ARITHMETIC PROGRESSION

DPP – 05

CLASS – 10th

TOPIC – N^{TH} TERM OF AN A.P

Q.1 Write the expression anak for the A.P. a, a + d, a + 2d,

Hence, find the common difference of the A.P. for which

(i) 11th term is 5 and 13th term is 79.

(ii) $a_{10} - a_5 = 200$

(iii) 20^{th} term is 10 more than the 18^{th} term.

Q.2 Find n if the given value of x is the nth term of the given A.P.

(i) 25, 50, 75, 100, ...; x 1000

(ii)-1, -3, -5, -7, ...; x= –151

(iii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots\ldots;$ X=550

(iv) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots\ldots;$ $X = \frac{171}{11}$

Q.3 The eighth term of an A.P. is half of its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.

Q.4 Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

Q.5 The 7^{th} term of an A.P. is 32 and its 13th term is 62. Find the A.P

Q.6 Which term of the A.P. 3, 10, 17, ... will be 84 more than its 13^{th} term?

Q.7 Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms?

Q.8 For what value of n, the nth terms of the arithmetic progressions 63, 65, 67,... and 3, 10, 17, ... are equal ?

Q.9 How many multiples of 4 lie between 10 and 250 ?

Q.10 How many three digit numbers are divisible by 7?

Q.11 Which term of the arithmetic progression 8, 14, 20, 26, ... will be 72 more than its 41^{st} term?

Q.12 Find the term of the arithmetic progression 9, 12, 15, 18, ... which is 39 more than its 36th term

Q.13 Find the 8th term from the end of the A.P. 7, 10, 13, ..., 184

Q.14 Find the 10th term from the end of the A.P. 8, 10, 12, ..., 126

Q.15 The sum of 4th and 8th terms of an A.P. is 24 and the sum of 6th and 10th terms is 44. Find the A.P.

Q.16 Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?

Q.17 The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, find the nth term

Q.18 Find the number of all three digit natural numbers which are divisible by 9

Q.19 The 19th term of an A.P. is equal to three times its sixth term. If its 9th term is 19, find the A.P

Q.20 The 9th term of an A.P. is equal to 6 times its second term. If its 5th term is 22, find the A.P.

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Sol.1

In the A.P. a, a + d, a + 2d, ...

$a_n = a + (n-1)d$ and $a_k = a + (k-1)d$

$\therefore a_n - a_k = [a + (n-1)d] - [a + (k-1)d]$

$= a + nd - d - (a + kd - d)$

$= a + nd - d - a - kd + d$

$= nd - kd = (n-k)d$

(*i*) 11th term = 5 and 13th term = 79

i.e., $a_{11} = 5$, $a_{13} = 79$

$\therefore a_{13} - a_{11} = (13-11)d$

$\Rightarrow 79 - 5 = 2d \Rightarrow 2d = 74$

$\Rightarrow d = \frac{74}{2} = 37$

$\therefore$ Common difference = 37

(*ii*) $a_{10} - a_5 = 200$

$\Rightarrow (10-5)d = 200 \Rightarrow 5d = 200$

$\Rightarrow d = \frac{200}{5} = 40$

$\therefore$ Common difference = 40]

(*iii*) 20th term is 10 more thehn the 18th term

i.e., $a_{20} - a_{18} = 10$

$\therefore (20-18)d = 10 \Rightarrow 2d = 10$

$\Rightarrow d = \frac{10}{2} = 5$

Hence common difference = 5

Sol.2

(*i*) The A.P. is 25, 50, 75, 100, ...; $x = 1000$

Here $a = 25$ and $d = 50 - 25 = 25$

and a_n = 1000

$\therefore a_n = a + (n-1)d$

$1000 = 25 + (n-1) \times 25$

$= 25 + 25n - 25 = 25n$

$\therefore n = \frac{1000}{25} = 40$

(*ii*) The A.P. is –1, –3, –5, –7, ...; $x = -151$

Here $a = -1$, $d = -3 - (-1) = -3 + 1 = -2$

and $a_n = -151$

$\therefore\ a_n = a + (n + 1)\,d$

$\Rightarrow -151 = -1 + (n - 1)(-2)$

$\Rightarrow -151 = -1 - 2n + 2$

$\Rightarrow -151 = 1 - 2n$

$-2n = -151 - 1 = -152$

$$\therefore\ n = \frac{-152}{-2} = 76$$

$\therefore\ n = 76$

(*iii*) The given A.P. is

$$5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots; x = 550$$

$$\text{Here } a = 5\frac{1}{2} = \frac{11}{2}$$

$$d = 11 - \frac{11}{2} = \frac{11}{2}$$

and $a_n = 550$

$a_n = a + (n - 1)\,d$

$$\Rightarrow 550 = \frac{11}{2} + (n - 1) \times \frac{11}{2}$$

$$\Rightarrow 550 = \frac{11}{2} + \frac{11}{2}n - \frac{11}{2} = \frac{11}{2}n$$

$$\therefore\ n = \frac{550 \times 2}{11} = 100$$

(*iv*) The given A.P. is

$$1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots; x = \frac{171}{11}$$

$$\text{Here } a = 1,\ d = \frac{21}{11} - 1 = \frac{21 - 11}{11} = \frac{10}{11}$$

$$\text{and } a_n = \frac{171}{11}$$

$\therefore\ a_n = a + (n - 1)\,d$

$$\Rightarrow \frac{171}{11} = 1 + (n-1) \times \frac{10}{11}$$

$$\Rightarrow \frac{171}{11} = 1 + \frac{10}{11}n - \frac{10}{11}$$

$$\Rightarrow \frac{171}{11} - 1 + \frac{10}{11} = \frac{10}{11}n \Rightarrow \frac{171-11+10}{11} = \frac{10}{11}n$$

$$\Rightarrow \frac{170}{11} = \frac{10}{11}n \Rightarrow n = \frac{170}{11} \times \frac{11}{10} = 17$$

Hence $n = 17$

Sol.3

Let a and d be the first term and common difference of an AP, respectively.

Now, by given condition, $a_8 = \frac{1}{2}a_2$

$a + 7d = \frac{1}{2}(a + d)$ $\quad [\because a_n = a + (n-1)d]$

$2a + 14d = a + d$

$a + 13d = 0$ ...(i)

and $a_{11} = \frac{1}{3}a_4 + 1$

$a + 10d = \frac{1}{3}[a + 3d] + 1$

$3a + 30d = a + 3d + 3$

$2a + 27d = 3$ ...(ii)

From Eqs. (i) and (ii),

$2(-13d) + 27d = 3$

$-26d + 27d = 3$

$d = 3$

From Eq. (i),

$a + 13(3) = 0 \Rightarrow a = -39$

$a_{15} = a + 14d = -39 + 14(3)$

$= -39 + 42 = 3$

Sol.4

Let a, a + d, a + 2d, a + 3d, ... be the A.P.

$a_n = a + (n - 1)\, d$

But $a_3 = 16$

$a_7 - a_5 = 12$

Now $a_3 = a + (3 - 1)\,d = a + 2d$

$a_5 = a + (5 - 1)\,d = a + 4d$

and $a_7 = a + (7 - 1)\,d = a + 6d$

$a + 2d = 16 \Rightarrow a = 16 - 2d$

and $a_7 - a_5 = a + 6d - a - 4d$

$12 = 2d \Rightarrow d = \frac{12}{2} = 6$

$a = 16 - 2d = 16 - 2 \times 6$

$a = 16 - 12 = 4$

Sequencing (A.P.) will be

4, 10, 16, 22, ...

Sol.5

Let a, a + d, a + 2d, a + 3d, be the A.P.

Here a is the first term and d is the common difference

$a_n = a + (n - 1)\,d$

Now $a_7 = a + (7 - 1)\,d = a + 6d = 32$ ....(i)

and $a_{13} = a + (13 - 1)\,d = a + 12d = 62$ ....(ii)

Subtracting (i) from (ii)

6d = 30

=> d = 5

a + 6 x 5 = 32

=> a + 30 = 32

=> a = 32 − 30 = 2

A.P. will be 2, 7, 12, 17, ...

Sol.6

The given A.P. is 3, 10, 17, ...

Whose first term $(a) = 3$

and common difference $(d) = 10 - 3 = 7$

$a_n = a + (n - 1)\,d$

Let kth term is greater than 13th term by 84

kth term$(a_k) = a + (k - 1)\,d$

and $a_{13} = a + (13 - 1) = a + 12d$

But $a_k - a_{13} = 84$

$a + (k - 1)\,d - (a + 12d) = 84$

$a + kd - d - a - 12d = 84$

$kd - 13d = 84$

$d\,(k - 13) = 84$

$7\,(k - 13) = 84 \Rightarrow 7k - 91 = 84$

$\Rightarrow 7k = 84 + 91 = 175$

$\Rightarrow k = \frac{175}{7} = 25$

$\therefore$ 25th term is greater

Sol.7

Let $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, ...$
and $a_2, a_2 + d, a_2 + 2d, a_2 + 3d, ...$
be the two A.P. is whose common difference is d and a_1, a_2 are their first terms respectively

Now a_1 $100 = a_1 + (100 - 1)\, d = a_1 + 99d$

and a_2 $100 = a_2 + 99d$

But a_1 $100 - a_2$ $100 = 100$

$(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 + 99d - a_2 - 99d = 100$

$a_1 - a_2 = 100$

Now a_1 $1000 = a_1 + 999d$

and a_2 $1000 = a_2 + 999d$

Their difference $= a_1 + 999d - a_2 - 999d$

$= a_1 - a_2 = 100$

Sol.8

In the A.P. 63, 65, 67, ...

a = 63 and d = 65 − 63 = 2

$a_n = a_1 + (n - 1)\, d = 63 + (n - 1) \times 2 = 63 + 2n - 2 = 61 + 2n$

and in the A.P. 3, 10, 17, ...

a = 3 and d = 10 − 3 = 7

$a_n = a + (n - 1)\, d = 3 + (n - 1) \times 7 = 3 + 7n - 7 = 7n - 4$

But both nth terms are equal

$61 + 2n = 7n - 4$

$\Rightarrow 61 + 4 = 7n - 2n$

$\Rightarrow 65 = 5n$

$\Rightarrow n = 13$

$n = 13$

Sol.9

All the terms between 10 and 250 are multiple of 4

First multiple (a) = 12
and last multiple (l) = 248
and d = 4

Let n be the number of multiples, then
$a_n = a + (n - 1) d$
=> 248 = 12 + (n − 1) x 4 = 12 + 4n − 4

=> 248 = 8 + 4n
=> 4n = 248 − 8 = 240
n = 60
Number of terms are = 60

Sol.10

First three digit number is 100 and last three digit number is 999
In the sequence of the required three digit numbers which are divisible by 7, will be between
a = 105 and last number l = 994 and d = 7

Let n be the number of terms, then
$a_n = a + (n - 1) d$
994 = 105 + (n − 1) x 7
994 = 105 + 7n − 7
=> 7n = 994 − 105 + 7
=> 7n = 896
=> n = 128
Number of terms =128

Sol.11

In the given A.P. 8, 14, 20, 26, ..

First term $(a) = 8$, $d = 14 - 8 = 6$
and $a_n = a + (n - 1) d$
Now $a_{41} = a + (41 - 1) d$
$= 8 + 40 \times 6 = 8 + 240 = 248$
Let a_n be the required term

$a_n = 8 + (n - 1) \times 6$

$a_n = 8 + 6n - 6 = 6n + 2$

But $a_n - a_{41} = 72$

$6n + 2 - 248 = 72$

$6n = 72 + 248 - 2 = 318$

$$n = \frac{318}{6} = 53$$

Required term is 53rd

Sol.12

In the given A.P 9,12,15,18

First term (a) = 9

and common difference (d) = 12 − 9 = 3

and $a_n = a + (n - 1) d$

Now $a_{36} = a + (36 - 1) d = 9 + 35 \times 3 = 9 + 105 = 114$

Let the an be the required term

$a_n = a + (n - 1) d$

$= 9 + (n - 1) \times 3 = 9 + 3n - 3 = 6 + 3n$

But their difference is 39

$a_n - a_{36} = 39$

=> $6 + 3n - 114 = 39$

=> $114 - 6 + 39 = 3n$

=> $3n = 147$

=> $n = 49$

Required term is 49th

Sol.13

The given A.P. is 7, 10, 13,..., 184

Here first term (a) = 7

and common difference (d) = 10 − 7 = 3

and last tenn (l) = 184

Let nth term from the last is $a_n = l - (n - 1) d$

$a_8 = 184 - (8 - 1) \times 3 = 184 - 7 \times 3 = 184 - 21 = 163$

Sol.14

The given A.P. is 8, 10, 12, ..., 126

Here first term (a) = 8

Common difference (d) = 10 – 8 = 2

and last tenn (l) = 126

Now nth term from the last is $a_n = l - (n - 1)d$

$a_{10} = 126 - (10 - 1) \times 2 = 126 - 9 \times 2 = 126 - 18 = 108$

Sol.15

Let the first term of A.P. be = a

and d be its common difference ($c.d$)

$a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$\{\because a_n = a + (n - 1)d\}$

$2a + 10d = 24$ ....(*i*)

and $a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$ ....(*ii*)

Subtracting (*i*) from (*ii*)

$4d = 20 \Rightarrow d = \frac{20}{4} = 5$

From (*i*) $2a + 10 \times 5 = 24 \Rightarrow 2a + 50 = 24$

$2a = 24 - 50 = -26 \Rightarrow a = \frac{-26}{2} = -13$

AP will be

-13, (-13 + 5), (−13 + 10), (−13 + 15)……

-13, -8, -3, 2,……

Sol.16

A.P. is given : 3, 15, 27, 39, …….

Here first term (a) = 3

and c.d. (d) = 15 – 3 = 12

Let nth term be the required term

Now 21st term = a + (n – 1) d = 3 + 20 x 12 = 3 + 240 = 243

According to the given condition,

nth term – 21 st term = 120

=> a + (n – 1) d – 243 = 120

=> 3 + (n – 1) x 12 = 120 + 243 = 363

=> (n – 1) 12 = 363 – 3 = 360

=> n – 1 = 30

=> n = 30 + 1 = 31

31 st term is the required term

Sol.17

17^{th} term of an A.P. = 5 + 2 × 8^{th} term

and 11^{th} term = 43

Now, $T_{17} = 5 + 2T_8$ and $T_{11} = 43$

$T_n = a + (n-1)d \Rightarrow T_8 = a + 7d$

$T_{17} = a + 16d$ and $T_{11} = a + 10d = 43$

$a = 43 - 10d$ ...(*i*)

a + 16d = 5 + 2 (a + 7d) -----> a + 16d = 5 + 2a + 14d

$16d - 14d = 5 + 2a - a$

$2d = 5 + a \Rightarrow a = 2d - 5$ ...(*ii*)

From (*i*) and $43 - 10d = 2d - 5$

$43 + 5 = 2d + 10d \Rightarrow 12d = 48$

$$d = \frac{48}{12} = 4$$

and $a = 2d - 5 = 2 \times 4 - 5 = 8 - 5 = 3$

$T_n = a + (n-1)d$

$= 3 + (n-1) \times 4 = 3 + 4n - 4 = 4n - 1$

Sol.18

First 3-digit number which is divisible by 9 = 108

and last 3-digit number = 999

d= 9

a + (n – 1) d = 999

=> 108 + (n – 1) x 9 = 999

=> (n – 1) d = 999 – 108

=> (n – 1) x 9 = 891

=> n – 1 = 99

=> n = 99 + 1 = 100

Number of terms = 100

Sol.19

Let a be the first term, d be the common difference, then

$T_{19} = 3T$ and $T_9 = 19$

$T = a + (n-1)d$

Now, $T_{19} = a + (19-1)d = a + 18d$

$T_6 = a + (6-1)d = a + 5d$

$T_9 = 19 \Rightarrow a + (9-1)d = 19$

$a + 8d = 19$ ...(*i*)

and $a + 18d = 3(a + 5d)$

$a + 18d = 3a + 15d$

$18d - 15d = 3a - a$

$2a = 3d$ ...(*ii*)

$a = \frac{3}{2}d$

From (*i*), $a + 8d = 19$

$\frac{3}{2}d + 8d = 19 \Rightarrow \frac{19}{2}d = 19$

$d = \frac{19 \times 2}{19} = 2$

and $a = \frac{3}{2}d = \frac{3}{2} \times 2 = 3$

A.P. will be 3, 5, 7, 9, 11, ...

Sol.20

Let a be the first term and d be the common difference and

$T_n = a + (n-1)d$

$T_9 = a + (9-1)d = a + 8d$

$T_2 = a + (2-1)d = a + d$

$a + 8d = 6(a + d)$

$a + 8d = 6a + 6d$

$8d - 6d = 6a - a \Rightarrow 5a = 2d$

$a = \frac{2}{5}d$ ....(1)

and $T_5 = a + (5-1)d = a + 4d$

$a + 4d = 22$

$\frac{2}{5}d + 4d = 22$ [From (*i*)]

$2d + 20d = 22 \times 5 \Rightarrow 22d = 22 \times 5$

$\Rightarrow d = \frac{22 \times 5}{22} = 5$

$\therefore a = \frac{2}{5}d = \frac{2}{5} \times 5 = 2$

$\therefore$ A.P. = 2, 7, 12, 17, ...