Sets

1. Write the following sets in the roaster from

   (i) $A = \{x : x \in \mathbf{R}, 2x + 11 = 15\}$ (ii) $B = \{x \mid x^2 = x, x \in \mathbf{R}\}$

   (iii) $C = \{x \mid x$ is a positive factor of a prime number $p\}$

2. Write the following sets in the roaster form :

   (i) $D = \{t \mid t^3 = t, t \in \mathbf{R}\}$ (ii) $E = \{w \mid \frac{w-2}{w+3} = 3, w \in \mathbf{R}\}$

   (iii) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in \mathbf{R}\}$

3. If $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number$\}$. Write Y in the roaster form.

4. State which of the following statements are true and which are false. Justify your answer.

   (i) $35 \in \{x \mid x$ has exactly four positive factors$\}$.

   (ii) $128 \in \{y \mid$ the sum of all the positive factors of y is $2y\}$

   (iii) $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$

   (iv) $496 \notin \{y \mid$ the sum of all the positive factors of y is $2y\}$.

5. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

   Verify that $L - (M \cup N) = (L - M) \cap (L - N)$

6. If A and B are subsets of the universal set U, then show that

   (i) $A \subset A \cup B$ (ii) $A \subset B \Leftrightarrow A \cup B = B$

   (iii) $(A \cap B) \subset A$

7. Given that $N = \{1, 2, 3, \dots, 100\}$. Then write

   (i) the subset of N whose elements are even numbers.

   (ii) the subset of N whose element are perfect square numbers.

8. If $X = \{1, 2, 3\}$, if n represents any member of X, write the following sets containing all numbers represented by

   (i) $4n$ (ii) $n + 6$ (iii) $\frac{n}{2}$ (iv) $n - 1$

9. If $Y = \{1, 2, 3, \ldots 10\}$, and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions.

(i) $a \in Y$ but $a^2 \notin Y$ (ii) $a + 1 = 6, a \in Y$

(iii) a is less than 6 and $a \in Y$

10. A, B and C are subsets of Universal Set U. If $A = \{2, 4, 6, 8, 12, 20\}$ $B = \{3, 6, 9, 12, 15\}$, $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

11. Let U be the set of all boys and girls in a school, G be the set of all girls in the school, B be the set of all boys in the school, and S be the set of all students in the school who take swimming. Some, but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationship among sets U, G, B and S.

12. For all sets A, B and C, show that $(A - B) \cap (C - B) = A - (B \cup C)$

Determine whether each of the statement in Exercises 13 – 17 is true or false. Justify your answer.

13. For all sets A and B, $(A - B) \cup (A \cap B) = A$

14. For all sets A, B and C, $A - (B - C) = (A - B) - C$

15. For all sets A, B and C, if $A \subset B$, then $A \cap C \subset B \cap C$

16. For all sets A, B and C, if $A \subset B$, then $A \cup C \subset B \cup C$

17. For all sets A, B and C, if $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.

Using properties of sets prove the statements given in Exercises 18 to 22

18. For all sets A and B, $A \cup (B - A) = A \cup B$

19. For all sets A and B, $A - (A - B) = A \cap B$

20. For all sets A and B, $A - (A \cap B) = A - B$

21. For all sets A and B, $(A \cup B) - B = A - B$

22. Let $T = \left\{x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Is T an empty set? Justify your answer.

23. Let A, B and C be sets. Then show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

24. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science; 4 in English and Science; 4 in all the three. Find how many passed

    (i) in English and Mathematics but not in Science

    (ii) in Mathematics and Science but not in English

    (iii) in Mathematics only

    (iv) in more than one subject only

25. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither?

26. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

27. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find

    (a) The number of families which buy newspaper A only.

    (b) The number of families which buy none of A, B and C

28. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows:

French = 17, English = 13, Sanskrit = 15

French and English = 09, English and Sanskrit = 4

French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study

    (i) French only

    (ii) English only

    (iii) Sanskrit only

    (iv) English and Sanskrit but not French

    (v) French and Sanskrit but not English

    (vi) French and English but not Sanskrit

    (vii) at least one of the three languages

    (viii) none of the three languages

Choose the correct answers from the given four options in each Exercises 29 to 43 (M.C.Q.).

29. Suppose $A_1, A_2, ..., A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, ..., B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j'S. then n is equal to

(A) 15 (B) 3 (C) 45 (D) 35

30. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively,

(A) 4, 7 (B) 7, 4 (C) 4, 4 (D) 7, 7

31. The set $(A \cap B')' \cup (B \cap C)$ is equal to

(A) $A' \cup B \cup C$ (B) $A' \cup B$ (C) $A' \cup C'$ (D) $A' \cap B$

32. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to

(A) $F_2 \cap F_3$ (B) $F_3 \cap F_4$

(C) $F_2 \cup F_5$ (D) $F_2 \cup F_3 \cup F_4 \cup F_1$

33. Let S = set of points inside the square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then

(A) $S \cap T \cap C = \phi$ (B) $S \cup T \cup C = C$

(C) $S \cup T \cup C = S$ (D) $S \cup T = S \cap C$

34. Let R be set of points inside a rectangle of sides a and b $(a, b > 1)$ with two sides along the positive direction of x-axis and y-axis. Then

(A) $R = \{(x, y) : 0 \le x \le a, 0 \le y \le b\}$

(B) $R = \{(x, y) : 0 \le x < a, 0 \le y \le b\}$

(C) $R = \{(x, y) : 0 \le x \le a, 0 < y < b\}$

(D) $R = \{(x, y) : 0 < x < a, 0 < y < b\}$

35. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Then, the number of students who play neither is

(A) 0 (B) 25 (C) 35 (D) 45

36. In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then the number of persons who read neither is

(A) 210 (B) 290 (C) 180 (D) 260

37. If $X = \{8^n - 7n - 1 \mid n \in \mathbf{N}\}$ and $Y = \{49n - 49 \mid n \in \mathbf{N}\}$. Then

(A) $X \subset Y$ (B) $Y \subset X$ (C) $X = Y$ (D) $X \cap Y = \phi$

38. A survey shows that 63% of the people watch a News Channel whereas 76% watch another channel. If x% of the people watch both channel, then

(A) $x = 35$ (B) $x = 63$ (C) $39 \le x \le 63$ (D) $x = 39$

39. If sets A and B are defined as

$A = \{(x, y) \mid y = \frac{1}{x}, 0 \ne x \in \mathbf{R}\}$ $B = \{(x, y) \mid y = -x, x \in \mathbf{R}\}$, then

(A) $A \cap B = A$ (B) $A \cap B = B$ (C) $A \cap B = \phi$ (D) $A \cup B = A$

40. If A and B are two sets, then $A \cap (A \cup B)$ equals

(A) A (B) B (C) ϕ (D) $A \cap B$

41. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$ $B = \{2, 4, \ldots, 18\}$ and $\mathbf{N}$ the set of natural numbers is the universal set, then $A' \cup ((A \cup B) \cap B')$ is

(A) ϕ (B) $\mathbf{N}$ (C) A (D) B

42. Let $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$

$P = \{x \mid x$ is a prime number less than 20$\}$. Then $n(S) + n(P)$ is

(A) 34 (B) 31 (C) 33 (D) 30

43. If X and Y are two sets and X′ denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to

(A) X (B) Y (C) ϕ (D) $X \cap Y$