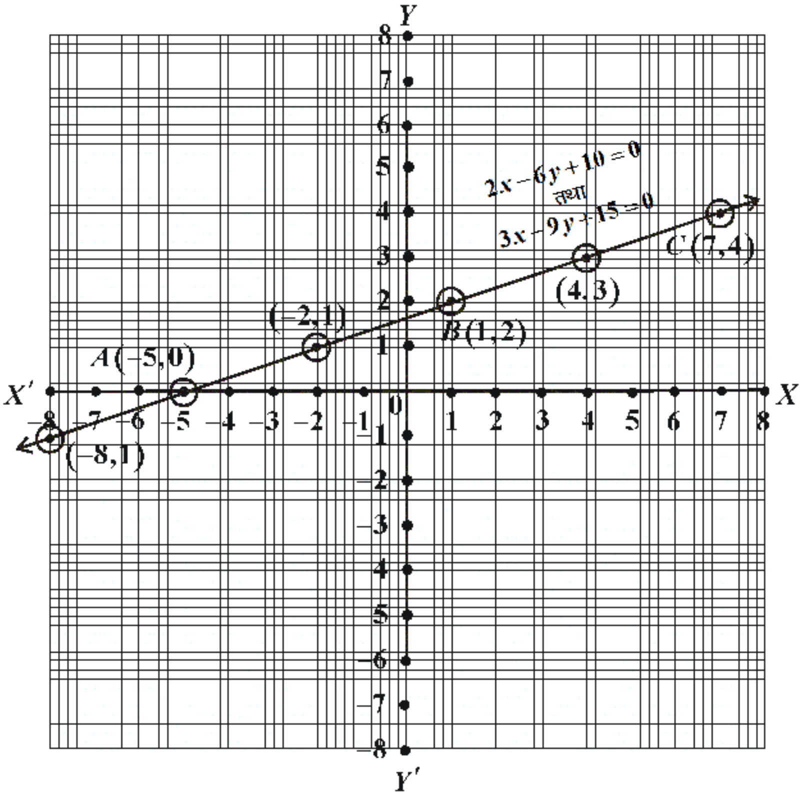

Fig. 4.07

Example 5. Solve the system of equations given below graphically. Also find the nature of system $2x+3y=12$; $2x+3y=6$.

Solution : We have two linear equations.

$$2x+3y=12 \quad \dots (1)$$

$$2x+3y=6 \quad \dots (2)$$

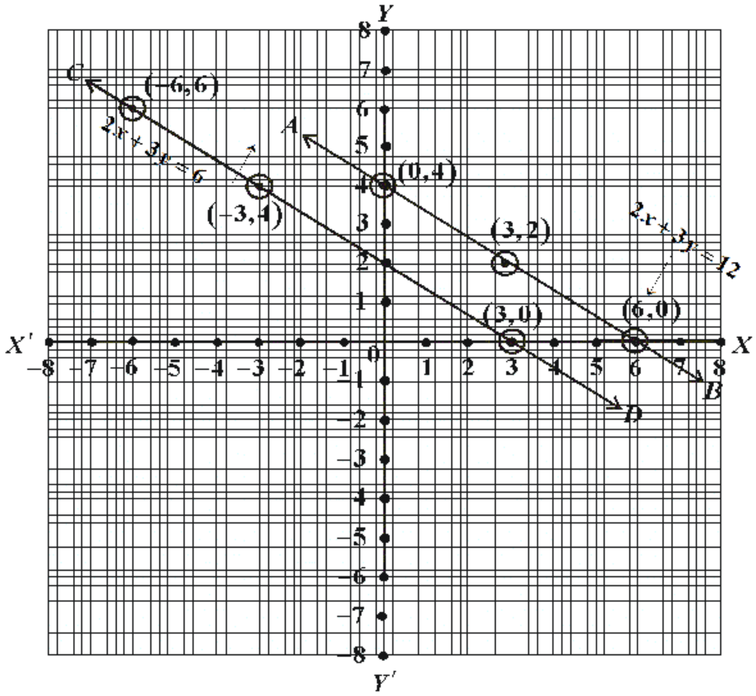

Fig.4.08

Now on plotting the points $(6,0),(3,2),(0,4)$. When we join these three points, we obtain a straight line AB. Which is a graph of equation $2x+3y-12=0$.

Again on plotting the points $(3,0),(-3,4)$ and $(-6,6)$. We get the graph of equation $2x+3y=6$ is a straight line CD by joining these points.

Now we see that the lines AB and CD which we obtained graphically are parallel it means the given system of linear equations is inconsistent. Hence, there is no solution of the given equations.

Exercise 4.1

Solve the following equations graphically.

1. $x+3y=6$
   $2x-3y=12$

2. $2x+y=6$
   $2x-y+2=0$

3. $x-2y=6$
   $3x-6y=0$

4. $x+y=4$
   $2x-3y=3$

5. $2x-3y+13=0$
   $3x-2y+12=0$

6. $3x-4y=1\,;-2x+\frac{8}{3}y=5$

7. $2x+\frac{y}{2}-5=0\,;\frac{x}{2}+y=-4$

8. $0\cdot 3x+0\cdot 4y=3\cdot 2\,;0\cdot 6x+0\cdot 8y=2\cdot 4$

9. $2x+3y=8\,;4x-\frac{3}{2}y=1$

10. $3x-y=2\,;6x-2y=4$

11. $3x+2y=0\,;2x+y=-1$

4.04. Algebraic Methods of Solving Simultaneous Linear Equations

Simultaneous equations is the pair of two linear equations in two variables. The values of both variables that satisfy both the equations are solutions of the simultaneous equations.

The solution of the given system of linear equations can be obtained by using following algebraic method.

(i) Method of elimination (by substitution)

(ii) Method of elimination (by equating the co-efficient)

(iii) Method of cross multiplication (General method)

(i) Method of Elimination (by Substitution) :

In this method, the value of a variable in an equation of simultaneous linear equation is expressed in the form of other variable of the equation. Now the value of the variable so obtained substituted in another equation of simultaneous linear equation. Consequently the second equation is equation in one variable. Solving this equation in one variable, we can easily find the value of the variable used in the equation. Then substituting this value in any of equation we obtain the value of other variable. The example given below will be helpful to understand this method.

Example 6. Solve the following system of equations by substuting method.

$$x+3y=11$$

$$4x-y=5$$

Solution : The given system of equations is :

$$x+3y=11 \quad \ldots(1)$$

$$4x-y=5 \quad \ldots(2)$$

From equation (1), we get

$$x=11-3y \quad \ldots(3)$$

Substuting this value of x in (2), we get.

$$4(11-3y)-y=5$$

or $\quad 44-12y-y=5$

or $\quad 44-13y=5$

or $\quad 13y=39$

or $\quad y=\frac{39}{13}$

$\therefore \quad y=3$

Putting $y=3$ in equation (1), we get :

$$x=11-3(3)$$

or $\quad x=11-9$

or $\quad x=2$

Hence, the solution of the given system of equations is $x=2,\ y=3$

(ii) Method of Elimilation (by Equating the Coefficient) :

In this method, one or both equations of the simultaneous equations are multiplied by such number so that the co-eficients of one variable in two equations may be equal. Now, according to the situation both the equations are added or subtracted, so that we can get an equation in one variable easily. Now this value is substituted in any of the two equations. In this way, the value of other variable is found.

Example 7. Solve the pair of equations using the method of elemination by equating the co-efficient.

$$4x+5y=31$$

$$7x-2y=22$$

Solution : The given system of equations is :

$$4x+5y=31 \quad \ldots(1)$$

$$7x-2y=22 \quad \ldots(2)$$

Let us eliminate y from the given equations. The coeficients of y in the given equations are 5 and 2 respectively. The LCM of 5 and 2 is 10. So, to make the coefficients of y equal, we multiply equation (1) by 2 and equation (2) by 5, we get

$$8x+10y=62 \quad \ldots(3)$$

$$35x-10y=110 \quad \ldots(4)$$

Adding the equations (3) and (4) we get :

$$43x=172 \quad \text{or} \quad x=\frac{172}{43}$$

$\therefore \qquad x = 4$

Substituting $x = 4$ in equation (1), we get

$$4(4) + 5y = 31 \quad \text{or} \quad 16 + 5y = 31$$

or $\quad 5y = 31 - 16 \quad \Rightarrow \quad y = \frac{15}{5}$

$\therefore \quad y = 3$

Hence, the solution of the given system of equations is $x = 4,\ y = 3$.

Using this method, we can solve such equations which are made of reciprocals of variables.

The method is so clear by the example given below.

Example 8. Solve the following equations :

$$\frac{20}{x} + \frac{2}{y} = 6, \ \frac{10}{x} - \frac{1}{y} = 2$$

Solution : Given , system of equations is :

$$\frac{20}{x} + \frac{2}{y} = 6 \qquad \ldots (1)$$

$$\frac{10}{x} - \frac{1}{y} = 2 \qquad \ldots (2)$$

Multiply equation (2) by 2, we get

$$\frac{20}{x} - \frac{2}{y} = 4 \qquad \ldots (3)$$

Adding equation (1) and (3), we get

$$\frac{40}{x} = 10 \quad \Rightarrow \quad x = \frac{40}{10}$$

or $\qquad x = 4$

Substituting $x = 4$ in equation (1), we get

$$\frac{20}{4} + \frac{2}{y} = 6 \quad \text{or} \quad 5 + \frac{2}{y} = 6$$

or $\quad \frac{2}{y} = 6 - 5 \quad$ or $\quad \frac{2}{y} = 1$

or $\quad y = 2$

Thus, the solution of the given system of equations is , $x = 4,\ y = 2$

Example 9. Find the solution of following system of equations :

$$5x + 6y = 3xy, \quad 10x + 9y = 5xy$$

Solution : The given system of equations :

$$5x + 6y = 3xy \qquad \dots(1)$$

$$10x + 9y = 5xy \qquad \dots(2)$$

Dividing the equation (1) and (2) both by xy, we have

$$\frac{5}{y} + \frac{6}{x} = 3 \qquad \dots(3)$$

$$\frac{10}{y} + \frac{9}{x} = 5 \qquad \dots(4)$$

Taking $\frac{1}{x} = m$ and $\frac{1}{y} = n$. The given system of equations become

$$5n + 6m = 3 \qquad \dots(5)$$

$$10n + 9m = 5 \qquad \dots(6)$$

Multiply equation (5) by 2, we get

$$10n + 12m = 6 \qquad \dots(7)$$

Substracting equation (6) from equation (7), we get

$$3m = 1 \quad \Rightarrow \quad m = \frac{1}{3}$$

Putting $m = \frac{1}{3}$ in equation (6), we get

$$10n + 9\left(\frac{1}{3}\right) = 5$$

or $\quad 10n + 3 = 5 \quad \Rightarrow \quad 10n = 5 - 3$

or $\quad 10n = 2 \quad \Rightarrow \quad n = \frac{2}{10}$

or $\quad n = \frac{1}{5}$

Now $m = \frac{1}{3} \Rightarrow \frac{1}{x} = \frac{1}{3} \Rightarrow x = 3$

and $n=\frac{1}{5}\Rightarrow\frac{1}{y}=\frac{1}{5}\Rightarrow y=5$

Thus, the solution of the equation is $x=3,\ y=5$

Exercise 4.2

Solve the following equations by the method of elimination (by substitution) :

1. $2x+3y=9$
   $3x+4y=5$

2. $x+2y=-1$
   $2x-3y=12$

3. $3x+2y=11$
   $2x+3y=4$

4. $8x+5y=9$
   $3x+2y=4$

5. $4x-5y=39$
   $2x-7y=51$

6. $5x-2y=19$
   $3x+y=18$

Solve the following equations by method of elemialtion by equating the coefficients :

7. $2x+y=13$
   $5x-3y=16$

8. $0.4x+0.3y=1.7$
   $0.7x-0.2y=0.8$

9. $\frac{x}{7}+\frac{y}{3}=5$
   $\frac{x}{2}-\frac{y}{9}=6$

10. $11x+15y=-23$
    $7x-2y=20$

11. $3x-7y+10=0$
    $y-2x=3$

12. $x+2y=\frac{3}{2}$
    $2x+y=\frac{3}{2}$

Solve the following equations :

13. $8v-3u=5uv$
    $6v-5u=-2uv$

14. $\frac{1}{2x}-\frac{1}{y}=-1$
    $\frac{1}{x}+\frac{1}{2y}=8$

15. $\frac{5}{(x+y)}-\frac{2}{(x-y)}=-1$
    $\frac{15}{(x+y)}+\frac{7}{(x-y)}=10$

Cross- Multiplication Method

Cross-multiplication is a general method to solve the simultaneous equations. Here we are going to clear this method.

Let the given system of equations be

$$a_1x + b_1y + c_1 = 0 \quad \ldots (1)$$

$$a_2x + b_2y + c_2 = 0 \quad \ldots (2)$$

Multiplying equation (1) by b_2 and (2) by b_1 respectively, we get

$$a_1b_2x + b_1b_2y + b_2c_1 = 0 \quad \ldots (3)$$

$$a_2b_1x + b_1b_2y + b_1c_2 = 0 \quad \ldots (4)$$

Substracting equation (4) from equation (3), we get

$$\left(a_1b_2 - a_2b_1\right)x + b_2c_1 - b_1c_2 = 0$$

or $$\left(a_1b_2 - a_2b_1\right)x = b_1c_2 - b_2c_1$$

$\Rightarrow$ $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \ldots (5)$$

Similarly multiplying equation (1) by a_2 and equation (2) by a_1, we get

$$a_1b_2x + a_2b_1y + c_1a_2 = 0 \quad \ldots (6)$$

$$a_1a_2x + a_1b_2y + c_2a_1 = 0 \quad \ldots (7)$$

Substracting equation (7) form equation (6)

$$\left(a_2b_1 - a_1b_2\right)y + c_1a_2 - c_2a_1 = 0$$

or $$\left(a_2b_1 - a_1b_2\right)y = -c_1a_2 + c_2a_1$$

or $$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad \ldots (8)$$

Hence, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

The above solution of equation can be written as :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

This result is shown by the following diagram, so that we can remembering the solution of given equation easily.

$$\frac{x}{\begin{array}{ccc} b_1 & & c_1 \\ & \times & \\ b_2 & & c_2 \end{array}} = \frac{y}{\begin{array}{ccc} c_1 & & a_1 \\ & \times & \\ c_2 & & a_2 \end{array}} = \frac{1}{\begin{array}{ccc} a_1 & & b_1 \\ & \times & \\ a_2 & & b_2 \end{array}}$$

In the above diagram, the direction of arrows shows the multiplication of the related numbers. First of all we multiply downwards and then subtract the multiplication upward from it.

In this method,the numbers are multiplied across, so it is called the cross multiplication method. In this system all the terms are taken to the left hand side and then right hand side become zero.

4.05. Condition for Solvability

In a pair of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, the solution of the equation depends on the ratio of corresponding coefficients.

1. First condition : if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.

2. Second condition : if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of equations is inconsistent, *i.e.*, it has no solution.

3. **Third condition :** if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of equations has infinitly many solutions.

Example 10. Solve the equations given below by cross multiplication method.

$$2x + 3y - 17 = 0 \; ; \; 3x - 2y - 6 = 0$$

Solution : By cross multiplication method, we get

$$\frac{x}{\begin{array}{ccc} 3 & & -17 \\ & \times & \\ -2 & & -6 \end{array}} = \frac{y}{\begin{array}{ccc} -17 & & 2 \\ & \times & \\ -6 & & 3 \end{array}} = \frac{1}{\begin{array}{ccc} 2 & & 3 \\ & \times & \\ 3 & & -2 \end{array}}$$

$$\Rightarrow \quad \frac{x}{(3)(-6)-(-2)(-17)} = \frac{y}{(-17)(3)-(-6)(2)} = \frac{1}{(2)(-2)-(3)(3)}$$

$$\Rightarrow \quad \frac{x}{-18-34}=\frac{y}{-51+12}=\frac{1}{-4-9}$$

$$\Rightarrow \quad \frac{x}{-52}=\frac{y}{-39}=\frac{1}{-13}$$

$$\Rightarrow \quad x=\frac{-52}{-13} \text{ and } y=\frac{-39}{-13}$$

$$\Rightarrow \quad x=4 \text{ and } y=3$$

Hence, the solution of the equations is $x = 4,\ y = 3$

Example 11. Check the consistency of the equations given below. If they are consistent, solve them.

$$2x+3y=7$$

$$6x+9y=15$$

Solution : The equations are

$$2x+3y=7$$

$$6x+9y=15$$

Taking all terms to the left side, we get

$$2x+3y-7=0$$

and $\quad 6x+9y-15=0$

here $\quad \frac{a_1}{a_2}=\frac{2}{6}=\frac{1}{3}$

and $\quad \frac{b_1}{b_2}=\frac{3}{9}=\frac{1}{3}$

also $\quad \frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$

Here, we see that $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Hence, the pair of equations is inconsistent and there is no solution.

Exercise 4.3

Check the equations given below, if they have a unique solution, no solution or infinitely many solutions. Incase there is a unique solution, find it;

1. $2x+y=35$ 2. $2x-y=6$

$3x+4y=65$ $\qquad$ $x-y=2$

3. $3x+2y+25=0$ $\qquad$ 4. $x+2y+1=0$

$2x+y+10=0$ $\qquad$ $2x-3y-12=0$

5. If the following system of equation has no solution, find the value of k.

(i) $2x+ky=1$, $3x-5y=7$

(ii) $kx+2y=5$, $3x+y=1$

6. Find the solution of system of equations $mx-ny=m^2+n^2$, $x+y=2m$

7. Find the value of λ if the system of equations $3x+\lambda y+1=0$, $2x+y-9=0$

(i) has a unique solution (ii) has no solution.

4.06 Application of linear equations in two variables

With the help of a pair of simultaneous linear equations we can solve the practical problems related to our daily life. In solving such problems, we may use the following steps :

(i) We use the variable for unknown quantities involved in the given problem.

(ii) Convert the conditions which are given in the verble from of problem in equation form by using the variables.

(iii) On solving these equation by appropriate method. We get the value of variables.

Example 12. 10 students of a class participated in a essay competition. If the number of boys is 4 more than the number of girls, find the number of boys and the girls separately participated in the competition.

Solution : Let the number of boys participating in the competition be x and number of girls be y

Given that the total number of students participating in the competition is 10. So number of boys + number of girls = 10

$\Rightarrow \qquad x+y=10$

It is also given that the number of boys is 4 more than the number of girls

Number of boys − number of girls = 4

$$x-y=4$$

According to the situation we get a pair of equations as :

$$x+y=10 \qquad \ldots(1)$$

$$x-y=4 \qquad \ldots(2)$$

Adding equations (1) and (2) , we get

$2x=14$

or $\quad x = 7$

Putting the value of x in equation (1), we get

$$7 + y = 10$$

or $\quad y = 10 - 7$

or $\quad y = 3$

Hence, the number of boys are 7 and the number of girls are 3.

Example 13. The ratio of the salaries of two persons is 9 : 7 and the ratio of their expenditure is 4 : 3. If each of the two persons saves ₹ 2000 per month, find out their salaries.

Solution : Let the salary of first person be ₹ x and the salary of second person by ₹ y.

According to the problem $x : y = 9 : 7$

or $\quad \dfrac{x}{y} = \dfrac{9}{7} \quad$ or $\quad 7x = 9y$

or $\quad 7x - 9y = 0 \quad \ldots(1)$

The two persons save ₹ 2000 per month separately.

Their monthly expenditure are ₹ $(x - 2000)$ and ₹ $(y - 2000)$ respectively.

The ratios of their expenditure is $4 : 3$

$\therefore \quad (x - 2000) : (y - 2000) = 4 : 3$

$$\frac{(x - 2000)}{(y - 2000)} = \frac{4}{3}$$

or $\quad 3(x - 2000) = 4(y - 2000)$

or $\quad 3x - 6000 = 4y - 8000$

or $\quad 3x - 4y + 2000 = 0 \quad \ldots(2)$

From equation (1), we get $x = \dfrac{9y}{7}$

Susbtituting $x = \dfrac{9y}{7}$ in equation (2), we get

$$3\left(\frac{9y}{7}\right) - 4y + 2000 = 0$$

or $\frac{27y}{7} - 4y + 2000 = 0$

or $27y - 28y + 14000 = 0$ or $-y = -14000$

or $y = 14000$

Putting the value of y in equation (1), we get

$7x - 9(14000) = 0$

or $7x - 126000 = 0$

or $7x = 126000$ or $x = \frac{126000}{7}$

or $x = 18000$

Hence the salaries of two persons are ₹ 18000 and ₹ 14000 respectively.

Example 14. The sum of the digits of a two digit number is 12. If 18 is subtracted form the number, the place of two digits is interchanged. Find digits the number.

Solution : Let the one's digit and ten's digits of the number of x and y respectively.

So the number is $x + 10y$

According to the problem, the sum of the digits of this number is 12. So

$$x + y = 12 \quad \ldots (1)$$

and $x + 10y - 18 = 10x + y$ or $10y - y + x - 10x = 18$

or $9y - 9x = 18$ or $9x - 9y = -18$

or $$x - y = -2 \quad \ldots (2)$$

Adding equation (1) and (2), we have

$2x = 10$ or $x = 5$

Putting the value of x in equation (1), we get

$5 + y = 12$

or $y = 12 - 5$

or $y = 7$

Thus, the number $10y + x = 10 \times 7 + 5 = 75$

Exercise 4.4

Solve the problems given below :

1. In a two-digit number, the one's digit is 3 times the ten's digit. If 10 is added to the 2 times of the number, its digits interchange their places in the new number. Find the number.
2. The perimeter of a rectangle is 56 cm. The ratio of length and its breadth is 4 : 3. Find the length and breadth of the rectangle.
3. The ratio of two numbers is 3 : 4. If 5 is substracted from each of the number, the ratio becomes 5 : 7. Find the numbers.
4. The age of a father is 5 years more than the age of 6 times of his son's age. After 7 years the age of father will be 3 more than the age of 3 times of his son's age. Find their present ages.
5. Ram said to Shyam, "If you give me ₹ 100, my money will be doubled to your money." Then Shyam said to Ram, "If you give me ₹ 10, my money will be 6 times to yours. Find how many rupees does each of them have?
6. The cost of 4 chairs and 3 tables is ₹ 2100 and the cost of 5 chairs and 2 tables is ₹ 1750. Find the cost of one chair and one table separately.
7. If 3 times of a larger number is divided by a smaller number the quotient is 4 and remainder is 3 and when 7 times of the smaller number is divided by the larger number, then quotient is 5 and remainder is 1. Find the two numbers.
8. A two-digit number is 4 times the sum of its digits and 2 times the product of it's digits. Find the number.
9. If one is added to the numerator and the denominator separately of a fraction, then the facrction becomes $\frac{4}{5}$ and if 5 is substracted form both numerator and denominator, then the fraction becomes $\frac{1}{2}$. Find the fraction.
10. 5 years ago Geeta's age was 3 times of Kamla's age. After 10 years Geeta's age will be 2 times of Kamla's age. Find their present ages.
11. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, he takes 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer, Find the speed of the train and car.

Important Points

1. A linear equation in two variables is in the form of $ax+by+c=0$, where a, b, and c are real numbers and $a \neq 0, b \neq 0$.
2. A linear equation in two variables has infinitely many solutions.
3. The nature of a pair of two linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x+b_2y+c_2=0$ is given as :

   (a) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are consistent and has a unique solution.

   (b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equations is consistent with infinitely many solutions.

   (c) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the pair of equation is inconsistent and have no solution.
4. If x and y are digits at one's and ten's places respectively in a number, then the number will be $10y+x$.

Miscellaneous Exercise-4

Choose the correct answer (Questions 1 to 10)

1. Find the value of x, if $y=2x-3$ and $y=5$.

   (a) 1 (b) 2 (c) 3 (d) 4

2. The pair of solution that satisfy the equation $2x+y=6$ is :

   (a) (1, 2) (b) (2, 1) (c) (2, 2) (d) (1, 1)

3. If $\frac{4}{x}+5y=7$ and $x=-\frac{4}{3}$, then the value of y is :

   (a) $\frac{37}{15}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

4. If $\frac{3}{x}+4y=5$ and $y=1$, then the value of x is :

   (a) 3 (b) $\frac{1}{3}$ (c) -3 (d) $-\frac{1}{3}$

5. If $x=1$, then find the value of y in the equation $\frac{4}{x}+\frac{3}{y}=5$:

   (a) 1 (b) $\frac{1}{3}$ (c) 3 (d) -3

6. If one's digit and ten's digit of a number are x and y respectively, then the number will be :

   (a) $10x+y$ (b) $10y+x$ (c) $x+y$ (d) xy

7. The son's age is one third of his mother's age. If the present age of the mother is x years, then after 12 years son's age will be :

   (a) $\frac{x}{3}+12$ (b) $\frac{x+12}{3}$ (c) $x+4$ (d) $\frac{x}{3}-12$

8. The coordinate of a point at the x- axis is :

(a) $(2,3)$ (b) $(2,0)$ (c) $(0,2)$ (d) $(2,2)$

9. The coordinates of origin are :

(a) $(0,0)$ (b) $(0,1)$ (c) $(1,0)$ (d) $(1,1)$

10. In which quadrant does the point $(3,-4)$ lie ?

(a) First (b) Second (c) Third (d) Fourth

11. In equation $5y-3x-10=0$, express y in terms of x , Find the point on the graph. Where the line represented by $5y-3x-10=0$ cuts y-axis.

12. Taking the values of x form -2 to 2 prepare a table of the equation $y=2x+1$. Also draw the graph of the equation.

13. Solve the simultaneous equations given below :

$0\cdot5x+0\cdot6y=2\cdot3;\ 0\cdot2x+0\cdot7y=2\cdot3$

14. Solve the system of the equations :

$$2x+3y=9\ ;\ 3x+4y=5$$

15. Solve the simultaneous equations

$$\frac{1}{2x}-\frac{1}{y}=-1\ ;\ \frac{1}{x}+\frac{1}{2y}=8\ ;\ \text{where } x\neq 0,\ y\neq 0$$

16. There are two numbers such that if 7 is added to the smaller number then sum becomes doubel to the larger number and if 4 is added to the larger number it becomes three times of the smaller one. Find the two numbers.

17. Numerator of a fraction is 4 less than it's denomilator. If 2 is subtracted form the numerator and 1 is added to the denominator, then the denomilator becomes 8 times of the numerator. Find the fraction.

18. The cost of 5 books and 7 pens is ₹ 79 and the cost of 7 books and 5 pens is ₹ 77. Find the cost of 1 book and 2 pens.

19. When a two digit number is multipled by 9, it becomes 2 times of the number that is obtained by interchanging the digits. If the difference of the digits is 7, find the number.

20. In a triangle ABC, $\angle A=x^\circ, \angle B=3x^\circ$ and $\angle C=y^\circ$. Now, if $5x^\circ-3y^\circ+30^\circ=0$, prove that given triangle is a right angled triangle.

21. Solve the following equations graphically :

(a) $x+y=4\ ;\ x=y$ (b) $x+y=3\ ;\ 2x+5y=12$

(c) $2x-3y-6=0\ ;\ 2x+y+10=0$ (d) $2x+y-3=0\ ;\ 2x-3y-7=0$

22. Solve the system of equations $2x-y=1; x+2y=8$ graphically and find the co-ordinates of the points where corresponding lines intersect y-axis.

Answer

Exercise 4.1

(1) $x = 6,\ y = 0$ (2) $x = 1,\ y = 4$ (3) No solution (4) $x = 3,\ y = 1$

(5) $x = -2,\ y = 3$ (6) inconsistent, no solution (7) $(4, -6)$

(8) inconsistent, no solution (9) $(1, 2)$

(10) coincide, infinitely many solutions (11) $(-2, 3)$

Exercise 4.2

(1) $x = -21,\ y = 17$ (2) $x = 3,\ y = -2$ (3) $x = 5,\ y = -2$ (4) $x = -2,\ y = 5$

(5) $x = 1,\ y = -7$ (6) $x = 5,\ y = 3$ (7) $x = 5,\ y = 3$ (8) $x = 2,\ y = 3$

(9) $x = 14,\ y = 9$ (10) $x = 2,\ y = -3$ (11) $x = -1,\ y = 1$ (12) $x = \frac{1}{2},\ y = \frac{1}{2}$

(13) $u = \frac{22}{31},\ v = \frac{11}{23}$ (14) $x = \frac{1}{6},\ y = \frac{1}{4}$ (15) $x = 3,\ y = 2$

Exercise 4.3

(1) $x = 15,\ y = 5$ (2) $x = 4,\ y = 2$ (3) $x = 5,\ y = -20$ (4) $x = 3,\ y = -2$

(5) (i) $k = -\frac{10}{3}$; (ii) $k = 6$ (6) $x = (m+n),\ y = m - n$

(7) (i) For unique solution $\lambda \neq \frac{3}{2}$ (ii) For no solution $\lambda = \frac{3}{2}$

Exercise 4.4

(1) 26 (2) length 16 cm, width 12 cm (3) 30 and 40

(4) age of father is 29 years and age of his son is 4 years.

(5) Ram's money is ₹ 40 and Shyam's money is ₹ 170.

(6) Cost of chair ₹ 150 and table is ₹ 500

(7) The larger number is 25 and smaller number is 18. (8) 36 (9) 7 / 9

(10) the age of Geeta is 50 years, age of Kamla is 20 years.

(11) Train 100 km/h and car 80 km/h.

Miscellaneous Exercise 4

1. (D) 2. (C) 3. (B) 4. (A) 5. (C) 6. (A) 7. (A)

8. (B) 9. (A) 10. (D) 11. $y = \frac{3x+10}{5}$, $(0, 2)$

12.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

13. $x = 1, y = 3$ 14. $x = -21, y = 17$

15. $x = \frac{1}{6}, y = \frac{1}{4}$ 16. 5,3 17. $\frac{3}{7}$ 18. ₹ 20 19. 18

21. (a) $x = 2, y = 2$ (b) $x = 1, y = 2$ (c) $x = -3, y = -4$ (d) $x = 2, y = -1$

22. $x = 2, y = 3$ first line meets the y-axis at point (0, -1) and second line meets the y-axis at point (0, 4).