Conic – Parabola

Q.1. Find the equation of the parabola having focus at (3, – 4) and directrix as $x + y = 2$.

Solution : 1

The focus is at F (3, – 4). Let P(x, y) be any point on the parabola and | MP | be perpendicular distance form P to the directrix, the by the definition of parabola,

| FP | = | MP | [for a parabola, e = 1.]

Or, $\sqrt{\{(x - 3)^2 + (y + 4)^2\}} = | x + y - 2|/\sqrt{(1^2 + 1^2)}$

Or, $2\{(x - 3)^2 + (y + 4)^2\} = (x + y - 2)^2$

Or, $2 (x^2 - 6x + 9 + y^2 + 8y + 16) = x^2 + y^2 + 4 + 2xy - 4x - 4y$

Or, $x^2 + y^2 - 2xy - 8x + 20y + 46 = 0$.

This is the required equation of the parabola.

Q.2. The equation of the directrix of the parabola is $3x + 2y + 1 = 0$. The focus is (2, 1). Find the equation of the parabola.

Solution : 2

$= 4x^2 - 12 x y + 9y^2 - 58 x - 30y + 64 = 0]$

Q.3. A straight line $2x + y + p = 0$ is a focal chord of the parabola $y^2 = - 8x$. Find the value of p.

Solution : 3

Equation of the parabola is

$y^{\mathbf{2}} = - 8x \Rightarrow 4a = - 8$, Therefore, $a = -2$ and focus is $(-2, 0)$.

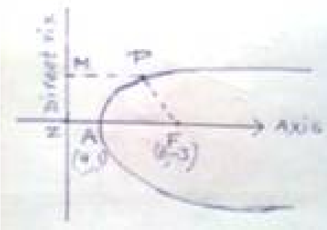
Line $2x + y + p = 0$, passes through $(-2, 0)$.

Therefore, $2 .(-2) + 0 + p = 0 \Rightarrow p = 4$

Q.4. The points (0, 4) and (0, 2) are the vertex and focus of a parabola. Find the equation of the parabola.

Solution : 4

Fig.



The vertex is A (0, 4) and focus is F (0, 2). The axis of the parabola is the line joining A and F.

Let the axis meet the directrix at Z (α, β). A is the mid-point of the line ZF.

Therefore, $(\alpha + 2)/2 = 0 \Rightarrow \alpha = 0$, and $(\beta + 2)/2 = 4 \Rightarrow \beta = 6$.

Or, co-ordinate of Z is (0, 6). Slope of the axis is $(4 - 2)/(0 - 0) = 2/0$.

Therefore, slope of the directrix $= -1/(2/0) = 0$.

Thus the equation of the directrix is $y - 6 = 0 (x - 0)$.

Or, $y - 6 = 0$. Let P (x, y) be any point on the parabola.

Therefore, $| FP | = | MP |$

Or, $\sqrt{[x^2 + (y - 2)^2]} = |y - 6|/\sqrt{(1^2 + 0^2)} = |y - 6|$

Or, $x^2 + (y - 2)^2 = (y - 6)^2$

Or, $x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$

Or, $x^2 + 8y - 3^2 = 0$.

Q.5. The equation $y^2 - 4y - 4x + 16 = 0$ represents a parabola. Find its vertex and focus.

Solution : 5

The equation of the parabola is

$y^2 - 4y - 4x + 16 = 0$.

Or, $(y - 2)^2 = 4x - 12$.

Or, $(y - 2)^2 = 4(x - 3)$.

Let Y2 = 4X, where, Y = y – 2 and X = x – 3.

Here 4a = 4 => a = 1.

Therefore, vertex is V (0, 0).

When Y = 0 => y – 2 = 0, or, y = 2.

When X = 0 => x – 3 = 0 , or, x = 3.

Therefore, vertex is V (3, 2).

Focus is F(a, 0) = F(1, 0).

Or, X = a = 1 => x – 3 = 1 => x = 4 and Y = 0 => y – 2 = 0 => y = 2.

Therefore, focus is F(4, 2).

Q.6. The two lines $t y = x + t^2$ and $y + t x = 2t + t^2$ intersect at the point P. Show that P lies on the curve whose equation is $y^2 = 4x$.

Solution : 6

Given lines are : $t y = x + t^2$ ------------------- (i)

and $y + t x = 2t + t^3$ ------------------- (ii)

Multiplying equation (i) by t and adding to (ii) we get

$t^2y + y + t x = t x + t^3 + 2t + t^3$

Or, $y(t^2 + 1) = 2t(t^2 + 1)$

Or, $y = 2t$. -------------------- (iii)

Putting $y = 2t$ in equation (i) we get

$2t^2 = x + t^2 \Rightarrow x = t^2$. --------------- (iv)

From (iii) and (iv) $y2 = 4t^2 = 4x$

Hence, P lies on $y^2 = 4x$. **[Proved.]**