Ch. 1 : Relations & Functions

Choose the correct answer

1. Let R be the relation in the set **N** given by $R = \{(a, b) : a = b - 2, b > 6\}$.

A) $(2, 4) \in R$ B) $(3, 8) \in R$ C) $(6, 8) \in R$ D) $(8, 7) \in R$ **(NCERT)**

2. Let R be the relation in the set $(1, 2, 3, 4\}$ given by

$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.

A) R is reflexive and symmetric but not transitive.

B) R is reflexive and transitive but not symmetric.

C) R is symmetric and transitive but not reflexive.

D) R is equivalence relation. **(NCERT)**

3. The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$ is _____

A) transitive but not reflexive B) symmetric but not transitive

C) reflexive but not symmetric D) neither symmetric nor transitive **(JEE-M 23)**

4. A set A has 4 elements. Then the number of relations on A is

A) 2^4 B) 2^{16} C) 16^2 D) 2^8

5. Let R be a relation on N, the set of all natural numbers given by $R = \{(a, b) : a \leq b\}$. Then,

A) R is reflexive and symmetric B) R is symmetric and transitive

C) R is reflexive and transitive but not symmetric D) R is an equivalence relation.

6. Divisibility relation on Z is

A) reflexive B) symmetric C) transitive D) equivalence relation

7. Let L denote the set of all straight lines in a plane. Let R be the relation on L defined by $R = \{(l, m) : l \text{ is perpendicular to } m\}$. Then R is

A) reflexive B) symmetric C) transitive D) equivalence relation

8. In the set of all integers z, which of the following relations is not an equivalence relation ?

A) $\{(x, y) : x \leq y\}$ B) $\{(x, y) : x = y\}$ C) $\{(x, y) : x - y \text{ is even integer}\}$ D) $Z \times Z$

9. The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is

A) reflexive B) symmetric C) transitive D) equivalence **(23-M, MQP)**

10. The function $f : Z \to Z$ given by $f(x) = x^2$ is

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

11. The greatest integer function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = [x]$, is

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

12. The modulus function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = |x|$, is

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

13. The Signum function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is.

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

14. Let $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^4$. Choose the correct answer.

A) f is one-one onto B) f is many-one onto

C) f is one-one but not onto D) f is neither one-one nor onto. **(NCERT, MQP)**

15. Let $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 3x$. Choose the correct answer.

A) f is one-one onto B) f is many-one onto

C) f is one-one but not onto D) f is neither one-one nor onto. **(NCERT)**

16. If f = {(5, 2), (6, 3)}, then which of the following is true?

A) Domain of f = N B) Domain of f is {2, 3, 5, 6}

C) Range of f = {2, 3} D) Range of f is {5, 6}

17. A set A has 3 elements and the set B has 4 elements. Then the number of injective functions that can be defined from A to B is

A) 4! B) 3! C) 12! D) 64!

18. The number of all one – one functions from the set A = {a, b, c} to itself is.

A) 3 B) 6 C) 27 D) 1

19. If A contains 3 elements and B contains 2 elements, then the number of one – one functions from A to B is___

A) 3 B) 0 C) 3^2 D) 3!

20. Relation R in the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$

Then R is

A) reflexive B) symmetric C) transitive D) none of these

21. If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$ is onto, then the interval S is

A) $[-1, 3]$ B) $[-1, 1]$ C) $[0, 1]$ D) $[0, 3]$ **(AIEEE 04)**

22. $f : R \to R$ given by $f(x) = x + \sqrt{x^2}$ is

A) one-one B) onto C) bijective D) many one-into

23. $f : R \to R$ given by $f(x) = 5x + |\cos x|$ is

A) one-one and onto B) one-one and into

C) many one and into D) many one and onto

24. In the set Z of all integers, which of the following relation R is not an equivalence relation ?

A) $x R y$ if $x \le y$ B) $x R y$ if $x = y$

C) $x R y$ if $x - y$ is an even integer D) $x R y$ if $|x| = |y|$

25. If $A = \{x, y, z\}$, then the relation $R = \{(x, y), (y, x), (x, x)\}$ on A is

A) reflexive B) symmetric and transitive

C) symmetric only D) transitive only

26. For $x, y \in R$, define a relation R by $x R y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then R is

A) an equivalence relation B) R is symmetric

C) R is reflexive D) R is transitive

27. Let $A = \{1, 2, 3\}$ and consider the relation

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then R is

A) reflexive but not symmetric B) reflexive but not transitive

C) symmetric and transitive D) neither symmetric nor transitive

28. Let $f : R \to R$ be defined by $f(x) = e^x - e^{-|x|}$. Then

A) the range of f is $(-\infty, 0]$ B) f is $1 - 1$

C) the range of f is $[0, \infty)$ D) f is onto

29. A is a set having 6 distinct elements. The number of distinct functions from A to A which are not bijections is **(CET 18)**

A) $6! - 6$ B) $6^6 - 6$ C) $6^6 - 6!$ D) $6!$

30. If $A = \{x \mid x \in \mathbb{N}, x \le 5\}$ $B = \{x \mid x \in \mathbb{Z}, x^2 - 5x + 6 = 0\}$, then the number of onto functions from A to B is **(CET 19)**

A) 30 B) 2 C) 32 D) 23

31. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then : **(JEE-M 21)**

A) $y = 273x$ B) $2y = 91x$ C) $y = 91x$ D) $2y = 273x$

32. If $f : R \to R$ defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $(f \circ f)(x) =$

A) $3 - x^3$ B) x C) x^3 D) $-x$

33. If $f : R \to R$ given by $f(x) = 7x + 8$ and $f^{-1}(12) = \frac{k}{7}$, then the value of k is

A) 7 B) 1 C) 4 D) 8

34. If $f(x) = \frac{3x+2}{5x-3}$, $x \in R - \left\{\frac{3}{5}\right\}$, then

A) $f^{-1}(x) = f(x)$ B) $f(f(x)) = -x$ C) $f^{-1}(x) = -f(x)$ D) Inverse does not exist

35. If a set A has m elements and set B has 7 elements and the number of injections from A to B is 2520, then the value of m is

A) 2 B) 7 C) 6 D) 5

36. For any two real numbers θ and ϕ, θ R ϕ iff $\sec^2\theta - \tan^2\phi = 1$. Then the relation R is

A) reflexive but not transitive B) symmetric but not reflexive

C) an equivalence relation D) both reflexive and symmetric but not transitive.

37. A function $f : [0, \infty) \to [0, \infty)$ defined by $f(x) = \frac{x}{1+x}$ is

A) one-one and onto B) one-one but not onto

C) onto but not one-one D) neither one-one nor onto

38. Let a function $f : N \to N$ be defined by $f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \ldots\ldots\ldots \\ n-1, & n = 3, 7, 11, 15, \ldots\ldots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \ldots\ldots \end{cases}$, then f is

A) one-one but not onto B) onto but not one-one

C) neither one-one nor onto D) one-one and onto

39. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}$, $\forall x \in R$, then f is .... **(CET 15)**

A) one – one B) onto C) bijective D) f is not defined

40. Let A = {x : x ∈ R; x is not a positive integer} Define $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is **(CET 21)**

A) Injective but not surjective B) surjective but not injective

B) bijective D) neither injective nor surjective

41. The function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one in the interval **(CET 21)**

A) $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$ B) $\left(\frac{\pi}{6}, \frac{-\pi}{3}\right]$ C) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ D) $\left[\frac{-\pi}{6}, \frac{-\pi}{3}\right)$

42. Let $f: R \to R$ be defined by $f(x) = 2x + 6$ which is a bijective mapping then $f^{-1}(x)$ is given by **(CET 16)**

A) $\frac{x}{2} - 3$ B) $2x + 6$ C) $x - 3$ D) $6x + 2$

43. If $f : R \to R$ is defined by $f(x) = 2x + 3$, then $f^{-1}(x)$ **(CET 12)**

A) is given by $\frac{x-3}{2}$ B) is given by $\frac{1}{2x+3}$

C) does not exist because 'f ' is not injective

D) does not exist because 'f ' is not surjective

44. The number of bijective functions from the set A to itself, if A contains 108 elements is **(COMEDK 15)**

A) 108 B) (108)! C) $(108)^2$ D) 2^{108}

45. The set A has 4 elements and the set B has 5 elements then the number of injective mappings that can be defined from A to B is **(CET 16)**

A) 144 B) 72 C) 60 D) 120

46. If the set x contains 7 elements and set y contains 8 elements, then the number of bijections from x to y is **(CET 22)**

A) 0 B) 7 ! C) $8\,P_7$ D) 8 !

47. If $f(x) = e^x$ and $g(x) = \log e^x$, then which of the following is TRUE ?

A) $f\{g(x)\} \neq g\{f(x)\}$ B) $f\{g(x)\} = g\{f(x)\}$

C) $f\{g(x)\} + g\{f(x)\} = 0$ D) $f\{g(x)\} - g\{f(x)\} = 1$

48. $f : R \to R$ and $g : [0, \infty) \to R$ are defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true ? **(CET 19, 23)**

A) (fog) (2) = 2 B) (gof) (4) = 4 C) (gof) (– 2) = 2 D) (fog) (– 4) = 4

49. Let f : R → R be defined by $f(x) = 3x^2 - 5$ and g : R → R by $g(x) = \frac{x}{x^2+1}$, then gof is **(CET 23)**

A) $\frac{3x^2}{x^4+2x^2-4}$ B) $\frac{3x^2-5}{9x^4-30x^2+26}$ C) $\frac{3x^2}{9x^4+30x^2-2}$ D) $\frac{3x^2-5}{9x^4-6x^2+26}$

50. Let f(x) = sin 2x + cos 2x and $g(x) = x^2 - 1$, then g(f(x)) is invertible in the domain **(CET 23)**

A) $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ B) $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ C) $x \in \left[0, \frac{\pi}{4}\right]$ D) $x \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right]$

Answers

1. **(C)**

$2 = 4 - 2$ but $4 < 6$ $\therefore (2, 4) \notin R$; $(3, 8) \notin R$ $[\because 3 \neq 8 - 2]$. $(6, 8) \in R$ because $8 > 6$ and $a = 8 - 2$.

2. **(B)**

$(a, a) \in R, \ \forall a \in \{1, 2, 3, 4\}$; $(1, 2) \in R$ but $(2, 1) \notin R$

$\therefore$ R is not symmetric ; it is trivially transitive.

3. **(D)**

4. **(B)**

No. of relations = $2^{n(A\times A)} = 2^{n(A).n(A)} = 2^{16}$

5. **(C)**

$a \leq a \ \forall a$; $2 \leq 3$ but $3 \not\leq 2$; $a \leq b$ and $b \leq c \Rightarrow a \leq c$

6. **(C)**

$0 \nmid 0$; $4 | 2$ but $2 \nmid 4$; $a | b$ and $b | c \Rightarrow a | c$

7. **(B)**

8. **(A)**

9. **(B)**

10. **(D)**

$f(2) = f(-2) = 4 \Rightarrow$ f is not 1 – 1

range of f = W ≠ Z ⇒ not on to

11. **(D)**

$f(1 . 2) = f(1 . 9) = 1 \Rightarrow$ f is not 1 – 1

range of f = Z ≠ R ⇒ not onto

12. **(D)**

$f(2) = f(-2) = 2 \Rightarrow$ f is not 1 – 1

range of $f = [0, \infty) \neq R \Rightarrow$ not onto

13. (D)

$f(1) = f(2) = 1 \Rightarrow$ not $1 - 1$

range of $f = \{-1, 0, 1\} \neq R$

14. (D)

$f(1) = f(-1)$; Range $= \mathbf{R}_* \neq \mathbf{R}$

15. (A)

$f(a) = f(b) \Rightarrow a = b;\ f\left(\frac{b}{3}\right) = b;\ \frac{b}{3} \in \mathbf{R}$ when $b \in \mathbf{R}$.

16. (C)

Domain $= \{5, 6\}$, Range $= \{2, 3\}$

17. (A)

Required $= {}^4P_3 = 4!$

18. (B)

Required $= {}^3P_3 = 3!$

19. (B)

If $n(A) > n(B)$ then no one-one functions.

20. (D)

$R = \{(x, y) : 3x - y = 0\}$ i.e. $R = \{(x, y) : 3x = y\}$

If R is to be reflexive, $(x, x) \in R,\ \forall\ x \in A$.

Now, $(x, x) \in R$ if $3x = x$, which is true only for $x = 0$.

In other words, $(1, 1) \notin R$ because $3.1 \neq 1$ $\therefore$ R is not reflexive.

If R is to be symmetric, then $(x, y) \in R \Rightarrow (y, x) \in R$.

Now, $(x, y) \in R \Rightarrow 3x = y \Rightarrow x = \frac{1}{3}y$ i.e. $3y \neq x \Rightarrow (y, x) \notin R$.

For example, $(1, 3) \in R$ but $(3, 1) \notin R$. $\therefore$ R is not symmetric

Let (x, y) and $(y, z) \in R$. Then $3x = y$ and $3y = z$

Then $3x = \frac{1}{3}z \Rightarrow 3x \neq z$ i.e. $(x, z) \notin R$.

For example, $(1, 3)$ and $(3, 9) \in R$ but $(1, 9) \notin R$ $\therefore$ R is not transitive.

21. (A)

Max. $f = 1 + 2$; Min. $f = 1 - 2$

22. (D)

$$f(x) = x + \sqrt{x^2} = x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

23. (A)

$f(x) = 5x \Rightarrow f'(x) = 5, \quad \forall\, x \in R$

$\Rightarrow$ f(x) is strictly increasing function

$\therefore$ $f(x) = 5x + |\cos x|$ is also strictly increasing function $\Rightarrow$ it is both 1 – 1 and onto

24. (A)

25. (C)

26. (C)

Since $x - x + \sqrt{2} = \sqrt{2}$ which is an irrational number, so xRx, $\forall$ x $\in$ R is possible. $\therefore$ R is reflexive.

But R is not symmetric, for, $\left(\sqrt{2},\ 1\right) \in R$ but $\left(1,\ \sqrt{2}\right) \notin R$

Also R is not transitive, for, $\left(\sqrt{2},\ 1\right) \in R$ and $\left(1,\ 2\sqrt{2}\right) \in R$

but $\left(\sqrt{2},\ 2\sqrt{2}\right) \notin R$

27. (1)

Clearly, R is reflexive, for, (1, 1) $\in$ R, (2, 2) $\in$ R, (3, 3) $\in$ R

But R is not symmetric, for, (2, 3) $\in$ R but (3, 2) $\notin$ R

28. (C)

When $x \geq 0$, $f(x) = e^x - e^{-x}$

When $x < 0$, $f(x) = e^x - e^{-(-x)} = 0$

Clearly f is not 1 – 1

When $x > 0$, $e^x > e^{-x}$ $\therefore$ $f(x) > 0, \forall\, x > 0$ $\therefore$ The range is $[0, \infty) \neq R$

$\therefore$ f is not onto.

29. (C)

Required = No. of functions – number of bijective functions = $6^6 - 6!$

30. (A)

A = {1, 2, 3, 4, 5} & B = {2, 3}

Use : If n(A) = n ($n \geq 2$) & n (B) = 2 , then the number of onto functions from A to B is

$2^n - 2 = 2^5 - 2 = 30$.

31. (B)

$x = {}^5P_3 = 5 \,.\, 4 \,.\, 3$

$y = {}^{15}P_3 = 15 \,.\, 14 \,.\, 13$ $\quad \therefore \ \frac{y}{x} = \frac{15.14.13}{5.4.3} = \frac{91}{2} \Rightarrow 2y = 91x$

32. (B)

33. (C)

$f^{-1}(x) = \frac{x-8}{7}$; $f^{-1}(12) = \frac{4}{7} \Rightarrow k = 4$

34. (A)

$$f^{-1}(x)=\frac{-3x-2}{-5x+3}=\frac{3x+2}{5x-3}=f(x)$$

35. (D)

$2520 = {}^7P_m \Rightarrow m = 5$

36. (C)

37. (B)

$$f(x)=\frac{x+1-1}{x+1}=1-\frac{1}{x+1}$$

$$f'(x)=\frac{1}{(x+1)^2}>0\Rightarrow f \text{ is } 1-1$$

Range of f = [0, 1) $\neq$ codomain $\Rightarrow$ f is not onto

38. (D)

39. (D)

$f(0) = \frac{1}{0}$, which is meaningless !

$\therefore$ f is not a well defined function.

40. (A)

Domain = R – N; $f(x) = \dfrac{2}{1-\frac{1}{x}}$

$a, b \in A \Rightarrow f(a) = f(b) \Rightarrow \dfrac{2}{1-\frac{1}{a}} = \dfrac{2}{1-\frac{1}{b}} \Rightarrow a = b \quad \therefore$ f is 1 - 1

Let y $\in$ R be such that y = f(x)

Then $y=\dfrac{2}{1-\frac{1}{x}} \Rightarrow 1-\dfrac{1}{x}=\dfrac{2}{y}$

$\dfrac{1}{x}=1-\dfrac{2}{y}=\dfrac{y-2}{y} \quad \therefore x=\dfrac{y}{y-2}\notin \textit{Domain if } y=2$

OR

$f(x)\neq 2 \;\because \dfrac{2x}{x-1}\neq 2 \textit{ and } 2\in R$

$\therefore$ Range $\neq$ R (codomain) $\quad\therefore$ It is not onto $\therefore$ (A) is the correct option

41. (A)

$$f(x) = \sqrt{3}\sin 2x - \cos 2x + 4 = 2\left(\frac{\sqrt{3}}{2}\sin 2x - \frac{1}{2}\cos 2x\right) + 4$$

$$= 2\left(\cos\frac{\pi}{6}\,.\,\sin 2x - \sin\frac{\pi}{6}\,.\,\cos 2x\right) + 4 = 2.\sin\left(2x - \frac{\pi}{6}\right) + 4$$

$$\sin\left(2x - \frac{\pi}{6}\right) \textit{ is } 1-1 \text{ in the interval: } \left(2x - \frac{\pi}{6}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore\ 2x \in \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \quad \therefore\ x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$

Aliter: The options in (B) and (D) are not intervals!!

$\therefore$ (A) or (C) is the correct answer

Take (C) : $f\left(\frac{\pi}{2}\right) = 5 = f\left(-\frac{\pi}{2}\right)$ $\quad\therefore$ f is not 1 – 1

$\therefore$ (A) should be the correct answer

42. (A)

We have, f(x) = 2x + 6. f is a bijective function $\Rightarrow f^{-1}$ exists.

Let $x \in R$ then there exists $y \in R$ such that $f(x) = y \Rightarrow 2x + 6 = y$

$$\Rightarrow x = \frac{y-6}{2} \Rightarrow f^{-1}(y) = \frac{y-6}{2} \quad \therefore\ f^{-1}(x) = \frac{x-6}{2} \text{ for all } x \in R$$

$x \neq 3$ because $[x]^2 - [x] - 6 = 0$ when $x = 3$ $\quad\therefore$ (A) is the correct answer

43. (A)

$$\text{Let } y = 2x + 3 \ \Rightarrow\ 2x = y - 3 \ \Rightarrow\ x = \frac{1}{2}(y-3) \ \therefore f^{-1}(x) = \frac{1}{2}(x-3)$$

44. (B)

From memory !

45. (D)

Set A has 4 elements and set B has 5 elements, hence the number of injective mappings from A to B = ${}^5P_4 = 120$

46. (A)

An $n(X) < n(Y)$, no onto function is possible and hence bijective function from $X \rightarrow Y$ is not possible.

47. (B)

$f(g(x)) = e^{g(x)} = f(x)$; $f(x) = x$; $g(f(x)) = f(x) = f(g(x))$

48. (D)

$(fog)(-4) = f(g(-4))$; but $g(-4) = \sqrt{-4}$ doesn't exist

49. (B)

$$(gof)(x) = g(f(x)) = \frac{f}{f^2+1} = \frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+26}$$

50. (D)

$g(f(x)) = (\sin 2x + \cos 2x)^2 - 1 = 2 \sin 2x \cdot \cos 2x = \sin 4x$;

It is invertible if $4x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ i.e., $x \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

Remark: sin x is bijective and hence invertible in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$