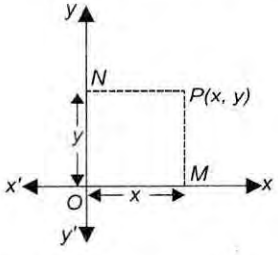
UNIT 11

Co-ordinate Geometry

Co-ordinate geometry is a branch of science which establishes a definite correspondence between the position of a point in a place and pair of algebraic numbers, called co-ordinates.

Cartesian Co-ordinates

In Cartesian co-ordinates the position of a point P is determined by knowing the distances from two perpendicular lines passing through the fixed point. Let O be the fixed point called the origin and XOX' and YOY', the two perpendicular lines through O, called Cartesian or Rectangular co-ordinates axes.



Draw PM and PN perpendiculars on OX and OY respectively. OM is called the x co-ordinate or abscissa of the point P. ON is called the y co-ordinate or the ordinate of the point P.

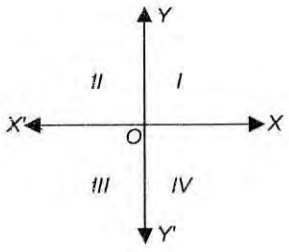
The abscissa and ordinate of a point taken together are known as co-ordinates of a point.

If $OM = x$, $ON = y$, the co-ordinates of the point P are (x, y)

In the figure OX and OY are called as x-axis and y-axis, respectively and both together are known as axes of co-ordinates.

- Origin is point O of intersection of the axes of co-ordinates. The co-ordinates of the point O are O (o, o)
- The distance of the point P from y-axis is called its abscissa. In the figure OM is the abscissa.
- The distance of the point P from x-axis is called its ordinate. ON is the ordinate in the figure.

Quadrant



A quadrant is $\frac{1}{4}$ part of a plane divided by the co-ordinate axes.

(*i*) XOY is called the first quadrant.

(*ii*) YOX' the second.

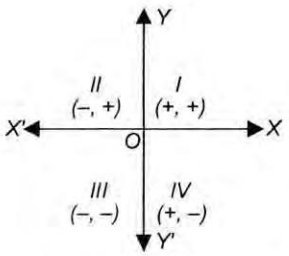
(*iii*) $X'OY'$ the third.

(*iv*) $Y'OX$ the fourth as marked in the figure.

Rules for Signs of Co-ordinates

(*i*) In the first quadrant, both co-ordinates *i.e.,* abscissa and ordinate of a point are positive.

(*ii*) In the second quadrant, for a point, abscissa is negative and ordinate is positive.



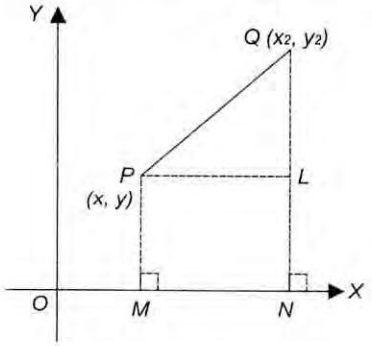
(*iii*) In the third quadrant, for a point, both abscissa and ordinate are negative.

(*iv*) In the fourth quadrant, for a point, the abscissa is positive ad the ordinate is negative.

Quadrant	*x*-co-ordinate	*y*-co-ordinate	Co-ordinates
First quadrant	+	+	(+,+)
Second quadrant	–	–	(–,+)
Third quadrant	–	–	(–,–)
Fourth quadrant	+	–	(+,–)

Distance between two points

From P and Q draw perpendiculars PM and QN on the x-axis. Also, from P draw PL perpendicular to QN.



We are required to find distance PQ in terms of co-ordinates of P we have,

$$OM = x_1,\ ON = x_2$$

$$MP = y_1,\ NQ = y_2$$

$$PL = MN,\ ON - OM$$

$$= x_2 - x_1$$

$$QL = QN - LN$$

$$= QN - PM$$

$$= y_2 - y_1$$

From the right-angled triangle PLQ,

$$PQ^2 = PL^2 + QL^2$$

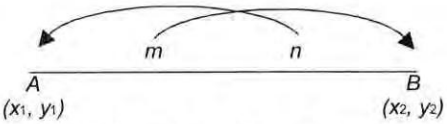
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore\ PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ratio formula for internal and external Division

The Co-ordinates of a point P which divides. The straight line joining two points $A\ (x_1 - y_1)$ and $B\ (x_2 - y_2)$ internally in the ratio m : n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



If P is the mid-point of AB, then it will divide AB in the ration of 1 : 1, and its co-ordinates are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

The co-ordinates of a point which divides. The straight line joining two points $(x_1,\ y_1)$ and $(x_2 - y_2)$ externally in the ratio m : n are

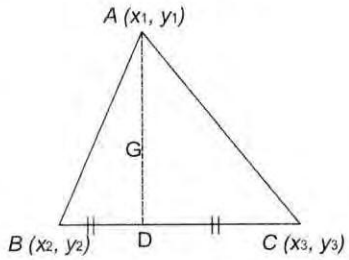
$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

Co-ordinates of the centroid of a triangle

Consider a triangle ABC, such that the co-ordinates of its vertices are given by $A(x_1, y_1)$, $B\ (x_2 - y_2)$ and $C\ (x_3 - y_3)$

The medians of a triangle are concurrent and the point of concurrence is called centriod. The centroid divides each median in the ratio of 1 : 2, G.

$$AG : GD = 2 : 1$$



The co-ordinates of the centriod are given as

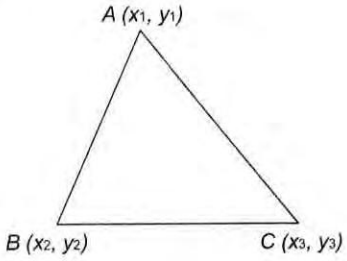
$$\left(\frac{x_2+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Area of a triangle in terms of co-ordinates of its vertices

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Area of ΔABC

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1 + x_3(y_1 - y_2)]$$



The area can also be written as,

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

Condition of Collinearity of three points

Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on a straight line of the area of the triangle formed by them is zero.

We can also write the condition as, $x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct choice amongst the following:

1. The point on the x–axis which is equidistant from the points (5, 4) and (–2, 3) is
   (a) (–2,0) (b) (2, 0)
   (c) (0, 2) (d) (2, 2)

2. If the distances of p (x, y) from A (–1, 5) and B (5, 1) are equal, then
   (a) $2x = y$ (b) $3x = 2y$
   (c) $3x = y$ (d) $2x = 3y$

3. $(1,-1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and $(1, 2)$ are the vertices of a/an _____ triangle.
   (a) equilateral (b) isosceles
   (c) right angled (d) scalene

4. If the point (x, y) is equidistant from the point $(a+b, b-a)$ and $(a-b, a+b)$, then which of the following is correct?
   (a) $ax = by$ (b) $ax^2 = by$
   (c) $ay = bx$ (d) $ay^2 = bx$

5. Which of the following points is equidistant from $(2,-3)$?
   (a) $(-1,0)$ (b) $(1,0)$
   (c) $(-2,0)$ (d) $(2,0)$

6. Which of the following point is equidistant from $(3, 2)$ and $(-5,-2)$?
   (a) $(0,2)$ (b) $(0,-2)$
   (c) $(2,0)$ (d) $(2,-2)$

7. Which of the following points are the vertices of an equilateral triangle?
   (a) $(a, a), (-a, -a), (2a, a)$
   (b) $(a, a), (-a,-a), (-a\sqrt{3}, a\sqrt{3})$
   (c) $(\sqrt{2}a, -a), (a, \sqrt{2}a), (a, -a)$
   (d) $(0, 0), (a, -a), (a, \sqrt{2}a)$

8. If the points $(-1, 3), (2, p)$ and $(5,-1)$ are collinear, the value of p is
   (a) 1 (b) -1
   (c) 0 (d) $\sqrt{2}$

9. The co–ordinates of the point which divides the line joining $(1,-2)$ and $(4, 7)$ interually in the ratio 1 : 2 are
   (a) $(1,2)$ (b) $(-1,-1)$
   (c) $(-1,2)$ (d) $(2,1)$

10. In what ratio is the line joining the points $A\,(4, 4)$ and $B\,(7, 7)$ divided by p $(-1,-1)$?
    (a) 8 : 5 (b) 5 : 8
    (c) 5 : 7 (d) 7 : 4

11. What is the ratio is which the point $P(n, 6)$ divides the join of $A\,(-4, 3)$ and $B\,(2, 8)$?
    (a) 1 : 3 (b) 2 : 3
    (c) 3 : 2 (d) 2 : 5

12. In question 11 above, the value of n is
    (a) -2 (b) 3
    (c) $-\frac{3}{2}$ (d) $-\frac{2}{5}$

13. The vertices of a triangle are $(2, 1), (5, 2)$ and $(3, 4)$. Then the co-ordinates of the centroid are
    (a) 10,3 (b) $\left(\frac{10}{3}, 7\right)$
    (c) $\left(\frac{10}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{7}{3}, \frac{10}{3}\right)$

14. If the area of the triangle formed by the three points is zero, then the points lie on a
    (a) straight line
    (b) curve pointing convex upwards
    (c) curve pointing convex downwards
    (d) all the above are wrong

15. What is the area of the triangle formed by the points $(a, b + c), (b, c + a)$ and $(c, a + b)$?
    (a) 1 (b) -1
    (c) 0 (d) $\frac{1}{2}(abc)^2$

16. What is the area of the triangle formed by the points (a, c + a), (a, c) and (–a, c – a)?
    (a) a^2 (b) $\frac{1}{a^2}$
    (c) $a^2 + a$ (d) zero

17. What is the area of the triangle formed by the points $(a, c + a), (a^2, c^2)$ and $(-a, c - a)$?
    (a) 1 (b) a^2
    (c) $\sqrt{a^2 + c^2}$ (d) None of these

18. What is the value of y if $(y, 3), (-5, 6)$ and $(-8, 8)$ are collinear?
    (a) -1 (b) 2
    (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

19. Which of the following points are collinear?
(a) $(2a, 0), (3a, 0), (a, 2a)$
(b) $(3a, 0), (0, 3b), (a, 2b)$
(c) $(3a, b), (a, 2b), (-a, b)$
(d) $(a, -6), (-a, 3b), (-2a, -2b)$

20. The mid-point of a line is $(-4, -2)$ and one end of the line is $(-6,4)$. The co-ordinates of the other end are
(a) $(2, -8)$ (b) $(-2, 8)$
(c) $(-2, -8)$ (d) $(2, 8)$

21. The ratio in which the line $3x + y = 9$ divides the line sequent joining the points $(1, 3)$ and $(2, 7)$ is given by
(a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 3 : 2

22. The area of a triangle whose vertices are $(-2, -2), (-1, -3)$ and $(p, 0)$ is 3 *sq*. units what is the value of p?
(a) -2 (b) 2
(c) 3 (d) -3

23. What are the co-ordinates of centre of a circle passing through the points whose co-ordinates are $(0, 0), (-2, 1)$ and $(-3, 2)$?
(a) $\left(\frac{3}{2}, 11\right)$ (b) $\left(3, \frac{11}{2}\right)$
(c) $\left(\frac{3}{2}, \frac{11}{2}\right)$ (d) $(-3, -11)$

24. In what ratio does the point $P\left(2, \frac{-5}{6}\right)$ divide the line request joining the points $A(-3, 5)$ and $B(3, -2)$?
(a) 1 : 5 (b) 5 : 1
(c) 2 : 3 (d) 3 : 5

25. If the vertices of a triangle are $(2, 4), (5, k)$ and $(3, 10)$ and its area is 15 sq. units, the value of k is
(a) 25 (b) 51
(c) 52 (d) $\frac{23}{2}$

26. The midpoints of sides of a triangle are $(3, 4), (4, 1)$ and $(2, 0)$. Which of the following does not devote the co-ordinates of its verities?
(a) 1, 3 (b) 5, 3
(c) 5, 5 (d) 3, -3

27. The point which lies on the perpendicular bisector of the line sequent joining the points $P(-2, 0)$ and $Q(2, 5)$ is
(a) $(0, 0)$ (b) $(0, 2)$
(c) $(2, 0)$ (d) $(-2, 0)$

28. The fourth vertex D of a parallelogram ABCD whose three vertices are A $(-2, 5)$ B$(6, 7)$ and C $(8, 3)$ is
(a) $(-1, 0)$ (b) $(1, 0)$
(c) $(0, -1)$ (d) $(0, 1)$

29. If $Q\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points A $(-6, 5)$ and B $(-2, 3)$, then the value of 'a' is
(a) 4 (b) -6
(c) -8 (d) -12

30. ΔOBC is a rectangle whose three vertices are A $(0, 3)$, $O(0, 0)$ and B $(5, 0)$. The length of its diagonal is
(a) 3 (b) 5
(c) $\sqrt{7}$ (d) $\sqrt{34}$

31. The perimeter of a triangle with vertices $(0, 4)$, $(0, 0)$ and $(3, 0)$ is
(a) $3 + \sqrt{5}$ (b) 11
(c) 12 (d) $\sqrt{13}$

32. Which point on x–axis is equidistant from $(7, 6)$ and $(-3, 4)$?
(a) $(2, 0)$ (b) $(3, 0)$
(c) $(-5, 0)$ (d) $(1, 0)$

33. In what ratio is the line segment joining the point $(-2, -3)$ and $(3, 7)$ divided by y–axis?
(a) $(-2, 3)$ (b) $(-3, 2)$
(c) $(2, 3)$ (d) $(6, 0)$

34. What is the perimeter of the triangle formed by the points (0, 0), (1, 0) and (0, 1)?

(a) $\sqrt{2}$ (b) 2

(c) $2-\sqrt{2}$ (d) $2+\sqrt{2}$

35. If points $(a, 0), (0, b)$ and $(1, 1)$ are collinear, then $\left(\frac{a+b}{ab}\right)$ equals

(a) 1 (b) –1

(c) 2 (d) $\sqrt{2}$

36. If the distance between the points $(4, y)$ and $(1, 0)$ is 5, then y equals.

(a) 4 only (b) –4 only

(c) ± 4 (d) 0

37. The distance of the point P(2, 3) from the x–axis is

(a) 5 (b) 3

(c) 2 (d) 1

38. The distance between the point (0, 5) and (–5, 0) is

(a) $2\sqrt{5}$ (b) $5\sqrt{2}$

(c) 5 (d) 10

39. Which of the following points lies in the fourth quadrant?

(a) (2, –7) (b) (–3, 5)

(c) (0, 0) (d) (–4, –7)

40. The points (–4, 0), (4, 0) and (0, 3) are the vertices of a

(a) Scalene triangle (b) Equilateral triangle

(c) Isosceles triangle (d) Right triangle

41. The point which divides the line segment joining the points (3, 4) and (7, –6) in the ratio 1: 2 internally lies in the _____quadrant.

(a) I (b) II

(c) III (d) IV

42. The co-ordinates of a point (x, y) which divides the straight line joining two points (x_1, y_1) and (x_2, y_2) internally in the ratio m_1 and m_2 are

(a) $x=\frac{m_1x_1+m_2x_2}{m_1+m_2}, y=\frac{m_1y_1+m_2y_2}{m_1+m_2}$

(b) $x=\frac{m_1x_2+m_2x_1}{m_1+m_2}, y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$

(c) $x=0, y=0$

(d) $x=\frac{m_1x_1-m_2x_2}{m_1-m_2}, y=\frac{m_1y_1-m_2y_2}{m_1-m_2}$

ANSWERS

1. (b)	2. (b)	3. (b)	4. (c)	5. (c)
6. (b)	7. (b)	8. (a)	9. (d)	10. (b)
11. (c)	12. (d)	13. (c)	14. (a)	15. (c)
16. (a)	17. (d)	18. (d)	19. (b)	20. (c)
21. (b)	22. (b)	23. (c)	24. (b)	25. (c)
26. (b)	27. (a)	28. (c)	29. (d)	30. (d)
31. (c)	32. (b)	33. (c)	34. (d)	35. (a)
36. (c)	37. (b)	38. (b)	39. (d)	40. (c)
41. (d)	42. (a)			