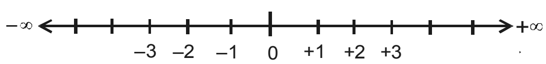
Number System

Real Numbers

The set of real numbers consists of all **rational** and **irrational numbers** i.e. all those numbers which can be represented on a number line are called **Real Numbers. e.g**: –1.3, +8.7, – 56.69, $\sqrt{2}$, $\sqrt{3}$, etc.

Number Line

It is a line on which all the positive and negative numbers can be marked in a sequence.



Rational Numbers

Numbers which can be expressed in the form $\frac{x}{y}$, where x and y are integers and $y \neq 0$, are known as **rational numbers.** These have a terminating or recurring decimal representation. Terminating decimal numbers can be converted into rational numbers by putting as many zeros after 1 in the denominator as the number of digits after decimal.

e.g: $\frac{1}{2}, \frac{3}{7}, -\frac{4}{5}, 1, 0,$ etc.

Irrational Numbers

Numbers which cannot be expressed in the form $\frac{x}{y}$, where x and y are integers and $y \neq 0$, are known as **irrational numbers**. **e.g**: π, $\sqrt{2}$ are irrational

Integers

The rational numbers which do not have fraction or decimal part are called integers.

e.g: –n,..., –3, –2, –1, 0, 1, 2, 3, …,

Whole Numbers

The numbers 0, 1, 2, 3, 4, ..., n are called **whole numbers**. The smallest whole number is 0. Fractions and negative numbers are not whole numbers.

Natural Numbers

The numbers 1, 2, 3, 4, ..., n are called **natural numbers**. 0 is not a natural number. The smallest natural number is 1. Fractions and negative numbers are not natural numbers.

Even Numbers

The numbers which are divisible by 2 are called **even numbers**. **e.g.:** 2, 4, 6, 8, ...

Odd Numbers

The numbers which are not divisible by 2 are called **odd numbers.** e.g. 1, 3, 5, 7, ...

Prime Numbers

If a number is divisible by 1 and itself only, then it is called a **prime number**. **e.g:** 2, 3, 5, 7, ...

Note:

1 is neither a prime number nor composite.

Composite Numbers

The numbers except 1, which are not prime, are called **composite numbers**. **e.g**: 4, 6, 8, 9, ...

Fractions

All rational numbers which are of $\frac{p}{q}$ form, where p, q are integers and p is not a multiple of q.

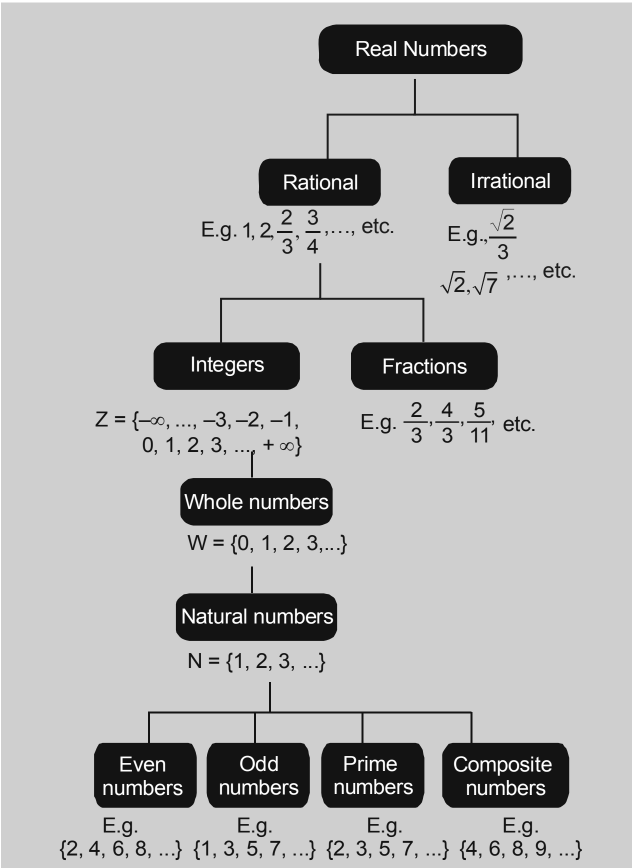
p is called numerator whereas q is known as denominator.

* Fractions are of the following types:

- **Proper:** $p < q$ e.g., $\frac{2}{7}, \frac{3}{8}$ etc.
- **Improper:** $p \geq q$ e.g., $\frac{6}{5}, \frac{5}{2}$ etc.
- **Mixed:** It is an integer plus a fraction

  e.g., $3\frac{1}{5}, 7\frac{1}{3}$, etc.

The family of Numbers can easily be learned by learning the following Number tree



Complex Number

A number of the form $a + ib$, where $i = \sqrt{-1}$, is called a **complex number** and such number can't be represented on a number line.

We will restrict ourselves only to **Real Numbers.**

BODMAS

Order of simplification of expression of numbers

B: Bracket	O: Of/Order
D: Division	M: Multiplication
A: Addition	S: Subtraction

In a given expression of numbers, the above order of operations has to be strictly followed.

Facts about numbers

Odd + Odd = Even	Odd + Even = Odd
Even + Even = Even	Odd × Odd = Odd
Odd × Even = Even	Even × Even = Even

Then, 9 is the divisor, 57 is the dividend, 6 is the quotient, and 3 is the remainder.

Dividend = (Divisor × Quotient) + Remainder

Divisibility Tests

1. A number is divisible by 2 if its units digit is even or 0.
2. A number is divisible by 3 if the sum of its digits is divisible by 3.
3. A number is divisible by 4 if the number formed by the last two digits is divisible by 4 or the last two digits are 0.
4. A number is divisible by 5 if its units digit is 5 or 0.
5. A number is divisible by 6 if it is divisible by 2 and 3 both.
6. A number is divisible by 8 if the number formed by the last three digits is divisible by 8, or when the last three digits are 0.
7. A number is divisible by 9 if the sum of its digits is divisible by 9.
8. A number is divisible by 10 if its units digit is 0.
9. A number is divisible by 11 if the difference between the sum of the digits at the odd and the even places is 0 or a multiple of 11.
10. A number is divisible by 12 if it is divisible by 3 and 4 both.

Conversion of a non terminating recurring decimal into $\frac{p}{q}$ form:

As seen from classification of numbers, non-terminating but recurring decimal numbers are rational numbers, they can be expressed in the form of $\frac{p}{q}$.

Method 1:

1. Convert 0.4444 into the form of $\frac{p}{q}$

Solution :

Let x = 0.4444 … (i)

multiply 10 on both sides.

10x = 4.444 … (ii)

Subtract (i) from (ii)

$$9x = 4 \Rightarrow x = \frac{4}{9}.$$

2. Convert 0.434343 into the form of $\frac{p}{q}$.

Solution :

Let x = 0.434343 … (i)

Multiply both sides by 100

$100x = 43.4343$ … (ii)

Subtract (i) from (ii)

$99x = 43 \Rightarrow x = \frac{43}{99}$

Method 2:

The $\frac{p}{q}$ form of any recurring number

$= \dfrac{\text{(The non-recurring and recurring part written once)} - \text{(The non-recurring part)}}{\text{As many 9's as the number of digits in the recurring part followed by as many 0's as digits in the non-recurring part.}}$

Factorial of a number:

The continued product of first 'n' natural numbers is called 'n factorial' of the number 'n' and is denoted by n! or $\underline{|n}$

$n! = 1 \times 2 \times 3 \times ... \times (n-1) \times n$

e.g: $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$

By definition 0! = 1.

LCM and HCF

Factor

A number is said to be the factor of another, if it divides the other number perfectly.

e.g: 7 and 9 are factors of 63.

Common factor

If one number perfectly divides two or more numbers, then it is called a common factor of the numbers.

e.g: 5 is a common factor of 10, 15, 20 and 25.

Highest common factor (HCF)

HCF of two or more numbers is the greatest number that perfectly divides each of them.

e.g: 5 is the HCF of 15 and 20.

Note:
HCF is also known as GCD (Greatest Common Divisor)

1. Find the HCF of 24 and 72.

Solution :

$24 = 2 \times 2 \times 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$HCF = 2 \times 2 \times 2 \times 3 = 24.$

Similarly, we can find the HCF of sets containing more than two numbers.

Common Multiple:

A common multiple of two or more numbers is a number which is perfectly divisible by each of the numbers.

e.g: 45 is a common multiple of 3, 5 and 15.

Lowest common multiple (LCM)

The LCM of two or more numbers is the least number which is perfectly divisible by each of the numbers.

e.g: 57 is a least common multiple of 3 and 19.

Let's consider the LCM of 18, 27 and 30.

$$\begin{array}{r|l} 2 & 18,\ 27,\ 30 \\ \hline 3 & 9,\ 27,\ 15 \\ \hline 3 & 3,\ 9,\ 5 \\ \hline & 1,\ 3,\ 5 \end{array}$$

$LCM = 2 \times 3 \times 3 \times 3 \times 5 = 270.$

Note:
LCM of two numbers × HCF of two numbers
= Product of the two numbers

LCM & HFC of Fractions

$$HFC\left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{HCF(a,\ c,\ e)}{LCM(b,\ d,\ f)}$$

$$LCM\left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{LCM(a,\ c,\ e)}{HCF(b,\ d,\ f)}$$

Cyclicity:

Take any two numbers – 46 and 57. If they are multiplied then the digit obtained at units place of the product will be the same as the units place digit of the product of 6 & 7.

$6 \times 7 = 4\boxed{2}$ & $46 \times 57 = 262\boxed{2}$

If this concept is applied to exponent of numbers, an interesting pattern emerges when we look at them.

The last digits of the exponents of all the numbers have a cyclicity. If after every N^{th} power of the base have the same digit at units place as the first power of the base, then N is the cyclicity of that number.

For instance,

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 3\boxed{2}$

Since 2^5 has units place digit to be the same as 2^1 then it is said to have a cyclicity of 4. i.e $2^{1+4} = 32$

Similarly, we can find out cyclicity of other digits also.

You can check for yourself the cyclicity of all the digits.

Digit	Cyclicity
0, 1, 5 and 6	1
2, 3, 7 and 8	4
4 and 9	2

Some important facts about squares

I. The square of an even number is always even.

II. The square of an odd number is always odd.

III. Square of an integer cannot end in 2, 3, 7, or 8.

IV. The square of an integer (negative or positive) is always positive.

V. The square of an integer is either a multiple of 3 or 4 or else exceeds a multiple of 3 or 4 by 1. It is of the form $3k \pm 1$ or $4k \pm 1$.

VI. If a square ends in 9, the preceding digit is even.

VII. The fifth power of any number has the same unit digit as the number itself.

Some important results

I. $(x^n + y^n)$ is divisible by $(x + y)$ when n is odd.

II. $(x^n - y^n)$ is divisible by $(x + y)$ when n is even and by $(x - y)$ always.

III. $(x^n - x)$ is divisible by n (if n is prime).

IV. $(p - 1)! + 1$ is divisible by p when p is prime.

Formulae

I. $(a + b)^2 = a^2 + b^2 + 2ab$

II. $(a - b)^2 = a^2 + b^2 - 2ab$

III. $(a + b)^2 = (a - b)^2 + 4ab$

IV. $(a - b)^2 = (a + b)^2 - 4ab$

V. $a^2 - b^2 = (a - b)(a + b)$

VI. $a^2 + b^2 = \frac{1}{2}\left[(a+b)^2 + (a-b)^2\right]$

VII. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

VIIII. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Rules on counting numbers

1. Sum of first n natural numbers = $\frac{n(n+1)}{2}$.
2. Sum of first n odd numbers = n^2.
3. Sum of first n even numbers = $n(n + 1)$.
4. Sum of the squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$.
5. Sum of the cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$.

Solved Examples

1. Simplify: $\frac{527 \times 527 \times 527 + 183 \times 183 \times 183}{527 \times 527 - 527 \times 183 + 183 \times 183}$

Solution :

Expression

$$\frac{(527)^3 + (183)^3}{(527)^2 - 527 \times 183 + (183)^2} = \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$\frac{(a+b)\left(a^2 - ab + b^2\right)}{a^2 - ab + b^2} = a + b$$

(since, a = 527 and b = 183)

= 527 + 183 = 710

2. Simplify: $\left(\frac{(614+168)^2 - (614-168)^2}{614 \times 168}\right)$

Solution :

Expression

$$\frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4$$

3. Find the square of 1605.

Solution :

$(1605)^2 = (1600 + 5)^2$

$= (1600)^2 + 2 \times 1600 \times 5 + (5)^2$

$= 2560000 + 16000 + 25 = 2576025$

4. Find the value of $896 \times 896 - 204 \times 204$

Solution :

$a^2 - b^2 = (a + b)(a - b)$,

where a = 896 and b = 204

$= (896 + 204)(896 - 204)$

$= 1100 \times 692 = 761200$

5. Evaluate: $(57)^2 + (43)^2 + 2 \times 57 \times 43$

Solution :

$(a^2 + b^2 + 2ab) = (a + b)^2$

$= (57 + 43)^2 = 100^2 = 10000$

6. Simplify: $(81)^2 + (68)^2 - 2 \times 81 \times 68$

Solution :

$a^2 + b^2 - 2ab = (a - b)^2$

$= (81 - 68)^2 = 13^2 = 169$

7. Evaluate: $(313 \times 313 + 287 \times 287)$

Solution :

$$a^2 + b^2 = \frac{1}{2}[(a+b)^2 + (a-b)^2],$$

where a = 313 and b = 287

$$= \frac{1}{2}\left[(313+287)^2 + (313-287)^2\right]$$

$$= \frac{1}{2}\left[\left(600^2 + 26^2\right)\right] = 180338$$

8. What is the divisor if dividend is 15968, quotient is 89, and remainder is 37?

Solution :

$$\text{Divisor} = \left(\frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}\right)$$

$$= \left(\frac{15968 - 37}{89}\right) = 179$$

9. Which least number must be subtracted from 2000 to get a number exactly divisible by 17?

Solution :

On dividing 2000 by 17, we get 11 as remainder.

$\therefore$ Required number to be subtracted = 11.

10. Which least number must be added to 3000 to obtain a number exactly divisible by 19?

Solution :

On dividing 3000 by 19, we get 17 as remainder.

$\therefore$ Number to be added = (19 – 17) = 2.

11. Find the number which is nearest to 3105 and exactly divisible by 21.

Solution :

On dividing 3105 by 21, we get 18 as remainder.

$\therefore$ Number to be added to 3105 is

(21 – 18) = 3.

$\therefore$ 3108 is the required number.

12. A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder?

Solution :

On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then number = 342k + 47

$= [(19 \times 18k) + (19 \times 2 + 9)]$

$= [19(18k + 2) + 9]$

$\therefore$ The given number when divided by 19, gives (18k + 2) as quotient and 9 as remainder.

Alternative method:

342 is a multiple of 19. Divide the remainder by the second dividend to get the remainder. 47, when divided by 19, gives 9 as remainder.

13. The HCF of two numbers is 11 and their LCM is 693. If one number is 77, find the other.

Solution :

$$\text{The other number} = \frac{11 \times 693}{77} = 99.$$

14. Find the largest number that can exactly divide 513, 783 and 1107.

Solution :

Required number = HCF of 513, 783 and 1107.

Now $513 = 3^3 \times 19$, $783 = 3^3 \times 29$,

$1107 = 3^3 \times 41$

$\therefore$ HCF = 3^3 = 27.

Hence, the required number is 27.

15. Find the least number exactly divisible by 12, 15, 20 and 27.

Solution :

Required number = LCM of 12, 15, 20, 27.

$\therefore$ LCM = 3 × 4 × 5 × 9 = 540.

$\therefore$ Required number = 540.

16. Find the least number, when divided by 6, 7, 8, 9 and 12, leaves remainder 1 in each case.

Solution :

Required number = (LCM of 6, 7, 8, 9, 12) + 1

$\therefore$ LCM = 3 × 2 × 2 × 7 × 2 × 3 = 504.

Hence, required number = (504 + 1) = 505.

17. Arrange the following rational numbers in ascending order.

$$\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$$

Solution :

LCM of (10, 8, 3) = 120

$$\therefore \frac{-7}{10} = \frac{-84}{120}$$

$$\frac{5}{-8} = \frac{-75}{120}$$

$$\frac{2}{-3} = \frac{-80}{120}$$

as $-84 < -80 < -75$

$$\therefore \frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$$

Alternate Method:

$$\frac{-7}{10} = -0.7,\ \frac{5}{-8} = -0.625 \text{ and } \frac{2}{-3} = -0.666$$

Clearly, –0.7 < –0.666 < –0.625.

So $\frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$.

18. If square root of 15 = 3.88, find the value of square root of $\frac{5}{3}$.

Solution :

$$\sqrt{\frac{5}{3}} = \sqrt{\frac{5\times 3}{3\times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} \approx 1.29$$

19. A four-digit number that is divisible by 7 becomes divisible by 3 when 10 is added to it. Find the largest such number.

Solution :

The largest four-digit number is 9999.

On dividing 9999 by 7, we get 3 as remainder.

The largest four-digit number divisible by 7 is 9996.

Let 9996 – x + 10 be divisible by 3.

By trial and error, we find that x = 7.

Hence, the required number = (9996 – 7) = 9989.

20. A three-digit number 4a3 is added to another three-digit number 984 to give the four-digit number 13b7 which is divisible by 11. Find the value of (a + b).

4a3
984
13b7

Solution :

$$a + 8 = b \Rightarrow b - a = 8$$

Also, 13b7 is divisible by 11.

So (7 + 3) – (b + 1) = 0 and b = 9.

Now, b – a = 8 and b = 9.

So, a = 1, and a + b = (1 + 9) = 10.

21. Express $0.\overline{643}$ into a fraction.

Solution :

Let x = $0.\overline{643}$. Thus, 1000x = $643.\overline{643}$

999x = 643 or x = $\frac{643}{999}$

22. Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55; and the sum of the third and thrice the first is 90. Find the third number.

Solution :

Let the numbers be x, y and z.

Then x + y = 45, y + z = 55 and 3x + z = 90.

y = 45 – x, and z = 55 – y = 55 – (45 – x)

= 10 + x

∴ 3x + 10 + x = 90 or x = 20

y = (45 – 20) = 25, and z = (10 + 20) = 30.

∴ The third number = 30.

23. The traffic lights at three different road-crossings change after every 24 sec, 72 sec and 120 sec respectively. If they all change simultaneously at 10.54.00 hr, then at what time will they change simultaneously for the second time?

Solution :

Interval of change

= LCM of (24, 72, 120) sec = 360 sec.

The lights will change simultaneously after every 360 sec, i.e. 6 min.

Next simultaneous change will take place at 11.00.00 hr.

24. The difference between a two-digit number and the number obtained by interchanging the digits is 72. What is the difference between the digits of the number?

Solution :

Let tens digit be x and units digit be y.

Then (10x + y) – (10y + x) = 72

$$\Rightarrow 9(x - y) = 72 \Rightarrow x - y = 8.$$

25. How many three-digit numbers are divisible by 6 in all?

Solution :

There are 16 numbers before 100, which are divisible by 6.

There are 166 numbers before 999, which are divisible by 6.

Total three-digit numbers divisible by 6

= (166 – 16) = 150

26. If $(5^{2n+1})^2 \times 5^6 = 1$, then what is the value of n?

Solution :

$(5^{2n+1})^2 \times 5^6 = 1$

$$\Rightarrow 5^{4n+2} \times 5^6 = 5^0 \Rightarrow 5^{4n+8} = 5^0$$

$$\Rightarrow 4n + 8 = 0 \Rightarrow n = -2.$$

Exercise

1. The LCM of two numbers is 4200 and their GCD is 20. If one number is 100, the other number is
   (a) 840 (b) 420
   (c) 210 (d) None of these
2. If $y = \sqrt{2} + 1$, then the value of $y + \frac{1}{y}$ is
   (a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$
   (c) $2\sqrt{2}$ (d) $\frac{3}{2}$
3. The largest fraction among $\frac{2}{5}, \frac{3}{5}, \frac{1}{5}, \frac{7}{15}$ and $\frac{4}{5}$ is
   (a) $\frac{2}{5}$ (b) $\frac{3}{5}$
   (c) $\frac{4}{5}$ (d) $\frac{7}{15}$
4. The sum of the squares of first ten natural numbers is
   (a) 281 (b) 402
   (c) 385 (d) 502
5. The value of factorial zero (i.e. 0!) is
   (a) 0 (b) 1
   (c) 8 (d) $\frac{1}{2}$
6. Zero is counted as
   (a) whole number (b) prime number
   (c) natural number (d) All of these
7. Two rational numbers lying between $\frac{4}{5}$ and $\frac{6}{7}$ are
   (a) $\frac{71}{84}, \frac{5}{6}$ (b) $\frac{29}{35}, \frac{5}{6}$
   (c) $\frac{29}{35}, \frac{57}{70}$ (d) All of these
8. The lowest four-digit number which is exactly divisible by 2, 3, 4, 5, 6 and 7 is
   (a) 1400
   (b) 1300
   (c) 1250
   (d) 1260
9. The least number when divided by 35 leaves a remainder 25; when divided by 45 leaves a remainder 35; and when divided by 55 leaves the remainder 45. The number is
   (a) 2515 (b) 3455
   (c) 2875 (d) None of these
10. The sum of two positive numbers is twice their difference. If their product is 48, the numbers will be
    (a) 3 and 16 (b) 6 and 8
    (c) 4 and 12 (d) None of these
11. Solve: $3 + \cfrac{3}{3 + \cfrac{1}{3 + \cfrac{1}{3}}}$
    (a) 1 (b) 3
    (c) $\frac{43}{11}$ (d) $\frac{63}{19}$
12. Find the units digit in the product $254 \times 361 \times 159 \times 18$.
    (a) 1 (b) 4
    (c) 6 (d) 8
13. A heap of coconuts is divided into groups of 2, 3 and 5, and each time one coconut is left out. The least number of coconuts in the heap is
    (a) 31 (b) 41
    (c) 51 (d) 61
14. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the units place is 3 more than the digit in the tens place, what is that number?
    (a) 69 (b) 25
    (c) 36 (d) None of these
15. The smallest number among the following is
    (a) $(7)^3$ (b) $(8.5)^3$
    (c) $(4)^4$ (d) $(6^5)^{3/5}$
16. The largest fraction among the following is
    (a) $\frac{17}{21}$ (b) $\frac{11}{14}$
    (c) $\frac{12}{15}$ (d) $\frac{5}{6}$

17. If $x = (6 - \sqrt{35})$, then the reciprocal of x is
(a) $\frac{1}{6+\sqrt{35}}$ (b) $6+\sqrt{35}$
(c) 1 (d) 12

18. The sum of the first 50 even numbers is
(a) 1275 (b) 2550
(c) 5100 (d) 10100

19. If x + 2y = 4 and $\left(\frac{x}{y}\right)$ = 2, then y = ?
(a) 1 (b) 0.5
(c) 2 (d) 8

20. If the product of three consecutive integers is 720, then their sum is
(a) 27 (b) 45
(c) 18 (d) 54

21. Solve: 18.18 ÷ 9 + 2.7 of 3
(a) 101.2 (b) 27.32
(c) 10.12 (d) None of these

22. Solve: 8127 – 5422 + 1614 – 808
(a) 3580 (b) 3058
(c) 3511 (d) 3088

23. Evaluate: $11^2 + 11^4 \div 11^3 - 11 + (0.5)11^2$
(a) 302.5 (b) 181.5
(c) 484.0 (d) 121

24. Which one of the following is incorrect?
(a) Square root of 5184 is 72
(b) Square root of 15625 is 125
(c) Square root of 1444 is 38
(d) Square root of 1296 is 34

25. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is
(a) 240 (b) 360
(c) 270 (d) 295

26. 243 has been divided into three parts such that half of the first part, one-third of the second part and one-fourth of the third part are equal. The largest part is
(a) 72
(b) 84
(c) 92
(d) 108

27. How many numbers between 200 and 600 are divisible by 4, 5 and 6?
(a) 5 (b) 6
(c) 7 (d) 8

28. The units digit of the product
(247 × 318 × 577 × 313) is
(a) 2 (b) 3
(c) 4 (d) 6

29. The sum of first 45 natural numbers is
(a) 2070 (b) 1035
(c) 1280 (d) 2140

30. Find the value of 'a' if the number '451a603' is exactly divisible by 9, such that 'a' is a single digit number.
(a) 2 (b) 5
(c) 8 (d) 7

31. Which least value must be assigned to 'a' so that the number 63576a2 is divisible by 8?
(a) 1 (b) 2
(c) 3 (d) 4

32. Which of the following is exactly divisible by 99?
(a) 114345 (b) 135792
(c) 3572404 (d) 913464

33. The number which is formed by writing any digit six times (e.g. 111111, 444444, etc.) is always divisible by
(a) 7 (b) 11
(c) 13 (d) All of these

34. The number nearest to 99547, which is exactly divisible by 687 is
(a) 100166 (b) 98928
(c) 99579 (d) 99615

35. Which largest number of five digits is divisible by 99?
(a) 99999 (b) 99981
(c) 99909 (d) 99990

36. Which smallest number of six digits is divisible by 111?
(a) 111111 (b) 110011
(c) 100011 (d) 111011

37. If n is any positive integer, then $(3^{4n} - 4^{3n})$ is always divisible by
(a) 7 (b) 17
(c) 112 (d) 145

38. Solve: 8756 × 99999
    (a) 815491244 (b) 796491244
    (c) 875591244 (d) None of these
39. Evaluate: 1399 × 1399
    (a) 1687401 (b) 1901541
    (c) 1943211 (d) 1957201
40. Find the value of 397 × 397 + 104 × 104 + 2 × 397 × 104.
    (a) 250001 (b) 251001
    (c) 260101 (d) 261001
41. The expression

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n\,(n+1)}$$

    for any natural number n, is
    (a) always greater than 1
    (b) always less than 1
    (c) always equal to 1
    (d) not definite
42. When ‘n’ is divided by 4, the remainder is 3. What is the remainder when ‘2n’ is divided by 4?
    (a) 1 (b) 2
    (c) 3 (d) 6
43. Six bells commence tolling together and toll at intervals of 3 sec, 6 sec, 9 sec, 12 sec, 15 sec and 18 sec respectively. If all bells rang together at 9 o’clock, for how many more times will they ring together till 9:32?
    (a) 4 (b) 10
    (c) 11 (d) 15
44. The units digit of 3^{33} is
    (a) 2 (b) 3
    (c) 4 (d) 5
45. The units digit of $24^{37} \times 37^{24}$ is
    (a) 4 (b) 3
    (c) 2 (d) 6

Answer Key

1. (a)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (c)
11. (c)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (a)
21. (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (b)	**28.** (d)	**29.** (b)	**30.** (c)
31. (c)	**32.** (a)	**33.** (d)	**34.** (d)	**35.** (d)	**36.** (c)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (b)
41. (b)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (a)					

Explanations

1. a LCM × GCD = Product of numbers = $N_1 \times N_2$

$\therefore N_2 = \frac{\text{Product}}{N_1} \Rightarrow N_2 = \frac{4200 \times 20}{100} = 840$

2. c $y = \sqrt{2} + 1$

$\Rightarrow y + \frac{1}{y} = \frac{y^2+1}{y} = \frac{\left(\sqrt{2}+1\right)^2 + 1}{\left(\sqrt{2}+1\right)}$

$= \frac{\left(4+2\sqrt{2}\right)}{\sqrt{2}+1} = \frac{2\sqrt{2}\left(1+\sqrt{2}\right)}{\left(\sqrt{2}+1\right)} = 2\sqrt{2}$

Short cut:

$\left(\sqrt{2}+1\right) + \frac{1}{\sqrt{2}+1} = \left(\sqrt{2}+1\right) + \left(\sqrt{2}-1\right) = 2\sqrt{2}$

3. c By observing, $\frac{4}{5} > \frac{3}{5} > \frac{2}{5} > \frac{1}{5}$.

Numerator is decreasing and denominator is same.

Now comparing $\frac{4}{5}$ and $\frac{7}{15}$.

As, $\frac{7}{15} = \frac{7}{3 \times 5}$ and $\frac{7}{3} < 4$

$\therefore \frac{4}{5}$ is the largest.

4. c $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow 1^2 + 2^2 + \ldots + 10^2 = \frac{10(11)\ (21)}{6} = 385$

5. b 0! = 1

6. a Whole number

7. d Change the fractions into decimals and then find

$\frac{71}{84} = 0.845$, $\frac{5}{6} = 0.833$, $\frac{29}{35} = 0.828$, $\frac{57}{70} = 0.814$.

Hence, the answer is all of these.

8. d LCM of 2, 3, 4, 5, 6 and 7 = 420

Smallest four-digit number divisible by LCM is 420 × 3 = 1260.

Short cut:

1260 is the only option divisible by 3.

9. b Difference between divisor and remainder

= 35 – 25 = 45 – 35 = 55 – 45 = 10.

LCM of 35, 45 and 55 = 5 × 7 × 9 × 11 = 3465.

Required number = 3465 – 10 = 3455.

Short cut:

3455 is the only option from which when we subtract 35, we get 3420 which is divisible by 45.

10. c x + y = 2(x – y)

$\Rightarrow$ x + y = 2(x – y)

$\Rightarrow$ x = 3y

Since, xy = 48, $\Rightarrow 3y^2 = 48$

$\Rightarrow y^2 = 16$

$\Rightarrow$ y = 4

Now, y = 4, $\therefore$ x = 12

11. c $3 + \frac{3}{3 + \frac{1 \times 3}{10}} = 3 + \frac{3 \times 10}{33} = 3 + \frac{10}{11} = \frac{43}{11}$

12. d Units digit = Units digit of (4 × 1 × 9 × 8 = 4 × 72) = 8.

Therefore, units digit of the number = 8.

13. a LCM of 2, 3, 5 = 30.

$\therefore$ Number of coconuts = 30 + 1 = 31.

14. c Let the tens digit be x. Then units digit = (x + 3).

Sum of the digits = x + (x + 3) = 2x + 3

Number = 10x + (x + 3) = 11x + 3

$\frac{11x+3}{2x+3} = \frac{4}{1} \Rightarrow 11x + 3 = 4(2x+3)$

$\therefore x = 3$

$\therefore$ Number = (11x + 3) = 36.

Short cut:

Pick up the option and check the conditions.

15. d $\because (6^5)^{3/5} = 6^3 = 216$.

16. d $\frac{5}{6}$ is largest among four.

17. b $x = 6 - \sqrt{35}$

$\therefore \frac{1}{x} = \frac{1}{6-\sqrt{35}} \times \frac{6+\sqrt{35}}{6+\sqrt{35}} = 6 + \sqrt{35}$

18. b 2 + 4 + 6 + 8 + ... + 100 (50 even numbers)

= 50 × (50 + 1) = 2550.

Sum of first n even numbers is n(n + 1).

19. a x + 2y = 4 and $\frac{x}{y} = 2 \Rightarrow x = 2y$

$\therefore$ 2y + 2y = 4

$\therefore$ y = 1

20. a Numbers are 8, 9, 10.

$\therefore$ Sum = 8 + 9 + 10 = 27.

21. c 18.18 ÷ 9 + 2.7 of 3 = 18.18 ÷ 9 + 8.1

= 2.02 + 8.1 = 10.12

22. c 8127 – 5422 + 1614 – 808 = 9741 – 6230 = 3511

23. b $11^2 + 11^4 \div 11^3 - 11 + (0.5)11^2$

= 121 + 11 – 11 + 0.5 × 121

= 121 + 60.5 = 181.5

24. d The square root of 1296 is 36 and not 34.

25. c Let the numbers be x and 1365 + x.

Then, 1365 + x = 6x + 15 or x = 270.

26. d $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x \Rightarrow A = 2x,\ B = 3\ x \text{ and } C = 4x$

$\Rightarrow$ A : B : C = 2 : 3 : 4

Largest part = $\left(243 \times \frac{4}{9}\right) = 108$.

27. b Every such number must be divisible by LCM of 4, 5 and 6, i.e. 60.

Such numbers are 240, 300, 360, 420, 480, 540.

Hence, there are 6 numbers.

28. d Units digit is governed by product of individual units digit only.

7 × 8 × 7 × 3 = 6.

29. b $\because 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

$\therefore 1 + 2 + 3 + ... + 45 = \frac{45 \times 46}{2} = 1035$

30. c $\frac{451a603}{9}$.

$\because$ **Sum** of all the digits must be divisible by 9,

$\therefore$ a = 8.

31. c Number formed by last 3 digits must be divisible by 8.

32. a Number should be divisible by 9 and 11 both.

33. d 111111 = 111 × 1001 = (37 × 3) × (7 × 11 × 13)

Hence, number of such forms are divisible by all its factors.

34. d When 99547 is divided by 687, remainder is 619.

$\therefore$ Nearest number = 99547 + 68 = 99615.

35. d Out of the given answer choices only 99990 is divisible by 99. Also, in order to a number being divisible by 99, it should be divisible by both 9 and 11, which is true in case of option (d) only.

36. c Any number divisible by 111 should also be divisible by 3 and 37. Out of the given answer choices, both options (a) and (c) are divisible by 111 but, option (c) is the smallest.

37. b $3^{4n} - 4^{3n} = (3^4)^n - (4^3)^n = 81^n - 64^n$

Since $a^n - b^n$ is always divisible by (a – b)

$81^n - 64^n$ will be always divisible by (81 – 64) = 17.

Shortcut:

Substitute n = 1.

38. c 8756 × 99999 = 8756 × [100000 – 1]

= 875600000 – 8756 = 875591244

39. d $1399 \times 1399 = (1400 - 1)^2$

$= (1400)^2 + (1)^2 - 2(1400)(1) = 1960000 + 1 - 2800$

= 1957201

40. b 397 × 397 + 104 × 104 + 2 × 397 × 104

$= (397 + 104)^2 = (501)^2 = (500 + 1)^2$

$= (500)^2 + (1)^2 + 2(500)(1)$

= 250000 + 1 + 1000 = 251001

41. b $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} ... + \frac{1}{n} - \frac{1}{(n+1)}$

$= 1 - \frac{1}{(n+1)} = \frac{n}{n+1}$

As, numerator is always less than denominator, thus the expression will always be less than 1.

42. b n = 4 × q + 3 (q = Quotient)

2n = 2 × 4 × q + 6

When 2n is divided by 4, quotient = 2q + 1 and remainder = 2.

43. b LCM of 3, 6, 9, 12, 15 and 18 is 180.

So the bells will toll together after every 180 s, i. e. 3 min.

In 32 min, they will toll together for $\frac{32}{3} \simeq 10$ times.

44. b 3 has a cyclicity of 4. 33 when divided by 4 has remainder 1. Hence unit digit is 3^1 i.e. 3.

45. a $24^{37} = 4^{37} = (4^2)^{18} \cdot 4^1 = 4$ (unit digit)

$37^{24} = 7^{24} = (7^4)^6 = (2401)^6 = 1$ (unit digit)

Hence Units digit $= 4 \times 1 = 4$.