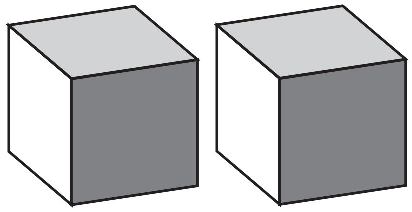

Activity 23

Algebraic identity (case I)

Objective

To verify the identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, for simple cases using a set of unit cubes.

Pre–requisite knowledge

1. Express the volume of an object as the number of unit cubes in it.
2. Knowledge of the identity $(a + b)(a^2 - ab + b^2) = a^2(a + b) - ab(a + b) + b^2 (a + b)$.

Material Required

36 unit cubes made of wood (dimension is 1 unit × 1 unit × 1 unit).

Procedure

For representing $a^2(a + b)$

1. Take a = 3 and b = 1. Make a cube of dimension $a^2(a + b)$ i.e. 3 × 3 × 4 using unit cubes as shown in Fig 23 (a).

For representing $a^3 + b^3$ as difference between $a^2(a + b)$ and $ab(a + b) + b^2(a + b)$

1. Remove a cuboid of dimension $ab(a + b)$ i.e. 3 × 1 × 4 [Fig 23 (b)] from Fig 23 (a) as shown in Fig 23 (c).
2. Add a cuboid of dimensions $b^2 (a + b)$ i.e. 1 × 1 × 4 [Fig 23 (d)] in Fig 23 (c) as shown in Fig 23 (e).
3. Number of cubes remaining is 28.
4. These 28 unit cubes can be rearranged as $27 + 1 = 3^3 + 1^3$ i.e. $a^3 + b^3$ as shown in Fig 23 (h).

Observations

Number of unit cubes in $a^2(a + b) = 36$
Number of unit cubes in $ab(a + b) = 12$
Number of unit cubes in $b^2 (a + b) = 4$
Remaining cubes = 36 – 12 + 4
= 28
= 27 + 1
$= 3^3 + 1^3$
Students verify that
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students will obtain the skill of adding and subtracting the volume of cuboids.

3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

Remark

1. Teacher can take any value of a and b and verify the result.
2. This activity can be done by taking the formula $a^3 + b^3 = (a + b)^3 – 3ab (a + b)$ also.
3. The dimensions of cuboid added and removed should be calculated by students..

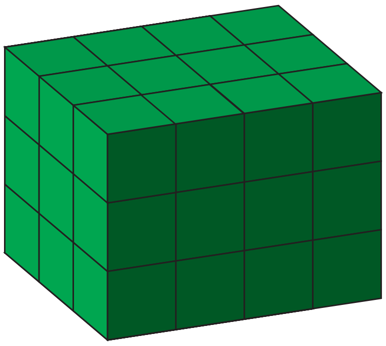

Fig 23 (a)

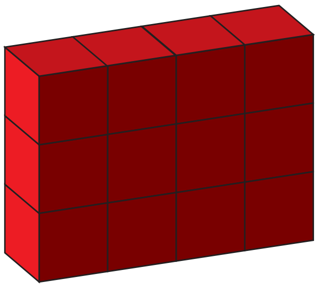

Fig 23 (b)

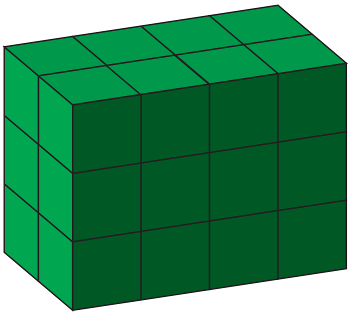

Fig 23 (c)

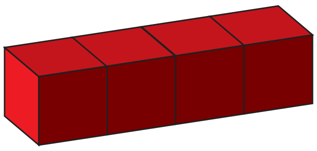

Fig 23 (d)

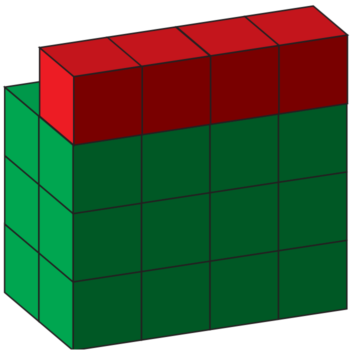

Fig 23 (e)

Algebraic identity (case II)

Objective

To verify the identity $a^3 + b^3 = (a + b)^3 – 3ab(a + b)$ using cuboids and unit cubes.

Pre–requisite knowledge

Express the volume of an object as the number of unit cubes in it.

Material Required

64 unit cubes made of wood (dimension is 1 unit × 1 unit × 1 unit).

Procedure

For representing $(a + b)^3$

1. Take a = 3 and b = 1. Make a cube of dimension 4 × 4 × 4 using 64 unit cubes as shown in Fig 23 (f).

For representing $(a + b)^3 – 3ab(a + b)$

1. Remove a cuboid of dimensions ab(a + b) i.e. 3 × 1 × 4 [Fig 23 (g)] three times from Fig 23 (f) as shown in Fig 23 (h).
2. Number of remaining cubes are 64 - 3 × (3 × 1 × 4) = 64 - 36 = 28.
3. These 28 unit cubes can be arranged as $27 + 1 = 3^3 + 1^3$ i.e. $a^3 + b^3$ as shown in Fig 23 (h).

Observations

1. Number of unit cubes in $(a + b)^3 = 64$
2. Number of unit cubes in $3ab(a + b) = 3 \times 4 \times 3 = 36$
3. Number of cubes remaining = 64 – 36 = 28
4. Number of cubes represented = $3^3 + 1^3$
5. It is verified that $a^3 + b^3 = (a + b)^3 – 3ab(a + b)$

Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students obtain the skill of adding and subtracting the volume of cuboids.
3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

Remark

1. Teachers can take any value of a and b and verify the result.
2. Students should find the volume of cuboid by measuring the length, breadth and height.

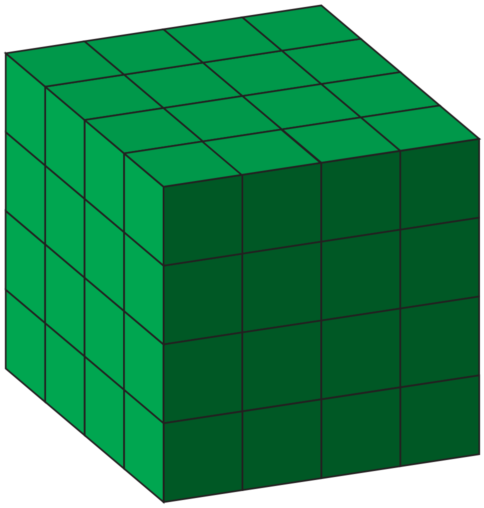

Fig 23 (f)

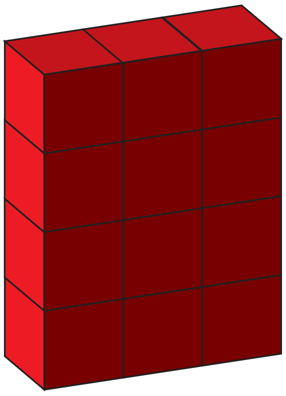

Fig 23 (g)

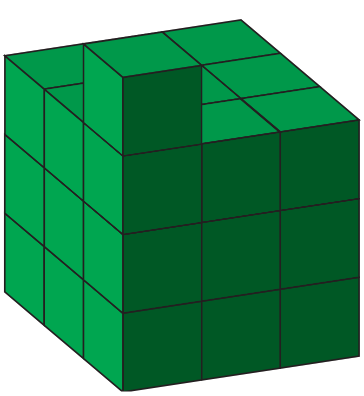

Fig 23 (h)