Areas Related to Circles

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1. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Ans. Let R be the radius of the circle which has area equal to the sum of areas of the two circles, then

According to the question,

$\pi R^2 = \pi(8)^2 + \pi(6)^2$

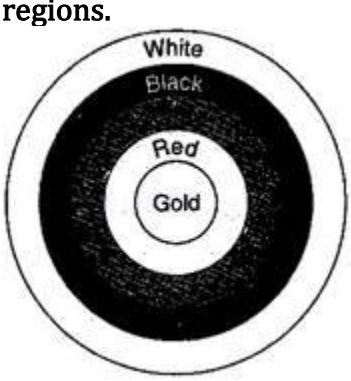
$\Rightarrow R^2 = (8)^2 + (6)^2$

$\Rightarrow R^2 = 64 + 36$

$\Rightarrow R^2 = 100$

$\Rightarrow R = 10$ cm

2. Figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of the five scoring regions.



Ans. Gold: Diameter = 21 cm

$\Rightarrow$ Radius = $\frac{21}{2}$ cm

Area of gold scoring region = $\pi\left(\frac{21}{2}\right)^2 = \frac{22}{7}\times\frac{21}{2}\times\frac{21}{2} = 346.5$ cm^2

Red: Area of red scoring region = $\pi\left(\frac{21}{2}+10.5\right)^2 - \pi\left(\frac{21}{2}\right)^2 = \pi(21)^2 - 346.5$

$= \frac{22}{7} \times 21 \times -346.5$ $= 1386 - 346.5 = 1039.5$ cm^2

Blue: Area of blue scoring region $= \pi(21+10.5)^2 - (1039.5 - 346.5) = \pi(31.5)^2 - 1386$

$= \frac{22}{7} \times 31.5 \times 31.5 - 1386$ $= 3118.5 - 1386 = 1732.5$ cm^2

Black: Area of black scoring region $= \pi(31.5+10.5)^2 - (1732.5 + 1039.5 + 346.5)$

$= \pi(42)^2 - 3118.5 = \frac{22}{7} \times 42 \times 42 - 3118.5$

$= 5544 - 3118.5 = 2425.5$ cm^2

White: Area of white scoring region $= \pi(42+10.5)^2 - (2425.5 + 1732.5 + 1039.5 + 346.5)$

$= \pi(52.5)^2 - 5544 = \frac{22}{7} \times 52.5 \times 52.5 - 5544$

$= 8662.5 - 5544 = 3118.5$ cm^2

3. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Ans. Diameter of wheel $= 80$ cm

$\Rightarrow$ Radius of wheel $(r) = 40$ cm

Distance covered by wheel in one revolution $= 2\pi r = 2 \times \frac{22}{7} \times 40 = \frac{1760}{7}$ cm

$\because$ Distance covered by wheel in 1 hour $= 66$ km $= 66000$ m $= 6600000$ cm

$\therefore$ Distance covered by wheel in 10 minutes $= \frac{6600000}{60} \times 10 = 1100000$ cm

$\therefore$ No. of revolutions $= \frac{\text{Total distance}}{\text{distance of one revolution}} = \frac{1100000 \times 7}{1760} = 4375$

4. Tick the correct answer in the following and justify your choice: If the perimeter and area of a circle are numerically equal, then the radius of the circle is:

(A) 2 units

(B) π units

(C) 4 units

(D) 7 units

Ans. (A) Circumference $=$ Area

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r = 2$ units

Unless stated otherwise, take $\pi = \frac{22}{7}$.

5. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

Ans. Here, r = 6 cm and $\theta = 60^\circ$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}$ cm^2

6. Find the area of a quadrant of a circle whose circumference is 22 cm.

Ans. Given, $2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2}$ cm

We know that for quadrant of circle, $\theta = 90^\circ$

$\therefore$ Area of quadrant = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{8}$ cm^2

7. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Ans. Here, r = 14 cm and $\theta = \frac{90^\circ}{3} = 30^\circ$

$\therefore$ Area swept = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= \frac{154}{3}$ cm^2

8. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment, (ii) major segment. $(\text{Use } \pi = 3.14)$

Ans. i) Here, r = 10 cm and $\theta = 90^\circ$

$\therefore$ Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 78.5 \text{ cm}^2$

Area of ΔOAB = $\frac{1}{2}$ x Base x Height

$= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

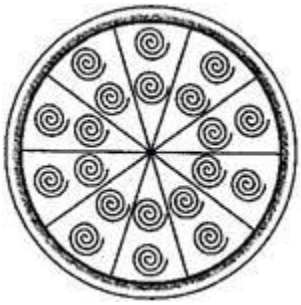
$\therefore$ Area of minor segment = Area of minor sector – Area of ΔOAB

$= 78.5 - 50 = 28.5 \text{ cm}^2$

(ii) For major sector, radius = 10 cm and $\theta = 360^\circ - 90^\circ = 270^\circ$

$\therefore$ Area of major sector = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{270^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 235.5 \text{ cm}^2$

9. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:
(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 cm. $(\text{Use } \pi = 3.14)$



Ans. (i) Area of quadrant with 5 m rope = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times 3.14 \times 5 \times 5 = 19.625 \text{ m}^2$

(ii) Area of quadrant with 10 m rope = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 78.5 \text{ m}^2$

$\therefore$ The increase in grazing area = $78.5 - 19.625 = 58.875 \text{ m}^2$

10. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:
(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.
Ans. (i) Diameter = 35 mm

$\Rightarrow$ Radius = $\frac{35}{2}$ mm

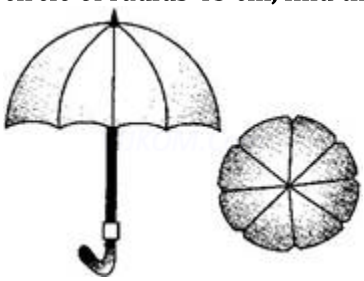
Circumference = $2\pi r = 2 \times \frac{22}{7} \times \frac{35}{2} = 110$ mm .....(i)
Length of 5 diameters = 35 x 5 = 175 mm .....(ii)
$\therefore$ Total length of the silver wire required = 110 + 175 = 285 mm

(ii) $r = \frac{35}{2}$ mm and $\theta = \frac{360^\circ}{10} = 36^\circ$

$\therefore$ The area of each sector of the brooch = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{36^\circ}{360^\circ} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{385}{4}$ mm^2

11. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Ans. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 = \frac{22275}{28}$ cm^2

12. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of $115°$. Find the total area cleaned at each sweep of the blades.

Ans. Here, r = 25 cm and $\theta = 115°$

The total area cleaned at each sweep of the blades = $2\times\left(\frac{\theta}{360°}\times\pi r^2\right)$

$= 2\times\left(\frac{115°}{360°}\times\frac{22}{7}\times 25\times 25\right) = \frac{158125}{126}$ cm^2

13. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle $80°$ to a distance of 16.5 km. Find the area of the sea over which the ships are warned. $(\text{Use } \pi = 3.14)$

Ans. Here, r = 16.5 km and $\theta = 80°$

The area of sea over which the ships are warned = $\frac{\theta}{360°}\times\pi r^2$

$= \frac{80°}{360°}\times 3.14\times 16.5\times 16.5$ = 189.97 km^2

14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is:

(A) $\frac{p}{180°}\times 2\pi r$

(B) $\frac{p}{180°}\times \pi r^2$

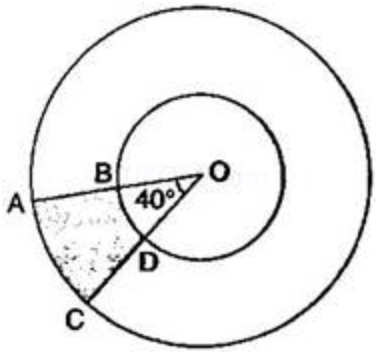
(C) $\frac{p}{360°}\times 2\pi r$

(D) $\frac{p}{360°}\times 2\pi r^2$

Ans. (D) Given, r = R and $\theta = p$

Area of sector = $\frac{\theta}{360°}\times\pi r^2 = \frac{p}{360°}\times\pi R^2 = \frac{p}{2\times 360°}\times 2\pi R^2 = \frac{p}{720°}\times 2\pi R^2$

15. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle$ AOC = $40°$.



Ans. Area of shaded region = Area of sector OAC – Area of sector OBD

$$= \frac{40°}{360°} \times \frac{22}{7} \times (14)^2 - \frac{40°}{360°} \times \frac{22}{7} \times (7)^2$$

$$= \frac{40°}{360°} \times \frac{22}{7} \left[(14)^2 - (7)^2 \right]$$

$$= \frac{40°}{360°} \times \frac{22}{7} (14-7)(14+7)$$

$$= \frac{40°}{360°} \times \frac{22}{7} \times 7 \times 21$$

$$= \frac{154}{3} \text{ cm}^2$$