5. Squares, Square-Roots, Cubes and Cube-Roots

Exercise 5.1

1. Question

Express the following statements mathematically:

(i) square of 4 is 16; (ii) square of 8 is 64; (iii) square of 15 is 225.

Answer

(i) square of 4 is 16;

$4^2 = 16$

(ii) square of 8 is 64;

$8^2 = 64$

(iii) square of 15 is 225

$15^2 = 225$

2. Question

Identify the perfect squares among the following numbers:

1, 2, 3, 8, 36, 49, 65, 67, 71, 81, 169, 625, 125, 900, 100, 1000, 100000.

Answer

Since,

$6^2 = 36$

$7^2 = 49$

$9^2 = 81$

$13^2 = 169$

$25^2 = 625$

$30^2 = 900$

$10^2 = 100$

$15^2 = 225$

Hence, 36, 49, 81, 169, 625, 900 and 100 are perfect squares.

3. Question

Make a list of all perfect squares from 1 to 500.

Answer

$1^2 = 1$

$2^2 = 49$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

$14^2 = 196$

$15^2 = 225$

$16^2 = 256$

$17^2 = 289$

$18^2 = 324$

$19^2 = 361$

$20^2 = 400$

$21^2 = 441$

$22^2 = 484$

So, perfect squares from 1 to 500 are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400,441, 484

4. Question

Write 3-digit numbers ending with 0, 1, 4, 5, 6, 9, one for each

digit, but none of them is a perfect square.

Answer

3-digit numbers ending with 0, 1, 4, 5, 6, 9, are 110, 111, 114, 115, 116, 119.

There are many other numbers too, satisfying the conditions mentioned.

5. Question

Find numbers from 100 to 400 that end with 0, 1, 4, 5, 6 or 9, which are perfect squares.

Answer

numbers from 100 to 400 that end with 0, 1, 4, 5, 6 or 9, which are perfect squares are:

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

$14^2 = 196$

$15^2 = 225$

$16^2 = 256$

$17^2 = 289$

$18^2 = 324$

$19^2 = 361$

$20^2 = 400$

Exercise 5.2

1. Question

Find the sum 1 + 3 + 5 + … + 51 (the sum of all odd numbers from 1 to 51) without actually adding them.

Answer

Total terms = $\frac{51 + 1}{2}$

= 26

As we know, that sum of first n odd natural numbers is n^2

⇒ Sum of above digits = 26^2

= 676

2. Question

Express 144 as a sum of 12 odd numbers.

Answer

As we know, that sum of first n odd natural numbers is n^2

Sum of above digits = 144

⇒ Number of digits = $\sqrt{144}$

= 12

So, digits are 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

3. Question

Find the 14^{th} and 15^{th} triangular numbers, and find their sum. Verify the **Statement 8** for this sum.

Answer

14^{th} triangular number = 1 + 2 + 3 + 4 + … + 14 = 105

15^{th} triangular number = 1 + 2 + 3 + 4 + … + 14 + 15 = 120

According to statement 8, the sum of n^{th} and $(n + 1)^{th}$ triangular number is $(n + 1)^2$

Here, 120 + 105 = 225

And $(14 + 1)^2 = 15^2$

= 225

Hence Statement 8 is verified

4. Question

What are the remainders of a perfect square when divided by 5?

Answer

Since all perfect squares terminate with digits 0, 1, 4, 5, 6, 9 , i.e., digits at unit place.

So, remainder when the perfect square is divided by 5 is either of 1, 4, and 0.

Exercise 5.3

1 A. Question

Find the squares of:

31

Answer

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

Here, $31^2 = (30 + 1)^2$

$= 30^2 + 1^2 + 2\times30\times1$

= 900 + 1 + 60

= 961

1 B. Question

Find the squares of:

72

Answer

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

Here, $72^2 = (70 + 2)^2$

$= 70^2 + 2^2 + 2\times70\times2$

= 4900 + 4 + 280

= 5184

1 C. Question

Find the squares of:

37

Answer

Using the identity $(a-b)^2 = a^2 + b^2- 2ab$

Here, $37^2 = (40 - 3)^2$

$= 40^2 + 3^2 - 2 \times 40 \times 3$

= 1600 + 9 - 240

= 1369

1 D. Question

Find the squares of:

166

Answer

Using the identity $(a-b)^2 = a^2 + b^2 - 2ab$

Here, $72^2 = (170 - 4)^2$

$= 170^2 + 4^2 - 2 \times 170 \times 4$

= 28900 + 16 - 1360

= 27556

2. Question

Find the squares of:

(i) 85

(ii) 115

(iii) 165

Answer

(i) 85

Since 85 terminates with 5.

⇒ 8 × (8 + 1)

8 × 9 = 72

So, $85^2 = 7225$

(ii) 115

Since 115 terminates with 5.

⇒ 11 × (11 + 1)

11 × 12 = 132

So, $115^2 = 13225$

(iii) 165

Since 165 terminates with 5.

$\Rightarrow 16 \times (16 + 1)$

$16 \times 17 = 272$

So, $165^2 = 27225$

3. Question

Find the square of 1468 by writing this as 1465 + 3.

Answer

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

Here, $72^2 = (1465 + 3)^2$

$= 1465^2 + 3^2 + 2\times1465\times3$ ..(i)

Since 1465 terminates with 5,

$1465^2 \Rightarrow 146 \times (146 + 1)$

$= 146 \times 147$

$= 21462$

$1465^2 = 2146225$

Substituting 1465^2 in equation (i)

$= 1465^2 + 3^2 + 2\times1465\times3$

$= 2146225 + 9 + 8790$

$= 2155024$

Exercise 5.4

1 A. Question

Find the square root of the following numbers by factorization:

196

Answer

$196 = 7 \times 7 \times 2 \times 2$

$196 = 7^2 \times 2^2$

$\Rightarrow \sqrt{196} = \sqrt{7^2 \times 2^2}$

$\Rightarrow \sqrt{196} = \sqrt{(7 \times 2)^2}$

$\Rightarrow \sqrt{196} = \sqrt{14^2}$

$\Rightarrow \sqrt{196} = 14$

∴ square root of 196 is 14

1 B. Question

Find the square root of the following numbers by factorization:

256

Answer

$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$256 = 2^8$

$\Rightarrow \sqrt{256} = \sqrt{2^8}$

$\Rightarrow \sqrt{256} = \sqrt{(2^4)^2}$

$\Rightarrow \sqrt{256} = \sqrt{16^2}$

$\Rightarrow \sqrt{256} = 16$

∴ square root of 256 is 16

1 C. Question

Find the square root of the following numbers by factorization:

10404

Answer

$10404 = 2 \times 2 \times 3 \times 3 \times 17 \times 17$

$10404 = 2^2 \times 3^2 \times 17^2$

$\Rightarrow \sqrt{10404} = \sqrt{2^2 \times 3^2 \times 17^2}$

$\Rightarrow \sqrt{10404} = \sqrt{(2 \times 3 \times 17)^2}$

$\Rightarrow \sqrt{10404} = \sqrt{102^2}$

$\Rightarrow \sqrt{10404} = 102$

∴ square root of 10404 is 102

1 D. Question

Find the square root of the following numbers by factorization:

1156

Answer

1156 = 2 × 2 × 17× 17

1156 = $2^2 \times 17^2$

$\Rightarrow \sqrt{1156} = \sqrt{2^2 \times 17^2}$

$\Rightarrow \sqrt{1156} = \sqrt{(2 \times 17)^2}$

$\Rightarrow \sqrt{1156} = \sqrt{34^2}$

$\Rightarrow \sqrt{1156} = 34$

∴ square root of 1156 is 34

1 E. Question

Find the square root of the following numbers by factorization:

13225

Answer

13225 = 5 × 5 × 23 × 23

13225 = $5^2 \times 23^2$

$\Rightarrow \sqrt{13225} = \sqrt{5^2 \times 23^2}$

$\Rightarrow \sqrt{13225} = \sqrt{(5 \times 23\ )^2}$

$\Rightarrow \sqrt{13225} = \sqrt{115^2}$

$\Rightarrow \sqrt{13225} = 115$

∴ square root of 13225 is 115

2 A. Question

Simplify:

$\sqrt{100} + \sqrt{36}$

Answer

$\sqrt{100} + \sqrt{36}$

$\sqrt{100} = 10$

$\sqrt{36} = 6$

$\Rightarrow \sqrt{100} + \sqrt{36} = 10 + 6$

$= 16$

$\therefore \sqrt{100} + \sqrt{36} = 16$

2 B. Question

Simplify:

$\sqrt{(1360 + 9)}$

Answer

$\sqrt{(1360 + 9)}$

$\sqrt{1360 + 9} = \sqrt{1369}$

$\sqrt{1369} = 37$

$\therefore \sqrt{1360 + 9} = 37$

2 C. Question

Simplify:

$\sqrt{2704} + \sqrt{144} + \sqrt{289}$

Answer

$\sqrt{2704} + \sqrt{144} + \sqrt{289}$

$\sqrt{2704} = 52$

$\sqrt{144} = 12$

$\sqrt{289} = 17$

$\Rightarrow \sqrt{2704} + \sqrt{144} + \sqrt{289} = 52 + 12 + 17$

$= 81$

$\therefore \sqrt{2704} + \sqrt{144} + \sqrt{289} = 81$

2 D. Question

Simplify:

$\sqrt{225} – \sqrt{25}$

Answer

$\sqrt{225} – \sqrt{25}$

$\sqrt{225} = 15$

$\sqrt{25} = 5$

$\Rightarrow \sqrt{225} - \sqrt{25} = 15 - 5$

$= 10$

$\therefore \sqrt{225} - \sqrt{25} = 10$

2 E. Question

Simplify:

$\sqrt{1764} – \sqrt{1444}$

Answer

$\sqrt{1764} – \sqrt{1444}$

$\sqrt{1764} = 42$

$\sqrt{1444} = 38$

$\Rightarrow \sqrt{1764} - \sqrt{1444} = 42 - 38$

$= 4$

$\therefore \sqrt{1764} - \sqrt{1444} = 4$

2 F. Question

Simplify:

$\sqrt{169} \times \sqrt{361}$

Answer

$\sqrt{169} \times \sqrt{361}$

$\sqrt{169} = 13$

$\sqrt{361} = 19$

$\Rightarrow \sqrt{163} \times \sqrt{361} = 13 \times 19$

$= 247$

$\therefore \sqrt{169} \times \sqrt{361} = 247$

3. Question

A square yard has area 1764 m^2. From a corner of this yard, another square part of area 784 m^2 is taken out for public utility. The remaining portion is divided in to 5 equal square parts. What is the perimeter of each of these equal parts?

Answer

Area of yard = 1764 m^2

Area of part taken out = 784 m^2

Area of remaining part = 1764 – 784

= 980 m^2

∴ Area of one part out of 5 equal parts = $\frac{980}{5}$ m^2

= 196 m^2

Side of the square of area 196 m^2 = $\sqrt{196}$

= 14 m

∴Perimeter of each small square part = 4 × 14

= 56 m

4. Question

Find the smallest positive integer with which one has to multiply each of the following numbers to get a perfect square:

(i) 847

(ii) 450

(iii) 1445

(iv) 1352

Answer

(i) 847

$847 = 7 \times 11 \times 11$

$847 = 7 \times 11 \times 11 \underline{\times 7}$

Hence required integer is 7

(ii) 450

450 = 5 × 5 × 6 × 3

450 = 5 × 5 × 6 × 3 <u>× 2</u>

Hence required integer is 2

(iii) 1445

1445 = 5 × 17 × 17

1445 = 5 × 17 × 17 <u>× 5</u>

Hence required integer is 5

(iv) 1352

1352 = 2 ×2 × 2 ×13 × 13

1352 = 2 ×2 × 2 ×13 × 13 <u>× 2</u>

Hence required integer is 2

5. Question

Find the largest perfect square factor of each of the following numbers:

(i) 48

(ii) 11280

(iii) 729

(iv) 1352

Answer

(i) 48

48 = 2 ×2 × 2 × 2 × 3

48 = 16 × 3

Hence largest perfect square factor is 16

(ii) 11280

11280 = 2 ×2 × 2 × 2 × 5 × 3 × 47

11280 = 16 × 705

Hence largest perfect square factor is 16

(iii) 729

729 = 3 ×3 × 3 × 3 × 3 × 3

729 = 729

Hence largest perfect square factor is 729

(iv) 1352

1352 = 2 ×2 × 2 × 13 × 13

1352 = 2 × 676

Hence largest perfect square factor is 676

Exercise 5.5

1. Question

Find the nearest integer to the square root of the following numbers:

(i) 232

(ii) 600

(iii) 728

(iv) 824

(v) 1729

Answer

(i) 232

Nearest perfect square < 232 = 225

Nearest perfect square > 232 = 256

⇒ Nearest integer to the square root of 232 = 15

(ii) 600

Nearest perfect square < 600 = 576

Nearest perfect square > 600 = 625

⇒ Nearest integer to the square root of 600 = 24

(iii) 728

Nearest perfect square < 728 = 676

Nearest perfect square > 728 = 729

⇒ Nearest integer to the square root of 232 = 27

(iv) 824

Nearest perfect square < 824 = 784

Nearest perfect square > 824 = 841

⇒ Nearest integer to the square root of 824 = 29

(v) 1729

Nearest perfect square < 1729 = 1681

Nearest perfect square > 1729 = 1764

⇒ Nearest integer to the square root of 1729 = 42

2. Question

A piece of land is in the shape of a square and its area is 1000 m^2. This has to be fenced using barbed wire. The barbed wire is available only in integral lengths. What is the minimum length of the barbed wire that has to be bought for this purpose?

Answer

Area of land = 1000 m^2

Length of side = $\sqrt{1000}$ m

⇒ Perimeter of land = 4 × $\sqrt{1000}$

= 127 m (approx.)

∴ Length of wire required is 127m

3. Question

A student was asked to final $\sqrt{961}$. He read it wrongly and found $\sqrt{691}$ to the nearest integer. How much small was his number from the correct answer?

Answer

$\sqrt{961}$ = 31

26^2 = 676 < 691 < 27 = 729

⇒ 691 is nearer to 676.

Thus, the nearest integer to is 26.

Difference = 31 − 26 = 5

∴ His number was smaller than the correct answer by 5.

Exercise 5.6

1. Question

Looking at the pattern, fill in the gaps in the following:

2	3	4	-5	—	8	—
$2^3 = 8$	$3^3 =$ —	— = 64	— = —	$6^3 =$ —	— = —	— = -729

Answer

2	3	4	-5	6	8	-9
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$(-5)^3 = 125$	$6^3 = 216$	$5^3 = 512$	$(-9)^3 = -729$

2. Question

Find the cubes of the first five odd natural numbers and the cubes of the first five even natural numbers. What can you say about the parity of the odd cubes and even cubes?

Answer

1^3	3^3	5^3	7^3	9^3	2^3	4^3	6^3	8^3	10^3
1	27	125	343	729	8	64	216	512	1000

The cube of even number is always even and that of odd be always odd.

3. Question

How many perfect cubes you can find from 1 to 100? How many from −100 to 100?

Answer

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

⇒ There are 4 cubes from 1 to 100.

0 = 0

$(-1)^3 = -1$

$(-2)^3 = -8$

$(-3)^3 = -27$

$(-4)^3 = -64$

∴ There are 9 cubes from −100 to 100.

4. Question

How many perfect cubes are there from 1 to 500? How many are perfect squares among these cubes?

Answer

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

Thus, there are 7 perfect cubes from 1 to 500 and 64 i.e., (8^2) is the only perfect square.

5. Question

Find the cubes of 10, 30, 100, 1000. What can you say about the zeros at the end?

Answer

$10^3 = 1000$

$30^3 = 27000$

$100^3 = 10,00,000$

$1000^3 = 1,00,00,00,000$

The number of zeroes is always a multiple of 3

6. Question

What are the digits in the unit's place of the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10? Is it possible to say that a number is not a perfect cube by looking at the digit in unit's place of the given number, just like you did for squares?

Answer

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

$9^3 = 729$

$10^3 = 1000$

The digits at units place are 1, 8, 7, 4, 5, 6, 3, 2 and 9.

Since, all digits are at end of some or other cube.

So, it is not possible to say that a number is not a perfect cube by looking at the digit in unit's place of the given number.

Exercise 5.7

1 A. Question

Find the cube root by prime factorization:

1728

Answer

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$1728 = 2^3 \times 2^3 \times 3^3$

$\Rightarrow \sqrt[3]{1728} = \sqrt[3]{2^3 \times 2^3 \times 3^3}$

$\Rightarrow \sqrt[3]{1728} = \sqrt[3]{(2 \times 2 \times 3)^3}$

$\Rightarrow \sqrt[3]{1728} = \sqrt[3]{(12)^3}$

$\Rightarrow \sqrt[3]{1728} = 12$

∴ cube root of 1728 is 12

1 B. Question

Find the cube root by prime factorization:

3375

Answer

3375 = 5 × 5 ×5 × 3 × 3 × 3

3375 = $5^3 \times 3^3$

$\Rightarrow \sqrt[3]{3375} = \sqrt[3]{5^3 \times 3^3}$

$\Rightarrow \sqrt[3]{3375} = \sqrt[3]{(5 \times 3)^3}$

$\Rightarrow \sqrt[3]{3375} = \sqrt[3]{(15)^3}$

$\Rightarrow \sqrt[3]{3375} = 15$

∴ cube root of 3375 is 15

1 C. Question

Find the cube root by prime factorization:

10648

Answer

10648 = 2 × 2 ×2 × 11 × 11 × 11 10648 = $2^3 \times 11^3$

$\Rightarrow \sqrt[3]{10648} = \sqrt[3]{2^3 \times 11^3}$

$\Rightarrow \sqrt[3]{10648} = \sqrt[3]{(2 \times 11)^3}$

$\Rightarrow \sqrt[3]{10648} = \sqrt[3]{(22)^3}$

$\Rightarrow \sqrt[3]{10648} = 22$

∴ cube root of 10648 is 22

1 D. Question

Find the cube root by prime factorization:

46656

Answer

46656 = 2 × 2 ×2 × 2 × 2 ×2 × 9× 9 ×9 46656 = $2^6 \times 9^3$

$\Rightarrow \sqrt[3]{46656} = \sqrt[3]{2^3 \times 2^3 \times 9^3}$

$\Rightarrow \sqrt[3]{46656} = \sqrt[3]{(2 \times 2 \times 9)^3}$

$\Rightarrow \sqrt[3]{46656} = \sqrt[3]{(36)^3}$

$\Rightarrow \sqrt[3]{46656} = 36$

∴ cube root of 46656 is 36

1 E. Question

Find the cube root by prime factorization:

15625

Answer

15625 = 5 × 5 × 5 × 5 × 5 × 5 15625 = 5^6

$\Rightarrow \sqrt[3]{15625} = \sqrt[3]{5^6}$

$\Rightarrow \sqrt[3]{15625} = \sqrt[3]{(25)^3}$

$\Rightarrow \sqrt[3]{15625} = \sqrt[3]{25^3}$

$\Rightarrow \sqrt[3]{46656} = 25$

∴ cube root of 15625 is 25

2 A. Question

Find the cube root of the following by looking at the last digit and using estimation.

91125

Answer

Let n^3 = 91125

Here, the unit's place digit is 5;

∴ The units digit of n must be 5.

Let us split the number 91125 as 91 and 125

Now, $4^3 < 91 < 5^3 = 125$

∴ $40^3 = 64000 < 91000 < 50^3 = 125000$

Since, the unit's place digit of n is 5, the only possible number is 45.

Also, $45^3 = 91125$

2 B. Question

Find the cube root of the following by looking at the last digit and using estimation.

166375

Answer

Let $n^3 = 166375$

Here, the unit's place digit is 5;

∴ Unit's place digit of n must be 5.

Let us split the number 166375 as 166 and 375

Now, $5^3 < 166 < 6^3$

∴ $50^3 = 125000 < 166000 < 60^3 = 216000$

Since, the unit's place digit of n is 5, the only possible number is 55.

Also, $55^3 = 166375$

2 C. Question

Find the cube root of the following by looking at the last digit and using estimation.

704959

Answer

Let $n^3 = 704959$

Here, the unit's place digit is 9;

∴ Unit's place digit of n must be 9.

Let us split the number 704959 as 704 and 959

Now, $8^3 < 704 < 9^3$

∴ $80^3 = 512000 < 166000 < 90^3 = 729000$

Since, the unit's place digit of n is 9, the only possible number is 89.

Also, $89^3 = 704969$

3. Question

Find the nearest integer to the cube root of each of the following:

(i) 331776

(ii) 46656

(iii) 373248

Answer

(i) 331776

Since, $6^3 = 216 < 331 < 7^3 = 343$

$\therefore 60^3 = 216000 < 331000 < 70^3 = 343000$

$\Rightarrow 68^3 = 314432, 69^3 = 328509, 70^3 = 343000$

So nearest integer to the cube root of 331776 is 69

(ii) 46656

Since, $3^3 = 27 < 46 < 4^3 = 64$

$\therefore 30^3 = 27000 < 46000 < 40^3 = 64000$

$\Rightarrow 31^3 = 29791, 32^3 = 32768, 35^3 = 42875, 36^3 = 46656$

So nearest integer to the cube root of 46656 is 36

(iii) 373248

Since, $7^3 = 343 < 373 < 8^3 = 512$

$\therefore 70^3 = 343000 < 373000 < 80^3 = 512000$

$\Rightarrow 71^3 = 357311, 72^3 = 357911, 73^3 = 389017$

So nearest integer to the cube root of 737248 is 72

Additional Problems 5

1. Question

Match the numbers in the column A with their squares in column B:

A		B		Answers	
(1)	5	(a)	25	(1)	____
(2)	8	(b)	144	(2)	____
(3)	2	(c)	36	(3)	____
(4)	−6	(d)	484	(4)	____
(5)	−22	(e)	64	(5)	____
(6)	12	(f)	4	(6)	____
		(g)	121		

Answer

(1) $5^2 = 5 \times 5 = 25$

(2) $8^2 = 8 \times 8 = 64$

(3) $2^2 = 2 \times 2 = 4$

(4) $(-6)^2 = -6 \times -6 = 36$

(5) $(-22)^2 = -22 \times -22 = 484$

(6) $12^2 = 12 \times 12 = 144$

A		B		Answers	
(1)	5	(a)	25	(1)	(a)
(2)	8	(b)	144	(2)	(e)
(3)	2	(c)	36	(3)	(f)
(4)	−6	(d)	484	(4)	(c)
(5)	−22	(e)	64	(5)	(d)
(6)	12	(f)	4	(6)	(b)
		(g)	121		

2 A. Question

The number of perfect squares from 1 to 500 is:

A. 1

B. 16

C. 22

D. 25

Answer

Let us write the square of numbers starting with the square of 1:

1, 4, 9,16,25,36,49,64,81,100

121,144,169,196,225,256,289,324,361,400, 441,484, 529

We see that 484 is the last square that appears from 1 to 500. Also, $(22)^2 = 22 \times 22 = 484$.

Hence, the number of perfect squares from 1 to 500 is 22.

2 B. Question

The last digit of a perfect square can never be

A. 1

B. 3

C. 5

D. 9

Answer

A. $1^2 = 1$, $11^2 = 121$, and so on. The last digit of these perfect squares is 1.

C. $5^2 = 25$, $15^2 = 225$, and so on. The last digit of these perfect squares is 5.

D. $3^2 = 9$, $13^2 = 169$, and so on. The last digit of these perfect squares is 9.

Hence, the last digit of a perfect square can never be 3.

2 C. Question

If a number ends in 5 zeros, its square ends in:

A. 5 zeros

B. 8 zeros

C. 10 zeros

D. 12 zeros

Answer

The number of zeroes of a number gets doubled in its perfect square. For example:

$10^2 = 100$, $100^2 = 10000$, and so on.

So, if a number ends in 5 zeroes, its square ends in 10 zeroes.

2 D. Question

Which could be the remainder among the following when a per feet square is divided by 8?

A. 1

B. 3

C. 5

D. 7

Answer

On dividing a perfect square by 8, we get 0, 1 and 4 as the remainder.

So, the remainder among when a per feet square is divided by 8 could be 1.

2 E. Question

The 6^{th} triangular number is:

A. 6

B. 10

C. 21

D. 28

Answer

The 6^{th} triangular number = 1 + 2 + 3 + 4 + 5 + 6 = 21

3. Question

Consider all integers from −10 to 5, and square each of them. How many distinct numbers do you get?

Answer

Squares of all integers from -10 to 5:

$(-10)^2 = -10 \times -10 = 100$

$(-9)^2 = -9 \times -9 = 81$

$(-8)^2 = -8 \times -8 = 64$

$(-7)^2 = -7 \times -7 = 49$

$(-6)^2 = -6 \times -6 = 36$

$(-5)^2 = -5 \times -5 = 25$

$(-4)^2 = -4 \times -4 = 16$

$(-3)^2 = -3 \times -3 = 9$

$(-2)^2 = -2 \times -2 = 4$

$(-1)^2 = -1 \times -1 = 1$

$0^2 = 0 \times 0 = 0$

$1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$4^2 = 4 \times 4 = 16$

$5^2 = 5 \times 5 = 25$

There are 11 distinct numbers.

4. Question

Write the digit in unit’s place when the following number are squared:

4, 5, 9, 24, 17, 76, 34, 52, 33, 2319, 18, 3458, 3453.

Answer

We know that if two numbers have same unit place then the unit place of their squares is also same.

$4^2 = 16$, unit place of $4^2 = 6$

$5^2 = 25$, unit place of $5^2 = 5$

$9^2 = 81$, unit place of $9^2 = 1$

24: $4^2 = 16$, unit place of $24^2 = 6$

17: $7^2 = 49$, unit place of $17^2 = 9$

76: $6^2 = 36$, unit place of $76^2 = 6$

34: $4^2 = 16$, unit place of $34^2 = 6$

52: $2^2 = 4$, unit place of $52^2 = 4$

33: $3^2 = 9$, unit place of $33^2 = 9$

2319: $9^2 = 81$, unit place of $2319^2 = 1$

18: $8^2 = 64$, unit place of $18^2 = 4$

3458: $8^2 = 64$, unit place of $3458^2 = 4$

3453: $3^2 = 9$, unit place of $3453^2 = 9$

5. Question

Write all numbers from 400 to 425 which end in 2, 3, 7 or 8. Check if any of these is a perfect square.

Answer

All the numbers from 400 to 425 which end in 2, 3, 7 or 8 are

402, 403, 407, 408, 412, 413, 417, 418, 422, 423

We know that every square has one of these as its unit place digit 1, 4, 9, 6, 5, 0. So, none of these is a perfect square.

6. Question

Find the sum of the digits of $(111111111)^2$.

Answer

We know that:

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

We see a pattern according to which

$(111111111)^2 = 12345678987654321$

whose digit sum is 81.

7. Question

Suppose $x^2 + y^2 = z^2$.

(i) if $x = 4$ and $y = 3$, find z;

(ii) if $x = 5$ and $z = 13$, find y;

(iii) if $y = 15$ and $z = 17$, find x.

Answer

(i) Given that $x^2 + y^2 = z^2$

where x = 4 and y = 3

⇒ $z^2 = 4^2 + 3^2$

⇒ $z^2 = 16 + 9$

⇒ $z^2 = 25$

⇒ $z = \pm 5$

(ii) Given that $x^2 + y^2 = z^2$

where x = 5 and z = 13

⇒ $y^2 = 13^2 - 5^2$

⇒ $y^2 = 169 - 25$

⇒ $y^2 = 144$

⇒ $y = \pm 12$

(iii) Given that $x^2 + y^2 = z^2$

where y = 15 and z = 17

⇒$x^2 = 17^2 - 15^2$

⇒$x^2 = 289 - 225$

⇒$x^2 = 64$

⇒ $x = \pm 8$

8. Question

A sum of Rs. 2304 is equally distributed among several people. Each gets as many rupees as the number of persons. How much does each one get?

Answer

Let the number of persons be x.

According to the question,

Money given to each person = Rs x

⇒ No. of persons × money given to each person = Total sum distributed

⇒$x \times x = 2304$

⇒$x^2 = 2304$

∵ $48^2 = 2304$

⇒ x = 48

Each person gets Rs 48.

9. Question

Define a new operation * on the set of all natural numbers by $m*n = m^2 + n^2$.

(i) Is ℕ closed under *?

(ii) Is * commutative on ℕ?

(iii) Is * associative on ℕ?

(iv) Is there an identity element in ℕ with respect to *?

Answer

Consider $m*n = m^2 + n^2$.(i)Yes, ℕ is closed under *. This is because, for any natural number m, m^2 is also a natural number.Further, on adding two natural numbers, we get a natural number only. So, if m and n belong to ℕ then $m^2 + n^2$ also belongs to ℕ.

(ii) Commutative means x*y = y*x where x and y belongs to ℕ

$\Rightarrow m*n = m^2 + n^2$

And $n*m = n^2 + m^2$

$\Rightarrow m^2 + n^2 = n^2 + m^2$

$\Rightarrow m*n = n*m$

Hence, * is commutative on ℕ

(iii) Associative means (x*y)*z = x*(y*z)where x, y and z belongs to ℕ

$m*n = m^2 + n^2$

Further,

$(m*n)*o = (m^2 + n^2)*o = (m^2 + n^2)^2 + o^2$

Similarly, $n*o = n^2 + o^2$

Further,

$m*(n*o) = m*(n^2 + o^2) = m^2 + (n^2 + o^2)^2$

$\Rightarrow (m^2 + n^2)^2 + o^2 \neq m^2 + (n^2 + o^2)^2$

$\Rightarrow (m*n)*o \neq m*(n*o)$

Hence, * is not associative.

(iv) An identity element is a special type of element of a set with respect to a binary operation on that set, which leaves other elements unchanged when combined with them.

that is for any element a if a * b = b * a = athen b is the identity of a.

For *, let k be the identity element then $m*k = m^2 + k^2 = k^2 + m^2 = m^2$

This is possible only when k = 0 but k needs to be a natural number.

Hence, * does not have an identity element.

10. Question

(**Exploration**) Find all perfect squares from 1 to 500, each of which is a sum of two perfect squares.

Answer

We know that the integers that satisfy

$A^2 + B^2 = C^2$

are called Pythagorean triplets.

So, all perfect squares from 1 to 500, each of which is a sum of two perfect squares are –

25 = 9 + 16

$\Rightarrow 5^2 = 3^2 + 4^2$

100 = 36 + 64

$\Rightarrow 10^2 = 6^2 + 8^2$

169 = 25 + 144

$\Rightarrow 13^2 = 5^2 + 12^2$

289 = 64 + 225

$\Rightarrow 17^2 = 8^2 + 15^2$

11. Question

Suppose the area of a square field is 7396 m^2. Find its perimeter.

Answer

Given area of a square = 7396 m^2

We know that the area of a square with side $s = s^2$

⇒ $s^2 = 7396$

⇒ $s = \sqrt{7396}$

⇒ s = 86 m

Also, the perimeter of a square = 4 × side

⇒ Perimeter = 4 × 86 m

⇒ Perimeter = 344 m

12. Question

Can 1010 be written as a difference of two perfect squares? [Hint: How many times 2 occurs as a factor of 1010?]

Answer

We are required to find two perfect squares such that 1010 can be written as a difference of two perfect squares.

This means $1010 = A^2 - B^2$

∵1010 is even number

∴ Either A and B are even numbers or odd numbers

So, $A^2 - B^2$ is divisible by 4 but 1010 is not divisible by 4 because 1010 = 10 × 101 = 2 × 5 × 101

Hence, 1010 cannot be expressed as a difference of two perfect squares.

13. Question

What are the remainders when a perfect cube is divided by 7?

Answer

Let us find the remainder by dividing each cube by 7.

Number	Cube	Remainder on dividing by 7
1	1	1
2	8	1
3	27	6
4	64	1
5	125	6
6	216	6
7	343	0
8	512	1
9	729	1
10	1000	6

Hence, the possible remainders are 0, 1, and 6.

14. Question

What is the least perfect square which leaves the remainder 1 when divided by 7 as well as by 11?

Answer

To find the least perfect square which leaves the remainder 1 when divided by 7 as well as by 11:

L.C.M. of 7 and 11 = 77

Then, required number is of the form = 77x + 1 where x = 1, 2, 3, 4, and so on.

Value of x	Required number	Whether a perfect square of not?
1	78	No
2	155	No
3	232	No
4	309	No
5	386	No
6	463	No
7	540	No
8	617	No
9	694	No
10	771	No
11	848	No
12	925	No
13	1002	No
14	1079	No
15	1156	Yes

$34^2 = 1156$

Hence, 1156 is the least perfect square which leaves the remainder 1 when divided by 7 as well as by 11.

15. Question

Find two smallest perfect squares whose product is a perfect cube.

Answer

Number	Cube	Whether a product of two squares
1	1	1 × 1
2	8	2 × 4
3	27	9 × 3
4	64	4 × 16

Hence, 4 and 16 are the two smallest perfect squares whose product is a perfect cube.

16. Question

Find a proper positive factor of 48 and a proper positive multiple of 48 which add up to a perfect square. Can you prove that there are infinitely many such pairs?

Answer

Proper Factor is a factor of a number other 1 and itself.

Proper factors of 48 = 2, 3, 4, 6, 8, 12, 16, 24

Proper Multiple is a multiple other than itself.

Proper Multiples of 48 = 96, 144, 192, 240, 288, 336, 384 ….

A proper positive factor of 48 and a proper positive multiple of 48 which add up to a perfect square are:

$4 + 96 = 100 = 10^2$

$4 + 192 = 196 = 14^2$

$16 + 240 = 256 = 16^2$

$16 + 384 = 400 = 20^2$

Hence, there are infinitely many such pairs