Chapter 4

Basic Geometrical Ideas

Introduction to Geometrical Ideas

Introduction

The term 'Geometry' is the English equivalent of the Greek word 'Geo-metron'. 'Geo' means Earth and 'metron' means Measurement. Therefore, Geometry means measurement of the earth.
In our daily life we observe and use objects having different shapes:

- Ruler, pencil, pen are straight

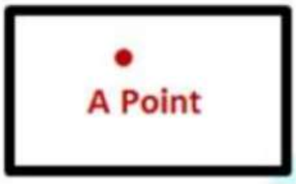


- Ball, bangle, coins, sun are round shaped.

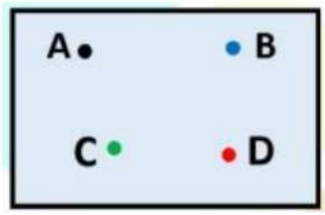



Point

A point determines a position. It has no length, breadth or thickness.



We can draw a point by the tip of a sharp pencil, tip of a compass or a pointed end of the needle.
A point is usually represented by a small dot and is named by a single capital letter of alphabet. These points will be read as point A, point B and point C.



Line Segment

A line segment corresponds to the shortest distance between two points.
Edge of a ruler, edge of a box are examples of line segment.

Let C and D be two points on a plane. The straight path from C to D is called the line segment CD and is denoted by $\overline{CD}$. The points C and D are called the endpoints of the line segment $\overline{CD}$.

- A line segment has a fixed length.
- The line segment $\overline{CD}$ is same as $\overline{DC}$

Ray

A ray is a portion of a line. It starts at one point (called starting point) and goes endlessly in a direction.

Examples of the ray are: Beam of light from a lighthouse and sun rays.

Thus a line segment $\bar{AB}$ extended endlessly in the direction from A to B, is a ray denoted by $\vec{AB}$

Ray $\overrightarrow{AB}$

The ray $\vec{AB}$ has one end (fixed) point called its initial point.

Rays $\vec{AB}$ and $\vec{BA}$ are two different rays.

Ray $\vec{BA}$ is a ray with initial point B and extends endlessly in the direction from B to A.

Ray $\overrightarrow{BA}$

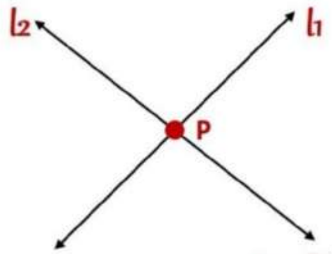
Line

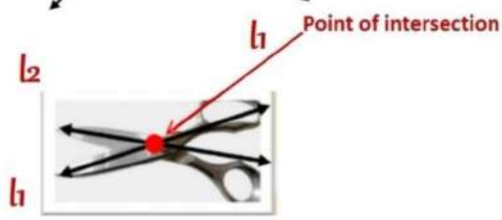
A line is straight and extends indefinitely on both directions.

A line segment $\overline{CD}$ extended on both sides and marked by arrows at the two ends represent a line, denoted by $\overleftrightarrow{CD}$ or $\overleftrightarrow{DC}$

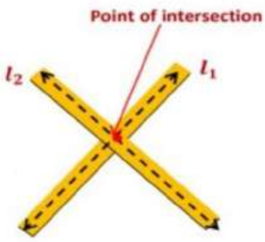
Intersecting and Parallel Lines

If two lines have one common point, they are called intersecting lines.
If there is a point P common to two lines $l1$ and $l2$, then the two lines intersect at the point P and this point P is called the point of intersection of the given lines.



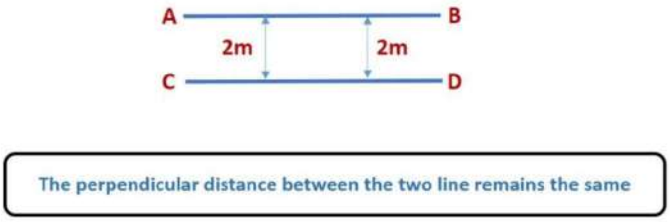


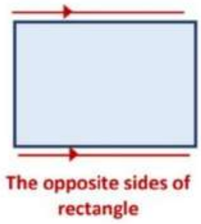
The blade of a scissors



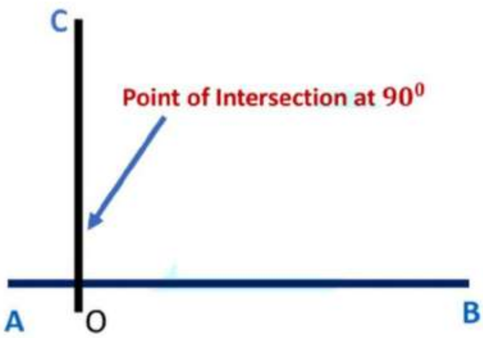
Letter X of the English alphabet

Parallel Lines
Two lines are said to be parallel if they are in the same plane and do not intersect each other.





Perpendicular Lines
Two lines are said to be perpendicular when they intersect at 90 degree (right angle).



Example: Use the figure to name:

a) Five points

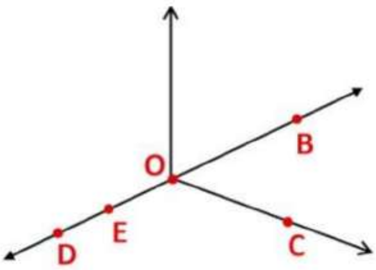
b) A line

c) Four rays

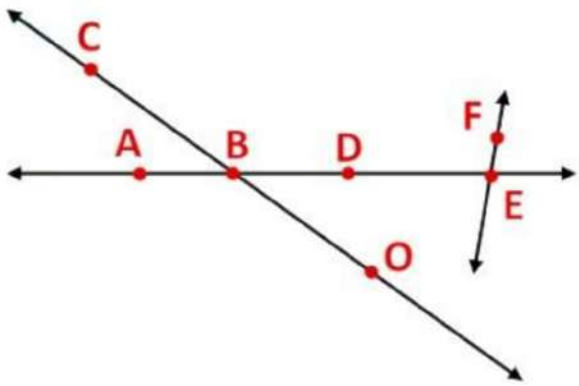
d) Five line segments

a) Five points are O, B, C, D and E.b) A line:

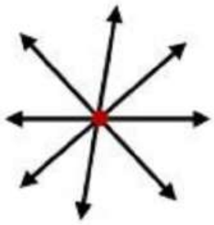
c) Four rays are , . and



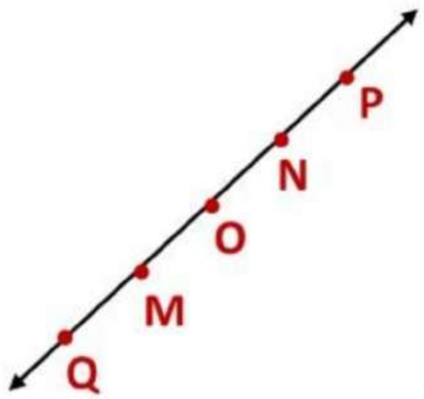
d) Five line segments are , , , , ,
Example: Use the figure to name:
a) Line containing point E.
b) Line passing through A.
c) Line on which O lies
d) Two pairs of intersecting lines.
a) Line containing point E is $\overline{AE}$
b) Line containing point A is $\overline{AE}$
c) Line on which O lies $\overline{CO}$
d) Two pairs of intersecting lines are $\overline{AD}$ and $\overline{CO}$, $\overline{AE}$ and $\overline{FE}$



Example: How many lines ca pass through
a) One given point?
b) Two given points?
a) Infinite number of lines can pass through a given point.



b) Only one line can pass through two given points.



Example: Consider the following figure of line MN. Say whether following statements are true or false in context of the given figure.

(a) Q, M, O, N, P are points on the line $\overleftrightarrow{MN}$

(b) M, O, N are points on a line segment $\overline{MN}$.
(c) M and N are end points of line segment $\overline{MN}$.
(d) O and N are end points of line segment $\overline{OP}$.
(e) M is one of the end points of line segment $\overline{QO}$.
(f) M is a point on ray $\overrightarrow{OP}$.
(g) Ray $\overrightarrow{OP}$ is different from ray $\overrightarrow{OQ}$.
(h) Ray $\overrightarrow{OP}$ is same as ray $\overrightarrow{OM}$.
(i) Ray $\overrightarrow{OM}$ is not opposite to ray $\overrightarrow{OP}$.
(j) O is not an initial point of $\overrightarrow{OP}$.

(k) N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.

a) True, points Q, M, O, N, P are points on the line MN. A line passing through two points when extends indefinitely can contain infinite number of points.

b) True, M, O, N are points on the line segment $\overline{MN}$.

c) True, M and N are end points of line segment $\overline{MN}$.

d) False, end points of line segment $\overline{OP}$ are O and P.

e) False, end points of line segment $\overline{QO}$ are Q and O and not M.

f) False, point M is not on ray $\overrightarrow{OP}$.

g) True, the end points of ray $\overrightarrow{OP}$ are O and P whereas the endpoints of ray $\overrightarrow{OQ}$ are O and Q in the opposite direction. Therefore the two rays $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are different.

h) False, the end points of ray $\overrightarrow{OP}$ are O and P whereas the endpoints of ray $\overrightarrow{OM}$ are O and M in the opposite direction.

Therefore the two rays $\overrightarrow{OP}$ and $\overrightarrow{OM}$ are different.

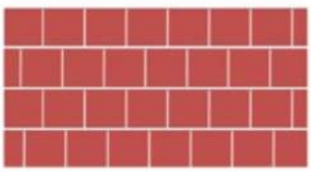
i) False, ray $\overrightarrow{OM}$ is opposite to ray $\overrightarrow{OP}$.

j) False, O is the initial point of ray $\overrightarrow{OP}$.

k) True, N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.

Plane

A plane is a flat surface that extends indefinitely in all directions. Surface of a wall, surface of a smooth blackboard are examples of a portion of a plane.

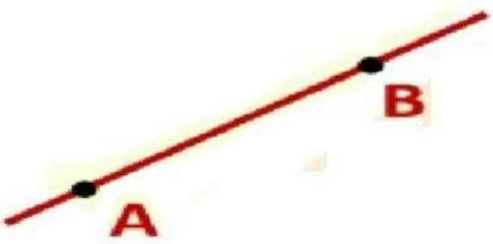


- A plane extends indefinitely in all directions and so cannot be drawn on paper.
- A plane is always represented by a minimum of three points.

Curves

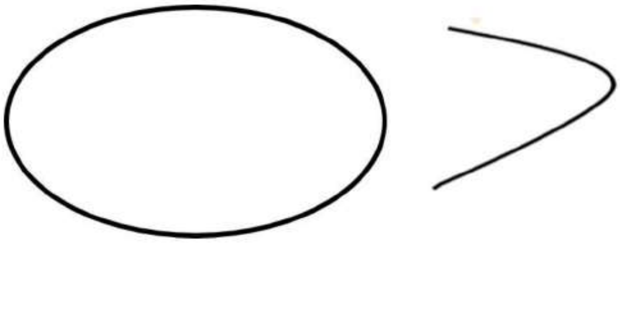
Curve can be defined as the continuous movement of points in any and every direction.

Curve can be anything which is drawn without lifting the pencil from the paper and without using a ruler.
'Curve' in everyday usage means "not straight". In Mathematics, a curve can be a straight line called a straight curve.

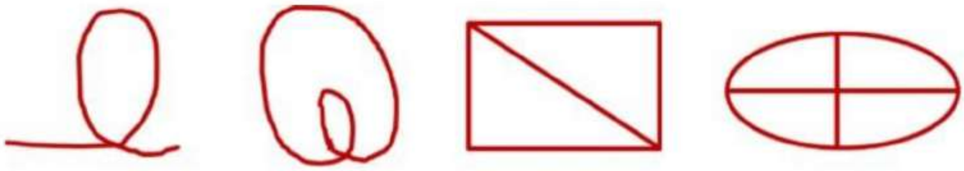


Types of Curve:

Simple Curve: If a curve does not cross itself, then it is called a simple curve.

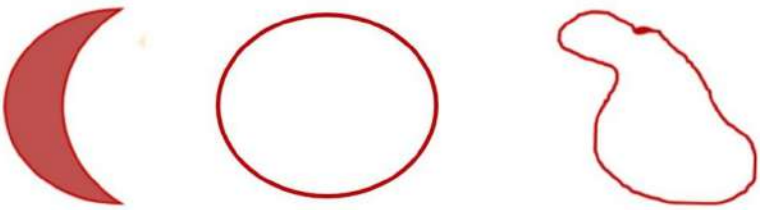


Not Simple Curves: These curves cross themselves.



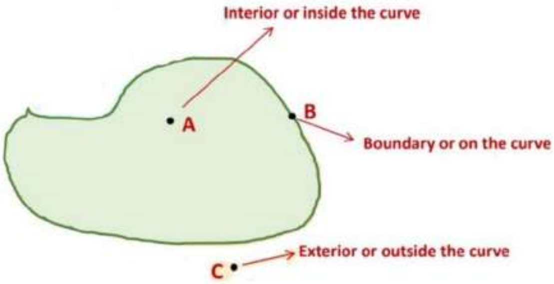
Open Curve: The curve which does not form a closed path is called an open curve.
In an open curve we can find at least one point where the curve begins or end.

Closed Curve: The curve which forms a closed path is called a closed curve.
In a closed curve we cannot find any point where the curve begins or ends.



Position in a figure

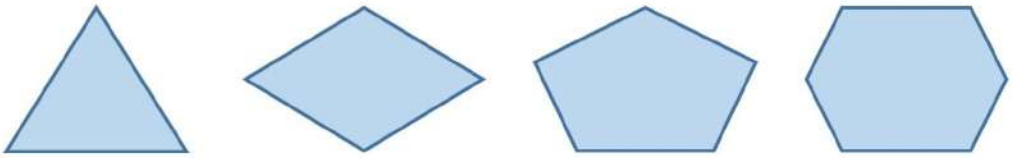
A court line in a tennis court divides it into three parts: inside the line, on the line and outside the line. You cannot enter inside without crossing the line.
Thus, in a closed curve, thus, there are three parts.
i) Interior ('inside') of the curve
ii) Boundary ('on') of the curve and
iii) Exterior ('outside') of the curve.

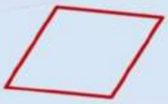
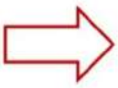


Polygons

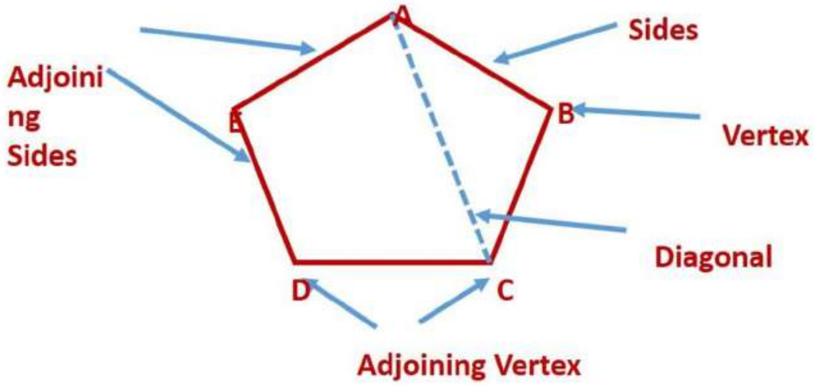
A polygon is a closed figure formed of three or more line segments.

Examples of polygons are triangle, quadrilateral, pentagon and hexagon.

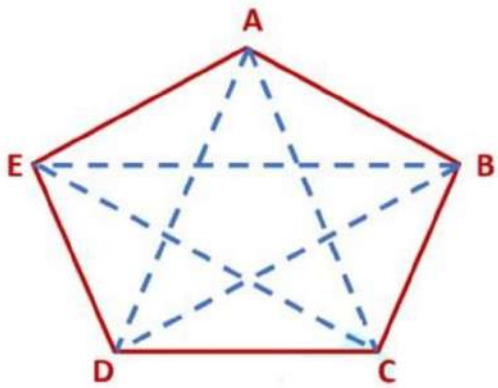


• Polygons are two dimensional. • They are bounded by straight lines and the shape is closed. • Minimum three line segments are required to make a closed figure, thus a triangle is a polygon with a minimum of three sides.	
	It is a polygon as the figure is closed and bounded by line segments.
	It is a polygon as the figure is closed and bounded by line segments.
	It is not a polygon as one of the sides is curved.
	It is not a polygon as one of the sides is curved.

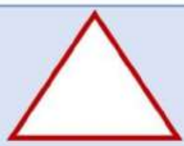
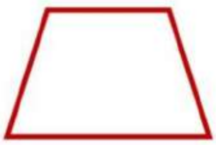
- The line segments forming a polygon are called its sides.
- The meeting point of a pair of sides is called its vertex.
- Any two sides with a common end point are called the adjacent sides of the polygon.
- The end points of the same side of a polygon are called the adjacent vertices.



Sides	$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$
Vertices	A, B, C, D and E
Adjacent Sides	$\overline{AB}$ and $\overline{BC}$, $\overline{BC}$ and $\overline{CD}$, $\overline{CD}$ and $\overline{DE}$, $\overline{DE}$ and $\overline{EA}$
Adjacent Vertices	A and B, B and C, C and D, D and E, E and A
Diagonals	$\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{BE}$ and $\overline{CE}$



Example: Classify the following curves as i) Open ii) Closed

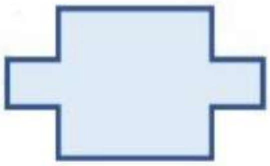
	It is an open curve.
	It is an open curve.
	It is a closed curve.
	It is a closed curve.

Example: Consider the given figure and answer the questions:
a) Is it a curve?
b) Is it closed?

a) Yes, it is a curve because a curve is drawn without lifting the pencil from the paper and without using a ruler.
b) Yes, it is closed because the curve forms a closed path.

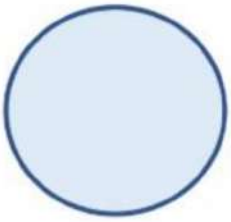
Example: Draw any polygon and shade its interior.

A polygon is a closed figure formed of three or more line segments.



Example: Illustrate, if possible, each one of the following with a rough diagram:
a) A closed curve that is not a polygon.
b) An open curve made up entirely of line segments.
c) A polygon with two sides.

a) A closed curve that is not a polygon.



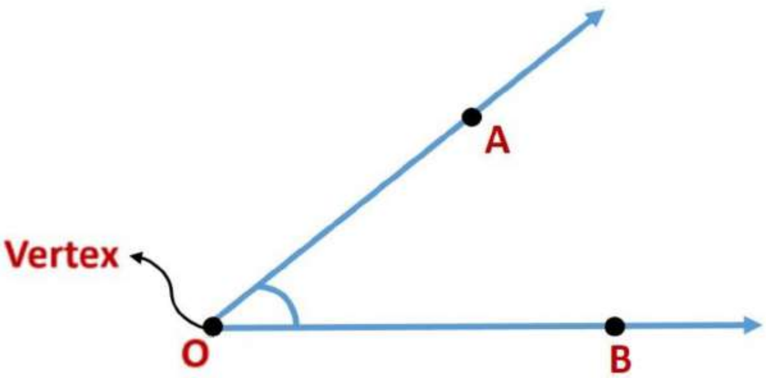
b) An open curve made up entirely of line segments.

c) A polygon with two sides cannot be drawn as minimum three line segments are required to make a polygon.

Angles

An angle is made up of two rays starting from a common end point.

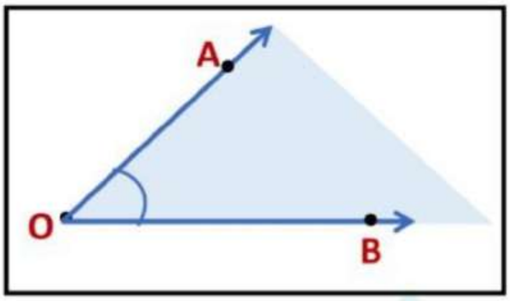
- The two rays forming the angle are called the arms or sides of the angle.
- The common end point is the vertex of the angle.



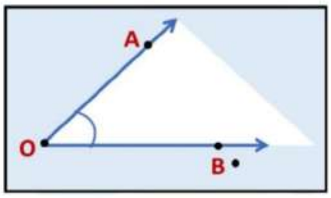
Here, angle is formed by the rays $\overline{OA}$ and $\overline{OB}$.

The name of the angle is ∠AOB or ∠BOA keeping the vertex in the middle.

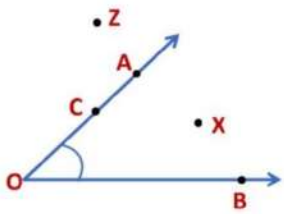
The interior of the angle is bounded by the arms of an angle.



The exterior of the angle is the region that lies outside the angle.



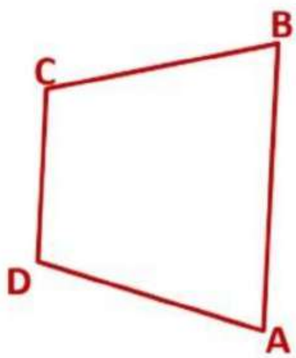
X is in the interior of the angle.
Z is in the exterior of the angle.
A, B and C are on ∠AOB



Example: Name the angles in the figure:

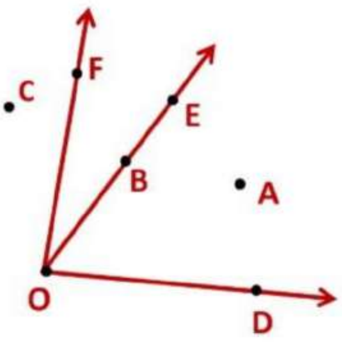
The four angles in the given figure are

∠DAB, ∠ABC, ∠BCD and ∠CDA



Example: In the given diagram, name the point(s)
a) In the interior of ∠DOE

b) In the exterior of ∠EOF
c) On ∠EOF

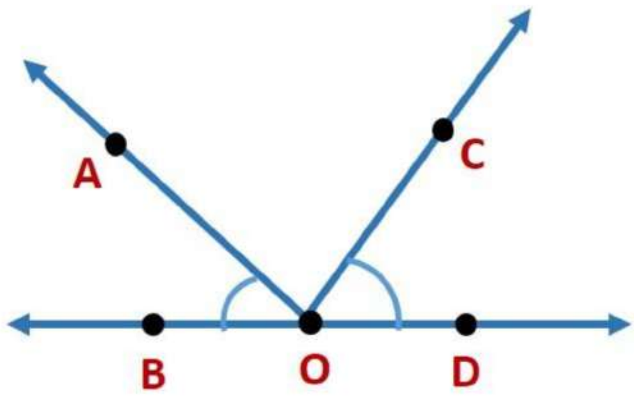


a)Point in the interior of ∠DOE is A.
b) Points in the exterior of ∠EOF is C, A and D.
c) Points on ∠EOF is E, B, O and F.

Example: Draw rough diagrams of two angles such that they have

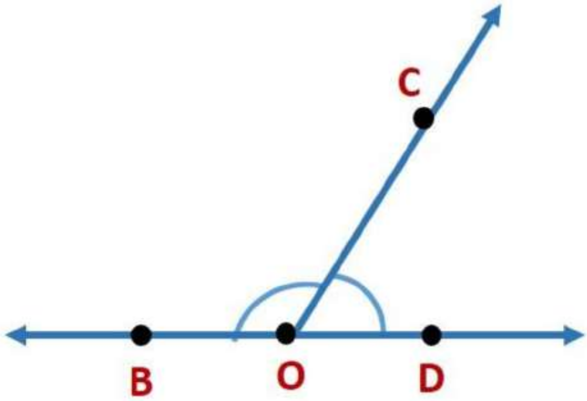
a) One point in common.
b) Two points in common.
c) Three points in common.
d) Four points in common.
e) One ray in common.

a) One point in common



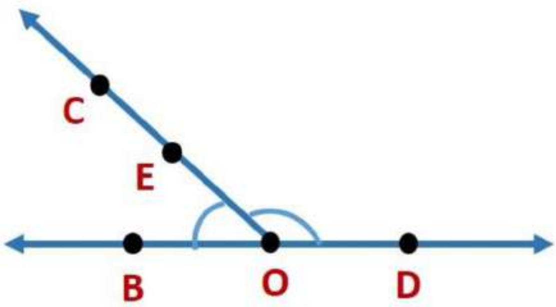
∠AOB and ∠COD have only one point in common, i.e. O

b) Two points in common



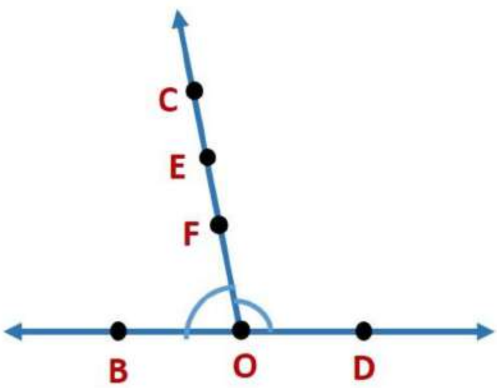
∠BOC and ∠COD have two points in common, i.e. O and C

c) Three points in common



∠BOC and ∠COD have three points in common, i.e. O, E and C.

d) Four points in common

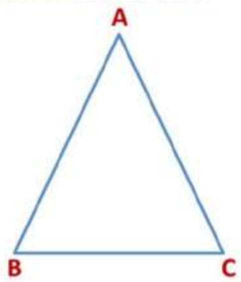


∠BOC and ∠COD have four points in common, i.e. O, C, E and F.

Triangles

A triangle is a three-sided polygon. It is the polygon with the least number of sides. It is denoted by the symbol Δ.

We see many triangle shaped objects in our daily life.

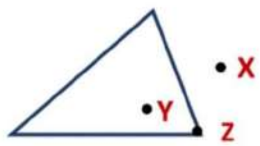


Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

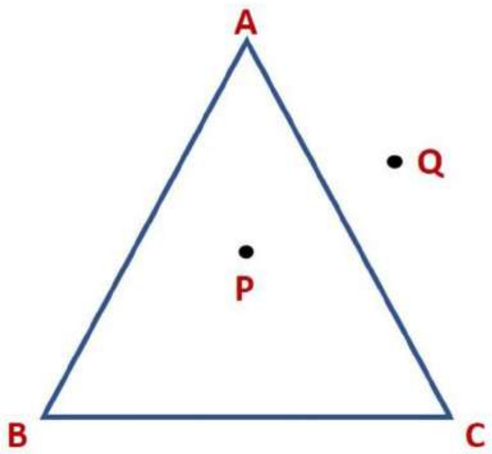
Angles: ∠BAC, ∠BCA, ∠ABC

Vertices: A, B and C

X is in the exterior of the triangle
Y is in the interior of the triangle
Z is on the triangle

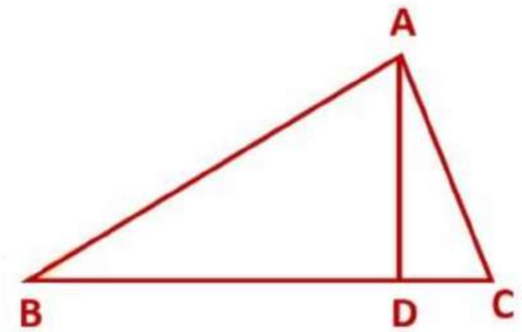


Example: Draw a rough sketch of a triangle ABC. Mark a point P in its interior and a point Q in its exterior. Is the point A in its exterior or in its interior?



Point A is not in the interior or exterior of Δ ABC as it is a vertex (a point where two line segments meet).

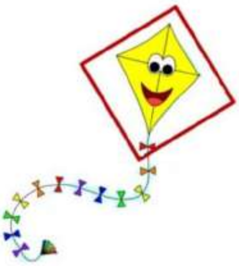
Example:
a) Identify three triangles in the figure.
b) Write the names of seven angles.
c) Write the names of six line segments.
d) Which two triangles have ∠B as common?

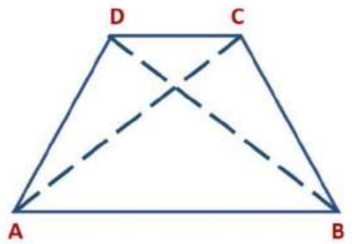


a) The three triangles are ΔABC, ΔABD and ΔACD
b) The angles are ∠BAC, ∠BAD, ∠DAC, ∠ABD, ∠ACD, ∠ADB and ∠ADC.
c) The line segments are AB, BC, CA, BD, DC and AD.
d) ΔABC and ΔABD have ∠B as common.

Quadrilaterals

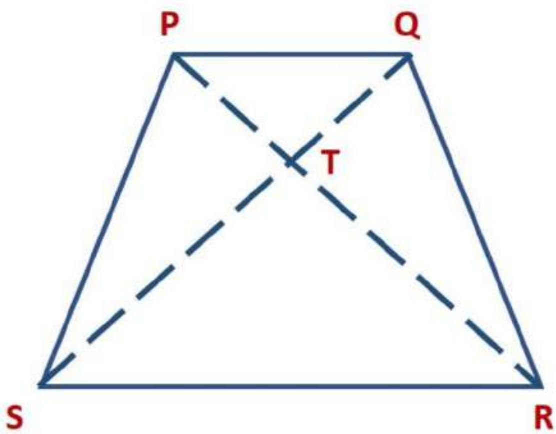
A four sided polygon is a quadrilateral. It has 4 sides and 4 angles.

Sides	$\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$
Angles	$\angle A$, $\angle B$, $\angle C$ and $\angle D$
Opposite Sides	$\overline{AB}$ and $\overline{DC}$, $\overline{BC}$ and $\overline{AD}$
Opposite Angles	$\angle A$ and $\angle C$, $\angle B$ and $\angle D$
Adjacent Sides	$\overline{AB}$ and $\overline{BC}$, $\overline{BC}$ and $\overline{CD}$, $\overline{CD}$ and $\overline{DA}$, $\overline{DA}$ and $\overline{AB}$
Adjacent Angles	$\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, $\angle D$ and $\angle A$

Example: Draw a rough sketch of a quadrilateral PQRS. Draw its diagonals. Name them. Is the meeting point of the diagonals in the interior or exterior of the quadrilateral?

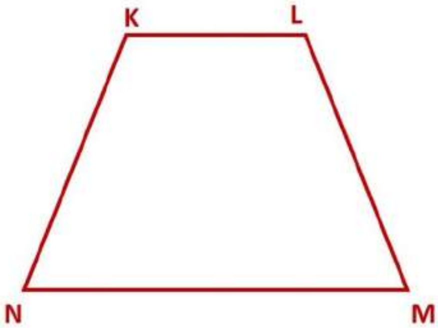


The two diagonals are PR and QS.

Diagonal PR and diagonal QS meet at point T which is in the interior of the quadrilateral PQRS.

Example: Draw a rough sketch of a quadrilateral KLMN. State,

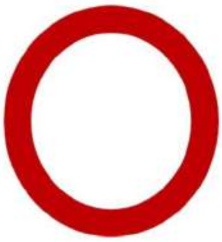
a) Two pairs of opposite sides,
b) Two pairs of opposite angles,
c) Two pairs of adjacent sides,
d) Two pairs of adjacent angles.



Opposite Sides	$\overline{KL}$ and $\overline{NM}$, $\overline{LM}$ and $\overline{KN}$
Opposite Angles	$\angle K$ and $\angle M$, $\angle L$ and $\angle N$
Adjacent Sides	$\overline{KL}$ and $\overline{LM}$, $\overline{KN}$ and $\overline{NM}$
Adjacent Angles	$\angle K$ and $\angle N$, $\angle M$ and $\angle L$

Circles

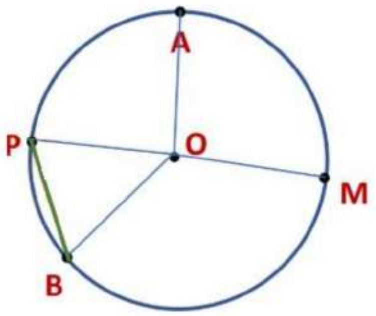
A circle is a simple closed curve which is not a polygon. We see many things that are round, a clock, a bangle, coin.



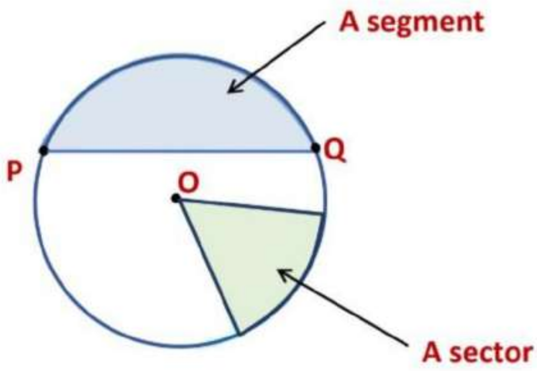
Parts of a circle

Here is a circle with center C.

A, P, B, M are points on the circle.

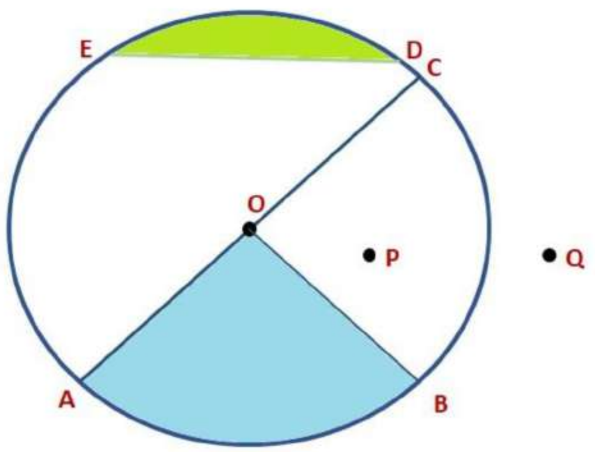


O is the centre of the circle.
The radius is a line segment that connects the center to a point on the circle. $\overline{OA}$, $\overline{OB}$, $\overline{OM}$ and $\overline{OP}$ are radii of the circle. OA = OB = OM = OP
PM is the diameter of the circle. Diameter = 2 x radius
The chord is the line segment joining any two points on the circle. $\overline{PB}$ is a chord.



Any part of a circle is called an arc. If we join points P and Q on the circle we get an arc PQ ($\overset{\frown}{PQ}$)
A region in the interior of a circle enclosed by an arc on one side and a pair of radii on the other two sides is called a sector.
A region in the interior of a circle enclosed by a chord and an arc is called a segment of the circle.
The distance around a circle is its circumference.

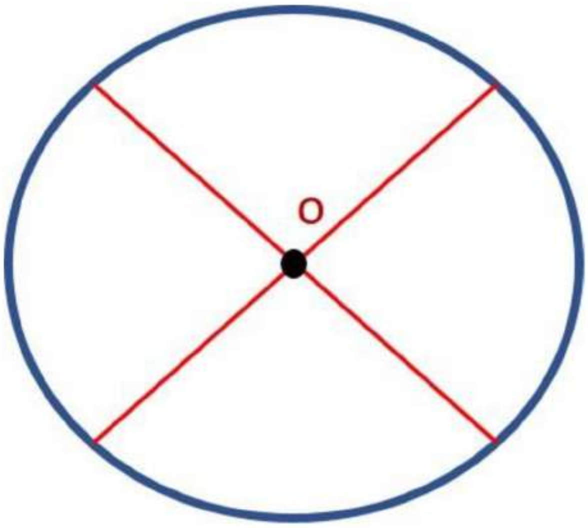
Example: From the figure, identify:
a) The center of circle
b) Three radii
c) A diameter
d) A chord
e) Two points in the interior
f) A point in the exterior
g) A sector
h) A segment



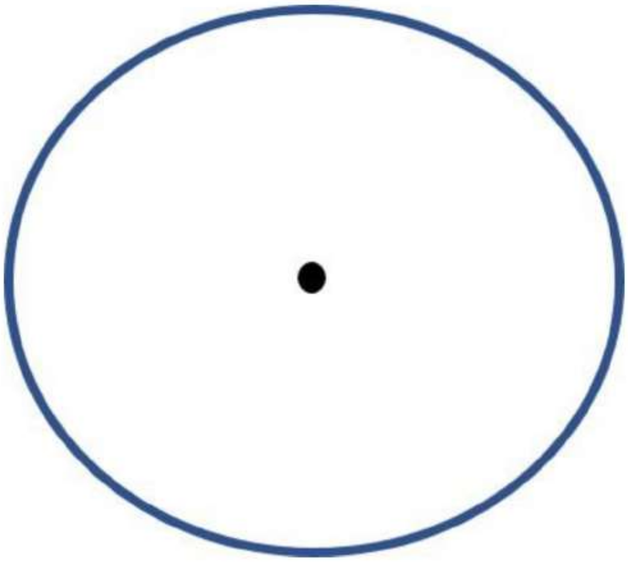
a) O is the center of the circle.
b) OA, OB and OC are the three radii
c) AC is the diameter of the circle

d) ED is a chord
e) O and P are the two points in the interior.
f) Q is the point in the exterior
g) OAB (shaded portion) is a sector
h) ED (shaded portion) is a segment

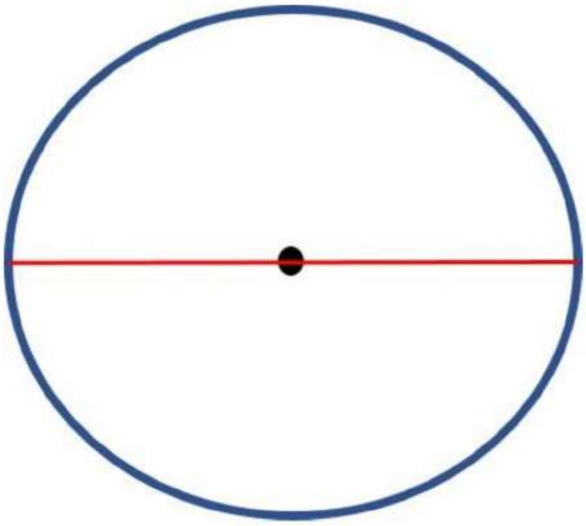
Example: Say true or false:
a) Two diameters of a circle will necessarily intersect.
b) The center of a circle is always in its interior.
a) The two diameters of the circle will always intersect at the center.



b) True, the center of a circle will always lie in its interior.



Example:
a) Is every diameter of a circle also a chord?
b) Is every chord of a circle also a diameter?
a) Yes, every diameter of a circle is a chord. A chord is the line segment joining any two points on the circle. Diameter is the longest possible chord of the circle.



b) No, every chord is not a diameter because a diameter always passes through the center, having its end points on the circle whereas a chord may not necessarily pass through the center.