9 Quadratic Equations

Multiple Choice Questions (MCQs)

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If the product of roots of the equation $x^3 - 3x + k = 10$ is -2, then the value of k is

(a) -2 (b) -8
(c) 8 (d) 12

2. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root, then the value of k is

(a) 0 (b) 1
(c) 2 (d) 5

3. If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is

(a) $x^2 - 6x + 6 = 0$ (b) $x^2 + 6x - 6 = 0$
(c) $x^2 - 6x - 6 = 0$ (d) $x^2 + 6x + 6 = 0$

4. If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots, then $k = ?$

(a) 1 or 4 (b) -1 or 4
(c) 1 or -4 (d) -1 or -4

5. If the roots of $5x^2 - kx + 1 = 0$ are real and distinct, then

(a) $-2\sqrt{5} < k < 2\sqrt{5}$
(b) $k > 2\sqrt{5}$ only
(c) $k < -2\sqrt{5}$ only
(d) either $k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

6. If $a - b$, $b - c$ are the roots of $ax^2 + bx + c = 0$, then find the value of $\frac{(a-b)(b-c)}{c-a}$

(a) $\frac{b}{c}$ (b) $\frac{c}{b}$
(c) $\frac{ab}{c}$ (d) $\frac{bc}{a}$

7. Find the product of the roots of $x^2 + 8x - 16 = 0$

(a) 8 (b) -8
(c) 16 (d) -16

8. If the roots of the equation $ax^2 + bx + c = 0$ are α and β, then the quadratic equation whose roots are $-\alpha$ and $-\beta$ is _____ .

(a) $ax^2 - bx - c = 0$ (b) $ax^2 - bx + c = 0$
(c) $ax^2 + bx - c = 0$ (d) $ax^2 - bx + 2c = 0$

9. If the equation

$(1 + m^2)\, x^2 + (2mc)\, x + (c^2 - a^2) = 0$ has equal roots, then

(a) $c^2 - a^2 = 1 + m^2$ (b) $c^2 = a^2 (1 + m^2)$
(c) $c^2a^2 = (1 + m^2)$ (d) $c^2 + a^2 = 1 + m^2$

10. Which of the following satisfy the equation

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

(a) $\frac{1}{a^2}$ (b) $\frac{1}{b^2}$
(c) $\frac{-1}{b^2}$ (d) None of these

11. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

(a) ± 0.2 (b) ± 0.02
(c) 0.4 (d) 2

12. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The correct roots are

(a) $3, -2$ (b) $-3, 2$
(c) $-6, -1$ (d) $6, -1$

13. The condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other, is

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$
(c) $c^2 = 4a + b^2$ (d) $c^2 = 9a - b^2$

14. If $\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2$, then x =

(a) 3 (b) 2
(c) 4 (d) None of these

15. If $x^2 + y^2 = 25$, $xy = 12$, then x =

(a) {3, 4} (b) {3, –3}
(c) {3, 4, –3, –4} (d) {–3, –3}

16. If $x=\sqrt{7+4\sqrt{3}}$, then $x+\frac{1}{x}$ =

(a) 4 (b) 6
(c) 3 (d) 2

17. If the roots of the equation $px^2 + 2qx + r = 0$ and $qx^2-2\sqrt{pr}x+q=0$ be real, then

(a) $p = q$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $r^2 = pq$

18. The equation $2x^2 + 2(p + 1) x + p = 0$, where p is real, always has roots that are

(a) Equal
(b) Equal in magnitude but opposite in sign
(c) Irrational
(d) Real

19. If the ratio of the roots of the equation $x^2 + bx + c = 0$ is the same as that of $x^2 + qx + r = 0$, then

(a) $r^2b = qc^2$ (b) $r^2c = qb^2$
(c) $c^2r = q^2b$ (d) $b^2r = q^2c$

20. The real roots of the equation $x^{2/3}+x^{1/3}-2=0$ are

(a) 1, 8 (b) –1, –8
(c) –1, 8 (d) 1, –8

21. Which of the following is not a quadratic equation?

(a) $x^2 – 2x + 2 (3 – x) = 0$
(b) $x (x + 1) + 1 = (x – 2) (x – 5)$
(c) $(2x – 1) (x – 3) = (x + 5) (x – 1)$
(d) $x^3 – 4x^2 – x + 1 = (x –2)^3$

22. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

(a) $b = c$ (b) $a = b$
(c) $ac = 1$ (d) $a = c$

23. The roots of the equation $x+\frac{1}{x}=3\frac{1}{3}$, $x \neq 0$, are

(a) 3, 1 (b) $3, \frac{1}{3}$
(c) $3, -\frac{1}{3}$ (d) $-3, -\frac{1}{3}$

24. If the equation $(m^2 + n^2) x^2 –2 (mp + nq) x + p^2 + q^2 = 0$ has equal roots, then

(a) $mp = nq$ (b) $mq = np$
(c) $mn = pq$ (d) $mq = \sqrt{np}$

25. Each root of $x^2 – bx + c = 0$ is decreased by 2. The resulting equation is $x^2 – 2x + 1 = 0$, then

(a) $b = 6, c = 9$ (b) $b = 3, c = 5$
(c) $b = 2, c = –1$ (d) $b = – 4, c = 3$

26. Two distinct polynomials $f(x)$ and $g(x)$ are defined as follows:

$f(x) = x^2 + ax + 2$; $g(x) = x^2 + 2x + a$.

If the equations $f(x) = 0$ and $g(x) = 0$ have a common root, then the sum of the roots of the equation $f(x) + g(x) = 0$ is

(a) $-\frac{1}{2}$ (b) 0
(c) $\frac{1}{2}$ (d) 1

27. If α and β are the roots of the quadratic equation $x^2 – 6x – 2 = 0$ and if $a_n = \alpha^n – \beta^n$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is

(a) 6.0 (b) 5.2
(c) 5.0 (d) 3.0

28. Consider the quadratic equation $nx^2+7\sqrt{n}x+n=0$, where n is a positive integer. Which of the following statements are necessarily correct?

I. For any n, the roots are distinct.
II. There are infinitely many values of n for which both roots are real.
III. The product of the roots is necessarily an integer.

(a) III only (b) I and III
(c) II and III (d) I, II and III

29. Two quadratic equations $x^2 – bx + 6 = 0$ and $x^2 – 6x + c – 0$ have a common root. If the remaining roots of the first and second equations are positive integers and are in the ration 3 : 4 respectively, then the common root is

(a) 1 (b) 2
(c) 3 (d) 4

30. The values of k, so that the equations $2x^2 + kx – 5 = 0$ and $x^2 – 3x – 4 = 0$ have one root in common, are

(a) $3, \frac{27}{2}$ (b) $9, \frac{27}{4}$
(c) $-3, \frac{-27}{4}$ (d) $3, \frac{4}{27}$

31. If $x = \frac{3+\sqrt{5}}{2}$ and $y = x^3$, then y satisfies the quadratic equation

(a) $y^2 - 18y + 1 = 0$ (b) $y^2 + 18y + 1 = 0$
(c) $y^2 - 18y - 1 = 0$ (d) $y^2 + 18y - 1 = 0$

32. Let b be a non-zero real number. Suppose the quadratic equation $2x^2 + bx + \frac{1}{b} = 0$ has two distinct real roots. Then

(a) $b+\frac{1}{b} > \frac{5}{2}$ (b) $b+\frac{1}{b} < \frac{5}{2}$
(c) $b^2 - 3b > -2$ (d) $b^2 + \frac{1}{b^2} < 4$

33. If the quadratic equations $2x^2 + 4x + (a + 5) = 0$ have equal roots and $(a + 4)x^2 + ax - 3b = 0$ have distinct real roots then which of the following is true:

(a) $a = -3, b < \frac{3}{4}$ (b) $a = 3, b > \frac{3}{4}$
(c) $a = -3, b > -\frac{3}{4}$ (d) $a = 3, b < \frac{3}{4}$

34. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:

(a) $\frac{15}{8}$ (b) 1
(c) – (d) 2

35. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is:

(a) 2 (b) 3
(c) 1 (d) 4

36. Which one of the following is not a quadratic equation?

(a) $(x + 2)^2 = 2(x + 3)$
(b) $x^2 + 3x = (-1)(1 - 3x)^2$
(c) $(x + 2)(x - 1) = x^2 - 2x - 3$
(d) $x^3 - x^2 + 2x + 1 = (x + 1)^3$

37. Which constant should be added and subtracted to solve the quadratic equation $4x^2 - \sqrt{3}x - 5 = 0$ by the method of completing the square?

(a) $\frac{9}{16}$ (b) $\frac{3}{64}$
(c) $\frac{3}{4}$ (d) $\frac{\sqrt{3}}{4}$

38. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$
(b) $x^2 + 3x - 12 = 0$
(c) $2x^2 - 7x + 6 = 0$
(d) $3x^2 - 6x - 2 = 0$

39. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(a) 0 only (b) 4 only
(c) 8 only (d) 0, 8

40. If α, β are roots of the equation $x^2 - 5x + 6 = 0$, then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is

(a) $2x^2 - 11x + 30 = 0$ (b) $-x^2 + 11x = 0$
(c) $x^2 - 11x + 30 = 0$ (d) $2x^2 - 22x + 40 = 0$

41. If equation $x^2 - (2 + m)x + 1(m^2 - 4m + 4) = 0$ has equal roots, then:

(a) $m = 0$ (b) $m = 6$
(c) $m = 2$ (d) $m = 3$

42. Which of the following equations have no real roots?

(a) $x^2 - 2\sqrt{3}x + 5 = 0$ (b) $2x^2 + 6\sqrt{2} + 8 = 0$
(c) $x^2 - 2\sqrt{3}x - 5 = 0$ (d) $2x^2 - 6\sqrt{2}x - 9 = 0$

43. Two numbers whose sum is 8 and the absolute value of whose difference is 10 are roots of the equation

(a) $x^2 - 8x + 9 = 0$ (b) $x^2 - 8x - 9 = 0$
(c) $x^2 + 8x - 9 = 0$ (d) $-x^2 + 8x + 9 = 0$

44. If α, β are roots of $x^2 + 5x + a = 0$ and $2\alpha + 5\beta = -1$, then

(a) $\alpha = 8$ (b) $\beta = -3$
(c) $\alpha = 9$ (d) $a = -24$

45. The value of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2, are

(a) 4 (b) 8
(c) 6 (d) – 4

46. If the roots of $x^2 + px + 12 = 0$ are in the ratio 1 : 3, then value(s) of p are

(a) 3 (b) 8
(c) 6 (d) – 3

47. Roots of quadratic equation $x^2 - 3x + 2 = 0$ are

(a) 3 (b) –1
(c) 2 (d) 4

48. If $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2px + 2q = 0$, then

(a) $p = \frac{2}{15}$ (b) $p = 15$
(c) $q = 9$ (d) $6p + 2q = 27$

Case/Passage Based Questions

DIRECTIONS : *Study the given Case/Passage and answer the following questions.*

Case/Passage-I

Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.

[From CBSE Question Bank-2021]

49. What will be the distance covered by Ajay's car in two hours?

(a) $2(x+5)$km (b) $(x-5)$km

(c) $2(x+10)$km (d) $(2x+5)$km

50. Which of the following quadratic equation describe the speed of Raj's car?

(a) $x^2-5x-500=0$ (b) $x^2+4x-400=0$

(c) $x^2+5x-500=0$ (d) $x^2-4x+400=0$

51. What is the speed of Raj's car?

(a) 20 km/hour (b) 15 km/hour

(c) 25 km/hour (d) 10 km/hour

52. How much time took Ajay to travel 400 km?

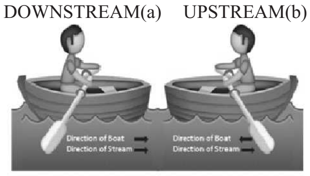
(a) 20 hour (b) 40 hour

(c) 25 hour (d) 16 hour

Case/Passage-II

The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boattook 1 hour more for upstream than downstream. **[From CBSE Question Bank-2021]**

DOWNSTREAM(a) UPSTREAM(b)



53. Let speed of the stream be x km/hr. then speed of the motorboat in upstream will be

(a) 20 km/hr (b) $(20+x)$ km/hr

(c) $(20-x)$ km/hr (d) 2 km/hr

54. What is the relation between speed ,distance and time?

(a) speed = (distance)/time (b) distance = (speed)/time

(c) time = speed x distance (d) speed = distance x time

55. Which is the correct quadratic equation for the speed of the current ?

(a) $x^2+30x-200=0$ (b) $x^2+20x-400=0$

(c) $x^2+30x-400=0$ (d) $x^2-20x-400=0$

56. What is the speed of current ?

(a) 20 km/hour (b) 10 km/hour

(c) 15 km/hour (d) 25 km/hour

57. How much time boat took in downstream?

(a) 90 minute (b) 15 minute

(c) 30 minute (d) 45 minute

Assertion & Reason

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

58. **Assertion :** If roots of the equation $x^2-bx+c=0$ are two consecutive integers, then $b^2-4c=1$.

Reason : If a, b, c are odd integer then the roots of the equation $4abc\,x^2+(b^2-4ac)\,x-b=0$ are real and distinct.

59. **Assertion :** $(2x-1)^2-4x^2+5=0$ is not a quadratic equation.

Reason : $x=0, 3$ are the roots of the equation $2x^2-6x=0$.

60. **Assertion :** The equation $9x^2+3kx+4=0$ has equal roots for $k=\pm 4$.

Reason : If discriminant 'D' of a quadratic equation is equal to zero then the roots of equation are real and equal.

61. **Assertion :** $4x^2-12x+9=0$ has repeated roots.

Reason : The quadratic equation $ax^2+bx+c=0$ have repeated roots if discriminant $D>0$.

62. **Assertion :** A quadratic equation $ax^2+bx+c=0$, has two distinct real roots, if $b^2-4ac>0$.

Reason : A quadratic equation can never be solved by using method of completing the squares.

63. **Assertion :** Sum and product of roots of $2x^2-3x+5=0$ are $\frac{3}{2}$ and $\frac{5}{2}$ respectively.

Reason : If α and β are the roots of $ax^2+bx+c=0$, $a\neq 0$, then sum of roots $=\alpha+\beta=-\frac{b}{a}$ and product of roots $=\alpha\beta=\frac{c}{a}$.

Match the Following

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column-I have to be matched with statements (p, q, r, s) in column-II.*

64. Column-II give roots of quadratic equations given in Column-I.

Column-I	Column-II
(A) $6x^2 + x - 12 = 0$	(p) $(-6, 4)$
(B) $8x^2 + 16x + 10 = 202$	(q) $(9, 36)$
(C) $x^2 - 45x + 324 = 0$	(r) $(3, -1/2)$
(D) $2x^2 - 5x - 3 = 0$	(s) $(-3/2, 4/3)$

65.

Column-I	Column-II
(A) $(x - 3)(x + 4) + 1 = 0$	(p) Forth degree polynomial
(B) $(x + 2)^3 = 2x(x^2 - 1)$	(q) Quadratic equation
(C) $(2x - 2)^2 = 4x^2$	(r) Non-quadratic equation
(D) $(2x^2 - 2)^2 = 3$	(s) Linear equation

66.

Column-I	Column-II
(A) If α, β are roots of $ax^2 + bx + c = 0$, then one of the roots of the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$	(p) $a < 0, b > 0$
(B) If the roots of $ax^2 + b = 0$ are real, then	(q) real and equal
(C) Roots of $4x^2 - 4x + 1 = 0$	(r) $\dfrac{\beta}{1+\beta}$
(D) Roots of $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always	(s) Real

Fill in the Blanks

DIRECTIONS : *Complete the following statements with an appropriate word/ term to be filled in the blank space(s).*

67. A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and a ......

68. A quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots, if $b^2 - 4ac$ ....

69. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, the other two sides are ....

70. The equation $ax^2 + bx + c = 0$, $a \neq 0$ has no real roots, if ....

71. The values of k for which the equation $2x^2 + kx + x + 8 = 0$ will have real and equal roots are ....

72. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $a\alpha + b$ and $a\beta + b$ is ....

73. If r, s are roots of $ax^2 + bx + c = 0$, then $\dfrac{1}{r^2} + \dfrac{1}{s^2}$ is ....

74. The quadratic equation whose roots are the sum and difference of the squares of roots of the equation $x^2 - 3x + 2 = 0$ is....

75. If a, b are the roots of $x^2 + x + 1 = 0$, then $a^2 + b^2 =$ ....

76. If α, β are the roots of $x^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $x^2 + qx + r = 0$, then $h =$ ....

77. A quadratic equation cannot have more than .... roots.

78. Let $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$, be a quadratic equation, then this equation has no real roots if and only if ....

True / False

DIRECTIONS : *Read the following statements and write your answer as true or false.*

79. A quadratic equation cannot be solved by the method of completing the square.

80. If we can factorise $ax^2 + bx + c$, $a \neq 0$, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

81. $(x - 2)(x + 1) = (x - 1)(x + 3)$ is a quadratic equation.

82 $(x^2 + 3x + 1) = (x - 2)^2$ is not a quadratic equation.

83. $x^2 + x - 306 = 0$ represent quadratic equation where product of two consecutive positive integer is 306.

84. The roots of the equation $(x - 3)^2 = 3$ are $3 \pm \sqrt{3}$

85. If sum of the roots is 2 and product is 5, then the quadratic equation is $x^2 - 2x + 5 = 0$

86. Sum of the reciprocals of the roots of the equation $x^2 + px + q = 0$ is $1/p$.

87. The nature of roots of equation $x^2 + 2x\sqrt{3} + 3 = 0$ are real and equal.

88. For the expression $ax^2 + 7x + 2$ to be quadratic, the possible values of a are non-zero real numbers.

ANSWER KEY & SOLUTIONS

1. **(c)** Given equation is $x^2 - 3x + (k-10) = 0$.
   $\therefore$ Product of roots = $(k - 10)$.
   So, $k - 10 = -2 \Rightarrow k = 8$.

2. **(d)** Let the roots be α and $\frac{1}{\alpha}$. Then,
   product of roots = $\left(\alpha \times \frac{1}{\alpha}\right) = 1$.
   So, $\frac{k}{5} = 1 \Rightarrow k = 5$.

3. **(a)** Required equation is $x^2 - 6x + 6 = 0$.

4. **(a)** Since the roots are equal, we have $D = 0$.
   $\therefore\ 4(k+2)^2 - 36k = 0 \Rightarrow (k+2)^2 - 9k = 0$
   $\therefore\ k^2 - 5k + 4 = 0 \Rightarrow k^2 - 4k - k + 4 = 0$
   $\Rightarrow k(k-4) - (k-4) = 0$
   $\Rightarrow (k-4)(k-1) = 0 \Rightarrow k = 4$ or $k = 1$.

5. **(d)** The roots of $5x^2 - kx + 1 = 0$ are real and distinct.
   $\therefore\ (k^2 - 4 \times 5 \times 1) > 0 \Rightarrow k^2 > 20$
   $\Rightarrow k > \sqrt{20}$ or $k < -\sqrt{20} \Rightarrow k > 2\sqrt{5}$ or $k < -2\sqrt{5}$.

6. **(b)** (i) $(a-b)(b-c)$ = product of the roots = $\frac{c}{a}$.
   (ii) $c - a = -(a - b + b - c) = -$ (sum of the roots)
   $= \frac{b}{a}$.

7. **(d)** $x^2 + 8x - 16 = 0$
   The product of the roots = $\frac{c}{a} = -16$.

8. **(b)** Use $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

9. **(b)** Since the equation has two equal roots, D = 0
   $\Rightarrow\ (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$
   $\Rightarrow\ 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$
   $\Rightarrow\ -4c^2 + 4a^2 + 4m^2a^2 = 0 \Rightarrow 4c^2 = 4a^2 + 4m^2a^2$
   $\Rightarrow\ 4c^2 = 4a^2(1 + m^2) \Rightarrow c^2 = a^2(1 + m^2)$

10. **(b)** $a^2b^2x^2 + b^2x - a^2x - 1 = 0$
   $\Rightarrow\ b^2x(a^2x + 1) - 1(a^2x + 1) = 0$
   $\Rightarrow\ (a^2x + 1)(b^2x - 1) = 0$
   $\Rightarrow\ x = -\frac{1}{a^2}, \frac{1}{b^2}$

11. **(a)** $x^2 - 0.04 = 0$
   $\Rightarrow\ x^2 = 0.04$
   $\Rightarrow\ x = \pm 0.2$

12. **(d)** Let α, β be the roots of the equation. Then $\alpha + \beta = 5$ and $\alpha\beta = -6$. So, the equation is $x^2 - 5x - 6 = 0$.
   The roots of the equation are 6 and –1.

13. **(b)** $\alpha + 2\alpha = -\frac{b}{a}$ and $\alpha \times 2\alpha = \frac{c}{a} \Rightarrow 3\alpha = -\frac{b}{a}$
   $\Rightarrow\ \alpha = -\frac{b}{3a}$ and $2\alpha^2 = \frac{c}{a} \Rightarrow 2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a}$
   $\Rightarrow\ \frac{2b^2}{9a^2} = \frac{c}{a} \Rightarrow 2ab^2 - 9a^2c = 0 \Rightarrow a(2b^2 - 9ac) = 0$
   Since $a \neq 0$, $\therefore\ 2b^2 = 9ac$
   Hence, the required condition is $2b^2 = 9ac$

14. **(c)** Use options or apply the formula
   $a^2 - b^2 = (a - b)(a + b)$, we get $x = 4$

15. **(c)** $x^2 + y^2 = 25$, $xy = 12$
   $\Rightarrow\ x^2 + \left(\frac{12}{x}\right)^2 = 25 \Rightarrow x^4 + 144 - 25x^2 = 0$
   $\Rightarrow\ (x^2 - 16)(x^2 - 9) \Rightarrow x^2 = 16$ and $x^2 = 9$
   $\Rightarrow\ x = \pm 4$ and $x = \pm 3$

16. **(a)** We have, $x = \sqrt{7 + 4\sqrt{3}}$
   $\therefore\ \frac{1}{x} = \frac{1}{\sqrt{7+4\sqrt{3}}} = \frac{\sqrt{7-4\sqrt{3}}}{\sqrt{7+4\sqrt{3}}.\sqrt{7-4\sqrt{3}}} = \sqrt{7-4\sqrt{3}}$
   $\therefore\ x + \frac{1}{x} = \sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}$
   $= (\sqrt{3} + 2) + (2 - \sqrt{3}) = 4$

17. **(b)** Equation $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}\,x + q = 0$ have real roots then from first
   $4q^2 - 4pr \geq 0 \Rightarrow q^2 \geq pr$ ... (i)
   and from second $4(pr) - 4q^2 \geq 0$ (for real root)
   $\Rightarrow\ pr \geq q^2$ ... (ii)
   From (i) and (ii), we get result $q^2 = pr$

18. **(d)** The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$.

$a = 2$, $b = 2(p + 1)$ and $c = p$

$b^2 - 4ac = [2(p + 1)]^2 - 4(2p) = 4(p + 1)^2 - 8p$

$= 4[(p + 1)^2 - 2p] = 4[(p^2 + 2p + 1) - 2p] = 4(p^2 + 1)$

For any real value of p, $4(p^2 + 1)$ will always be positive as p^2 cannot be negative for real p.

Hence, the discriminant $b^2 - 4ac$ will always be positive.

When the discriminant is greater than '0' or is positive, then the roots of a quadratic equation will be real.

19. **(d)** Let 1, 2 be the roots of equations (i), 2 and 4 be the roots of equation (ii).

$\therefore$ equations are $x^2 - 3x + 2 = 0$ and $x^2 - 6x + 8 = 0$.

Comparing with $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$, we get $b = -3$, $c = 2$, $q = -6$ and $r = 8$.

Putting these values in the options, we find that option (d) is satisfied.

20. **(d)** The given equation is $x^{2/3} + x^{1/3} - 2 = 0$

Put $x^{1/3} = y$, then $y^2 + y - 2 = 0$

$\Rightarrow (y - 1)(y + 2) = 0 \Rightarrow y = 1$ or $y = -2$

$\Rightarrow x^{1/3} = 1$ or $x^{1/3} = -2$

$\therefore x = (1)^3$ or $x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, –8.

21. **(b)** $x(x + 1) + 1 = (x - 2)(x - 5)$

$\Rightarrow x^2 + x + 1 = x^2 - 7x + 10$

$\Rightarrow 8x - 9 = 0$, which is not a quadratic equation.

22. **(d)** If one root is α, then the other is $\frac{1}{\alpha}$

$\therefore \alpha . \frac{1}{\alpha} =$ product of roots $= \frac{c}{a} \Rightarrow 1 = \frac{c}{a} \Rightarrow a = c$

23. **(b)** $x + \frac{1}{x} = \frac{10}{3} \Rightarrow \frac{x^2 + 1}{x} = \frac{10}{3} \Rightarrow 3x^2 - 10x + 3 = 0$

$\Rightarrow (x - 3)(3x - 1) = 0 \Rightarrow x = 3, x = \frac{1}{3}$

24. **(b)** '$b^2 = 4ac$'

$\Rightarrow 4(mp + nq)^2 = 4(m^2 + n^2)(p^2 + q^2)$

$\Rightarrow m^2q^2 + n^2p^2 - 2mnpq = 0$

$\Rightarrow (mq - np)^2 = 0 \Rightarrow mq - np = 0$.

25. **(a)** $\alpha + \beta = b$, $\alpha\beta = c$

Sum of roots of resulting equation $= (\alpha - 2) + (\beta - 2)$

$\Rightarrow (\alpha + \beta - 4) = b - 4$;

Product of roots resulting equation

$= (\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$

$= c - 2b + 4$

Now, $2 = b - 4$; $1 = c - 2b + 4$ etc.

26. **(c)** Given,

$f(x) = x^2 + ax + 2$ and $g(x) = x^2 + 2x + a$

Let α be the common root of $f(x) = 0$ and $g(x) = 0$.

$\therefore \quad \alpha^2 + a\alpha + 2 = 0 \qquad ...(i)$

and $\alpha^2 + 2\alpha + a = 0 \qquad ...(ii)$

Using elimination method,

$$\begin{array}{rl} & \alpha^2 + a\alpha = -2 \\ \text{and} & \alpha^2 + 2\alpha = -a \\ & -\quad +\qquad + \\ \hline & a\alpha - 2\alpha = -2 + a \end{array}$$

$\Rightarrow \alpha(-2 + a) = -2 + a \Rightarrow \alpha = \frac{-2 + a}{-2 + a} = 1$

Substitute value of α in (i) eqn., we get

$\therefore \quad -(a + 2) = 1 \Rightarrow a + 2 = -1 \Rightarrow a = -3$

Now $f(x) + g(x) = 0$

$\therefore \quad x^2 - 3x + 2 + x^2 + 2x - 3 = 0 \Rightarrow 2x^2 - x - 1 = 0$

So, sum of roots $= \frac{1}{2}$

27. **(d)** $x^2 - 6x - 2 = 0$

α and β are the roots of the above equation.

So, $\alpha^2 - 2 = 6\alpha$

Similarly, $\beta^2 - 2 = 6\beta$

We can see that, $\alpha + \beta = 6$ and $\alpha\beta = -2$

Given: $a_n = \alpha^n - \beta^n$

So, $\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2\left(\alpha^8 - \beta^8\right)}{2\left(\alpha^9 - \beta^9\right)}$

$= \frac{\alpha^{10} - \beta^{10} + \alpha\beta\left(\alpha^8 - \beta^8\right)}{2\left(\alpha^9 - \beta^9\right)}$

$= \frac{\alpha^{10} - \alpha^9\beta - \left(\alpha\beta^9 + \beta^{10}\right)}{2\left(\alpha^9 - \beta^9\right)}$

$= \frac{\alpha^9(\alpha + \beta) - \beta^9(\alpha - \beta)}{2\left(\alpha^9 - \beta^9\right)} = \frac{(\alpha + \beta)\left(\alpha^9 - \beta^9\right)}{2\left(\alpha^9 - \beta^9\right)}$

$= \frac{6}{2} = 3 \qquad (\because a + b = 6)$

28. **(b)** The given quadratic equation is, $nx^2 + 7\sqrt{n}\,x + n = 0$

Now, the discriminant,

$D = 49n - 4n^2 = n(49 - 4n)$

$D \neq 0; \quad \therefore \forall\, n \in I^+$

$\Rightarrow$ Roots of the quadratic equation are distinct.

For real roots $D \geq 0$

$\Rightarrow n(49 - 4n) \geq 0 \Rightarrow n \leq \dfrac{49}{4}$

So, $n \in \{1, 2, 3, 4, ..., 12\}$

So, x have only finite value.

Product of roots $= \dfrac{n}{n} = 1$

$\Rightarrow$ Products of root is necessarily integer.

Hence, option (b) is correct.

29. **(b)** Let α, β be the roots of $x^2 - bx + 6 = 0$ and α, γ be the roots of $x^2 - 6x + c = 0$

$x^2 - bx + 6 = 0;\ \alpha + \beta = b, \alpha + \gamma = 6$

$x^2 - 6x + c = 0;\ \alpha\beta = 6, \alpha\gamma = c$

Given, $\dfrac{\beta}{\gamma} = \dfrac{3}{4}$

$\dfrac{\alpha\beta}{\alpha\gamma} = \dfrac{6}{c}$

$\dfrac{\beta}{\gamma} = \dfrac{6}{c} \Rightarrow \dfrac{3}{4} = \dfrac{6}{c}\ \therefore\ c = 8$

$\alpha\beta = 6;\ \alpha\gamma = 6$

HCF $(\alpha\beta, \alpha\gamma) = \alpha$

HCF $(6, 8) = 2$

$\alpha = 2$

30. **(c)** Let the common root be t

Then, the equation becomes

$2t^2 + kt - 5 = 0$ ...(i)

$t^2 - 3t - 4 = 0$ ...(ii)

Multiply equation (ii) by 2 and then subtract from equation (i)

$2t^2 + kt - 5 = 0$

$2t^2 - 6t - 8 = 0$

$-\quad +\quad +$

$(k + 6)t + 3 = 0$

$t = -\dfrac{3}{k+6}$

Now, put the value of t in equation (i)

$2\left(\dfrac{-3}{k+6}\right)^2 + k\left(\dfrac{-3}{k+6}\right) - 5 = 0$

$\dfrac{18}{(k+6)^2} + \dfrac{-3}{k+6} - 5 = 0$

$18 - 3k(k + 6) - 5(k + 6)2 = 0$

$18 - 3k2 - 18k - 5k2 - 180 - 60k = 0$

$-8k2 - 78k - 162 = 0$

$8k2 + 78k + 162 = 0$

$4k2 + 39x + 81 = 0$

$4k2 + 27k + 12k + 81 = 0$

$k(4k + 27) + 3(4k + 27) = 0$

$(k + 3)(4k + 27) = 0$

$k = -3, -\dfrac{27}{4}$

31. **(a)** $x = \dfrac{3+\sqrt{5}}{2}$

$\Rightarrow x^3 = \left(\dfrac{3+\sqrt{5}}{2}\right)^3 = \dfrac{27 + 5\sqrt{5} + 9\sqrt{5}(3+\sqrt{5})}{8} = 9 + 4\sqrt{5}$

$\Rightarrow y = x^3 = 9 + 4\sqrt{5}$

$\Rightarrow$ One root is $9 + 4\sqrt{5}$ $\therefore$ other is root $9 - 4\sqrt{5}$

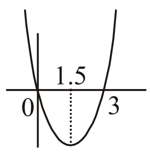
$\therefore$ Sum of roots $= 9 + 4\sqrt{5} + 9 - 4\sqrt{5} = 18$

Product of roots $= (9 + 4\sqrt{5})(9 - 4\sqrt{5}) = 1$

$\therefore$ Required equation is: $y^2 - 18y + 1 = 0$

32. **(c)** $D > 0$ (for real roots)

$\Rightarrow b^2 - 4 \times 2 \times \dfrac{1}{b} > 0 \Rightarrow \dfrac{b^3 - 8}{b} > 0$



$\Rightarrow \dfrac{(b-2)(b^2 + 2b + 4)}{b} > 0$

$\Rightarrow b \in (-\infty, 0) \cup (2, \infty)$

Clearly options A and B are wrong

Let $f(b) = b^2 - 3b$

range of $f(b)$ when $b \in (-\infty, 0) \cup (2, \infty)$

$= (f(2), \infty) = (-2, \infty)$

$\Rightarrow b^2 - 3b > -2$ is correct

So, $b \in (-\infty, 0) \cup (2, \infty)$ is subset of solution set of $b^2 - 3b + 2 > 0$

Also D is wrong as $b^2 + \dfrac{1}{b^2} \in (0, \infty)$

33. **(c)** (i) For equal roots,

$D = 0 \Rightarrow b^2 - 4ac = 0$

$16 - 8(a + 5) = 0$

$a + 5 = 2$

$a = -3$

(ii) For distinct real roots,

$D > 0 \Rightarrow b^2 - 4ac > 0$

$a^2 + 12b(a + 4) > 0$

$9 + 12b > 0$

$b > -\frac{9}{12}$

$b > -\frac{3}{4}$

34. **(d)** The given quadratic equation is

$x^2 + (3 - \lambda)x + 2 = \lambda$

Sum of roots $= \alpha + \beta = \lambda - 3$

Product of roots $= \alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (\lambda - 3)^2 - 2(2 - \lambda)$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1$

For least $(\alpha^2 + \beta^2)$, $\lambda = 2$.

35. **(a)** $\because \alpha + \beta = 64, \alpha\beta = 256$

$$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$$

36. **(c)** $(x+2)(x-1) = x^2 - 2x - 3$

$\Rightarrow \quad x^2 - x + 2x - 2 = x^2 - 2x - 3$

$\Rightarrow \quad x - 2 = -2x - 3 \Rightarrow 3x = -1 \Rightarrow x = \frac{-1}{3}$

37. **(b)** $4x^2 - \sqrt{3}x - 5 = 0$

$\Rightarrow \quad x^2 - \frac{\sqrt{3}}{4}x - \frac{5}{4} = 0$

$\Rightarrow \quad x^2 - 2\cdot\frac{\sqrt{3}}{8}.x + \left(\frac{\sqrt{3}}{8}\right)^2 - \left(\frac{\sqrt{3}}{8}\right)^2 - \frac{5}{4} = 0$

$\Rightarrow \quad \left(x - \frac{\sqrt{3}}{8}\right)^2 - \frac{3}{64} - \frac{5}{4} = 0 \Rightarrow \left(x - \frac{\sqrt{3}}{8}\right)^2 - \frac{83}{64} = 0$

Hence, the required constant that should be added and subtracted is $\frac{3}{64}$.

38. **(c)** $2x^2 - 7x + 6 = 2(2)^2 - 7(2) + 6$

$= 8 - 14 + 6 = 14 - 14 = 0$

39. **(d)** Compare the quadratic equation $2x^2 - kx + k = 0$ with the standard form of quadratic equation

$ax^2 + bx + c = 0$; $a = 2, b = -k, c = k$

For equal roots, discriminant, $D = 0$

$\Rightarrow \quad b^2 - 4ac = 0 \Rightarrow (-k)^2 - 4(2)(k) = 0$

$\Rightarrow \quad k^2 - 8k = 0 \Rightarrow k = 0, 8$

40. **(c)** Let $\alpha + 3 = x \;\therefore\; \alpha = x - 3$ (replace x by $x - 3$)

So the required equation

$(x - 3)^2 - 5(x - 3) + 6 = 0$

$\Rightarrow \quad x^2 - 6x + 9 - 5x + 15 + 6 = 0 \Rightarrow x^2 - 11x + 30 = 0$

$(x^2 - 11x + 30) \times 2 = 0 \Rightarrow 2x^2 - 22x + 60 = 0$

41. **(b)** **42. (a)**

43. **(b)** Let the roots be α and β.

$\alpha + \beta = 8$, $|\alpha - \beta| = 10$

$(\alpha - \beta)^2 = 100 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 100 \Rightarrow \alpha\beta = -9$

$\therefore \quad x^2 - 8x - 9 = 0 \Rightarrow (x^2 - 8x - 9) = 0$

or $-(-x^2 + 8x + 9) = 0$

44. **(d)** Since, α and β are roots of $x^2 + 5x + a = 0$

$\therefore \quad \alpha + \beta = -5$ and $\alpha\beta = a$

Consider $2\alpha + 5\beta = -1$

$\Rightarrow \quad 2\alpha + 5(-5 - \alpha) = -1 \Rightarrow 2\alpha - 25 - 5\alpha = -1$

$\Rightarrow -3\alpha = 24 \Rightarrow \alpha = -8$ and $\beta = -5 + 8 = 3$

Also, $\alpha\beta = (3)(-8) = -24 = a$

45. **(c)** Let α and β be the roots of $x^2 + px + 8 = 0$

$\Rightarrow \quad \alpha + \beta = -p$ and $\alpha\beta = 8$

$\Rightarrow \quad \alpha(-p - \alpha) = 8 \Rightarrow -p\alpha - \alpha^2 = 8$

$\Rightarrow \quad \alpha^2 + p\alpha + 8 = 0 \quad$ ... (i)

Also, given $\alpha - \beta = 2$

$\therefore \;\alpha + \beta = -p$ and $\alpha - \beta = 2$ together gives

$2\alpha = 2 - p \Rightarrow p = 2 - 2\alpha$

Put value of 'p' in equation (i), we get

$\alpha^2 + (2 - 2\alpha)\alpha + 8 = 0 \Rightarrow \alpha^2 + 2\alpha - 2\alpha^2 + 8 = 0$

$\Rightarrow -\alpha^2 + 2\alpha + 8 = 0$

On solving this, we get, $\alpha = -2, 4$

$\therefore \; p = \pm 6$ (when $\alpha = -2, 4$)

46. **(b)** Let the roots of the given equation be α and 3α.

Now, $\alpha + 3\alpha = -p$ and $\alpha(3\alpha) = 12$

$\Rightarrow 4\alpha = -p$ and $\alpha^2 = 4 \Rightarrow \alpha = \pm 2$

Now, $4(2) = -p$ and $4(-2) = -p \Rightarrow p = \pm 8$

47. **(c)** Given equation is

$x^2 - 3x + 2 = 0$

$\Rightarrow \quad x^2 - 2x - x + 2 = 0 \Rightarrow x(x-2) - 1(x-2) = 0$

$\Rightarrow \quad (x-1)(x-2) = 0 \Rightarrow x = 1, 2$

48. **(c)** Since $x = 2$ and $x = 3$ are roots of given equation

$\therefore \quad 3(2)^2 - 2p(2) + 2q = 0$

$\Rightarrow \quad 12 - 4p + 2q = 0 \Rightarrow -2p + q = -6 \quad \ldots (i)$

and $\quad 3(3)^2 - 2p(3) + 2q = 0$

$\Rightarrow \quad 27 - 6p + 2q = 0 \Rightarrow -6p + 2q = -27 \quad \ldots (ii)$

On solving (i) and (ii), we get

$p = \frac{15}{2}$ and $q = -6 + 15 = 9$

49. **(a)** Speed of Ajay's car = $(x + 5)$ km/h.

Distance = Speed × Time = $2(x + 5)$ km

50. **(c)** $\frac{400}{x} - \frac{400}{x+5} = 4$

$\Rightarrow x^2 + 5x - 500 = 0$

51. **(a)** $(x + 25)(x - 20) = 0$

$\Rightarrow x = 20$ km/hour

52. **(d)** Speed of Ajay = $x + 5 = 25$ km/h.

Time = $\frac{400}{25}$ = 16 hours.

53. **(c)** (20 – x) km/hr

54. **(a)** Speed = Distance/Time

55. **(c)** $\frac{15}{20-x} - \frac{15}{20+x} = 1$

$\Rightarrow x^2 + 30x - 400 = 0$

56. **(b)** $(x - 10)(x + 40) = 0$

$\Rightarrow \quad x = 10$ km/hour

57. **(c)** Speed in downstream = 20 + 10 = 30 km/h

Time = $\frac{\text{Distance}}{\text{Speed}} = \frac{15}{30}$ = 30 minutes

58. **(b)** Assertion : Given equation $x^2 - bx + c = 0$

Let α, β be two consecutive roots such that $|\alpha - \beta| = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \Rightarrow b^2 - 4c = 1$

Reason : Given equation :

$4abc\, x^2 + (b^2 - 4ac)\, x - b = 0$

$D = (b^2 - 4ac)^2 + 16ab^2c$

$D = (b^2 + 4ac)^2 > 0$

Hence, roots are real and unequal.

59. **(b)** Assertion and Reason both are true statements. But Reason is not the correct explanation.

Assertion : $(2x-1)^2 - 4x^2 + 5 = 0 \Rightarrow -4x + 6 = 0$

Reason : $2x^2 - 6x = 0 \Rightarrow 2x(x - 3) = 0$

$\Rightarrow x = 0$ and $x = 3$.

60. **(a)** Assertion : $9x^2 + 3k\,x + 4 = 0$

$\Rightarrow D = b^2 - 4ac = (3k)^2 - 4(9)(4) = 9k^2 - 144$

For equal roots : $D = 0 \Rightarrow 9k^2 = 144$

$\Rightarrow k = \pm\frac{12}{3} \Rightarrow \quad k = \pm 4$

61. **(c)** Reason is false.

Assertion : $4x^2 - 12x + 9 = 0$

$\Rightarrow \quad D = b^2 - 4ac = (-12)^2 - 4(4)(9) = 144 - 144 = 0$

$\Rightarrow$ Roots are repeated.

62. **(c)** Assertion is correct. Reason is incorrect.

63. **(a)** Assertion and Reason both are correct and Reason is correct explanation.

Assertion : $2x^2 - 3x + 5 = 0$

$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$ and $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

64. (A) → s; (B) → p; (C) → q; (D) → r

(A) $6x^2 + x - 12 = 0$

$6x^2 + 9x - 8x - 12 = 0$

$3x(2x + 3) - 4(2x + 3) = 0$

$(3x - 4)(2x + 3x) = 0$

$x = \frac{4}{3}, \frac{-3}{2}$

(B) $8x^2 + 16x - 192 = 0$

$8x^2 + 48x - 32x - 192 = 0$

$8x(x + 6) - 32(x + 6) = 0$

$x = 4, -6$

(C) $x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0$

$x(x - 36) - 9(x - 36) = 0$

$2x(x - 3) + 1(x - 3) = 0$

$x = 9, 36.$

(D) $2x^2 - 5x - 3 = 0$

$2x^2 - 6x + x - 3 = 0$

$x = \frac{-1}{2}, 3$

65. (A) → q; (B) → r; (C) → s; (D) → p

66. (A) → (r,); (B) → (p,); (C) → (q) ; (D) → (s)

67. $\neq 0$

68. > 0

69. 5 cm, 12 cm.

70. $b^2 < 4ac$

71. 7 and –9

72. $x^2 - bx + ca = 0$

73. $\dfrac{b^2 - 2ac}{c^2}$

74. $x^2 - 8x + 15 = 0$

75. –1

76. $\dfrac{1}{2}(b - q)$

77. two

78. $b^2 < 4ac$

79. False

80. True

81. False

82. True

83. True

84. True

85. True

86. False

87. True

88. True