CHAPTER

7 ALTERNATING CURRENT

Syllabus

- *Alternating currents, peak and RMS value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only).*
- *LCR series circuit, resonance; power in A.C. circuits, power factor, wattless current.*
- *AC generator and transformer.*

Chapter Analysis

List of Topics	2016		2017		2018
	D	OD	D	OD	D/OD
Alternating Current	–	–	–	–	1 Q (5 marks)
LCR series Circuit	1 Q (5 marks)	2 Q (3 marks)	–	–	–
AC Generator and Transformer	1 Q (5 marks)	1 Q (5 marks)	2 Q (5 marks)	1 Q (5 marks)	

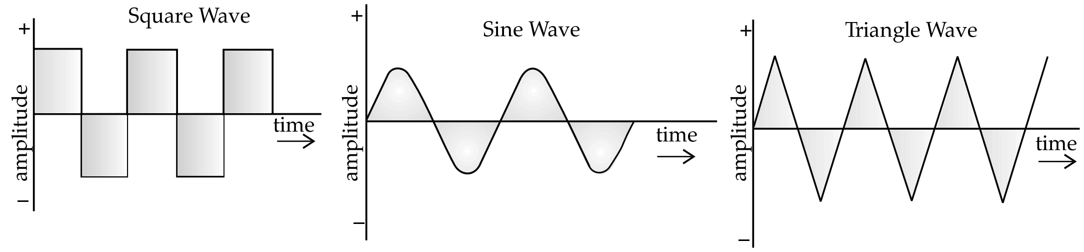
TOPIC-1
Alternating Current

Revision Notes

Alternating current

- Alternating current is that which changes continuously in magnitude and periodically in direction.
- It is represented by sine curve or cosine curve as $I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$ where, I_0 is peak value of current and I is instantaneous value of current.
- Frequency of an alternating current supply f, is the number of cycles which gets completed per second which is measured in Hertz (Hz). In India, it is 50 Hz.
- The time period T, of an alternating supply is time taken to complete one cycle.
- The behaviour of ohmic resistance R in *ac* circuit is the same as in *dc* circuit.
- Alternating current can be produced by using a device called an alternator.

- ***AC* waveforms are :**



Peak and *rms* value of alternating current/voltage

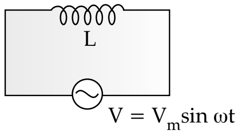
- Root mean square or *rms* is the root of mean of square of voltage or current in an *ac* circuit for one complete cycle denoted by V_{rms} or I_{rms}
- *rms* value is the standard way of measuring alternating current and voltage as it gives the *dc* equivalent values.
- *rms* value of *ac* is also called effective value or virtual value of *ac* represented as I_{rms}, I_{eff} or I_v shown as

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

- *rms* voltage value is the square root of averages of the squares of instantaneous voltages in a time varying waveform.

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\ V_0$$

- **AC voltage applied to pure inductive circuits :**



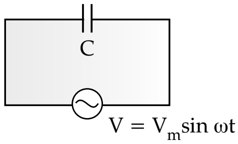
$$V = V_m \sin \omega t$$

$$i = i_m\left(\sin \omega t - \frac{\pi}{2}\right)$$ [Which shows current lags the voltage by $\frac{\pi}{2}$]

Average $P_L = \frac{i_m V_m}{2}[\sin (2\omega t)] = 0$ [Since average of sin $2\omega t$ over a complete cycle is zero]

Thus the average power supplied to an inductor over one complete cycle is zero.

AC applied to pure capacitive circuit :



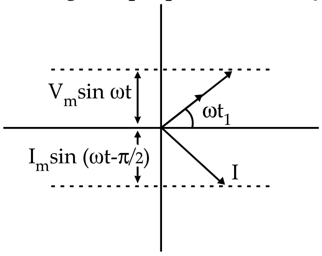
$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$ [Which shows current leads the voltage by $\frac{\pi}{2}$]

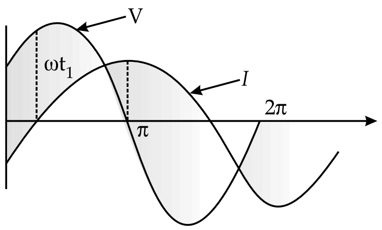
Average $P_C = \frac{I_m V_m}{2}\sin(2\omega t) = 0$ [Since average of sin $2\omega t$ over a complete cycle is zero]

Thus the average power supplied to an capacitor over one complete cycle is zero.

- **Phasor-diagram :** A phasor diagram represents sinusoidal *ac* current and sinusoidal voltage in a circuit, and the phase difference between current and voltage. The length of phasor is proportional to the instantaneous of *V* & *I* and the maximum length is proportional to V_0 & I_0.



Phasor diagram of pure Inductive circuit



Graphical representation of *V* & *i* versus ω*t*.

Reactance and impedance

- When an *ac* current is passed through a resistance, a voltage drop is produced which is in phase with the current and is measured in ohms (Ω).
- Reactance is the inertia against the motion of electrons where an alternating current after passing through it produces a voltage drop which is 90° out of phase with the current.
- Reactance is shown by "*X*" and is measured in ohms (Ω).
- Reactance is of two types : inductive and capacitive.

- Inductive reactance is linked with varying magnetic field that surrounds a wire or a coil carrying a current.
- Inductive reactance (X_L) is the resistance offered by an inductor and is given by $X_L = \omega L = 2\pi fL$
- Through a pure inductor, alternating current lags behind the alternating *emf* by phase angle of 90°.
- Capacitive reactance is linked with changing electric field between two conducting surfaces separated from each other by an insulating medium.
- Capacitive reactance (X_C) is the resistance offered by a capacitor and is given by

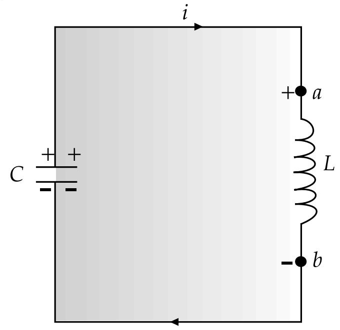
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

- Through a pure capacitor, alternating current leads the alternating *emf* by a phase angle of 90°.
- Impedance is the comprehensive expression of all forms of opposition to electron flow, including resistance and reactance, where an alternating current after passing through it produces a voltage drop between 0° and 90° which will be out of phase with current given as,

$$Z = \sqrt{R^2 + X^2}$$

where, Z = impedance of circuit, R = resistance, X = reactance

***LC* Oscillations (qualitative treatment only)**



- *LC* circuit comprises of inductor and capacitor connected in series where energy from the cell is given to capacitor which will keep on oscillating between *L* & *C*.
- When *ac* voltage is applied to the capacitor, it will keep on charging and discharging continuously.
- When capacitor is fully charged, it starts discharging and charge gets transferred to the inductor which is connected to capacitor.
- Due to change in current, there will be change in magnetic flux of the inductor in the circuit which induces an *emf* in the inductor.
- The *emf* is given by $e = -L\frac{dI}{dt}$ which try to oppose the growth of the current.
- When capacitor gets completely discharged, all the energy stored in it will now store in an inductor as a result of which, inductor will start charging the capacitor and energy stored in the capacitor will start increasing.
- As there is no current in the circuit, energy in the inductor is zero, so total energy of *LC* circuit will be

$$U_E = \frac{1}{2}.\frac{q^2}{C}$$

- **Band Width :** It is the range of angular frequencies over which the average power is greater than ½ the maximum value of average power.
- Impedance : Is an *ac* analog to resistance in a *dc* circuit that measures the combined effect of resistance, capacitive reactance, and inductive reactance.

Key Formulae

- $$I_{rms} = \frac{I_0}{\sqrt{2}}$$
- $$V_{rms} = \frac{V_0}{\sqrt{2}}$$
- $$P = V_{rms}I_{rms}$$

- In pure inductive circuit if, $V = V_m \sin \omega t$

  $i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$, where $i_m = \frac{V_m}{X_L}$ and $X_L = \omega L$

  $(P_{avg})_L = 0$
- In pure capacitive circuit if, $V = V_m \sin \omega t$

  $i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$ where, $i_m = \frac{V_m}{X_C}$ and $X_C = \frac{1}{\omega C}$
- Average Power $= \frac{1}{2} V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi$ (where, $\cos\phi = \frac{R}{Z}$ is power factor)
- $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- $emf = e = -L\frac{dI}{dt}$
- Energy in LC circuit, $U_E = \frac{1}{2}\frac{q^2}{C}$

Objective Answer Type Questions **(1 mark each)**

Q. 1. If the *rms* current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is

(a) $5\sqrt{2}\,A$. **(b)** $5\sqrt{\frac{3}{2}}A$.

(c) $\frac{5}{6}A$. **(d)** $\frac{5}{\sqrt{2}}A$.

[NCERT Exemplar]

Ans. Correct option : (b)

Explanation :

Here, $I_{rms} = 5$ A, $\nu = 50$ Hz and $t = \frac{1}{300}$ s

I_0 = Peak value $= \sqrt{2} I_{rms} = \sqrt{2} \times 5 = 5\sqrt{2}\ A$

Now, $I = I_0 \sin \omega t = 5\sqrt{2}\ \sin 2\pi\nu t$

$= 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300} = 5\sqrt{\frac{3}{2}}$ A

Q. 2. An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to

(a) zero. **(b)** X_g.

(c) $-X_g$. **(d)** R_g.

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : As internal resistance of generator is already equal to external resistance R_g. So to deliver maximum power, *i.e.*, to make reactance equal to zero, the reactance in external circuit will be $-X_g$. In order to deliver maximum power, the generator to the load, the total reactance must be equal to zero, *i.e.*, $X_L + X_g = 0$ or $X_L = -X_g$.

Q. 3. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. this means

(a) Input voltage cannot be AC voltage, but a DC voltage.

(b) Maximum input voltage is 220 V.

(c) The meter reads not V but $<V^2>$ and is calibrated to read $\sqrt{<V^2>}$.

(d) The pointer of the meter is stuck by some mechanical defect. **[NCERT Exemplar]**

Ans. Correct option : (c)

Explanation : The voltmeter in AC circuit reads value $<V^2>$ and meter is calibrated to rms value $<V^2>$ which is multiplied by $\sqrt{2}$ to get V_{rms}. In other words, voltmeter connected to the AC main read root mean square value of AC voltage, *i.e.*, $\sqrt{<V^2>}$.

Q. 4. To reduce the resonant frequency in an *L-C-R* series circuit with a generator

(a) the generator frequency should be reduced.

(b) another capacitor should be added in parallel to the first.

(c) the iron core of the inductor should be removed.

(d) dielectric in the capacitor should be removed.

[NCERT Exemplar]

Ans. Correct option : (b)

Explanation : The resonant frequency of *L-C-R* series circuit is $\nu_o = \frac{1}{2\pi\sqrt{LC}}$

So to reduce resonant frequency, we have either to increase L or to increase C.

To increase capacitance another capacitor must be connected in parallel with the first.

Very Short Answer Type Questions (1 mark each)

Q. 1. Define capacitive reactance. Write its S.I. unit. R **[Delhi I, II, III 2015]**

Ans. It is defined as the opposition to the flow of current in ac circuits offered by a capacitor.

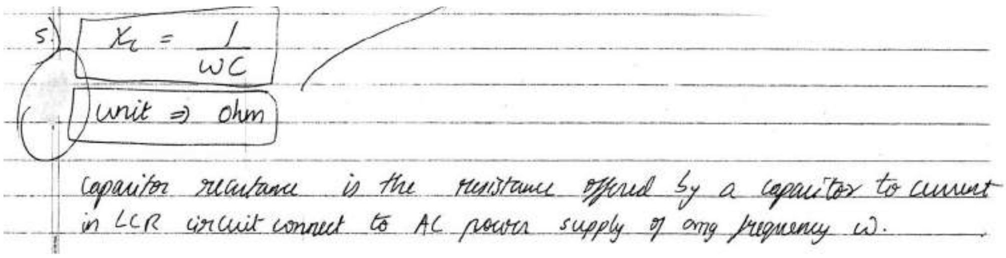
Alternatively,

$$X_c = \frac{1}{\omega C}$$ ½

S.I. unit : ohm (Ω). **[CBSE Marking Scheme 2015]** ½

OR

Ans.


$X_c = \frac{1}{\omega C}$

unit ⇒ ohm

Capacitor reactance is the resistance offered by a capacitor to current in LCR circuit connect to AC power supply of any frequency ω.

[Topper's Answer 2015]

Q. 2. State which of the two a capacitor or an inductor, tends to become SHORT when the frequency of the applied alternating voltage has a high value. U **[CBSE SQP 2014]**

Ans. The capacitor. **[CBSE Marking Scheme 2014] 1**

Detailed Answer :

Between inductor and capacitor, the capacitor will act short at high frequencies while will act open at low frequencies. So at high frequencies, capacitor has time to get a very less amount of charge while at low frequencies, there is lot of time for capacitor to collect considerable amount of charge which shows presence of large reverse voltage. Hence the sum of the voltages of capacitor and *ac* source will be nearly zero which shows current also to be nearly zero.

AI **Q. 3. Why is the use of *ac* voltage preferred over *dc* voltage ? Give two reasons.** R **[OD. I, II, III 2014]**

Ans. Any two of the following (or any other correct) reasons :

(i) *ac* can be transmitted with much lower energy losses as compared to *dc*.

(ii) *ac* voltage can be adjusted (stepped-up or stepped down) as per requirement.

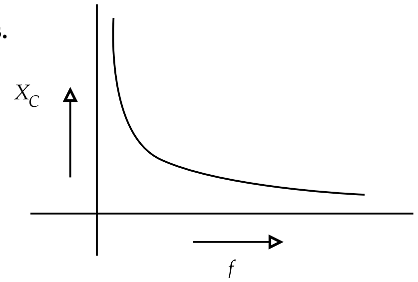
(iii) *ac* current in a circuit can be controlled using (almost) wattless devices like choke coil.

(iv) *ac* is easier to generate.

[CBSE Marking Scheme 2014] ½ + ½

AI **Q. 4. Plot a graph showing variation of capacitive reactance with the change in the frequency of the *ac* source.** U

Ans.



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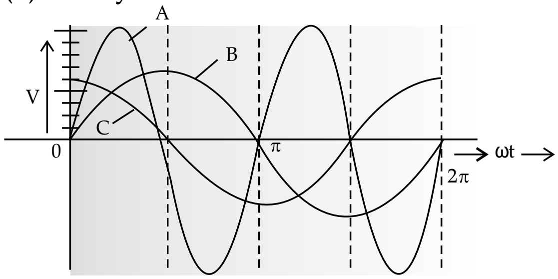
Q. 5. A device '*X*' is connected to an *ac* source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one complete cycle is shown in the following figure. U **[CBSE SQP 2015-2016]**

(i) Which curve shows power consumption over a full cycle?

(ii) Identify the device '*X*'.



Ans. (i) *A* ½

(ii) Capacitor **[CBSE Marking Scheme 2016]** ½

Detailed Answer :

(i) Power's frequency is double of current or voltage frequency.

(ii) From the graph, the phase difference between *V* and *I* is $\frac{\pi}{2}$ so device '*X*' may be an inductor (*L*) or capacitor (*C*) but since current (graph *C*) leads the voltage (graph *B*), device is capacitor.

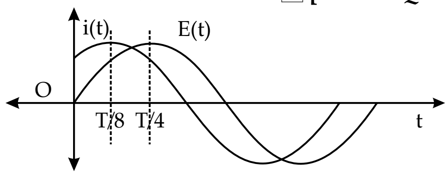
Q. 6. Why is choke coil needed in the use of fluorescent tubes with *ac* mains ? U **[Delhi I, II, III 2014]**

Ans. Choke coil reduces the voltage across the fluorescent tube without wastage of power. **1**

[CBSE Marking Scheme 2014]

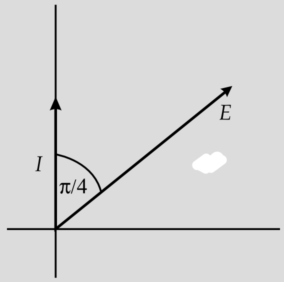
Short Answer Type Questions-I (2 marks each)

Q. 1. The figure shows two sinusoidal curves representing oscillating supply voltage and current in an *ac* circuit. U **[CBSE SQP 2018-19]**



Draw a phasor diagram to represent the current and supply voltage appropriately as phasors. State the phase difference between the two quantities. A

Ans.



Equal length of phasors ½

Current leads voltage ½

phase difference is $\frac{\pi}{4}$ **1**

[CBSE Marking Scheme, 2018]

AI **Q. 2. A lamp is connected in series with a capacitor. Predict your observation for *dc* and *ac* connections. What happens in each if the capacitance of the capacitor is reduced ?** U **[SQP II 2017]**

Ans. When a *dc* source is connected to a capacitor, the capacitor gets charged and after charging, no current flows in the circuit and the lamp will not glow. There will be no change even if *C* is reduced.

With an *ac* source, the capacitor offers capacitive reactance and the current flows in the circuit. Consequently the lamp will shine. Reducing *C* will increase the reactance and the lamp will shine less brightly than before. **2**

[CBSE Marking Scheme 2017]

Q. 3. Show that the current leads the voltage in phase by $\frac{\pi}{2}$ in an *ac* circuit containing an ideal capacitor. U **[Foreign 2014]**

Ans. If $V = V_0 \sin \omega t$ ½

$q = CV = CV_0 \sin \omega t$ ½

$\therefore \quad I = \frac{dq}{dt} = \omega CV_0 \cos \omega t$ ½

or $\quad I = \omega CV_0 \sin (\omega t + \frac{\pi}{2})$

So, the current leads the applied voltage, in phase by $\frac{\pi}{2}$.

[CBSE Marking Scheme 2014] ½

AI **Q. 4. An alternating voltage given by $V = 140 \sin 314\, t$ is connected across a pure resistor of 50 Ω. Find :**

(i) the frequency of the source.

(ii) the *rms* current through the resistor.

A **[O.D. I, II, III 2012]**

Ans. (i) $2\pi f = \omega$

$\Rightarrow \quad 2\pi f = 314 \text{ rad } s^{-1}$ ½

$\Rightarrow \quad f = 50 \text{ Hz}$ ½

(ii) $I_{rms} = \frac{V_{rms}}{R}$ where, $V_{rms} = \frac{V_0}{\sqrt{2}}$ ½

$I_{rms} = \frac{140}{\sqrt{2} \times 50} = 1.98\, A \approx 2\, A$ ½

[CBSE Marking Scheme 2012]

Short Answer Type Question-II (3 marks)

Q. 1. Show that in the free oscillations of an *LC* circuit, the sum of energies stored in the capacitor and the inductor is constant in time. U **[CBSE SQP 2018-19]**

Ans. At an instant *t*, charge *q* on the capacitor and the current *i* are given by :

$$q(t) = q_0 \cos \omega t$$
$$i(t) = -q_0 \omega \sin \omega t$$

Energy stored in the capacitor at time *t* is

$$U_K = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{q_0^2}{2C}\cos^2(\omega t) \quad \textbf{1}$$

Energy stored in the inductor at time *t* is

$$U_M = \frac{1}{2}Li^2$$
$$= \frac{1}{2}Lq_0^2\omega^2 \sin^2(\omega t)$$
$$= \frac{q_0^2}{2C}\sin^2(\omega t) \quad \left(\because \omega = 1/\sqrt{LC}\right) \quad \textbf{1}$$

Sum of energies

$$U_E + U_M = \frac{q_0^2}{2C}(\cos^2 \omega t + \sin^2 \omega t)$$

$$= \frac{q_0^2}{2C}$$

This sum is constant in time as q_0 and C, both are time-independent. **1**

[CBSE Marking Scheme 2017]

AI **Q. 2. Obtain the expression for the energy density of magnetic field B produced in the inductor.**

U **[Delhi Comptt. 2016]**

Ans. Instantaneous Induced *emf* in an inductor when current changes through it

$$e = -L\frac{dI}{dt}$$

Hence, instantaneous applied voltage

$$e = V = L\frac{dI}{dt} \qquad ½$$

Work done, $dW = V.\,dq = VIdt$

$\therefore \quad dW = LIdI \qquad ½$

$$\Rightarrow \quad \int dW = \int_0^I LIdI$$

$$W = \frac{1}{2}LI^2 \qquad ½$$

Energy density, $U = \dfrac{\text{total energy stored}}{\text{volume}} \qquad ½$

$$U = \frac{\left(\frac{1}{2}\right)LI^2}{Al} = \frac{\frac{1}{2}(LI)I}{Al}$$

$$\text{Flux} = NBA = LI$$

and $B = \dfrac{\mu_0 NI}{l} \Rightarrow I = \dfrac{Bl}{\mu_0 N} \qquad ½$

$$\therefore \quad U = \frac{\frac{1}{2}(NBA).\frac{Bl}{\mu_0 N}}{Al} = \frac{B^2}{2\mu_0} \qquad ½$$

[CBSE Marking Scheme 2016]

Long Answer Type Questions

(5 marks each)

Q. 1. (i) Prove that current flowing through an ideal inductor connected across *ac* source, lags the voltage in phase by $\frac{\pi}{2}$.

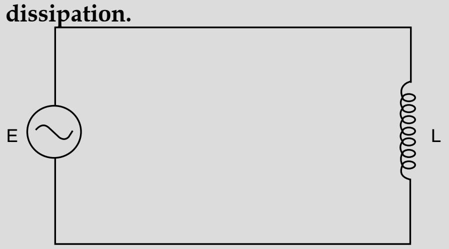
(ii) An inductor of self inductance 100 mH, and a bulb are connected in series with *ac* source of *rms* voltage 10 V, 50 Hz. It is found that effective voltage of the circuit leads the current in phase by $\frac{\pi}{4}$. Calculate the inductance of the inductor used and average power dissipated in the circuit, if a current of 1 A flows in the circuit.

U **[Delhi Comptt. I, II, III 2017]**

Ans. (i) Showing that current lags voltage by an angle $\frac{\pi}{2}$ in an ideal inductor **3**

(ii) Calculation of induction and average power dissipation. **2**

(i)



½

Induced *emf*, $e = -L\dfrac{dI}{dt} \qquad ½$

Hence, Net voltage in the circuit

$$= V - L\frac{dI}{dt} \qquad ½$$

According to Kirchhoff's Rule

$$V - L\frac{dI}{dt} = 0 \qquad ½$$

$$V_m \sin \omega t = L\frac{dI}{dt}$$

$$dI = \frac{V_m}{L}\sin \omega t dt$$

$$I = -\frac{V_m}{\omega L}\cos \omega t \qquad ½$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\therefore \quad I = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ½$$

Hence current lags by $\frac{\pi}{2}$

(ii) Inductance of the inductor = 100 mH

Average power dissipation,

$$P = V_{rms} I_{rms} \cos\phi \qquad ½$$

$$= 10 \times 1 \times \cos\frac{\pi}{4}$$

$$= \frac{10}{\sqrt{2}}W \qquad ½$$

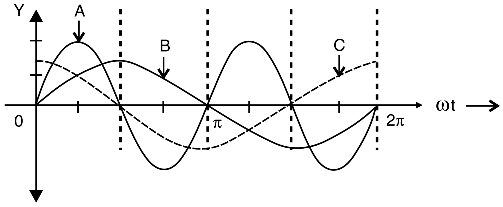
$$= 5\sqrt{2} \text{ Watts or } (7.05 \text{ W}) \qquad 1$$

[CBSE Marking Scheme, 2017]

Answering Tip

- Self inductance and inductance are the same terms. Hence, inductance of the inductor = 100 mH

Q. 2. A device '*X*' is connected to an *ac* source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph :



(i) Identify the device 'X'.

(ii) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit ? Justify your answer.

(iii) How does its impedance vary with frequency of the *ac* source ? Show graphically.

(iv) Obtain an expression for the current in the circuit and its phase relation with *ac* voltage.

U **[CBSE SQP 2017]**

Ans. (i) Identification ½

(ii) Identifying the curves ½

Justification ½

(iii) Variation of Impedance with frequency ½

Graph ½

(iv) Expression for current 2

Phase relation ½

(i) The device X is a capacitor ½

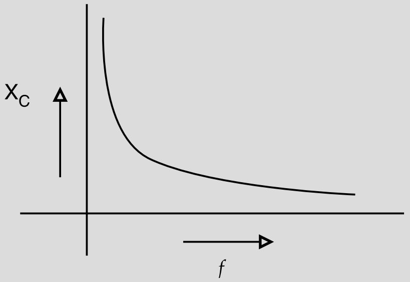
(ii) Curve $B \rightarrow$ voltage

Curve $C \rightarrow$ current

Curve $A \rightarrow$ power ½

Reason : The current leads the voltage in phase, by $\frac{\pi}{2}$, for a capacitor. ½

(iii) $X_c = \frac{1}{\omega C}$ or $\left(X_c \propto \frac{1}{\omega}\right)$ ½



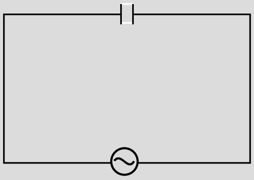
½

(iv)

$$V = V_0 \sin \omega t$$

$$q = VC = CV_0 \sin \omega t \quad ½$$

$$I = \frac{dq}{dt} = \omega C V_0 \cos \omega t \quad ½$$



$$V = V_0 \sin \omega t \quad ½$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad ½$$

Current leads the voltage, in phase, by $\frac{\pi}{2}$ ½

[**Note :** If the student identifies the device X as an Inductor but writes correct answer to parts (c) and (d) (in terms of an inductor), the student be given full marks for (only) these two parts]

[CBSE Marking Scheme 2017]

AI **Q. 3. (i) An *ac* source generating a voltage $V = V_0 \sin \omega t$ is connected to a capacitor of capacitance C. Find the expression of the current I flowing through it. Plot a graph of V and I versus ωt to show that the current is $\frac{\pi}{2}$ ahead of the voltage.**

(ii) A resistor of 200 Ω and a capacitor of 15 μF are connected in series to a 220 V, 50 Hz *ac* source. Calculate the current in the circuit and the *rms* voltage across the resistor and the capacitor. Why the algebraic sum of these voltages is more than the source voltage ? A **[CBSE SQP 2016]**

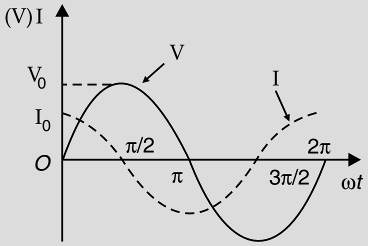
Ans. (i)

$$V = V_0 \sin \omega t, \; V = \frac{Q}{C} \quad ½$$

$$I = \frac{dQ}{dt} \quad ½$$

$$I_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \quad ½$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad ½$$



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(ii)

$$X_C = \frac{1}{2\pi f C} = 212.3\ \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = 291.5\ \Omega \quad ½$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{291.5} = 0.755\ \text{A} \quad ½$$

$V_R\,(rms) = 151$ V

$V_C\,(rms) = 160.3$ V ½

Two voltages are out of phase, hence they are added vectorially. ½

[CBSE Marking Scheme 2016]

Q. 4. A device X is connected across an *ac* source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

(a) Identify the device X and write the expression for its reactance.

(b) Draw graphs showing variation of voltage and current with time over one cycle of *ac*, for X.

(c) How does the reactance of the device X vary with frequency of the *ac* ? Show this variation graphically.

(d) Draw the phasor diagram for the device *X*.

U **[Delhi & O.D. 2018]**

Ans. (b) Graphs of voltage and current with time 1+1

(c) Variation of reactance with frequency ½

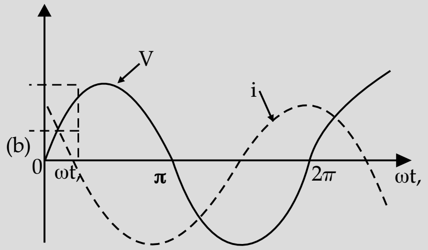
(Graphical variation) ½

(d) Phasor Diagram 1

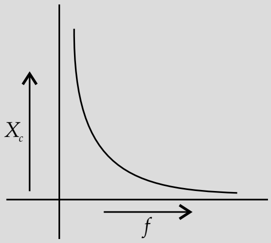
(a) X : Capacitor ½

Reactance, $X_c = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ ½

(b) ½+½



(c) Reactance of the capacitor varies in inverse proportion to the frequency *i.e.*, $X_c \propto \dfrac{1}{f}$ **1**



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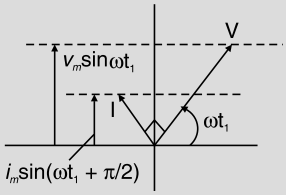

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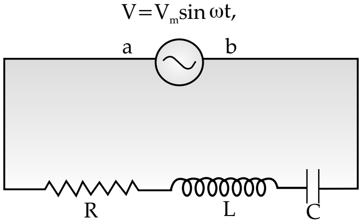
[CBSE Marking Scheme 2018]

TOPIC-2
LCR Series Circuit

Revision Notes

***LCR* series circuit**

➢ In an *LCR* series circuit with resistor, inductor and capacitor, the expression for the instantaneous potential difference between the terminals *a* and *b* is given as



➢ The potential difference in this will be equal to the sum of the potential differences across *R*, *L* and *C* elements as

$$V = V_m \sin \omega t = RI + L\frac{dI}{dt} + \frac{1}{C}q$$

where, q is the charge on capacitor.

➢ The steady state solution will be

$$i = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi) \text{ and } i_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{C}\right)}}$$

where, $\phi = \tan^{-1} \dfrac{\omega L - \frac{1}{\omega C}}{R}$

➢ From the equation, steady-state current like terminal voltage, varies sinusoidal with time, so steady-state current can be written as $I = I_m \sin(\omega t - \phi)$

➢ In an *LCR* circuit :

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = R^2 + X^2$$

$$I_m = \frac{V_m}{R^2 + (X_L - X_C)^2} = \frac{V_m}{R^2 + X^2} = \frac{V_m}{Z}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + X^2}} = \frac{V_m}{Z}$$

Here, Z = impedance of the circuit, X = reactance, X_L and X_C = inductive and capacitive reactance.

- For steady-state currents, maximum current I_m is related to maximum potential difference V_m by

$$I_m = \frac{V_m}{Z}$$

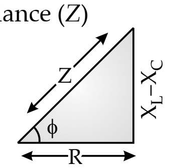
- Total effective resistance of *RLC* circuit is called Impedance (Z) of the circuit given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The angle by which alternating voltage leads the alternating current in *RLC* circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$

- In an *RLC* circuit, impedance triangle is a right-angled triangle in which base is ohmic resistance R, perpendicular is reactance ($X_L - X_C$) and hypotenuse is impedance (Z)



- When a condenser of capacity C charged to certain potential is connected to inductor L, energy stored in C oscillates between L and C where frequency of energy oscillations is given by

$$X_L = X_C \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

- In *LCR* circuit, if there is no loss of energy, then total energy in L and C at every instant will remain constant.
- Sign for phase difference (ϕ) between I and E for a series *LCR* circuit :

ϕ is positive, when $X_L > X_C$.
ϕ is negative, when $X_L < X_C$.
ϕ is zero, when $X_L = X_C$.
$\phi = \pi/2$, when $\omega = \infty$.
$\phi = -\pi/2$, when $\omega = 0$.

Resonance

- Circuit in which inductance L, capacitance C and resistance R are connected in series and the circuit admits maximum current, then such circuit is called as series resonant circuit.
- The necessary condition for resonance in *LCR* series circuit is : $V_C = V_L$

$X_L = X_C$ which gives $\omega^2 = \frac{1}{LC}$ or $f = \frac{1}{2\pi\sqrt{LC}}$

- In this, frequency of *ac* fed to circuit will be equal to natural frequency of energy oscillations in the circuit under conditions,

$$Z = R$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$$

- The sharpness of tuning at resonance is measured by Q factor or quality factor of the circuit given as

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- At series *LCR* resonance or acceptor circuit, current is maximum.

$$I_{\max} = \frac{E}{R}$$

Power in AC circuits

- When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives only what is known as apparent power.
- If. the instantaneous values of the voltage and current in an *ac* circuit are given by

$$E = E_0 \sin \omega t$$
$$i = i_0 \sin(\omega t - \phi)$$

where ϕ is the phase difference between voltage and the current. Then the instantaneous power

$$P_{in} = E \times i = E_0 i_0 \sin \omega t . \sin (\omega t - \phi)$$

or average power $$P_{avg} = \frac{1}{2} E_0 i_0 \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi = V_{rms} \times I_{rms} \times \cos \phi$$

where, cos ϕ is known as power factor.

- Power factor (cos ϕ) is important in power systems as it shows how closely the effective power equals the apparent power given as :

$$\cos \phi = \frac{\text{effective power}}{\text{apparent power}}$$

- The value of power factor varies from 0 to 1.
- The instantaneous rate at which energy is supplied to an electrical device by *ac* circuit is

$$P = VI$$

- Average power in *RLC* where, $X_L = X_C$ over a complete cycle in a non inductive circuit or pure resistive circuit is given as $$P = V_0 I_0 \text{ or } I_0^2 R$$

Wattless Current

- The average power associated over a complete cycle with pure inductor or pure capacitor is zero which makes current through *L* and *C* as wattless or idle current.
- In *LCR* circuit at resonance, the power loss is maximum, so
- Wattless component of current = $I_{rms} \sin \phi$
- Power component of current = $I_{rms} \cos \phi$
- **Phase angle :** It is the amount by which the voltage and current are out of phase with each other in a circuit.
- **Power factor :** It is the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase.
- **Quality factor :** It is a dimensionless quantity that shows sharpness of the peak of bandwidth.
- **Resonant frequency :** It is the frequency at which the amplitude of the current is maximum where circuit oscillates when not driven by voltage source.

Know the Formulae

- Impedance for a series *LCR* circuit,

$$Z = \sqrt{R^2 + X^2} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}.$$

- Average power, $$P = \frac{E_0 I_0}{2} \cos \phi = V_{rms} I_{rms} \cos \phi$$
- Power factor, $$PF = \cos \phi = \frac{\text{Resistance}}{\text{Impedance}} = \frac{R}{Z} = \frac{\text{True power}}{\text{Apparent power}}$$
- Quality factor $$\phi = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Objective Type Question (1 mark)

Q. 1. Which of the following combinations should be selected for better tuning of an *L-C-R* circuit used for communication?

(a) $R = 20\ \Omega, L = 1.5\ H, C = 35\ \mu F$.
(b) $R = 25\ \Omega, L = 2.5\ H, C = 45\ \mu F$.
(c) $R = 15\ \Omega, L = 3.5\ H, C = 30\ \mu F$.
(d) $R = 25\ \Omega, L = 1.5\ H, C = 45\ \mu F$.

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : Quality factor (*Q*) of an *L-C-R* circuit is given by, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Tuning of an *L-C-R* circuit depends on quality factor of the circuit. Tuning will be better when quality factor of the circuit is high.

As, quality factor (*Q*) of an *L-C-R* circuit is given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

For *Q* to be high, *R* should be low, *L* should be high and *C* should be low.

Very Short Answer Type Questions (1 mark each)

Q. 1. For an ideal conductor, connected across a sinusoidal *ac* voltage source. State which one of the following quantity is zero :

(i) Instantaneous power

(ii) Average power over full cycle of the *ac* voltage source. U **[Foreign 2016]**

Ans. Average power over full cycle of *ac* voltage source is zero. **1**

[CBSE Marking Scheme 2016]

Q. 2. Define 'Wattless current'.

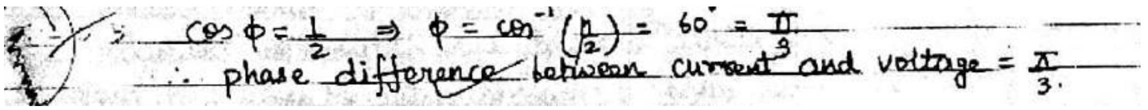
R **[Delhi Comptt. I, II, III 2014, Delhi I, II, III 2011]**

Ans. Current flowing in a circuit without any net dissipation of power, is called wattless current. **1**

[CBSE Marking Scheme 2014]

Q. 3. The power factor of an *ac* circuit is 0.5. What is the phase difference between voltage and current in the circuit ? A **[O.D. I, 2016]**

Ans.

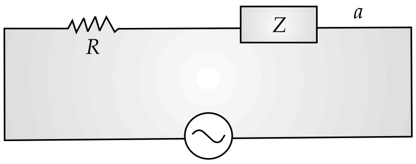

$\therefore \cos\phi = \frac{1}{2} \Rightarrow \phi = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$

$\therefore$ phase difference between current and voltage $= \frac{\pi}{3}$.

[Topper's Answer 2016]

Short Answer Type Questions-I (2 marks each)

Q. 1. An alternating voltage $E = E_0 \sin \omega t$ is applied to the circuit containing a resistor R connected in series with a black box. The current in the circuit is found to be $I = I_0 \sin\left(\omega t + \frac{\pi}{4}\right)$



$E = E_0 \sin \omega t$

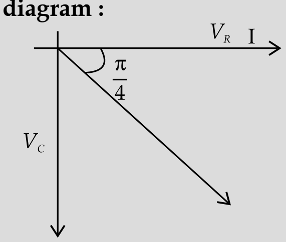
(i) State whether the element in the black box is a capacitor or inductor.

(ii) Draw the corresponding phasor diagram and find the impedance in terms of *R*.

U **[CBSE SQP 2017-18]**

Ans. (i) As the current leads the voltage by $\frac{\pi}{4}$, the element used in black box is a 'capacitor'. ½

(ii) Phasor diagram : ½



$$\tan\frac{\pi}{4} = \frac{V_C}{V_R}$$

$$V_C = V_R$$

$$X_C = R$$

Impedance, $Z = \sqrt{(X_C^2 + R^2)}$ ½

$Z = R\sqrt{2}$ ½

[CBSE Marking Scheme 2017]

AI Q. 2. In a series *LCR* circuit, obtain the conditions under which (i) the impedance of the circuit is minimum, and (ii) wattless current flows in the circuit. U **[Foreign 2014]**

Ans. (i) The impedance of a series *LCR* circuit is given by

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ ½

or For Z to be minimum, $X_L = X_C$ (or $\omega = \frac{1}{\sqrt{LC}}$) ½

(ii) For wattless current to flow, circuit should not have any ohmic resistance, *i.e.* $R = 0$.

Alternatively, Power $= V_{rms} I_{rms} \cos\phi$

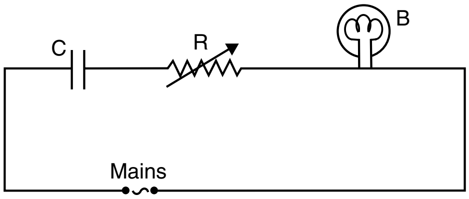
for $\phi = 90^\circ = \frac{\pi}{2}$

Power $= 0$

$\therefore$ Wattless current flows when the impedance of the circuit is purely inductive/capacitive or the combination of the two. **1**

[CBSE Marking Scheme 2014]

Q. 3. A capacitor 'C', a variable resistor 'R' and a bulb 'B' are connected in series to the *ac* mains in circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be the same; (ii) the resistance R is increased keeping the same capacitance ? U



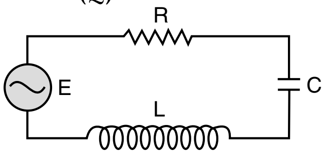
Ans. (i) When a dielectric slab is introduced in between the plates of capacitor, its capacitance will increase which tends to decrease the potential drop across the capacitor, *i.e.*, $V = \frac{Q}{C}$. As a result of this, potential drop across bulb will increase as they are connected in series, so the brightness of the bulb will increase. **1**

(ii) As the resistance R increases, potential drop across the resistor will also increase which lead to decrease in potential drop across the bulb, since it is connected in series. Hence, the brightness of the bulb will decrease. **1**

Q. 4. The figure shows a series LCR circuit connected to a variable frequency of 200 V source with L = 50 mH, C = 80 μF and R = 40 Ω find.

(i) the source frequency which drives the circuit in resonance;

(ii) the quality factor (Q) of the circuit.



A **[O.D. Comptt. I, II, III 2014]**

Ans. (i)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{50\times10^{-3}\times80\times10^{-6}}} = 500 \text{ rad/s}$$ ½

i.e., $$\nu_0 = \frac{500}{2\pi} = \frac{250}{\pi} \text{ Hz} \approx 80 \text{ Hz}$$ ½

(ii) $$Q = \frac{\omega_0 L}{R}$$

$$Q = \frac{500\times50\times10^{-3}}{40}$$ ½

$$Q = 0.625$$ ½

[CBSE Marking Scheme 2014]

Short Answer Type Questions-II

(3 marks each)

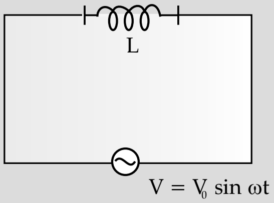
AI **Q. 1. A source of *ac* voltage $V = V_0 \sin \omega t$, is connected across a pure inductor of inductance L. Derive the expressions for the instantaneous current in the circuit. Show that average power dissipated in the circuit is zero.** U **[Foreign I, II, III 2017]**

Ans. Derivation of instantaneous current 2

Derivation of average power dissipated 1

Given : $V = V_0 \sin \omega t$

$$V = L\frac{di}{dt} \Rightarrow di = \frac{V}{L}dt$$



$$\therefore \quad di = \frac{V_0}{L}\sin \omega t dt$$ ½

Integrating, $$i = -\frac{V_0}{\omega L}\cos \omega t$$ ½

$$\therefore i = -\frac{V_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right) = I_0 \sin\left(\frac{\pi}{2} - \omega t\right)$$ ½

where, $$I_0 = \frac{V_0}{\omega L}$$

Average power,

$$P_{av} = \int_0^T Vi dt$$ ½

$$= \frac{-V_0^2}{\omega L}\int_0^T \sin \omega t \cos \omega t \, dt$$

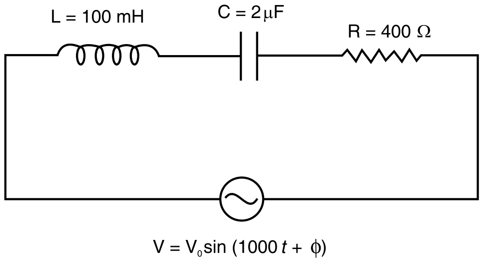
$$= \frac{-V_0^2}{2\omega L}\int_0^T \sin(2\omega t) dt$$

$$= 0$$ 1

[CBSE Marking Scheme 2017]

Q. 2. (i) Find the value of the phase difference between the current and the voltage in the series LCR circuit shown below. Which one leads in phase : current or voltage ?

(ii) Without making any other change, find the value of the additional capacitor C_1, to be connected in parallel with the capacitor C, in order to make the power factor of the circuit unity.



U **[Delhi I, II, III 2017]**

Ans. (i) Calculation of phase difference between current and voltage **1**

Name of quantity which leads **½**

(ii) Calculation of value of '*C*', is to be connected in parallel **1½**

(i) $X_L = \omega L = (1000 \times 100 \times 10^{-3})\ \Omega$

$= 100\ \Omega$

$$X_C = \frac{1}{\omega C} = \left(\frac{1}{1000 \times 2 \times 10^{-6}}\right)\Omega$$

$= 500\ \Omega$ ½

Phase angle

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{100 - 500}{400} = -1$$

$$\phi = -\frac{\pi}{4}$$ ½

As $X_C > X_L$, (phase angle is negative), hence current leads voltage ½

(ii) To make power factor unity ½

$$X_C' = X_L$$

$$\frac{1}{\omega C'} = 100$$

$C' = 10\ \mu F$ ½

$$C' = C + C_1$$

$$10 = 2 + C_1$$

$C_1 = 8\ \mu F$ ½

[CBSE Marking Scheme 2017]

AI **Q. 3. A sinusoidal voltage of peak value 10 V is applied to a series *LCR* circuit in which resistance, capacitance and inductance have values of 10 Ω, 1 μF and 1 H respectively. Find (i) the peak voltage across the inductor at resonance (ii) quality factor of the circuit.**

U **[CBSE SQP 2018-19]**

Ans. $I_0 = V_0/R = 10/10 = 1\ A$ ½

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1 \times 1 \times 10^{-6})}}$$ ½

$= 10^3$ rad/s

$V_0 = I_0 X_L = I_0 \omega_r L$ ½

$= 1 \times 10^3 \times 1 = 10^3$ V ½

$Q = \omega_r L/R$ ½

$= (10^3 \times 1)/10 = 100$ ½

[CBSE Marking Scheme 2018]

Q. 4. A capacitor of unknown capacitance, a resistor of 100 Ω and an inductor of self inductance $L = (4/\pi^2)$ henry are connected in series to an *ac* source of 200 V and 50 Hz. Calculate the value of the capacitance and impedance of the circuit when the current is in phase with the voltage. Calculate the power dissipated in the circuit.

U **[O.D. I, 2016]**

Ans. $R = 100\,\Omega$, $L = \frac{4}{\pi^2}$ H, C, $V_{rms} = 200$ V, f = 50 Hz, C = ?, Z = ?

$\phi = 0°$, power = ?

When current and voltage are in phase,

$$L\omega = \frac{1}{C\omega} \Rightarrow \left(\frac{4}{\pi^2}\right)(2 \times \pi \times 50) = \frac{1}{C(2\pi \times 50)}$$

$$\Rightarrow 8 \times 50 \times 100 = \frac{1}{C} \Rightarrow 4 \times 10^4 = \frac{1}{C} \Rightarrow C = \frac{10^{-4}}{4}$$

$\therefore C = 2.5 \times 10^{-5}$ F $= 25\ \mu F$

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} = \sqrt{R^2} = R \quad \left(\because L\omega = \frac{1}{C\omega}\right)$$

$\therefore Z = 100\ \Omega$

$$i_{rms} = \frac{200}{100} = 2\ A$$

$\therefore$ Power $= 200 \times 2 \times \cos 0 = 400$ W.

[Topper's Answer 2016]

Q. 5. (i) When an *ac* source is connected to an ideal inductor shows that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with an inductor and an *ac* source. What happens to the brightness of the lamp when the key is plugged in and an iron rod is inserted inside the inductor ? Explain.

U [O.D. I, II, III 2016]

Ans. (i) $P_{av} = I_{av} \times V_{av} \cos\phi$ ½

For an ideal inductor, $\phi = \frac{\pi}{2}$ ½

$\therefore \quad P_{av} = I_{av} \times V_{av} \cos\frac{\pi}{2}$ ½

$P_{av} = 0$ ½

(ii) Brightness decreases. Because as iron rod is inserted, inductance increases, hence the current will decrease with decrease in brightness. **1**

[CBSE Marking Scheme 2016]

Detailed Answer :

(i) The average power supplied by the source over a complete cycle is

$$P_{av} = E_{rms} \times I_{rms} \times \cos\phi$$

When the circuit carries an ideal inductor, then the phase difference between the current and voltage is $\frac{\pi}{2}$

In case of pure inductive circuit, $\phi = \frac{\pi}{2} \Rightarrow \cos\frac{\pi}{2} = 0$

So power dissipated = 0 **2**

Hence, when an *ac* source is connected to an ideal inductor, the average power supplied by the source over a complete cycle is zero.

AI Q. 6. An inductor *L* of inductance X_L is connected in series with a bulb *B* and an *ac* source. How would brightness of the bulb change when (i) number of turns in the inductor is reduced, (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your answer in each case.

U [Delhi I, II, III 2015]

Ans. (i) Increases :

$$X_L = \omega L$$ ½

As number of turns decreases, L decreases, hence current through bulb increases / voltage across bulb increases. ½+½

∴ Brightness increases.

(ii) Decreases :

Iron rod increases the inductance, which increases X_L, hence current through the bulb decreases / voltage across bulb decreases.

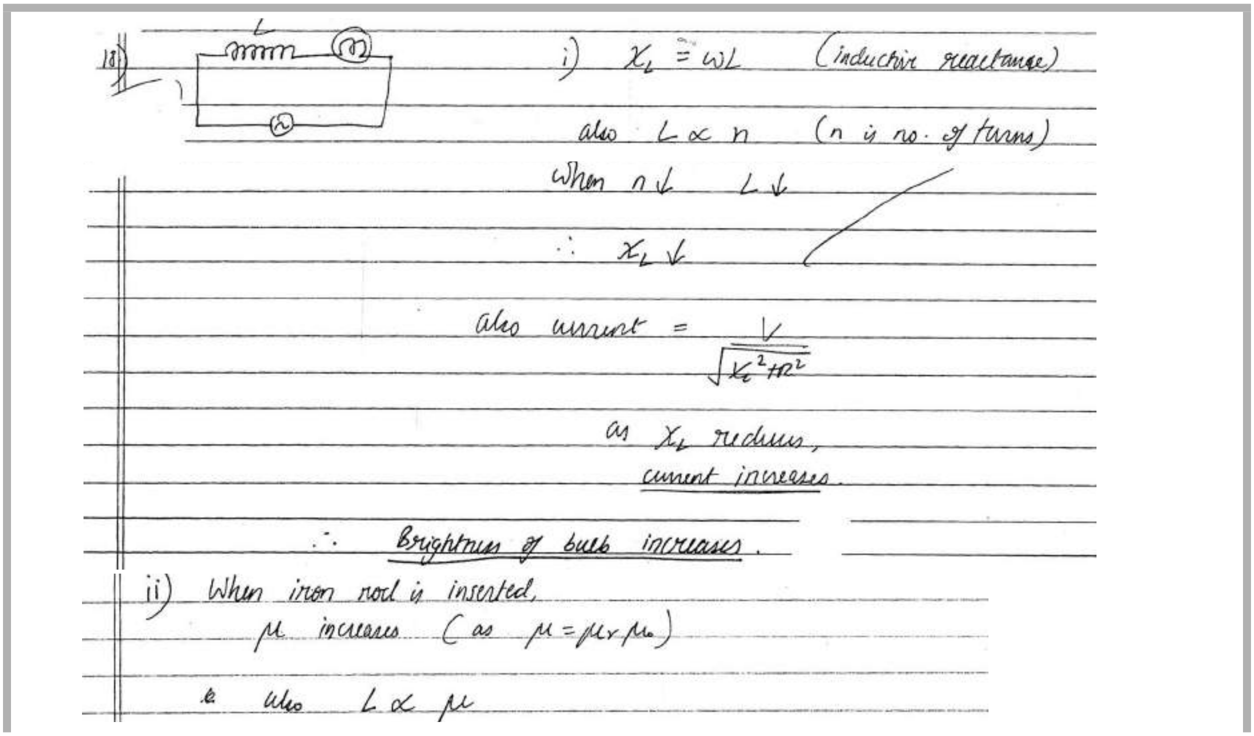
∴ Brightness decreases. ½+½

(iii) Increases :

Under this condition ($X_C = X_L$) the current through the bulb will become maximum / increase. ½

∴ Brightness increases. **[CBSE Marking Scheme 2015]**

OR



∴ L also increases

⇒ X_L increases

$i = \frac{V}{\sqrt{X_L^2 + R^2}}$

∴ i reduces

⇒ hence bulb will glow less brightly.

(ii) In an LCR circuit

current = $\frac{V}{\sqrt{(X_C - X_L)^2 + R^2}}$

initially, $i = \frac{V}{\sqrt{X_L^2 + R^2}}$

∵ $X_C = X_L$

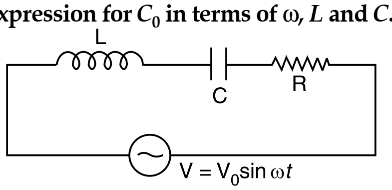
finally, $i = \frac{V}{R}$

∴ current increases

hence brightness also increases

[Topper's Answer 2015]

Q. 7. The current, in the *LCR* circuit shown in the figure is observed to lead the voltage in phase. Without making any other change in the circuit, a capacitor, of capacitance C_0, is (appropriately) joined to the capacitor *C*. This results in making the current, in the 'modified' circuit, flow in phase with the applied voltage.
Draw a diagram of the 'modified' circuit and obtain an expression for C_0 in terms of ω, *L* and *C*.

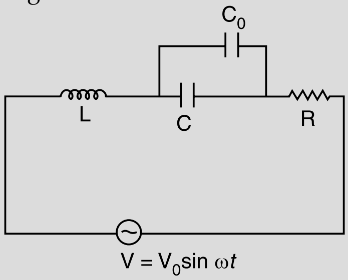


U **[Foreign 2016]**

Ans. The current leads the voltage in phase.
Hence, $X_c > X_L$
For resonance, we must have ½
New value of $X_c = X_L$

We, therefore, need to decrease $X_c = \left(\frac{1}{\omega C}\right)$. This requires an increase in the value of *C*. Hence, capacitor C_0 should be connected in parallel across *C*. ½

The diagram of the modified circuit is as shown.



1

For resonance, we have ½

$$\frac{1}{\omega(C + C_0)} = \omega L$$

$$\therefore \quad C_0 = \left[\frac{1}{\omega^2 L} - C\right]$$ ½

[CBSE Marking Scheme 2016]

Q. 8. A 200 mH (pure) inductor and a 5 μF (pure) capacitor are connected one by one, across a sinusoidal *ac* voltage source of *V* = [70.7sin (1000 *t*)] voltage. Obtain the expression for the current in each case. A **[Foreign, 2016]**

Ans. For the applied voltage
$V = 70.7\sin(1000\,t)$, we have
$V_0 = 70.7$ volts ½

$\omega = 1000$ rad/s ½

For the inductor

$$i_o = \frac{V_0}{\omega L} = \frac{70.7}{1000 \times 200 \times 10^{-3}} \text{ A}$$

$= 35.35 \times 10^{-2}$ A ½

$= 0.3535$ A

∴ Expression for current is ½

$$i = (0.3535)\sin\left(1000\,t - \frac{\pi}{2}\right)$$

For the capacitor

$$i_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)} = V_0 \,.\, \omega C$$

$= 70.7 \times 1000 \times 5 \times 10^{-6}$ A ½

$= 353.5 \times 10^{-3}$ A $= 0.3535$ A

∴ Expression for current is

$$I = 0.3535\sin\left(1000t + \frac{\pi}{2}\right) \quad ½$$

[CBSE Marking Scheme, 2016]

AI **Q. 9. A circuit containing an 80 mH inductor and a 250 μF capacitor in series connected to a 240 V, 100 rad/s supply. The resistance of the circuit is negligible.**

(i) Obtain *rms* value of current.

(ii) What is the total average power consumed by the circuit ? A **[Delhi I, II, III 2015]**

Ans. (i) $X_L = \omega L = 100 \times 80 \times 10^{-3} = 8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 250 \times 10^{-6}}\ \Omega \quad ½$$

$= 40\ \Omega$ ½

Total Impedance $(Z) = X_C - X_L$ ½

$= 32\ \Omega$ ½

$$I_{rms} = \frac{240}{32} \text{ A} = 7.5 \text{ A} \quad ½$$

(ii) Average power consumed = 0

(As there is no ohmic resistance in the current.) ½

Q. 10. Derive the expression for the average power dissipated in a series *LCR* circuit for an *ac* source of a voltage, $V = V_m \sin \omega t$, carrying a current, $i = i_m \sin(\omega t + \phi)$

Hence define the term "Wattless current". State under what condition it can be realized in a circuit. U **[Delhi Comptt. 2016]**

Ans. $V = V_m \sin \omega t$

$i = i_m \sin(\omega t + \phi)$

Power at any instant

$P = Vi = V_m i_m \sin \omega t \sin(\omega t + \phi)$

$$P = \frac{V_m i_m}{2}[\cos\phi - \cos(2\omega t + \phi)] \quad ½$$

The average of second term in the above expression is zero over a full cycle. ½

$$\therefore \text{Average Power} = \bar{P} = \frac{V_m i_m}{2}\cos\phi$$

$$\bar{P} = \frac{V_m}{\sqrt{2}} \times \frac{i_m}{\sqrt{2}}\cos\phi \quad ½$$

$$\therefore \quad \bar{P} = V_{rms} I_{rms} \cos\phi \quad ½$$

Wattless current is that which flows in the circuit but no power dissipation occurs. ½

It is realized only when circuit is purely inductive or capacitive, *i.e.*, when $\cos\phi = 0$ or $\phi = \frac{\pm\pi}{2}$ ½

[CBSE Marking Scheme 2016]

Q. 11. A source of *ac* voltage $V = V_0 \sin \omega t$ is connected to a series combination of a resistor '*R*' and a capacitor '*C*'. Draw the phasor diagram and use it to obtain the expression for (i) impedance of the circuit and (ii) phase angle. U **[O.D. I, II, III 2015]**

Ans. The Pythagoras theorem gives

$$V_m^2 = V_{rm}^2 + V_{cm}^2$$

Substituting the values of V_{rm} and V_{cm} into this equation, gives

$$V_m^2 = (i_m R)^2 + (i_m X_C)^2$$

$$= i_m^2 (R^2 + X_C^2)$$

$$\therefore \quad i_m = \frac{V_m}{\sqrt{R^2 + X_C^2}} \quad ½$$

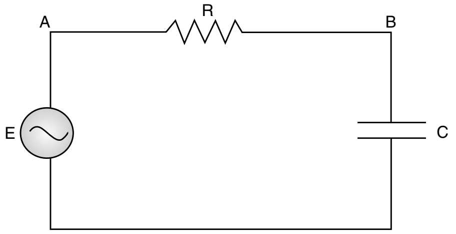
∴ The impedance of the circuit is given by :

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

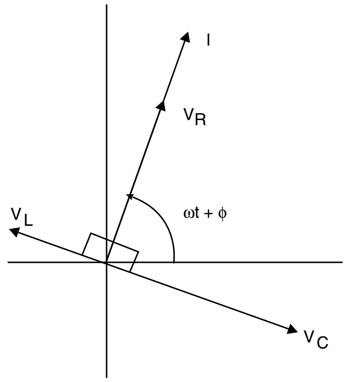
The phase angle is the angle between V_R and V. Hence

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} \quad ½$$

The circuit diagram and the phasor diagram, for the given circuit, are as shown.



½



½

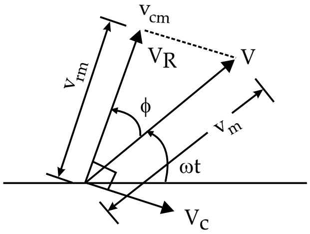
Derivation,

The voltage equation, for the circuit, can be written as :

$$V_R + V_C = V \quad ½$$

The phasor relation, whose vertical component gives the above equation, is

$$\vec{V}_R + \vec{V}_C = \vec{V}$$



½

Q. 12. A voltage $V = V_0 \sin \omega t$ is applied to a series *LCR* circuit. Derive the expression for the average power dissipate over a cycle.
Under what conditions is (i) no power dissipated even though the current flows through the circuit, (ii) maximum power dissipated in the circuit ?
U **[O.D. I, II, III 2014]**

Ans. Applied voltage $= V_0 \sin \omega t$ ½

Current in the circuit $= I_0 \sin(\omega t - \phi)$

where, ϕ is the phase lag of the current with respect to the voltage applied.

Hence instantaneous power dissipation

$$= V_0 \sin \omega t \times I_0 \sin(\omega t - \phi)$$

$$= \frac{V_0 I_0}{2} [2 \sin \omega t . \sin(\omega t - \phi)]$$

$$= \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t - \phi)]$$ ½

Therefore, average power for one complete cycle= average of $[\frac{V_0 I_0}{2} \{\cos \phi - \cos(2\omega t - \phi)\}]$

The average of the second term over a complete cycle is zero. ½

Hence, average power dissipated over one complete cycle $= \frac{V_0 I_0}{2} \cos \phi$ ½

Conditions :

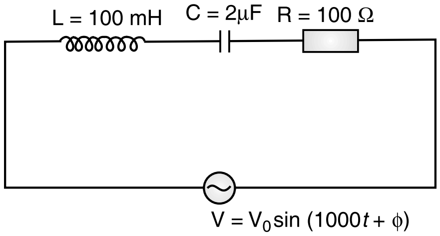
(i) No power is dissipated when $R = 0$ (or $\phi = 90°$)½
[**Note :** Also accept if the student writes 'This condition cannot be satisfied for a series *LCR* circuit'.]

(ii) Maximum power is dissipated when $X_L = X_C$. ½

Or $\omega L = \frac{1}{\omega C}$ (or $\phi = 0$)

[CBSE Marking Scheme 2014]

Q. 13. Find the value of the phase lag/lead between the current and voltage in the given series *LCR* circuit. Without making any other change, find the value of the additional capacitor, such that when 'suitably joined' to the capacitor (C = 2μF) as shown. What would make the power factor of this circuit as unity ?

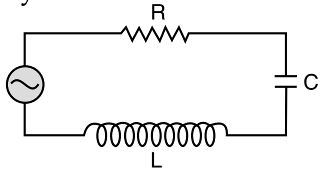


A **[CBSE SQP 2014]**

Ans. Try yourself, Similar to Q. 2 (i) in Short Answer Type Questions-II

[CBSE Marking Scheme, 2014]

AI **Q. 14. The figure shows a series *LCR* circuit with *L* = 5.0 H, *C* = 80 μF, *R* = 40 Ω connected to a variable frequency of 240 V source. Calculate**



(i) The angular frequency of the source which drives the circuit at resonance.
(ii) The current at the resonating frequency.
(iii) The *rms* potential drop across the capacitor at resonance. A **[Delhi I, II, III 2012]**

Ans. (i) $$\omega = \frac{1}{\sqrt{LC}}$$ ½

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad/s}$$ ½

(ii) Current at resonance

$$I_{rms} = \frac{V_{rms}}{R} = \frac{240}{40} \text{ A} = 6 \text{ A}$$ ½ + ½

(iii) V_{rms} across a capacitor

$$V_{rms} = I_{rms} X_C$$ ½

$$= 6 \times \frac{1}{50 \times 80 \times 10^{-6}} \text{ V}$$

$$= \frac{6 \times 10^6}{4 \times 10^3} \text{ V} = \frac{6000}{4} \text{ V} = 1500 \text{ V}$$ ½

[CBSE Marking Scheme, 2012]

Q. 15. (i) When an *ac* source is connected to an ideal capacitor show that the average power supplied by the source over a complete cycle is zero.
(ii) A lamp is connected in series with a capacitor. Predict your observation when the system is connected first across a *dc* and then an *ac* source. What happens in each case if the capacitance of the capacitor is reduced ?
A&E **[Delhi Comptt. I, II, III 2013]**

Ans. (i) When an ideal capacitor is connected with *ac* source, the current flow continuously but due to dielectric in between the plates of capacitor, there is no current, *i.e.*,

$$I_{avg} = 0$$

$$\therefore \quad P_{avg} = V \times I_{avg}$$

$$P_{avg} = 0$$ **1**

(ii) For *dc*, lamp will not shine as capacitor blocks *dc* even if we reduce the capacitance, the lamp will not shine. **1**

Lamp will shine, if *ac* is used on reducing *C*, with increase of impedance.

Hence, lamp will shine less brightly. **1**

Long Answer Type Questions (5 marks each)

Q. 1. (a) Draw graphs showing the variations of inductive reactance and capacitive reactance with frequency of the applied *ac* source.

(b) Draw the phasor diagram for a series *RC* circuit connected to an ac source.

(c) An alternating voltage of 220 V is applied across a device *X*, a current of 0.25 A flows, which lag behind the applied voltage in phase by $\frac{\pi}{2}$ radian.

If the same voltage is applied across another device *Y*, the same current flows but now it is in phase with the applied voltage.

(i) Name the devices *X* and *Y*.

(ii) Calculate the current flowing in the circuit when the same voltage is applied across the series combination of *X* and *Y*. U **[CBSE Comptt. 2018]**

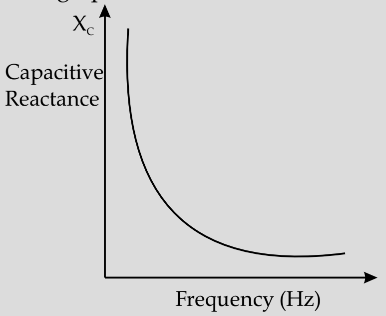
Ans. (a) Drawing the two graphs ½+½

(b) Drawing the phaser diagram 1

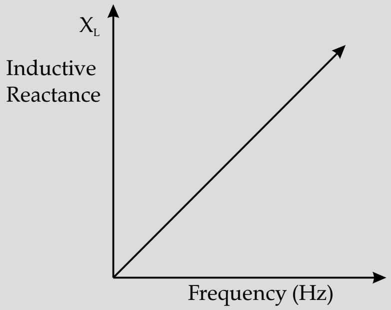
(c) (i) Naming the devices ½+½

(ii) Calculating the current flowing 2

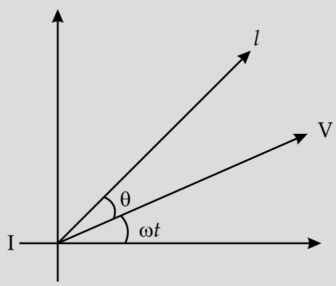
(a) The two graphs are as shown



½+½



(b) (the current leads the voltage by an angle θ where $0 < \theta < \frac{\pi}{2}$). The required phaser diagram is as shown.



1

[Here, $\theta = \tan^{-1}\left[\frac{1}{\omega CR}\right]$

(c) In device *X* :

Current lags behind the voltage by $\frac{\pi}{2}$

∴ *X* is an inductor.

In device *Y* : ½

Current in phase with the applied voltage.

∴ X is resistor. ½

We are given that

$$0.25 = \frac{220}{X_L}$$

$$\text{or } X_L = \frac{220}{0.25}\,\Omega = 880\,\Omega \quad ½$$

$$\text{Also } 0.25 = \frac{220}{X_R}$$

$$\therefore \quad X_R = \frac{220}{0.25}\,\Omega = 880\,\Omega \quad ½$$

For the series combination of *X* and *Y*,

$$\text{Equivalent impedance} = \sqrt{X_L^2 + X_R^2} = \left(880\sqrt{2}\right)\Omega \quad ½$$

$$\therefore \quad \text{Current flowing} = \frac{220}{880\sqrt{2}}\text{ A} = 0.177\text{ A} \quad ½$$

[CBSE Marking Scheme 2018]

AI **Q. 2. An *ac* voltage $V = V_0 \sin \omega t$ is applied to a pure inductor *L*. Obtain an expression for the current in the circuit. Prove that the average power supplied to an inductor over one complete cycle is zero.**

A **[SQP I 2017]**

Ans. Expression for current 3

Proof 2

(i) If an alternating voltage $V = V_0 \sin \omega t$ is applied across pure inductor of inductance *L*, then the magnitude of induced emf will be equal to applied voltage,

$$E = L\frac{dI}{dt}$$

For the circuit, magnitude of induced emf = applied voltage

$$L\frac{dI}{dt} = V_0 \sin \omega t$$

$$dI = \frac{V_0}{L}\sin \omega t\, dt$$

Integrating both the sides, we get

$$I = \frac{V_0}{L}\int \sin \omega t\, dt = \frac{V_0}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$I = -\frac{V_0}{\omega L}\cos \omega t = -\frac{V_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$I = \frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{if } X_L = \omega L$$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad 3$$

(ii) The average power supplied by the source over a complete cycle is

$$P_{av} = E_{rms} I_{rms} \cos \phi$$

When the circuit carries an ideal inductor, then the phase difference between the current and voltage is $\frac{\pi}{2}$

In case of pure inductive circuit,

$$\phi = \frac{\pi}{2}$$

But $\cos \frac{\pi}{2} = 0$

So power dissipated $= 0$ **2**

Hence, when an *ac* source is connected to an ideal inductor, the average power supplied by the source over a complete cycle is zero.

[CBSE Marking Scheme 2017]

Q. 3. (i) Prove that an ideal capacitor in an *ac* circuit does not dissipate power.

(ii) An inductor of 200 mH, capacitor of 400 μF and a resistor of 10 Ω are connected in series to ac source of 50 V of variable frequency. Calculate the

(a) angular frequency at which maximum power dissipation occurs in the circuit and the corresponding value of the effective current, and

(b) value of Q-factor in the circuit.

A [O.D. Comptt I, II, III 2017]

Ans. (i) Average Power dissipation is zero **2**

(ii) Numerical **3**

(i) Try yourself, Similar to Q. 1, Short Answer Type Questions-II **2**

(ii) (a) $\omega_0 = \frac{1}{\sqrt{LC}}$ ½

$$= \frac{1}{(200 \times 10^{-3} \times 400 \times 40^{-6})^{1/2}}$$

$$= \frac{1}{\sqrt{8 \times 10^{-5}}} \text{rad/s}$$

$$= \frac{10^3}{\sqrt{80}} \text{rad/s}$$

$\cong 111$ rad/s. ½

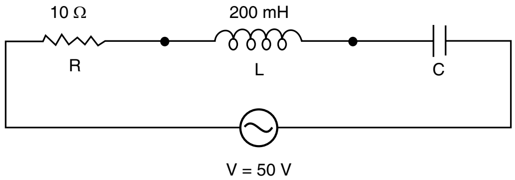
$I = \frac{V}{R} = \frac{50}{10} = 5\text{A}$ **1**

(b) $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{400 \times 10^{-6}}} = \sqrt{5}$ **1**

[CBSE Marking Scheme 2017]

Q. 4. In the following circuit, calculate (i) the capacitance of the capacitor, if the power factor of the circuit is unity, (ii) the Q-factor of this circuit. What is the significance of the Q-factor in *ac* circuit ? Given the angular frequency of the *ac* source to be 100 rad/s. Calculate the average power dissipated in the circuit. **A [O.D. Comptt I, II, III 2017]**



Ans. (i) Calculation of Capacitance **1**

(ii) *Q*-factor of circuit and its importance **2**

Calculation of average power dissipated **2**

(i) As power factor is unity,

$\therefore$ $X_L = X_C$ ½

$\Rightarrow$ $\omega = \frac{1}{\sqrt{LC}}$ ½

$100 = \frac{1}{\sqrt{200 \times 10^{-3} \times C}}$ ½

$10^4 \times 2 \times 10^2 \times 10^{-3} \times C = 1$ ½

$$C = \frac{1}{2 \times 10^3} \text{ F}$$

$= 0{\cdot}5 \times 10^{-3}$ F ½

$= 0{\cdot}5$ mF ½

(ii) Quality factor, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ ½

$$= \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{0{\cdot}5 \times 10^{-3}}}$$

$= \frac{1}{10} \times 20 = 2$ ½

Significance : It measures the sharpness of resonance.

Average Power dissipated,

$P = V_{rms} I_{rms} \cos \phi$ **1**

$$= 50 \times \frac{50}{10} \times 1 \text{ W}$$

$= 250$ watts

[CBSE Marking Scheme 2017]

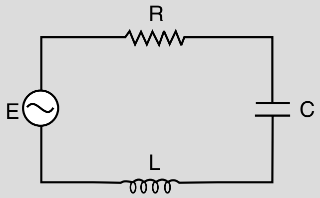
AI Q. 5. An *ac* voltage $V = V_m \sin \omega t$ is applied to a series *LCR* circuit. Obtain an expression for the current in the circuit and the phase angle between the current and voltage. What is resonance frequency.

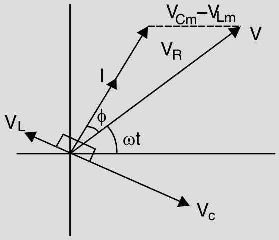
A [SQP I 2017]

Ans. (i) Expression for Current and Phase angle **4**

(ii) Resonance Frequency **1**

(i) In a series *LCR* circuit shown,





1

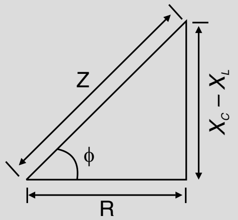
From the phasor relation, voltages $V_L+V_R+V_C=V$, as V_C and V_L are along the same line and in opposite directions, so they will combine in single phasor (V_C+V_L) having magnitude $|V_{Cm}-V_{Lm}|$. Since voltage V is shown as hypotenuse of right angled triangle with sides as V_R and $(V_C + V_L)$, so the Pythagoras Theorem results as :

$$V_m^2 = V_R^2 + (V_{Cm} - V_{Lm})^2$$
$$V_m^2 = (I_m R)^2 + (I_m X_C - I_m X_L)^2$$
$$V_m^2 = I_m^2 (R^2 + (X_C - X_L)^2)$$

Now current in the circuit :

$$I_m = \frac{V_m}{\sqrt{(R^2 + (X_C - X_L)^2}}$$

$$I_m = \frac{V_m}{Z} \text{ as } Z = \sqrt{\left[R^2 + (X_C - X_L)^2\right]}$$ 1



As phasor I is always parallel to phasor V_R, the phase angle φ is the angle between V_R and V and can be determined from figure.

$$\tan\phi = \frac{V_{Cm} - V_{Lm}}{V_{Rm}}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$ 2

(ii) Resonance Frequency

Frequencies at which the response amplitude is relative maximum are known as system's resonant frequencies. It is shown as :

$$V_{Cm} = V_{Lm}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$ 1

[CBSE Marking Scheme 2017]

Q. 6. (i) An *ac* source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of *L, C* and *R*. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called ?

(ii) In a series *LR* circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance *C* such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate $\frac{P_1}{P_2}$. U [Delhi I, II, III 2016]

Ans. (i) Try yourself similar Q. 5 Long Answer Type Question.

(ii) Power factor :

$$P_1 = \frac{R}{Z} = \frac{R}{\sqrt{R^2+R^2}} \quad (\text{as } X_L = R)$$

$$= \frac{1}{\sqrt{2}}$$ ½

Power factor when capacitor C of Reactance $X_C = X_L$ is put in series in the circuit

as $Z = R$ (at resonance)

$$P_2 = \frac{R}{Z} = \frac{R}{R} = 1$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}$$ ½

[CBSE Marking Scheme 2016]

AI **Q. 7. (i) With the help of a diagram, explain the principle and working of a device which produces current that reverses its direction after regular intervals of time.**

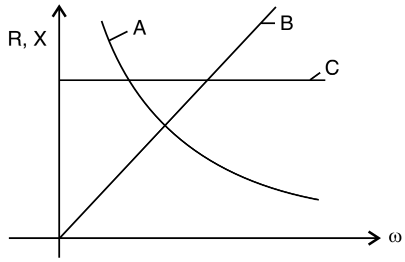
(ii) If a charged capacitor *C* is short circuited through an inductor *L*, the charge and current in the circuit oscillate simple harmonically.

(a) In what form the capacitor and the inductor store energy ?

(b) Write two reasons due to which the oscillations become damped. R [CBSE SQP 2015]

OR

(i) Figure shows the variation of resistance and reactance versus angular frequency. Identify the curve which corresponds to inductive reactance and resistance.



(ii) Show that series *LCR* circuit at resonance behaves as a purely resistive circuit. Compare the phase relation between current and voltage in series LCR circuit for (i) $X_L > X_C$, (ii) $X_L = X_C$ using phasor diagrams.

(iii) What is an acceptor circuit and where it is used ?

U [CBSE SQP 2015]

Ans. (i) AC generator

Diagram 1

Principle 1

Working 1

(ii) (a) Capacitor – electric field ½

Inductor – magnetic field ½

(b) resistance of the circuit

Radio tuning ½

Radiation in the form of EM waves ½

OR

(i) B : inductive reactance ½

C : resistance ½

(ii) At resonance $X_L = X_C$ **1**

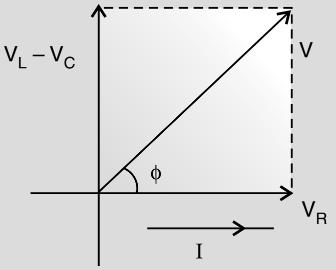
Increase

$Z = [(X_L - X_C)^2 + R^2]^{1/2}$, $Z = R$

Phasor diagrams

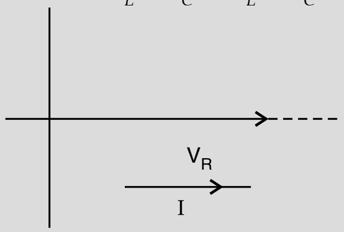
phase difference is ϕ

for $X_L > X_C \Rightarrow V_L > V_C$



1

$X_L = X_C \Rightarrow V_L = V_C$



1

same phase, *i.e.* $\phi = 0$

(iii) Acceptor circuit : Series LCR circuit ½

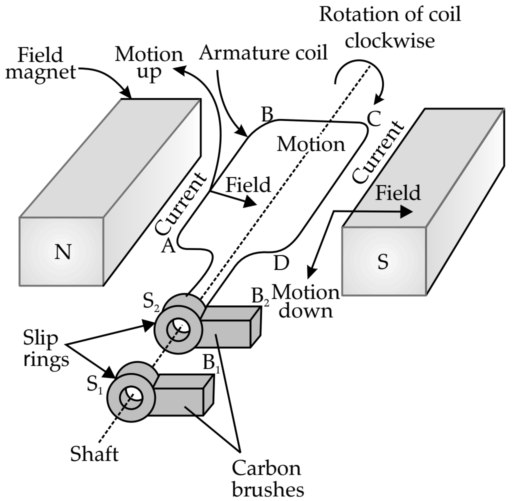
Radio tuning ½

Detailed Answer :

(i) AC generator

Basic elements of an AC generator :

- **Rectangular coil :** Also called as armature
- **Strong permanent magnets :** Magnetic field is perpendicular of the axis of rotation of coil.
- Slip rings
- Brushes



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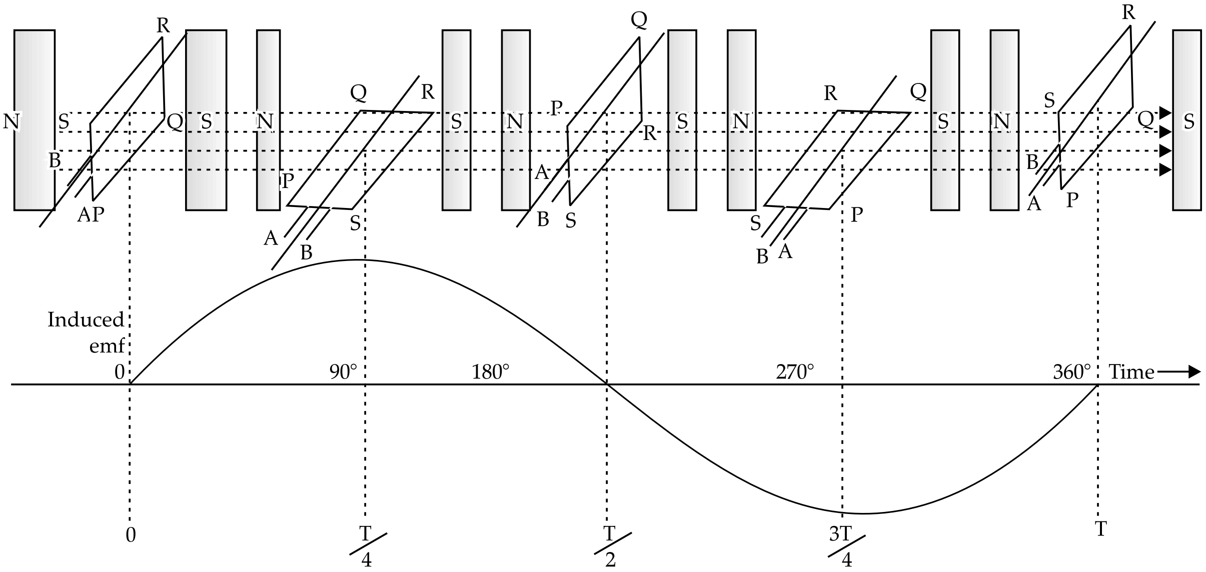
AC generator

Principle : It is based on the principle of electromagnetic induction. That is, when a coil is rotated about an axis perpendicular to the direction of uniform magnetic field, an induced *emf* is produced across it. **1**

Working of AC Generator



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(ii) (a) The capacitor stores energy in the form of electric field and the inductor stores energy in the form of magnetic field.

(b) Oscillations become damped due to : ½+½

- Resistance of the circuit ½
- Radiation in the form of EM waves ½

OR

(i) Curve B corresponds to inductive reactance and curve C corresponds to resistance. ½+½

(ii) At resonance,

$$X_L = X_C$$ **1**

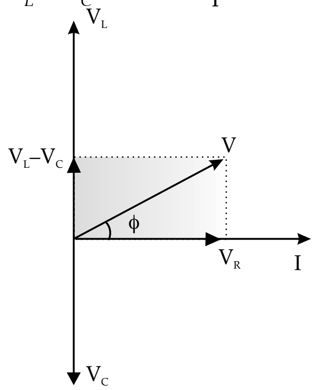
Therefore, impedance is given as :

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = R$$

Thus, series LCR circuit at resonance behaves as a purely resistive circuit.

For $X_L > X_C$, $V_L > V_C$. Therefore phasor diagram is :



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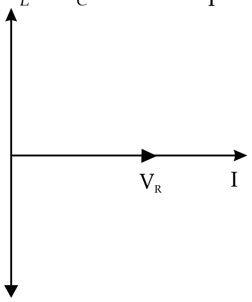
Here, ϕ is phase difference.

For $X_L = X_C$, $V_L = V_C$. Therefore phasor diagram is :



1

(iii) Series resonance *LCR* circuit is called an acceptor circuit. ½

They are widely used in the tuning mechanism of a radio or a TV. ½

Q. 8. (i) A series *LCR* circuit is connected to an *ac* source of variable frequency. Draw a suitable phasor diagram to deduce the expressions for the amplitude of the current and phase angle.

(ii) Obtain the condition at resonance. Draw a plot showing the variation of current with the frequency of a.c. source for two resistances R_1 and R_2 ($R_1 > R_2$). Hence define the quality factor, Q and write its role in the tuning of the circuit.

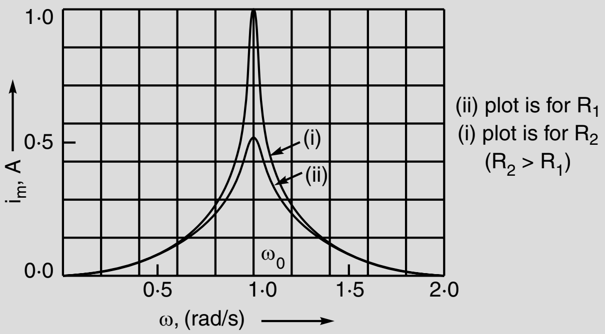
U [Delhi Comptt. I, II, III 2014]

Ans. (i) Try yourself similar Q. 5 Long Answer Type Question.

(ii) At resonance, I_m is maximum.

$\Rightarrow \quad X_L = X_C,$

[Alternatively : $\omega_0 = \dfrac{1}{\sqrt{LC}}$ **]** ½



½ + ½

Quality factor of *LCR* circuit is defined as

$$\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$ 1

A larger value of quality factor corresponds to a sharper resonance. ½

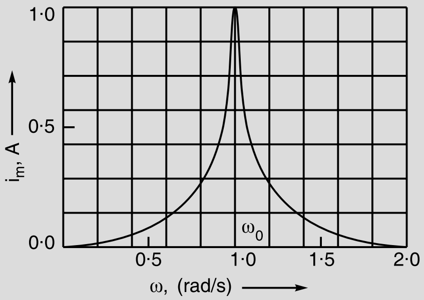
[CBSE Marking Scheme, 2014]

Q. 9. Derive an expression for the impedance of a series *LCR* circuit connected to an *ac* supply of variable frequency.

Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set. U [O.D. Comptt. I, II, III 2013, O.D. I, II, III 2012, Delhi I, II, III 2011]

Ans. (i) Try yourself similar Q. 5 Long Answer Type Question.

(ii)



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The capacitance of a capacitor in the tuning circuit is varied such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal to be received. When this happens, the amplitude of the current becomes maximum in the receiving circuit. 1

[CBSE Marking Scheme 2013]

TOPIC-3
AC Generator and Transformer

Revision Notes

***AC* generator**

- An alternator is an electrical machine which converts mechanical energy into alternating electrical energy.
- Alternator or a synchronous generator has a stator and rotor.
- It is similar to the basic working principle of a *dc* generator.

- It works on the principle of electromagnetic induction where a coil gets rotated in uniform magnetic field, sets an induced emf given as :

$$e = e_0 \sin \omega t = NBA\omega \sin \omega t$$

Transformer

- Transformer is an electrical device used for changing the alternating voltages.
- It is based on the phenomenon of mutual induction.
- The main use of transformer is in transmission of *ac* over long distances at extremely high voltages which reduces the energy losses in transmission.
- It comprises of two sets of coils which are insulated from each other and are wound on soft-iron core.
- In this, one of the coil is called as primary (input coil) having N_p turns while other coil is secondary (output coil) having N_s turns, so we have

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- **Transformer Ratio :**

$$E_s = \left(\frac{N_s}{N_p}\right)E_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$ is defined as the transformer ratio.

The value of turns ratio of a transformer $\frac{N_p}{N_s} = \frac{V_p}{V_s} = n$

- **Step-up transformer :** If secondary coil has more number of turns than primary ($N_s > N_p$), voltage gets stepped up ($V_s > V_p$).

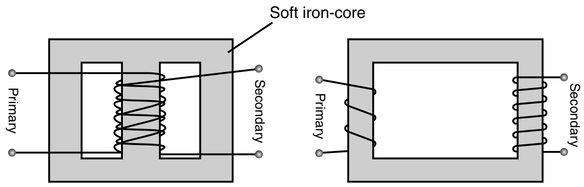
  In this, there is less current in secondary as compared to primary ($\frac{N_s}{N_p} > 1$ and $I_s < I_p$).

  The value of transformer ratio $k > 1$
- **Step-down transformer :** In this, the secondary coil has less number of turns than primary ($N_s < N_p$). In this, $V_s < V_p$ and $I_s > I_p$ as voltage gets stepped down or reduced with increase in current.

  In this, value of transformer ratio $k < 1$
- The main use of transformers is in stepping up voltage for power transmission.
- Electric power can be transmitted much more efficiently at high voltages than at low voltages due to less I^2R heat loss in a high voltage / low current transmission.
- **Efficiency of transformer :**

$$\eta = \frac{\text{output power}}{\text{input power}}$$

$$\eta = \frac{E_s I_s}{E_p I_p}$$



- Inspite of heavy power losses, the efficiency in a transformer is usually above 90%.
- An ideal transformer is 100% efficient as it delivers all energy it receives.
- Real transformer is not 100% efficient and at full load, its efficiency lies between 94% to 96%.
- A transformer operating with constant voltage and frequency with very high capacity, efficiency results as 98%.
- **Energy losses in transformers :**

  1. Flux Leakage
  2. Resistance of windings
  3. Eddy currents
  4. Hysteresis

Know the Formulae

- $\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$

➢ $V_s = \left(\frac{N_s}{N_p}\right)V_p$ and $I_s = \left(\frac{N_p}{N_s}\right)I_p$

➢ The value of transformer ratio is greater than 1 for step up transformer and less than 1 for step-down transformer.

➢ $\eta = \frac{E_s I_s}{E_p I_p}$

➢ $\%\text{efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\%$

$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \times 100\%$$

➢ $e = e_0 \sin \omega t = NBA\omega \sin \omega t$

➢ $I = \frac{e}{r}$ $I = \frac{NBA\omega \sin \omega t}{R}$

Objective Answer Type Questions (1 mark each)

Q. 1. An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6 *V* (*rms*) AC source. The power dissipated in the circuit is

(a) 8 W. **(b) 12 W.**
(c) 14.4 W. **(d) 18 W.**

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation :

Here, $E_{rms} = 6\text{ V}, X_L = 1\,\Omega, R = 2\,\Omega, P_{av} = ?$

We know that, average power dissipated in the circuit is given by,

$$P_{av} = E_{rms} I_{rms} \cos\phi$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_{rms}}{Z}$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{4+1} = \sqrt{5}\,\Omega$$

$$\therefore \quad I_{rms} = \frac{6}{\sqrt{5}}\text{ A}$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = 14.4\text{ W}$$

Q. 2. The output of a step-down transformer is measured to be 24 V when connected to a 12 W Light bulb. The value of the peak current is

(a) $1/\sqrt{2}$ A **(b)** $\sqrt{2}$ A
(c) 2 A **(d)** $2\sqrt{2}$ **A**

[NCERT Exemplar]

Ans. Correct option : (a)

Explanation : Given,

Power associated with secondary, P_s = 12 W

Secondary voltage, V_s=24 V

Current in the secondary, $I_s = \frac{P_s}{V_s} = \frac{12}{24} = 0.5\ A$

Peak value of the current in the secondary, $I_0 = I_s\sqrt{2} = (0.5)(1.414) = 0.707$ or $\frac{1}{\sqrt{2}}$ A.

Very Short Answer Type Questions (1 mark each)

Q. 1. Mention the two characteristic properties of the material suitable for making core of a transformer.

R [O.D. I, II, III 2012]

Ans. Any two of the following :

(i) Low coercivity / Low retentivity ½

(ii) Low hysteresis loss ½

OR

(i) High magnetic susceptibility / High Permeability ½

(ii) High resistivity ½

[CBSE Marking Scheme 2012]

AI Q. 2. Why is the core of a transformer laminated ?

R [Delhi Comptt. I 2013]

Ans. To reduce the iron loss, the iron core of a transformer is laminated. 1

[CBSE Marking Scheme 2013]

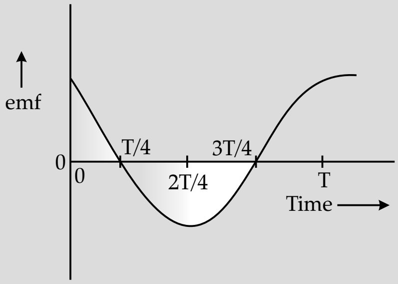
Detailed Answer :

The main purpose of transformer core is to transfer the flux from primary winding to secondary winding. During the transfer of flux, there results some leakage of flux which causes the eddy current to flow in transformer core. In order to minimise the eddy current, transformer core is laminated that will help in increasing the resistance with decrease in area of cross section.

Q. 3. Show a plot of variation of alternating emf versus time generated by a loop of wire rotating in a magnetic field. U **[Delhi I, II, III 2014]**

$$\text{emf} = NBA\cos\left(T - \frac{\pi}{4}\right)$$

Ans.



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[CBSE Marking Scheme 2014]

Short Answer Type Questions-I

(2 marks each)

Q. 1. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers ? R **[O.D. I, II, III 2012]**

Ans. A transformer is based on the principle of mutual induction which states that due to continuous change in the current in the primary coil, an emf gets induced across the secondary coil. **1**

Electric power generated at the power station, is stepped-up to very high voltages by means of a step-up transformer and transmitted to a distant place. At receiving end, it is stepped down by a step-down transformer. **1**

[CBSE Marking Scheme, 2012]

AI **Q. 2. An athlete peddles a stationary tricycle whose pedals are attached to a coil having 100 turns each of area 0·1 m². The coil, lying in the *X* – *Y* plane, is rotated, in this plane, at the rate of 50 rpm, about the Z-axis, in a region where a uniform magnetic field, $\vec{B} = (0.01)\hat{k}$ tesla, is present. Find the**

(i) maximum emf

(ii) average emf generated in the coil over one complete revolution. A **[CBSE SQP 2013]**

Ans. (i) The maximum emf 'ε_0' generated in the coil is,

$\varepsilon_0 = NBA\omega$ ½

$= NBA2\pi f$

$(\because 50 \text{ rpm} = \frac{50}{60} \text{ revolution} = \frac{5}{6})$

$= [100 \times 0.01 \times 0.1 \times 2\pi \frac{(5)}{6}]\, V$

$= \frac{\pi}{6} \text{ V} = 0.52 \text{ V}$ **1**

(ii) The average emf generated in the coil over one complete revolution = 0 ½

Short Answer Type Question-II

(3 marks)

AI **Q. 1. (a) What is the principle of transformer?**

(b) Explain how laminating the core a transformer helps to reduce eddy current losses in it

(c) Why the primary and secondary coils of a transformer are preferably wound on the same core U **[CBSE SQP 2018-19]**

Ans. (a) Try yourself Similar to Q.2 (a) Long Answer Type Questions.

(b) Laminations are thin, making the resistance higher. Eddy currents are confined within each thin lamination. This reduces the net eddy current.

(c) For maximum sharing of magnetic flux and magnetic flux per turn to be the same in both primary and secondary.

[CBSE Marking Scheme 2018]

Long Answer Type Questions

(5 marks each)

AI **Q. 1. (i) Describe, with the help of a suitable diagram, the working principle of a step-up transformer. Obtain the relation between input and output voltages in terms of the number of turns of primary and secondary windings and the currents in the input and output circuits.**

(ii) Given the input current 15 A and the input voltage of 100 V for a step-up transformer having 90% efficiency, find the output power and the voltage in the secondary if the output current is 3 A.

R **[Foreign I, II, III 2017]**

Ans. (i) Diagram ½

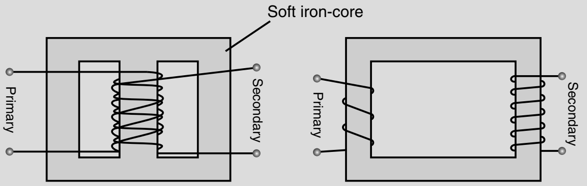
Principle ½

Relation between voltage, number of turns, and Currents 2½

(ii) Input power ½

Output power ½

Output voltage ½



(i) (ii)

Working principle

Whenever current in one coil changes an emf gets induced in the neighbouring coil (Principle of mutual induction) **1**

Voltage across secondary

$$V_s = e_s = -N_s \frac{d\phi}{dt}$$ ½

Voltage across primary

$$V_p = e_p = -N_p \frac{d\phi}{dt}$$ ½

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{(Here, } N_s > N_p\text{)}$$ ½

In an ideal transformer

Power Input = Power output

$$I_p V_p = I_s V_s$$ ½

$$\therefore \quad \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$ ½

(ii) Input power, $P_i = I_i \times V_i = 15 \times 100 = 1500$ W ½

Power output, $P_0 = P_i \times \frac{90}{100} = 1350$ W ½

$\Rightarrow \quad I_0 V_0 = 1350$ W

Output voltage, $V_0 = \frac{1350}{3}$ V $= 450$ V ½

[CBSE Marking Scheme, 2017]

Q. 2. (a) State the principle of working of a transformer.
(b) Define efficiency of a transformer.
(c) State any two factors that reduce the efficiency of a transformer.
(d) Calculate the current drawn by the primary of a 90% efficient transformer which steps down 220 V to 22 V, if the output resistance is 440 Ω.

U **[CBSE Comptt. I, II, III 2018]**

Ans. (a) Principle of working 1

(b) Defining efficiency 1

(c) Any two factors ½+½

(d) Calculating the current drawn 2

(a) A tansformer works on the principle of mutual induction.

(Alternatively-an emf is induced in the secondary coil when the magnetic flux, linked with it changes with time due to a (time) changing magnetic flux linked with the primary coil).

(b) The efficiency of a transformer equals the ratio of the output power to the input power.

Alternatively :

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{Efficiency} = \frac{V_s I_s}{V_p I_p}$$

(c) (i) Eddy current losses
(ii) joule heat losses
(iii) hysteresis losses
(iv) magnetic flux leakage losses **(Any two)**

(d) We have

$$\frac{V_s I_s}{V_p I_p} = 90\% = 0.9$$

$$\therefore \quad \frac{22 I_s}{220 I_p} = 0.9$$

$$\text{or} \quad \frac{I_s}{I_p} = \frac{0.9}{0.1} = 9$$

$$\therefore \quad I_p = \frac{I_s}{9} = \frac{\left(\frac{22}{440}\right)}{9}\text{A}$$

$$= \frac{1}{180}\text{A}$$

$= 0.0056$ A

[CBSE Marking Scheme, 2018]

AI **Q. 3. (i) Draw a labelled diagram of an *ac* generator. Obtain the expression for the emf induced in the rotating coil of *N* turns each of cross-sectional area *A*, in the presence of a magnetic field $\vec{B}$.**

(ii) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angles to the horizontal component of the Earth's magnetic field, 0.3 × 10^{-4} Wb m^{-2}. Find the instantaneous value of the emf induced in the rod. U **[O.D. Set III 2017]**

Ans. (i) Consider a coil rotating in a uniform magnetic field. The flux associated with the coil

$\phi = NBA\cos\theta$

The emf induced due to the flux change.

$\therefore \varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}(NBA\cos\theta)$

$-NBA \cdot \frac{d(\cos\theta)}{dt}$

But the coil consists of N turns and also θ is a function of time $\theta = \omega t$

So $\varepsilon = -NBA \frac{d}{dt}(\cos\omega t)$

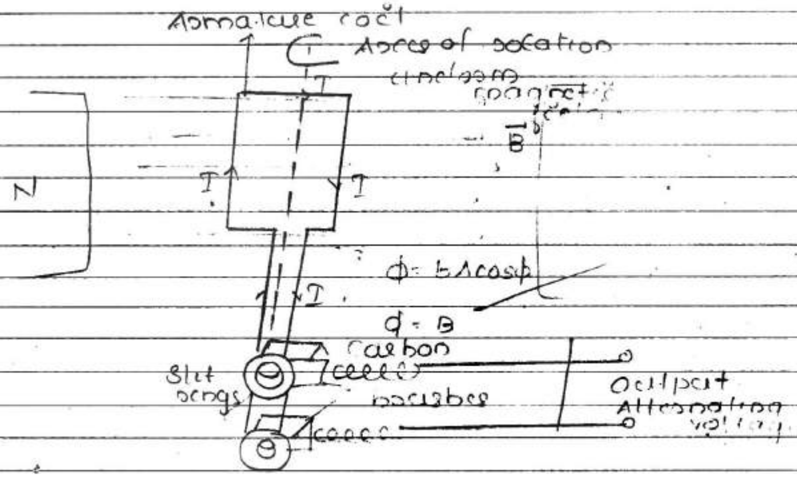
$-NBA \times -\sin\omega t \times \omega$

$= NBA\omega\sin\omega t$

$= \varepsilon_0 \sin\omega t$

$\varepsilon = \varepsilon_0 \sin\omega t$ where ε_0 is the maximum induced voltage or peak voltage

$\varepsilon_0 = NBA\omega$



(ii) The rod is moving perpendicular to the magnetic field

so

$\varepsilon = Blv$

$= 0.3\times10^{-4}\ \text{Wb m}^{-2} \times 10\ \text{m} \times 5\ \text{m s}^{-1}$

$= 0.3\times10^{-4}\times10\times5\ \text{V}$

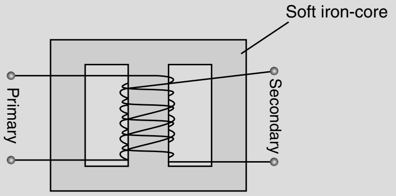
$= 1.5\times10^{-3}\ \text{V}$

[Topper's Answer 2017]

Q. 4. (i) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(ii) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. U **[Delhi I, III 2017]**

Ans. (i) Labelled diagram of a step up transformer 1½
Derivation of ratio of secondary and primary voltage 2
(ii) Calculation of number of turns in the secondary. 1½



(i) Try yourself, Similar to Q. 1 (a) Long Answer Type Questions. 3½

(ii) $$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$ ½

$$\frac{N_s}{3000} = \frac{220}{2200}$$ ½

$\therefore$ $N_s = 300$ ½

[CBSE Marking Scheme 2017]

Q. 5. (i) Draw a labelled diagram of *ac* generator. Derive the expression for the instantaneous value of the emf induced in the coil.

(ii) A circular coil of cross-sectional area 200 cm² and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude 3.0 × 10^{-2} *T*. Calculate the maximum value of the current in the coil.

U **[Delhi I, II, III 2017]**

Ans. (i) Labelled diagram of *ac* generator 1½

Expression for instantaneous value of induced emf. 1½

(ii) Calculation of maximum value of current 2

(i) Labelled diagram :

Try yourself, Similar to Q. 3(a), Long Answer Type Questions 3

(ii) Maximum value of emf

$$e_0 = NBA\omega$$ ½

$$= 20 \times 200 \times 10^{-4} \times 3 \times 10^{-2} \times 50 \text{ V}$$ ½

$$= 600 \text{ mV}$$ ½

Maximum induced current,

$$i_0 = \frac{e_0}{R} = \frac{600}{R} \text{mA}$$ ½

[**Note 1 :** If the student calculates the value of the maximum induced emf and says that "since *R* is not given, the value of maximum induced current cannot be calculated", the ½ mark, for the last part, of the question, can be given.]

[**Note 2 :** The direction of magnetic field has not been given. If the student takes this direction along the axis of rotation and hence obtains the value of induced emf and, therefore, maximum current, as zero, award full marks for this part.]

[CBSE Marking Scheme 2017]

Detailed Answer :

(i) Expression :

If at any moment, *t* be the perpendicular vector to the plane of coil that makes an angle θ with direction of magnetic field (*B*), so flux passing through the coil will be :

$$\phi = nBA \cos \theta$$
$$\theta = \omega t$$

[ω=angular velocity of the coil]

Now,

$$\phi = nAB\cos \omega t$$ 1

If *e* is instantaneous induced emf produced in the coil, then

$$e = -\frac{d\phi}{dt}$$
$$= -\frac{d(nAB\cos \omega t)}{dt}$$
$$= -nAB\omega(-\sin \omega t)$$ ½

Maximum value or peak value of instantaneous induced emf *e* which is attained when sin ωt = ±1

$\therefore$ $e_{max} = n\omega AB$

So, $e = e_{max}\sin \omega t$

As value of sine function varies from +1 to −1, polarity of emf changes with time. Also the output voltage is sinusoidal in nature. 1

Q. 6. Draw an arrangement for winding of primary and secondary coils in a transformer with two coils on a separate limb of the core.

State the underlying principle of a transformer. Deduce the expression for the ratio of secondary voltage to the primary voltage in terms of the ratio of the number of turns of primary and secondary winding. For an ideal transformer, obtain the ratio of primary and secondary currents in terms of the ratio of the voltages in the secondary and primary voltages.

Write any two reasons for the energy losses which occur in actual transformers.

U **[Delhi Comptt. 2016]**

Ans. Try yourself, Similar to Q. 1 (i), Long Answer type Questions. 4

Two reasons for energy losses

Flux leakage / joule heat losses in the windings / Eddy currents / hysteresis **(Any two)** ½ + ½

[CBSE Marking Scheme 2016]

AI **Q. 7. (i) Write the function of a transformer. State its principle of working with the help of a diagram. Mention various energy losses in this device.**

(ii) The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are respectively 220 V and 1100 W. Calculate

(a) number of turns in secondary

(b) current in primary

(c) voltage across secondary

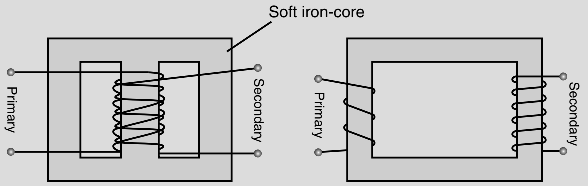
(d) current in secondary

(e) power in secondary R **[Delhi I, II, III 2016]**

Ans. (i) Conversion of *ac* of low voltage into *ac* of high voltage & vice versa.

Working Principle :

Mutual induction : When alternating voltage is applied to primary windings, emf is induced in the secondary windings. 2



(a) **(b)**

(Any one of the above diagram)

Energy losses :

- Leakage of magnetic flux
- Eddy currents
- Hysteresis loss ½ + ½
- Copper loss **(Any two)**

(ii) $N_p = 100$

Transformation ratio = 100

(a) Number of turns in secondary coil

$N_S = 100 \times 100 = 10000$ ½

(b) Input Power = Input voltage × current in primary

$1100 = 220 \times I_p$ ½

$\Rightarrow \quad I_p = 5$ A

(c) $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$\frac{V_S}{220} = 100$

$\Rightarrow \quad V_S = 2.2 \times 10^4$ volts ½

(d) $\frac{I_P}{I_S} = \frac{N_S}{N_P}$

$\frac{5}{I_S} = 100$

$\Rightarrow \quad I_s = \frac{5}{100} = 0.05$ A ½

(e) Power in secondary = Power in Primary

= 1100 W

[CBSE Marking Scheme 2016]

Q. 8. (i) Draw a labelled diagram of a step-down transformer. State the principle of its working.

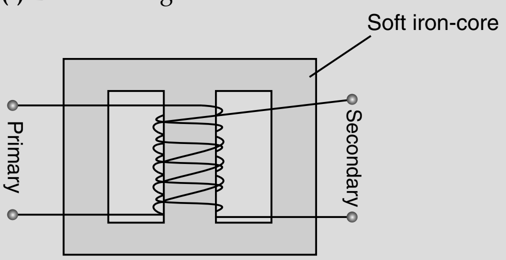
(ii) Express the turn ratio in terms of voltages.

(iii) Find the ratio of primary and secondary currents in terms of turn ratio in an ideal transformer.

(iv) How much current is drawn by the primary of a transformer connected to 220 V supply when it delivers power to a 110 V – 550 W refrigerator ?

U **[O. D. I, II, III 2016]**

Ans. (i) Labelled diagram :



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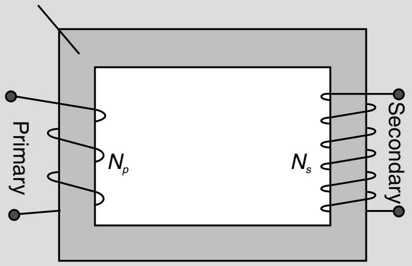


Principle : ½ + ½

When the current flowing through the primary coil changes, an emf is induced in the secondary coil due to the change in magnetic flux linked with it.

[**Note :** Give ½ mark to the student who writes only mutual induction.]

(ii) $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ 1

(iii) For an ideal transformer,

$i_p V_p = i_s Vs$

$\therefore \quad \frac{i_p}{i_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}$ 1

(iv) We have

$i_p V_p = i_s V_s = 550$ W

$V_p = 220$ V

$i_p = \frac{550}{220} = \frac{5}{2} = 2.5$ A 1

[CBSE Marking Scheme 2016]

Q. 9. (i) Draw a labelled diagram of an *ac* generator and state its working principle. R **[Delhi I, II, III 2014]**

(ii) How is magnetic flux linked with the armature coil changed in a generator ?

(iii) Derive the expression for maximum value of the induced emf and state the rule that gives the direction of the induced emf.

(iv) Show the variation of the emf generated versus time as the armature is rotated with respect to the direction of the magnetic field.

U **[Delhi Compt. I, II, III 2014, O.D. Compt. I, II, III 2012]**

Ans. (i) Try yourself, Labelled Diagram: Refer to Q. 3 (a), Long Answer Type Questions. 1

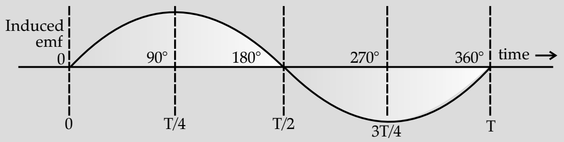
Principle :

It works on the principle of electromagnetic induction, *i.e.*, when a coil continuously rotates in a magnetic field, the magnetic flux associated with it keeps on changing; thus an emf is induced in it. 1

(ii) When the coil rotates in a magnetic field, its effective area, *i.e.*, A cos θ, (*i.e.*, area normal to the magnetic field) keeps on changing. Hence magnetic flux $\phi = NBA\cos\theta$, keeps on changing.

(iii) Try yourself, Expression : Refer to Q. 3(a), Long Answer Type Questions. 2

(iv)



[CBSE Marking Scheme, 2014] 1

Q. 10. (ii) Draw a schematic arrangement for winding of primary and secondary coil in a transformer when the two coils are wound on top of each other.

(ii) State the underlying principle of a transformer and obtain the expression for the ratio of secondary to primary voltage in terms of the

(a) number of secondary and primary windings and

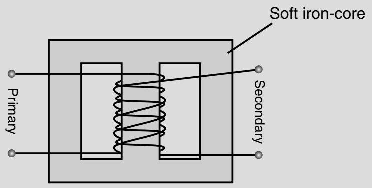
(b) primary and secondary currents.

(iii) Write the main assumption involved in deriving the above relations.

(iv) Write any two reasons due to which energy losses may occur in actual transformers.

U [O.D. I, II, III 2014]

Ans.(i)



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(ii) Principle of a transformer : When an alternating current flows through the primary coil, an emf is induced in the neighbouring (secondary) coil. ½

(a) Let $\frac{d\phi}{dt}$ be the rate of change of flux through each turn of the primary and the secondary coil

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{-N_1\frac{d\phi}{dt}}{-N_2\frac{d\phi}{dt}} = \frac{N_1}{N_2} \qquad ½$$

or

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad ...(i)$$

(b) But for an ideal transformer,

$$V_1I_1 = V_2I_2$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} \qquad ...(ii)\ ½$$

From equation (i) and (ii)

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \qquad ½$$

(iii) Main assumptions : ½

(a) The primary resistance and current are small.

(b) The flux linked with the primary and secondary coils is same / there is no leakage of flux from the core.

(c) Secondary current is small. ½

(Any one)

(iv) Reason due to which energy losses may occur :

Flux leakage/Resistance of the coils/Eddy currents/ Hysteresis. **(Any two)** ½ + ½

[CBSE Marking Scheme 2014]

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