5 Quadratic Equations

STUDY NOTES

- An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a *quadratic equation.*
- A number α is called a root of $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- Let a and b be two real numbers, then $ab = 0 \Rightarrow a = 0$ or $b = 0$.
  This is called zero product rule.
- *To solve a quadratic equation by factorization*
  (i) Make the given equation free from fractions and radicals and put it into the standard form $ax^2 + bx + c = 0$.
  (ii) Factorize $ax^2 + bx + c$ into two linear factors.
  (iii) Put each linear factor equal to 0 (zero product rule).
  (iv) Solve these linear equations and get two roots of the given quadratic equation.
- The roots of the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in R$ and $a \neq 0$ are given by
  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Shridharacharya's formula)
- *To solve a quadratic equation by the use of formula*
  (i) Clear all fractions and square roots (if any).
  (ii) Write the equation in the form $ax^2 + bx + c = 0$, $a \neq 0$.
  (iii) Compare the equation obtained in step (ii) with the equation $ax^2 + bx + c = 0$.
  (iv) Use the formula : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- $D = b^2 - 4ac$ is called the discriminant of the quadratic equation $ax^2 + bx + c = 0$; where $a, b, c \in R$ and $a \neq 0$.
- If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the nature of the roots of the equation depends upon the value of its discriminant $D = b^2 - 4ac$, as given below :
  (i) If $D = 0$, then, the roots are real and equal, each equal to $-\dfrac{b}{2a}$.
  (ii) If $D > 0$ and not a perfect square, then the roots are irrational and unequal.
  (iii) If $D > 0$ and a perfect square, then the roots are unequal and rational., if $a, b, c \in Q$.
  (iv) If $D > 0$ and a perfect square, then the roots are unequal and irrational, if b is irrational.
  (v) If $D < 0$, the roots are unequal and imaginary.
  (vi) If $D \geq 0$, the roots are real.

QUESTION BANK

A. Multiple Choice Questions [1 Mark]

Choose the correct option:

1. If $x^2 = 3x$, then :
(a) $x = 0$ (b) $x = 0$ or $x = 3$ (c) $x = 3$ (d) $x = 0$ and $x = 3$

2. If $3x^2 + 8 = 10x$, then :
(a) $x = 2$ or $\dfrac{4}{3}$ (b) $x = 2$ or 3 (c) $x = 3$ or $\dfrac{4}{3}$ (d) $x = 1$ or $\dfrac{1}{3}$

3. The quadratic equation whose solution set is $\{-2, 3\}$, is :

(a) $x^2 - 3x - 6 = 0$ (b) $x^2 - x + 6 = 0$ (c) $x^2 + x - 6 = 0$ (d) $x^2 - x - 6 = 0$

4. One of the roots of $2x^2 - 7x + 6 = 0$, is :

(a) -2 (b) 2 (c) $-\frac{3}{2}$ (d) 4

5. The discriminant of the quadratic equation $2x^2 - x + 3 = 0$ is :

(a) 24 (b) 25 (c) -23 (d) -20

6. On solving $x^2 + 4x - 21 = 0$, we get :

(a) $x = -3$ or -7 (b) $x = 7$ or -3 (c) $x = 7$ or 3 (d) $x = -7$ or 3

7. The roots of the quadratic equation $2x^2 + x - 1 = 0$ are :

(a) $\frac{1}{2}$ or -1 (b) $-\frac{1}{2}$ or 1 (c) $-\frac{1}{2}$ or -1 (d) $\frac{1}{2}$ or 1

8. The discriminant of $x^2 - 4x - 7 = 0$ is :

(a) 44 (b) $\sqrt{44}$ (c) 12 (d) -12

9. The quadratic equation whose roots are $-1, -5$ is :

(a) $x^2 + 6x + 5 = 0$ (b) $x^2 - 6x + 5 = 0$ (c) $x^2 + 6x - 5 = 0$ (d) $x^2 - 6x - 5 = 0$

10. For the equation $3x^2 - 4x - 2 = 0$, the roots are :

(a) real and equal (b) real and unequal (c) imaginary (d) both (a) and (b)

11. For the quadratic equation $2x^2 - 4x + 1 = 0$, the discriminant is :

(a) 0 (b) +ve (c) –ve (d) none of these

12. For the quadratic equation $2x^2 - 3x + 1 = 0$, the discriminant is :

(a) 1 (b) -1 (c) 0 (d) 9

13. For the quadratic equation $9x^2 + 6x + 1 = 0$, the discriminant is :

(a) +ve (b) –ve (c) 0 (d) imaginary

14. For the quadratic equation $2x^2 + ax - a^2 = 0$, the sum of the roots is :

(a) $\frac{a}{2}$ (b) $-\frac{a}{2}$ (c) $2a$ (d) $-a$

15. Given a quadratic equation $mx^2 + 8x - 2 = 0$, $m \neq 0$. For this quadratic equation the value of discriminant is :

(a) $64 - 8m$ (b) $\sqrt{64+8m}$ (c) $64 + 8m$ (d) $\sqrt{8m-64}$

16. For the quadratic equation $3x^2 + 7x + 8 = 0$, the roots are :

(a) real and distinct (b) real and equal (c) imaginary (d) both (a) and (b)

17. The roots of the quadratic equation $2x^2 - kx + k = 0$ are equal. If $k \in \mathrm{N}$, then k is equal to :

(a) 0 (b) 8 (c) 6 (d) -6

18. Given the quadratic equation $x^2 + 2\sqrt{2}x + 1 = 0$. The roots of the quadratic equation are :

(a) $\frac{-\sqrt{2} \pm 1}{2}$ (b) $\frac{\sqrt{2} \pm 1}{2}$ (c) $\sqrt{2} \pm 1$ (d) $-\sqrt{2} \pm 1$

19. The quadratic equation $(m + 1)x^2 + 2(m + 3)x + (m + 8) = 0$ has equal roots. The value of m is :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) 3 (d) -3

20. The quadratic equation $x^2 + m(2x + m - 1) + 2 = 0$ has equal roots. The value of m is :

(a) 1 (b) 2 (c) -2 (d) 0

Answers

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (b)

B. Short Answer Type Questions [3 Marks]

1. Find the value of k for which the following equation has equal roots.
$x^2 + 4kx + (k^2 - k + 2) = 0$

Sol. Given: $x^2 + 4kx + (k^2 - k + 2) = 0$

For equal roots, D = 0 i.e.; $b^2 - 4ac = 0$

$\Rightarrow (4k)^2 - 4\,(k^2 - k + 2) = 0 \Rightarrow 16k^2 - 4\,(k^2 - k + 2) = 0$

$\Rightarrow 4k^2 - k^2 + k - 2 = 0$

$\Rightarrow 3k^2 + k - 2 = 0 \Rightarrow (k + 1)\,(3k - 2) = 0$

$\Rightarrow k = -1$ or $k = \frac{2}{3}$.

2. Find the values of p for which the equation $px^2 - 5x + p = 0$ has real and equal roots.

Sol. Given $px^2 - 5x + p = 0$

$\because$ Roots are equal and real, then $b^2 - 4ac = 0$

$\Rightarrow (-5)^2 - 4 \times p \times p = 0 \Rightarrow 4p^2 = 25 \Rightarrow p = \pm \frac{5}{2}$.

3. For what values of m the equation $2x^2 + mx + 2 = 0$ has real roots?

Sol. Given: $2x^2 + mx + 2 = 0$

For real roots, $b^2 - 4ac \geq 0$

$\Rightarrow (m)^2 - 4 \times 2 \times 2 \geq 0 \Rightarrow (m)^2 - 16 \geq 0 \Rightarrow (m + 4)(m - 4) \geq 0.$

$m \geq 4$ or $m \leq -4$.

4. Find the values of p for which the equation $px^2 + 2x + 1 = 0$ has distinct real roots.

Sol. Given : $px^2 + 2x + 1 = 0$

For distinct real roots $b^2 - 4ac > 0$

$\Rightarrow (2)^2 - 4 \times p \times 1 > 0 \Rightarrow 4p < 4 \Rightarrow p < 1$

5. Show that the equation $2(p^2 + q^2)\,x^2 + 2(p + q)x + 1 = 0$ has no real roots when $p \neq q$.

Sol. Given : $2\,(p^2 + q^2)\,x^2 + 2\,(p + q)x + 1 = 0$

$D = b^2 - 4ac$

$= [2(p + q)]^2 - 4 \times 2(p^2 + q^2) \times 1$

$= 4(p^2 + q^2 + 2pq) - 8(p^2 + q^2)$

$= -4\,(p^2 + q^2 - 2pq) = -4\,(p - q)^2$

So for $p \neq q$, $D < 0$, i.e., it has no real roots. **Proved**.

6. Solve the quadratic equation $x^2 - 4x - 8 = 0$ for x. Give your answer correct to three significant figures.

Sol. Given : $x^2 - 4x - 8 = 0$

Using quadratic formula,

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 + 4 \times 1 \times 8}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$= 2\,(1 \pm \sqrt{3}\,) = 2\,(1 + 1.732)$ or $2\,(1 - 1.732)$

$= 2 \times 2.732$ or $2\,(-0.732)$

$\Rightarrow x = 5.46$ or -1.46.

7. Solve $3x^2 + 11x + 10 = 0$, when $x \in I$ by factorisation.

Sol. Given : $3x^2 + 11x + 10 = 0$

$\Rightarrow (x + 2)\,(3x + 5) = 0$

$\Rightarrow (x + 2) = 0$ or $(3x + 5) = 0$

$\Rightarrow x = -2$ or $x = -\frac{5}{3} \Rightarrow x = -2$ as $x \in I$

8. Solve $\sqrt{2x+9} = 13 - x$ by factorisation.

Sol. Given $\sqrt{2x+9} = 13 - x$

Squaring on both sides, we get

$2x + 9 = 169 + x^2 - 26x \Rightarrow x^2 - 28x + 160 = 0 \Rightarrow (x - 20)(x - 8) = 0$

$\Rightarrow x = 20$ or $x = 8$.

$\Rightarrow x = 8$,

[Rejecting $x = 20$, as it does not satisfy the equation.]

9. Solve the equation $2x^2 + \sqrt{5}x - 5 = 0$ using formula, write your answer correct to 2 decimal places.

Sol. Given : $2x^2 + \sqrt{5}x - 5 = 0$

Using quadratic formula,

$$x = \frac{-\sqrt{5} \pm \sqrt{(\sqrt{5})^2 + 4 \times 2 \times 5}}{2 \times 2} = \frac{-\sqrt{5} \pm 3\sqrt{5}}{4}$$

$$\Rightarrow x = \frac{-\sqrt{5} + 3\sqrt{5}}{4} \text{ or } \frac{-\sqrt{5} - 3\sqrt{5}}{4} \Rightarrow x = \frac{2\sqrt{5}}{4} \text{ or } \frac{-4\sqrt{5}}{4} \Rightarrow x = \frac{2.23}{2} \text{ or } -2.23$$

$\Rightarrow x = 1.115$ or -2.23

$\Rightarrow x = 1.12$ or $- 2.23$.

10. Solve for x and give your answer correct to 2 decimal places.

$\frac{x + 1}{2x + 5} = \frac{x + 3}{3x + 4}$.

Sol. Given : $\frac{x + 1}{2x + 5} = \frac{x + 3}{3x + 4}$.

By using cross multiplication, we get, $(x + 1)(3x + 4) = (2x + 5)(x + 3)$

$\Rightarrow x^2 - 4x - 11 = 0$

By using quadratic formula,

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 + 4 \times 1 \times 11}}{2 \times 1} = 2 \pm \sqrt{15}$$

$\Rightarrow x = 2 + 3.87$ or $2 - 3.87$

$\Rightarrow x = 5.87$ or $- 1.87$.

C. Long Answer Type Questions [4 Marks]

1. For what value of k will the following quadratic equation $(k + 1)x^2 - 4kx + 9 = 0$ have real and equal roots. Solve the equations.

Sol. Given: $(k + 1)x^2 - 4kx + 9 = 0$

For real and equal roots, we have

$b^2 - 4ac = 0$

$\Rightarrow (-4k)^2 - 4(k + 1) \times 9 = 0 \Rightarrow 4k^2 - 9k - 9 = 0$

By using factorisation method, we get

$(k - 3)(4k + 3) = 0$

$\Rightarrow k = 3$ or $k = \frac{-3}{4}$

For $k = 3$, the equation becomes $4x^2 - 12x + 9 = 0 \Rightarrow (2x - 3)^2 = 0$

$\Rightarrow x = \frac{3}{2}, \frac{3}{2}$.

For $k = \frac{-3}{4}$, the equation becomes

$\frac{1}{4}x^2 + 3x + 9 = 0 \Rightarrow x^2 + 12x + 36 = 0$

$\Rightarrow (x + 6)^2 = 0 \Rightarrow x = -6, -6$

2. Solve : $\sqrt{2x^2 - 2x + 21} = 2x - 3$

Sol. Given : $\sqrt{2x^2 - 2x + 21} = 2x - 3$

Squaring on both sides, we get

$\Rightarrow 2x^2 - 2x + 21 = 4x^2 + 9 - 12x \Rightarrow x^2 - 5x - 6 = 0$

$\Rightarrow (x + 1)(x - 6) = 0$

$\Rightarrow x = -1$ or $x = 6$

$\Rightarrow x = 6$ [Rejecting $x = -1$, as it does not satisfy the equation]

3. Solve : $2\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 2 = 0,\ x \neq -1$

Sol. Given : $2\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 2 = 0$

Let $\frac{x}{x+1} = y$, then

$2y^2 - 5y + 2 = 0$

$\Rightarrow (y - 2)(2y - 1) = 0$

$\Rightarrow y = 2$ or $y = \frac{1}{2}$

So, $\frac{x}{x+1} = 2 \Rightarrow 2x + 2 = x \Rightarrow x = -2$

And $\frac{x}{x+1} = \frac{1}{2} \Rightarrow 2x = x + 1 \Rightarrow x = 1$.

4. If a, b, c are rationals, prove that the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are also rational.

Sol. Given $(b - c)x^2 + (c - a)x + (a - b) = 0$

Now, $b^2 - 4ac = (c - a)^2 - 4(b - c)(a - b)$

$= c^2 + a^2 - 2ac - 4(ab - b^2 - ac + bc) = a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac$

$= (a - 2b + c)^2$

Since, the discriminant is a perfect square, so roots are rational.

5. The difference of the squares of two natural numbers is 84. The square of the larger number is 25 times the smaller number. Find the numbers.

Sol. Let the smaller number be x. Then the larger number be $\sqrt{25x}$.

According to question,

$(\sqrt{25x})^2 - x^2 = 84 \Rightarrow x^2 - 25x + 84 = 0$

$\Rightarrow (x - 4)(x - 21) = 0$

$\Rightarrow x = 4$ or $x = 21$

If $x = 4$, then the larger number = $\sqrt{25x} = \sqrt{100} = 10$

For $x = 21$, larger number = $\sqrt{25 \times 21}$, which is not a natural number.

Hence, smaller number = 4 and larger number = 10.

6. The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.

Sol. Let Vivek's age be x and his younger brother's age be $47 - x$.

According to questions,

$x \times (47 - x) = 550 \Rightarrow x^2 - 47x + 550 = 0$

$\Rightarrow (x - 25)(x - 22) = 0$

$\Rightarrow x = 25$ or $x = 22$

If $x = 25$, i.e. Vivek's age = 25 years

Then his younger brother's age = 22 years.

We reject $x = 22$ as it does not satisfy the given condition.

7. An aeroplane travelled a distiance of 400 km at an average speed of x km/h. On the return journey, the speed was increased by 40 km/h. Write down an expression for the time taken for:

(a) the onward journey

(b) the return journey

If the return journey took 30 minutes less than the onward journey, write an equation in x and find the value of x.

Sol. Let the time taken for the onward journey be T_1 and the time taken for the return journey be T_2.

(a) Since, speed $= \frac{\text{distance}}{\text{time}}$

$\Rightarrow x = \frac{400}{T_1} \Rightarrow T_1 = \frac{400}{x}$

(b) For the return journey, speed $= \frac{\text{distance}}{\text{time}}$

$x + 40 = \frac{400}{T_2} \Rightarrow T_2 = \frac{400}{x + 40}$

Now, $T_1 - T_2 = \frac{30}{60}$ [Given]

$\Rightarrow \frac{400}{x} - \frac{400}{x + 40} = \frac{1}{2} \Rightarrow x^2 + 40x - 32000 = 0$

$\Rightarrow (x + 200)(x - 160) = 0$

$\Rightarrow x = -200$, which not possible, or $x = 160$.

8. The hotel bill for a number of people for overnight stay is ₹14,400. If there were 4 more people, the bill each person had to pay would have reduced by ₹600. Find the number of people staying overnight.

Sol. Let the number of people staying overnight be x.

According to question,

$\frac{14400}{x} - \frac{14400}{x + 4} = 600 \Rightarrow \frac{1}{x} - \frac{1}{x + 4} = \frac{600}{14400} = \frac{1}{24}$

$\Rightarrow x^2 + 4x - 96 = 0 \Rightarrow (x + 12)(x - 8) = 0$

$\Rightarrow x = -12$, which not possible or $x = 8$.

Hence, required number of people is 8.

9. A fruitseller bought x apples for ₹1200.

(a) Write the cost price of each apple in terms of x.

(b) If 10 of the apples were rotten and he sold rest at ₹3 more than the cost price of each, write the selling price of $(x - 10)$ apples.

(c) If he made a profit of ₹60 in this transaction, form an equation in x and solve it to evaluate x.

Sol. (a) Cost price of each apple $= \frac{1200}{x}$

(b) 10 of the apples are rotten.

$\therefore$ Number of good apples $= x - 10$

Selling price of each apple $= \frac{1200}{x} + 3$

Selling price of $(x - 10)$ apples $= (x - 10)\left(\frac{1200}{x} + 3\right)$

(c) Selling price – cost price = profit.

$\Rightarrow (x - 10)\left(\frac{1200}{x} + 3\right) - 1200 = 60$

$\Rightarrow x^2 - 30x - 4000 = 0 \Rightarrow (x - 80)(x + 50) = 0$

$\Rightarrow x = 80$ or $x = -50$ not possible.

$\therefore$ Total number of apples = 80.

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