Chapter – 11

Constructions

Division of a Line Segment

Introduction

In geometry ‘Construction’ means drawing of shapes, angles, and lines accurately, using a compass, ruler, and pencil.

Division of a Line Segment

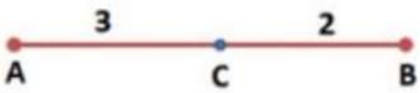
Suppose a line segment is given and we have to divide it in the ratio of 2 : 3. One way of doing it is to measure the length of the line segment and then mark the points on it that divides it in the given ratio. If we do not have any way of measuring it precisely then we can do it by using construction methods.

Construction 1: To divide a line segment in a given ratio.

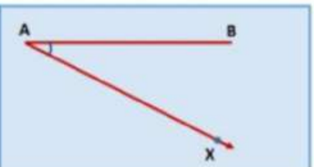
We will divide a line segment AB into two segments AC and CB, such that C divides AB in the ratio m : n, where both m and n are positive integers.

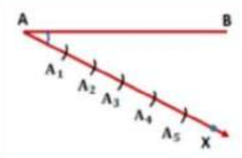
If AC : CB = 3 : 2, then C divides AB in the ratio 3 : 2

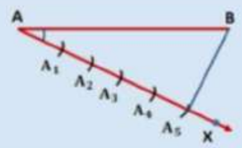
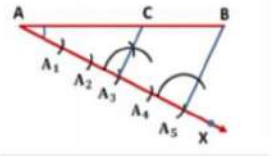
Let m = 3 and n = 2.



Steps of Construction

Draw any ray AX, such that the ray AX makes an acute angle with AB.	

Locate 5 points $A_1, A_2, A_3, A_4$and A_5 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$ (because $m + n = 3 + 2 = 5$)	

Join BA_5. ($m + n = 5$)	
Through the point A_3 ($m = 3$), draw a line parallel to A_5B (making an equal to$\angle AA_5B$) intersectingAB at the point C. Hence, point C divides ABin the ratio3: 2. **AC : CB = 3 : 2.**	

Now, $A_3C \parallel A_5B$

$\therefore \frac{AA_3}{A_3A_5} = \frac{AC}{CB}$ (By Basic Proportionality Theorem)

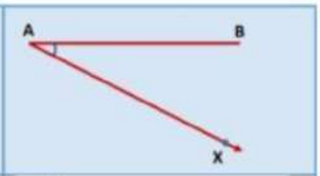
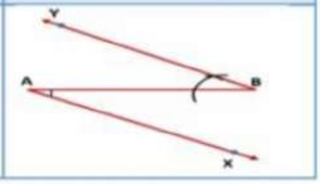
By Construction, $\frac{AA_3}{A_3A_5} = \frac{3}{2}$

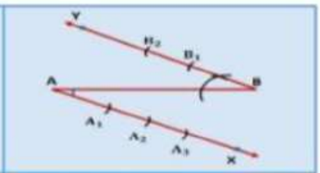
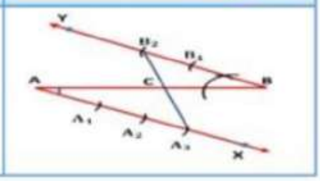
$$\therefore \frac{AC}{CB} = \frac{3}{2}$$

This shows that C divides AB in the ratio 3 : 2.

Alternative method

Steps of Construction

Draw any ray AX, such that the ray AX makes an acute angle with AB.	
Draw a ray BY parallel to AX, such that $\angle ABY = \angle BAX$.	

Locate the points A_1, A_2, A_3 ($m = 3$) on AX and B_1, B_2 ($n = 2$) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$	
Join A_3B_2, intersecting AB at the point C. Then $AC : CB = 3 : 2$	

Here, ΔAA_3C is similar to ΔBB_2C

$$\frac{AA_3}{BB_2} = \frac{AC}{BC}$$

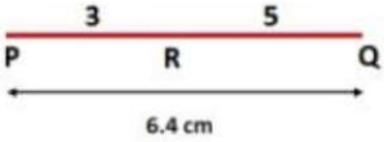
By construction $\frac{AA_3}{BB_2} = \frac{3}{2}$, therefore,

$$\frac{AC}{BC} = \frac{3}{2}$$

Example: PQ is a line segment of length 6. 4 cm. Geometrically obtain point R on PQ such that $\frac{QR}{PQ} = \frac{5}{8}$.

Now, $\frac{QR}{PQ} = \frac{5}{8}$

$$\frac{PQ}{QR} = \frac{8}{5} \Rightarrow \frac{PQ}{QR} - 1 = \frac{8}{5} - 1$$



$$\frac{PQ - QR}{QR} = \frac{8-5}{5} \Rightarrow \frac{PR}{QR} = \frac{3}{5}$$

Here, m = 3, and n = 5

Steps of Construction

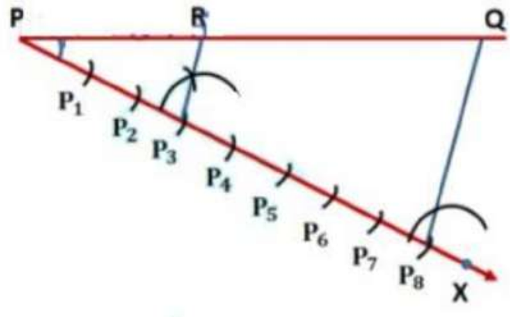
1. Draw a line segment PQ of length 6.4 cm and a ray PX making an acute angle with the line segment PQ.

2. Mark m + n = 3 + 5 = 8 points P_1, P_2, P_3, P_4, P_5, P_6, $P_7$and P_8 on PX such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = P_5P_6 = P_6P_7 = P_7P_8$.

3. Join QP_8.

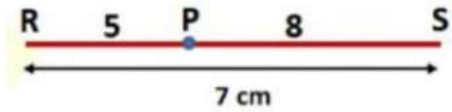
4. Through the point P_3 (m = 3), draw a line parallel to QP_8 (making an equal to $\angle PP_8Q$) intersecting PQ at the point R. Hence, point R divides PQ internally in ratio 3 : 5.

PR : RQ = 3 : 5



Example: Draw a line segment of length 7 cm. Find a point P on it, which divides it in the ratio 5: 8.

RP : PS = 5 : 8



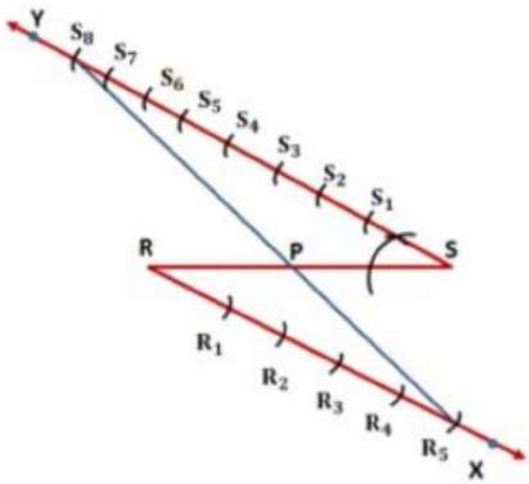
Steps of Construction

1. Draw a line segment RS of length 7 cm and a ray RX making an acute angle with the line segment RS.

2. Draw another ray SY ∥ RX, such that $\angle RSY = \angle SRX$.

3. Locate the points R_1, R_2, R_3, R_4, R_5 ($m = 5$) on RX and $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8$ ($n = 8$) on SY such that $RR_1 = R_1R_2 = R_2R_3 = R_3R_4 = R_4R_5 = SS_1 = S_1S_2 = S_2S_3 = \dots \dots \dots .\ . S_7S_8$

4. Join R_5S_8, intersecting RS at the point P. Then $RP : PS = 5 : 8$



To Construct a Triangle Similar to Given Triangle as per given Scale Factor

Construction 2

To construct a triangle similar to a given triangle as per the given scale factor.

Scale Factor means the ratio of the sides of the triangle constructed with the corresponding sides of the given triangle.

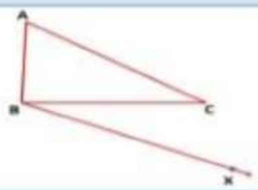
A scale factor is generally written as $\frac{m}{n}$ which may be less than 1 or greater than 1.

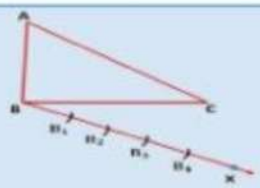
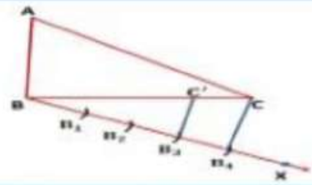
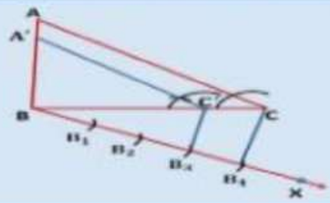
i) If $\frac{m}{n} < 1$ or m < n, then the sides of the triangle to be constructed are smaller than the corresponding sides of the given triangle.

ii) If $\frac{m}{n} > 1$ or m > n, then the sides of the triangle to be constructed are larger than the corresponding sides of the given triangle.

1) Case 1: We have to construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle. ($\frac{m}{n} < 1$)

Steps of Construction

1. Construct the given Δ ABC	
2. Draw any ray BX making an acute angle with BCon the side opposite to the vertex A.	

3. Locate 4 (greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$	
4. Join B_4C and draw a line through B_3 (3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B_4C and intersecting BC at C'.	
5. Draw a line through C' parallel to the line CA to intersect BA at A'.	
6. Then $\Delta\, A'BC'$ is the required triangle.	

By construction,

$$\frac{BC'}{C'C} = \frac{3}{1} \ (\because B_3C' \parallel B_4C)$$

Therefore, $\frac{BC}{BC'} = \frac{BC' + C'C}{BC'} = 1 + \frac{C'C}{BC'} = 1 + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$

$$\frac{BC'}{BC} = \frac{3}{4}$$

In $\Delta\, A'BC'$ and $\Delta\, ABC$

$\angle A'C'B = \angle ACB \ (\because A'C' \parallel AC)$

$\angle A'BC' = \angle ABC$ (Common)

$\Delta\, A'BC' \sim \Delta\, ABC$ (By AA Similarity Criterion)

So, $\frac{AB'}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$

Example: Draw a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60^{o}$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

(REFERENCE: NCERT)

In Δ ABC,

BC = 6 cm, AB = 5 cm, $\angle ABC = 60°$ and Scale factor = $\frac{3}{4}$

Steps of construction:

i) Draw a line segment AB of length 5 cm

ii) At point B, draw $\angle ABY = 60°$ and cut off BC = 6 cm from BY.

iii) Join AC, Δ ABC is the given triangle.

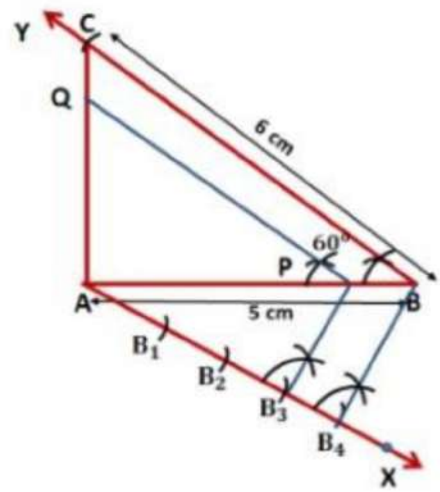
iv) Draw any ray AX making an acute angle with AB on the side opposite to the vertex A.

v) Locate 4 (greater of 3 and 4 in $\frac{3}{4}$) points B_1,B_2,B_3 and B_4 on AX such that $AB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

vi) Join B_4B and draw a line through B_3 (3^{rd} point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B4B and intersecting AB at P.

vii) From point P draw PQ ∥ BC intersecting AC at Q.

viii) Δ APQ is the required triangle.



By construction,

$\frac{AP}{PB} = \frac{3}{1}$ ($\because B_3P \parallel B_4B$)

Therefore, $\frac{AB}{AP} = \frac{AP + PB}{AP} = \frac{AP}{AP} + \frac{PB}{AP}$

$= 1 + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$

$\frac{AP}{PB} = \frac{3}{4}$

In Δ CAB and Δ QAP

$\angle ABC = \angle APQ$($\because PQ \parallel BC$)

$\angle CAB = \angle QAP$ (Common)

Δ CAB ∼Δ QAB (By AA Similarity Criterion)

So, $\frac{QA}{CA} = \frac{QP}{CB} = \frac{AP}{PB} = \frac{3}{4}$

Example: Construct a triangle similar to a given ΔABC such that each of its sides is $\frac{2}{3}$ rd of the corresponding sides of ΔABC. It is given that AB = 4 cm, BC = 5 cm and AC = 6 cm.

(REFERENCE: NCERT)

In Δ ABC,

AB = 4 cm, BC = 5 cm, AC = 6 cm and Scale factor = $\frac{2}{3}$

Steps of construction:

i) Draw a line segment BC of length 5 cm.

ii) With B as the centre, cut an arc of radius 4 cm and with C as the centre, cut an arc of radius 6 cm. Name the point of intersection as A.

iii) Join AB and AC, Δ ABC is the given triangle.

iv) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

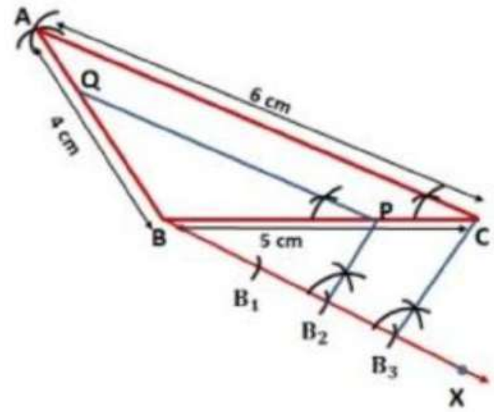
v) Locate 3 (greater of 2 and 3 in $\frac{2}{3}$) points B_1,B_2 and B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$.

vi) Join B_3C and draw a line through B_2 (2nd point, 2 being smaller of 2 and 3 in $\frac{2}{3}$ parallel to B_3C and intersecting BC at P.

vii) From point P draw PQ ∥ CA intersecting AB at Q.

viii) Δ QBP is the required triangle.

By construction,



$\frac{BP}{PC} = \frac{BP}{BC - BP} = \frac{2}{3 - 2} = \frac{2}{1}$ ($\because B_3C \parallel B_2P$)

Therefore, $\frac{BC}{BP}\frac{BP + PC}{BP} = \frac{BP}{BP} = \frac{PC}{BP}$

$= 1 + \frac{1}{2} = \frac{2 + 1}{2} = \frac{3}{2}$

$\frac{BP}{BC} = \frac{2}{3}$

In Δ ABC and Δ QBP

$\angle ACB = \angle QPB$ ($\because AC \parallel QP$)

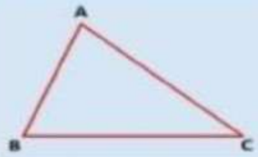
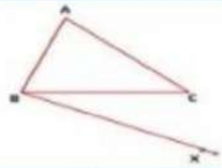
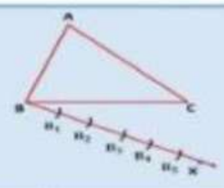
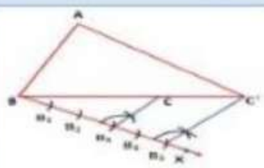
$\angle ABC = \angle QBP$ (Common)

$\Delta ABC \sim \Delta QBP$ (By AA Similarity Criterion)

So, $\frac{QB}{AB} = \frac{QP}{AC} = \frac{BP}{BC} = \frac{2}{3}$

Case 2: We have to construct a triangle similar to the given triangle ABC whose sides are $\frac{5}{3}$ of the corresponding sides of the triangle ABC. ($\frac{m}{n} > 1$)

Steps of Construction

1. Construct the given Δ ABC	
2. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.	
3. Locate 5(greater of 5 and 3 in $\frac{5}{3}$) points $B_1, B_2, B_3, B_4$and B_5on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$	
4. Join B_3C(3[rd] point, 3 being smaller of 5 and 3 in $\frac{5}{3}$) and draw a line through B_5 parallel to B_3C and intersecting the extended line segmentBC at C'.	
5. Draw a line through C' parallel to the line CA, intersecting the extended line segment BA atA'.	
6. Then Δ $A'BC'$ is the required triangle.	

In $\Delta\ A'BC'$ and Δ ABC

$\angle A'C'B = \angle ACB$ ($\because A'C' \parallel AC$)

$\angle A'BC' = \angle ABC$ (Common)

$\Delta\ A'BC' \sim \Delta\ ABC$ (By AA Similarity Criterion)

So, $\frac{AB}{A'B} = \frac{AC}{A'C'} = \frac{BC}{BC'}$

But, $\frac{BC'}{BC} = \frac{5}{3}$

Hence, $\frac{A'B}{AB} = \frac{A'C}{AC} = \frac{BC'}{BC} = \frac{5}{3}$

Example: Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. (REFERENCE: NCERT)

In Δ ABC,

AB = 5 cm, BC = 6 cm, AC = 7 cm and Scale factor = $\frac{7}{5}$

Steps of construction:

i) Draw a line segment BC of length 6 cm.

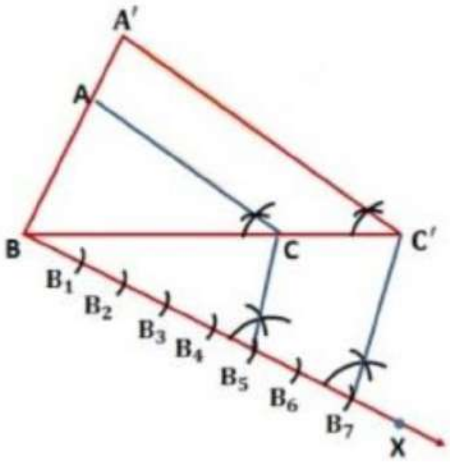
ii) With B as the centre, cut an arc of radius 5 cm and with C as the center, cut an arc of radius 7 cm. Name the point of intersection as A.

iii) Join AB and AC, Δ ABC is the given triangle.

iv) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

v)Locate 7 (greater of 7 and 5 in $\frac{7}{5}$) points B^1,B^2,B_3,B_4, B_5,B_6,B_7on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

vi)Join B_5C and draw a line through B_7 (5^{th} point, 5 being smaller of 5 and 7 in $\frac{7}{5}$) parallel to B_5C and intersecting the extended line segment BC at C ′



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vii) From point C draw AC ∥ A′C′ intersecting A ′B at A.

viii) Δ ABC is the required triangle.

In Δ ABC and Δ A′BC′

∠ACB = ∠A ′C′B(∵ AC ∥ ′C′)

∠ABC = ∠A′BC′(Common)

Δ ABC ∼Δ A′BC′(By AA Similarity Criterion)

So, $\frac{AB}{A'B} = \frac{AC}{A'C'} = \frac{BC}{BC'}$

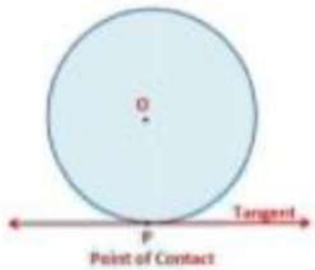
But, $\frac{BC'}{BC} = \frac{7}{5}$

Hence, $\frac{A'B}{AB} = \frac{A'C}{AC} = \frac{BC'}{BC} = \frac{7}{5}$

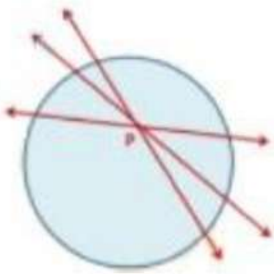
Construction of Tangents to a Circle

Constructions of Tangents to a Circle

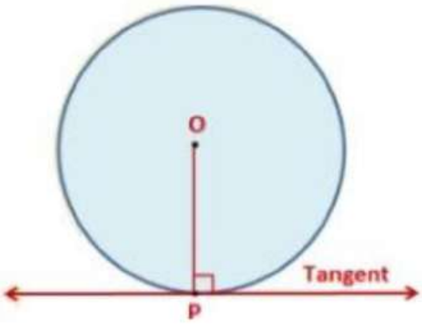
• A tangent to a circle is a line that intersects the circle at one point only. The common point of the tangent and the circle is called the point of contact and the tangent touches the circle at the common point.



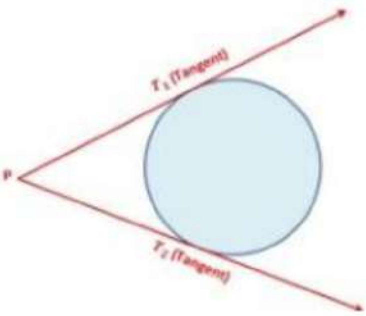
• If a point lies inside a circle, there cannot be a tangent to the circle through this point.



• If a point lies on the circle, then there is only one tangent to the circle at this point and it is perpendicular to the radius through this point.

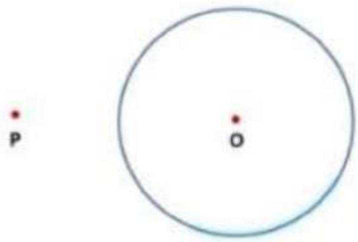


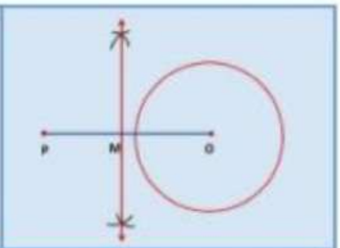
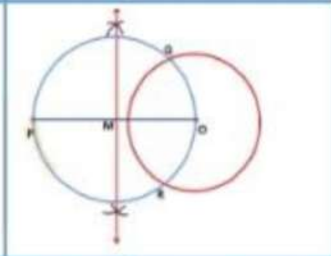
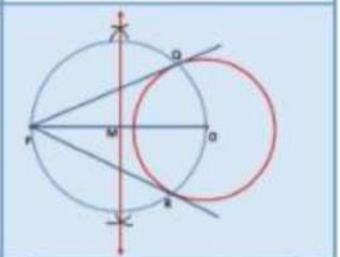
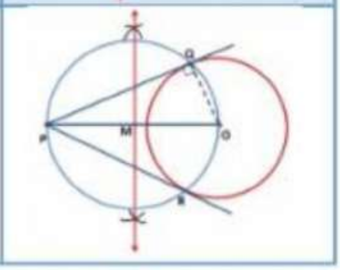
• If the point lies outside the circle, there will be two tangents to the circle from this point.



To construct the tangents to a circle from a point outside it.

Given, a circle with center O and a point P outside it. We have to construct the two tangents from the P to the circle.



1. Join PO and bisect it. Let M be the midpoint of PO.	
2. Taking M as center and MO as radius, draw a circle. Let it intersect the given circle at point Q and R.	
3. Join PQ and PR. They are the required two tangents.	
4. Join OQ. Then $\angle PQO$ is an angle in the semicircle and, therefore, $\angle PQO = 90^{0}$	

As OQ is a radius of the given circle, PQ has to be a tangent to the circle. Similarly, PR is also a tangent to the circle.

Example: Construct a pair of tangents PQ and PR to a circle of radius 4 cm from a point P outside the circle 8 cm away from the center.

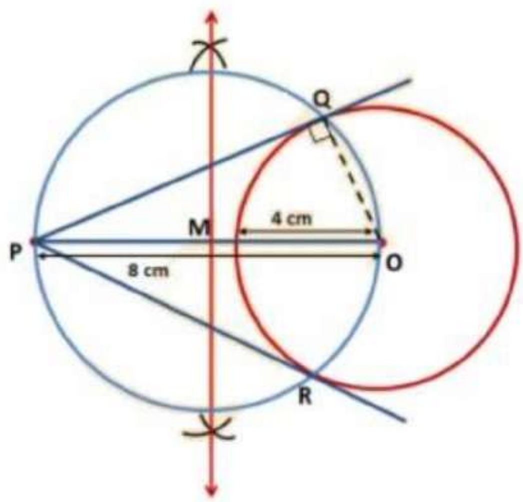
Steps of Construction

i) Draw a circle with O as center and radius ON = 4 cm. Take a point P such that OP = 8 cm.

ii) Draw the bisectors of OP which intersects OP at M.

iii) Taking M as center and MO as radius draw a circle. Let it intersect the given circle at Q and R.

iv) Join PQ and PR. They are the required two tangents.



Example: Construct a pair of tangents to the circle of radius 4 cm from a point on the concentric circle of radius 9 cm.

(REFERENCE: NCERT)

Steps of Construction

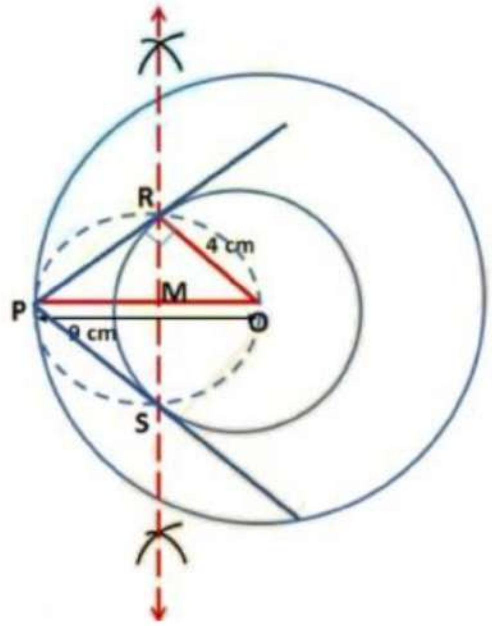
i) Draw two concentric circles with a common center O and radii 4 cm and 9 cm respectively.

ii) Take any point P on the outer circle and join OP.

iii) Draw the bisector of OP which bisects OP at M.

iv)Taking M as center and OM as radius, draw a circle that intersects the inner circle at two points R and S.

v) Join PR and PS. Thus PR and PS are the required tangents.



Example: Draw a pair of tangents to a circle of radius 5 cm which is inclined to each other at an angle of 60°.

(REFERENCE: NCERT)

Steps of Construction

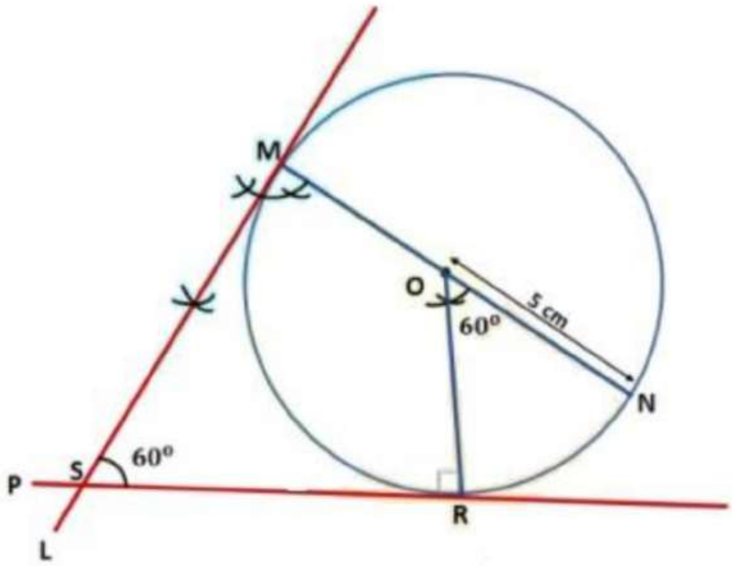
i) Draw a circle with O as centre and radius 5 cm.

ii) Draw any diameter MON of this circle.

iii) Draw a radius OR such that $\angle$ NOR = 60°

iv) Draw LM $\perp$MN and PR $\perp$ OR.

v) Let the point of intersection of LM and PR be S. Then MS and SR are the required tangents inclined to each other at an angle of 60°.



M
O
5 cm
60°
N
P
S
60°
R
L