ALL INDIA TEST SERIES

TEST – 26

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

<u>General Instructions:</u>

- The test consists of total **54** questions.
- Each subject (PCM) has **18** questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section – B & Section-C.**

Section-A (01 – 06, 19 – 24, 37– 42) *this section contains **18 multiple** choice questions.*
*Each question has FOUR options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:
Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **: –2** In all other cases

Section-B (07 – 12, 25 – 30, 43– 48) *contains **18 Numerical** based questions with **Single digit integer** as answer, ranging from **0 to 9** and each question carries **+3 marks** for correct answer and **–1 mark** for wrong answer.*

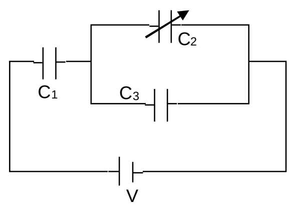
Section-C (13 – 18, 31 – 36, 49– 54) *contains **18 Numerical** answer type questions with answer XXXXX.XX and each question carries **+4 marks** for correct answer and **0 marks** for wrong answer.*

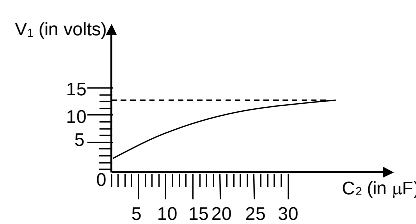
Physics PART – I

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. In the given circuit C_2 is a variable capacitor. The graph represents the variation in potential difference (V_1) across C_1 when the capacitance C_2 is gradually increased from 0 to ∞. It is also observed that as $C_2 \to \infty, V_1$ approaches 12 V.





(A) Potential difference (V) across the battery is 12 volt.

(B) $\frac{C_1}{C_3}$ has a value of 3.

(C) As C_2 is increased the energy stored in C_1 increases

(D) As C_2 is increased the energy sored in capacitor C_3 increases.

2. The relation between internal energy U, pressure P and Volume V of a gas in an adiabatic process U = a + b PV, where a and b are positive constants. Then

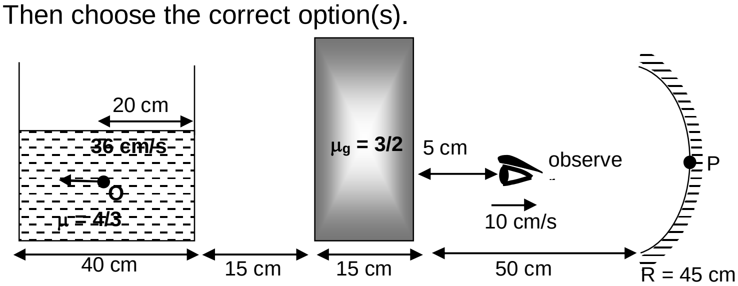
(A) Equation for the process is $PV^{\frac{b+2}{b}}$ = constant

(B) Equation for the process is $PV^{\frac{b+1}{b}}$ = constant

(C) The gas must be monoatomic

(D) The gas must be diatomic

3. A point object 'O' lies inside a transparent cuboidal container (of negligible thickness) having water as shown in the figure. An observer is situated between the glass slab and the concave mirror. At t = 0 observer starts moving with a velocity 10 cm/s towards right side and object 'O' also starts moving towards left side with a velocity 36 cm/s on the optic axis of concave mirror. Then choose the correct option(s).



(A) Distance of image formed by the mirror from its pole is 30 cm left.

(B) Distance of object 'O' as seen by observer from the observer is 45 cm.

(C) The magnitude of velocity of the object 'O' with respect to observer when the observer directly sees the object is 37 cm/s

(D) The magnitude of velocity of the image of object 'O' formed by the concave mirror with respect to observer is 13 cm/s.

4. A charged particle 'P' leaves the origin with speed $v = v_o$ at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. The particle 'P' strikes a fixed target 'T' on the x-axis for a minimum value of $B = B_o$. The particle 'P' will also strike the target T, if

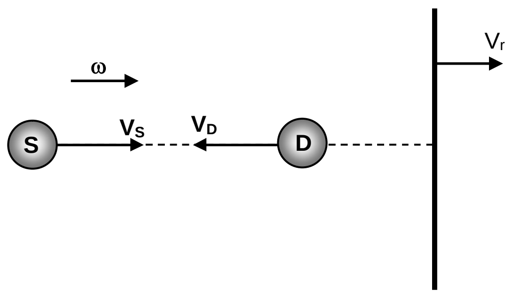
(A) $B = 2B_0, v = 2v_0$

(B) $B = 2B_0, v = v_0$

(C) $B = B_0, v = 2v_0$

(D) $B = \frac{B_0}{2}, v = 2v_0$

5. In the arrangement shown in the figure, a reflector is moving towards right with velocity v_r = 20 m/s. Source and detector are moving towards each other with the velocities V_S = 30 m/s and $V_D = 10$ m/s respectively. The wind is blowing with a velocity ω = 10 m/s towards the reflector. The frequency of sound emitted by the source is f = 527 Hz and the velocity of sound with respect to air is V = 330 m/s. Then choose the correct option(s).



Reflector

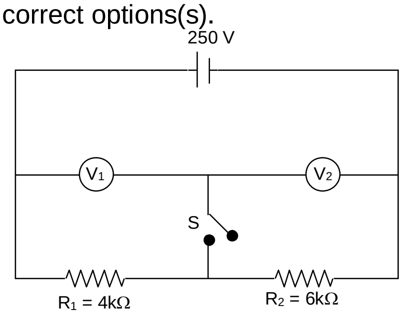
(A) The frequency of the reflected wave received by the detector is 595 Hz.

(B) The frequency of the reflected wave received by the detector is 496 Hz

(C) The wavelength of the reflected wave received by the detector is $\left(\frac{5}{8}\right)$ meter.

(D) The wavelength of the reflected wave received by the detector is $\left(\frac{10}{17}\right)$ meter.

6. In the circuit shown in figure, V_1 and V_2 are two voltmeters having resistances 6 KΩ and 4 KΩ respectively. The emf of the battery is 250 V, having negligible internal resistance. The resistances R_1 = 4KΩ and R_2 = 6 KΩ are also connected in the circuit as shown. Choose the correct options(s).



(A) When switch S is open, then readings of voltmeters are V_1 = 100 V and V_2 = 150 V.

(B) When switch S is open, then readings of voltmeters are V_1 = 150 V and V_2 = 100 V.

(C) When switch S is closed, then readings of voltmeter are V_1 = 100 V and V_2 = 150 V.

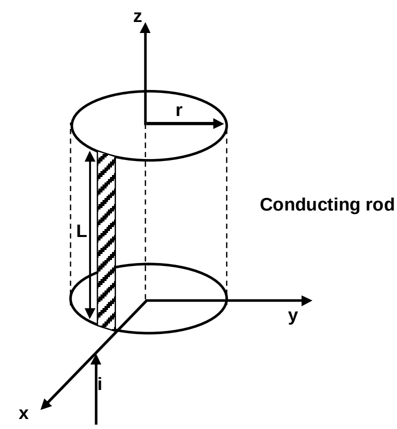
(D) When switch S is closed, then readings of voltmeters are V_1 = 125 V and V_2 = 125 V.

SECTION – B
(Single Digit Integer Type)

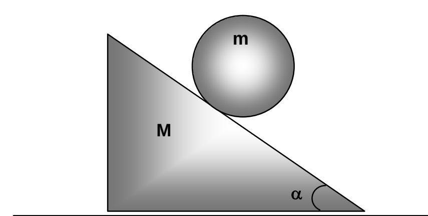
This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

7. A soap bubble of radius r and surface tension T is formed in vacuum. It is slowly sprayed with some amount of charge, as a result of which it slowly expands and finally stops expanding when radius becomes 2r. If the amount of charge given to the bubble is $N\sqrt{12\pi^2 r^3 T\varepsilon_0}$, then find N. (Assume temperature of the gas to be constant during expansion)

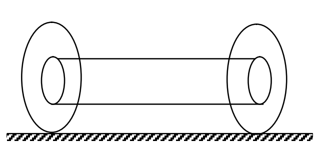
8. A conducting rod of length $L = \frac{1}{4}$ m is carrying a current I = 1A as shown in the diagram. It is made to move in a circular path of radius $r = \frac{2}{\pi}$ m in a magnetic field $\vec{B} = B_0\hat{a}_r$ with 10 revolutions per minute (Where $B_0 = 3T$ and $\hat{a}_r$ is a unit vector is radial direction). If the power (in watt) required to make this rod move with a constant angular speed is P, find the value of 10P.



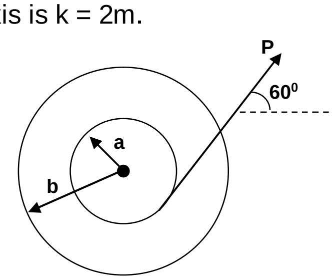
9. A wedge of mass M = 10 kg and inclination $\alpha = 30^0$ is free to move on a smooth horizontal plane. A uniform solid cylinder of mass M = 20 kg is placed on the rough inclined face of the wedge. Find the acceleration (in m/s^2) of the centre of the cylinder relative to the wedge down the face. There is no slipping between the cylinder and the wedge.



10. Rope is coiled round a drum of radius a = 1.5 m. Two wheels each of radius b = 2 m are fitted to the ends of the drum, and the wheels and drum form a rigid body having a common axis. The system stands on the rough horizontal surface and a free end of the rope, after passing under the drum, is inclined at an angle of 60^0 to the horizontal. If a force P = 10 N be applied to the roe, find the magnitude of acceleration (in m/s^2) of centre of drum. Where mass of the system is 0.25 kg and its radius of gyration about the axis is k = 2m.

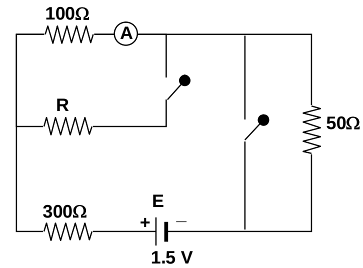


Front view

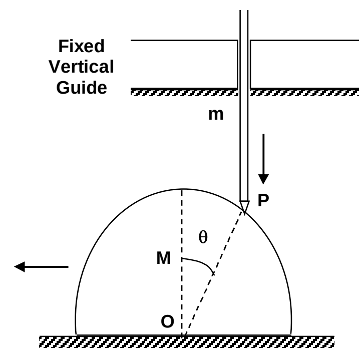


Side view

11. In the circuit shown, the reading of the ammeter (ideal) is the same with both switches open as with both switches closed. Find the value of resistance $\frac{R}{100}$ in ohm.



12. A smooth hemisphere, of mass M = 4kg and radius a = 2m, is placed with its plane base on a smooth table. A rod, of mass m = 2kg is constrained to move in a vertical line with one end P on the curved Surface of the hemisphere. Find the angular speed (in rad/s) of point P with respect to centre O of the hemisphere when OP makes an angle $\theta = 60^0$ with the vertical. Initially the rod was at the topmost point of the hemisphere.

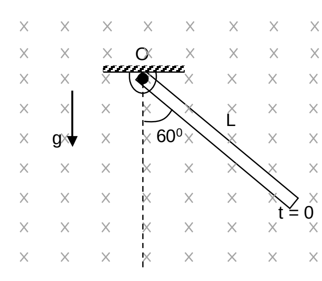


SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

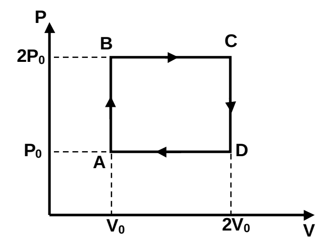
13. Suppose that one of the slits in YDSE is wider than the other, so the amplitude of the light reaching on screen from one slit, acting alone is thrice that from the other slit, acting alone. The intensity for Central Bright Fringe is $I_0 = 5$ W/m^2. Then find the intensity I (W/m^2) on the screen for (angular position) $\theta = 37^0$. (Take $d/\lambda = 5/6$, where d is the separation between the slits and λ is the wavelength of the light)

14. A non-conducting uniform rod of mass 1 kg and length 1 m is hinged at one end as shown in the figure. A charge q = 1 C is uniformly distributed over its length. The rod is released from rest in a uniform magnetic field of $2\sqrt{15}$ T at time t = 0 from the position shown. Find the hinge reaction (in Newton) when the rod becomes vertical. (Take g = 10 m/s^2)

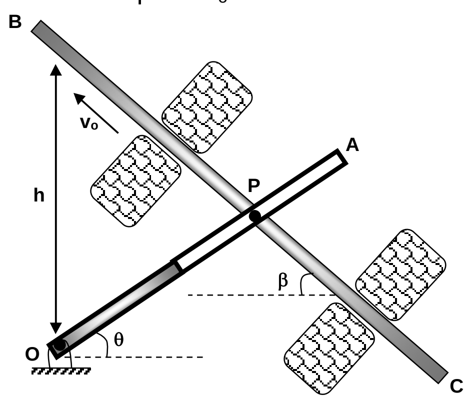


15. A ball is thrown with a speed $u = 10\sqrt{3}$ m/s so that the area enclosed by the balls trajectory with the horizontal surface is maximum. Taking g = 10 m/s^2 and $\sqrt{3} - 1.732$, the maximum area (in m^2) enclosed by the balls trajectory with the horizontal surface is closest to the whole number……..

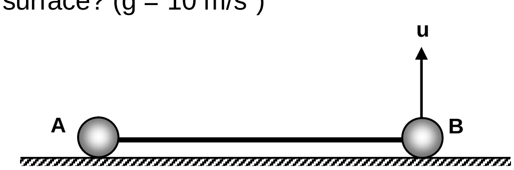
16. The given figure shows the indicator diagram corresponding to n moles of an ideal monoatomic gas taken along the cyclic process ABCDA. Find the efficiency (in percentage) of the cycle.



17. Pin P is attached to BC and slides freely in the slot of OA. Determine the rate of change $d\theta/dt$ (in rad/s) of the angle θ at the instant when h = 4 m, $\beta = 60^0$, and $\theta = 30^0$, knowing that BC moves at a constant speed v_o = 5 m/s.



18. Two identical small balls A and B each of mass m connected by a light inextensible string of length $\ell = 67.5$ cm are placed on a smooth horizontal surface. With what minimum velocity u (in m/s) should the ball B be projected vertically upwards so that the ball A leaves the horizontal surface? (g = 10 m/s^2)



Chemistry

PART – II

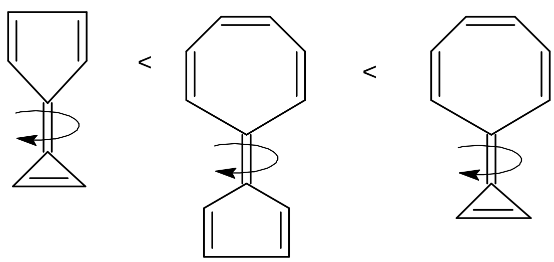
SECTION – A
(One or More than one correct type)

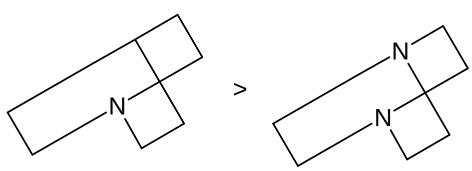
This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

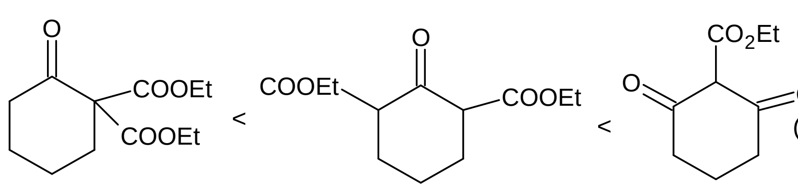
19.

(A)  (inversion barrier)

$\triangle G^{+}$ = 7.9 KCal/mol $\triangle G^{+}$ = 22.9 KCal/mol

(B)  (order of C - C bond rotation)

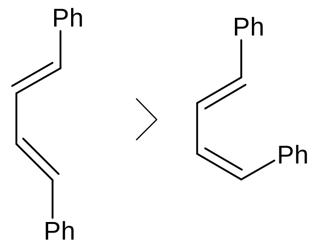
(C)  > Et_3N (order of basic strength)

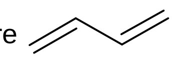
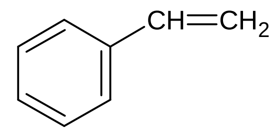
(D)  (order of Acidic strength)

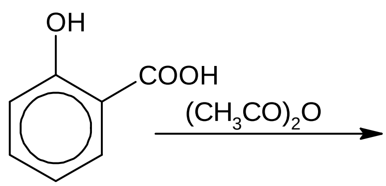
The correct option(s) is/are

20. A metal 'X' crystallizes into a lattice containing a sequence of layers of ABABAB……… any packing of spheres leaves out voids in the lattice. The correct option(s) is/are:
    (A) The co-ordination number of lattice point is 12.
    (B) The volume of unit cell is $24\sqrt{2}\ r^3$.
    (C) The atomic packing fraction is $\pi / 3\sqrt{2}$.
    (D) The number of atoms per unit cell is six.

21. Select the correct statement(s):
    (A) When size of charge at intermediate species is greater than reactant stage, the increase in polarity of solvent increases rate of reaction.

(B)  (rate of diels alder reaction)

(C) The monomeric units of Buna - s are  and 

(D)  A, A is an antipyretic

22. $\underset{\text{alloy}}{(Ag+Pb)} \xrightarrow[\text{Zn is added}]{\text{Melt and}} (Ag+Pb+Zn)\text{melt} \xrightarrow{\text{Cool}} \xrightarrow[\text{Layer Y}]{\text{Layer X}}$

Select the correct statement(s) based on above scheme:

(A) Layer X contains zinc and silver

(B) Layer Y contains Lead and Silver but amount of silver in this layers is smaller than in the layer X.

(C) X and Y are immiscible layers

(D) X and Y are miscible layers.

23. $ZnCl_2 \xrightarrow{NaOH} P \xrightarrow{CH_3COOH} Q \xrightarrow{K_4Fe(CN)_6} 'R'$

Select correct statements:

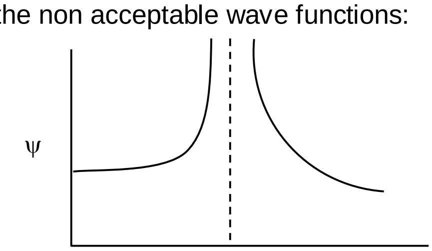
(A) Compound ‘P’ gives white precipitate with H_2S

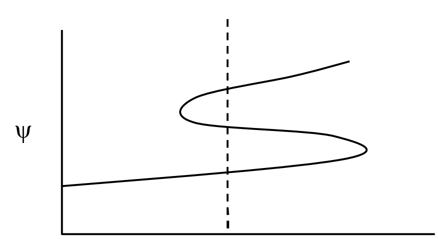
(B) Compound ‘R’ is a white precipitate

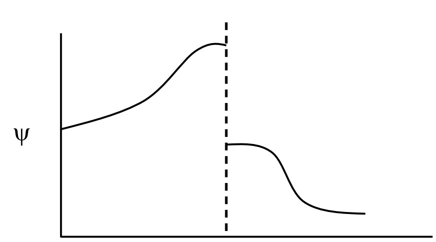
(C) the reaction $P \rightarrow Q \rightarrow R$ is a non redox process

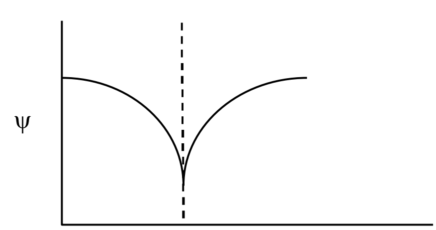
(D) $P \rightarrow Q$ is a non redox process but $Q \rightarrow R$ is a redox process

24. Select the non acceptable wave functions:

(A) 

(B) 

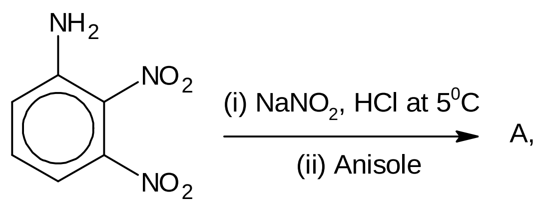
(C) 

(D) 

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

25. **Statement – 1** The spin multiplicity of singlet carbene is one.
Statement – 2 The degeneracy of electron in $H^{\ominus}$ in 2^{nd} excited state is 3 (Not considering the electronic spin)
Statement – 3 Simple vicinal dihalides do not form Grignard reagent with Mg in dry ether.
Statement – 4 $[AgF_4]^-$ and $[AuCl_4]^-$ both are square planer complexes.
How many statement(s) are correct?

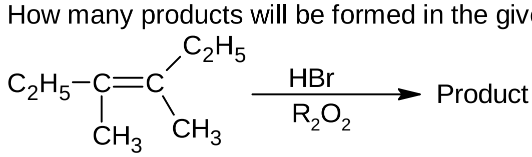
26.


The number of oxygen atoms present in the product 'A' is

27. Solution of equal strength of XOH and QOH are prepared. The I.P's of X and Q are 5.1 and 13.0 eV respectively. Whereas their E.N's are 0.9 and 3.2 respectively. The statements based on the above given information are
S_1 – Reaction of X OH and NH_4Cl will produce NH_3.
S_2 – Solution of QOH will give effervescence with $NaHCO_3$
S_3 – Phenolphthalein will give pink colour with XOH solutions.
S_4 – The pH of QOH solution will be more than 7.
How many statements are correct?

28. S_1 – Xe-F bond has high bond energy
S_2 – F_2 has exceptionally low bond energy
S_3 – PtF_6 is a strong oxidant
S_4 – O_2 molecule and Xe atom have very similar ionisation energies.
The number of statements, which are not prompted Neil Bartlett to prepare the first noble gas are

29. $(NH_4)_2Cr_2O_7 \longrightarrow N_2 + Cr_2O_3 + 4H_2O$
The n factor of $(NH_4)_2Cr_2O_7$ in the given reaction is

30. How many products will be formed in the given reaction (including stereoisomers)?


SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

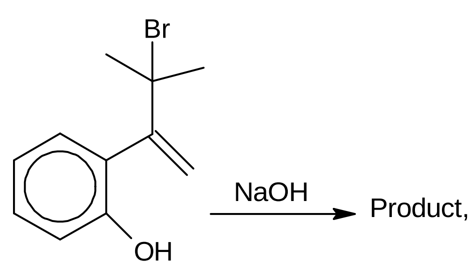
31. The EMF of standard Weston cell, written as
$Cd(Hg), CdSO_4.8/3H_2O(s) || CdSO_4(Sat.), Hg_2SO_4(s), Hg$
in which the cell reaction is
$Cd(Hg) + Hg_2SO_4(s) + 8/3H_2O(\ell) \rightleftharpoons CdSO_4.8/3H_2O(s) + 2Hg(\ell)$ is 1.0185 V at 25^o C. If the value of $(dE^o/dT)_p$ for the cell is 5×10^{-5} VK^{-1}, the ΔS^o for the cell at 25^o C is ……….. JK^{-1}.

32. One mole of an ideal gas (monoatomic) at 27^oC expands adiabatically against a constant external pressure of 1 atm from a volume of 10 dm^3 to a volume of 20 dm^3. The value of ΔH for the process in KJ (magnitude only)?

33. How many moles of water vapour will be contained in a cylindrical vessel having diameter 2 m and height 10 m, if the relative humidity is 80% and the temperature is 300 K, if we assume the V.P of water is 26 mm of Hg?

34. A liquid mixture of benzene and toluene is composed of 1 mol of benzene and 1 mol of toluene. If the pressure over the mixture is reduced, the vapour pressure of first vapour form is 'a' mm of Hg. If the pressure reduced further, the vapour pressure of the system when last trace of liquid will disappear is 'b' mm of Hg. Calculate the value of 'b' + a'? ($P_{Toluene}$ = 32.05 mm of Hg, $P^0_{Benzene} = 103$ mm of Hg)

35. Total number of geometrical isomers of Diamminebis(glycinato)platinum(IV)chloride, is 'x'. Then, value of $\frac{x+100}{2}$ is

36. 

If the molecular weight of Product is 'M', then value of $\frac{M}{100}$ is

Mathematics PART – III

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. If variable line $x(3+\lambda)+2y(2-\lambda)+\lambda-7=0$ always passes through a fixed point (a, b) where λ is a parameter and $\ell = \lim\limits_{x\to(a-b)^-}\dfrac{[\sin x-2]+\{\cos x\}}{x-[x]-1}$, where [.] is greatest integers function and {.} is fractional part function, then
(A) $a+2b=3$
(B) $a-b+2\ell=2$
(C) $\ell=1$
(D) ℓ not defined

38. If $\mathrm{In}=\int_0^1\dfrac{dx}{\left(1+x^2\right)^n};n\in N$ then which of the following statements hold good?
(A) $2n\,I_{n+1}=2^{-n}+(2n-1)I_n$
(B) $I_2=\dfrac{\pi}{8}+\dfrac{1}{4}$
(C) $I_2=\dfrac{\pi}{8}-\dfrac{1}{4}$
(D) $I_3=\dfrac{\pi}{16}-\dfrac{5}{48}$

39. In ΔABC, a = 4, b = c = $2\sqrt{2}$. A point P moves within the triangle such that the square of its distance from side BC is half the area of rectangle contained by its distances from the other two sides. If D be the centre of locus of P, then
(A) locus of P is an ellipse with eccentricity $\sqrt{\dfrac{2}{3}}$
(B) locus of P is a hyperbola with eccentricity $\sqrt{\dfrac{3}{2}}$
(C) area of quadrilateral ABCD $=\dfrac{16}{3}$ sq. units
(D) area of quadrilateral ABCD $=\dfrac{32}{3}$ sq. units

40. Let $\dfrac{dy}{dx}+y=f(x)$ where y is a continuous function of x with y(0) = 1 and $f(x)=\begin{cases}e^{-x}, & 0\le x\le 2\\ e^{-2}, & x>2\end{cases}$.
Which of the following holds good?
(A) $y(1)=2/e$
(B) $y'(1)=\dfrac{-1}{e}$

(C) $y(3) = \frac{-2}{e^3}$

(D) $y'(3) = \frac{-2}{e^3}$

41. Let $f_1(x) = ax^2 + bx + c = 0$ has imaginary roots and $f_2(x) = 2ax + b$ and $f_3(x) = 2a.$. Then which of the following equations will have imaginary roots, where $a \neq 0$?

(A) $f_1(x) + f_2(x) + f_3(x) = 0$

(B) $f_1(x) + f_2(x) = 0$

(C) $f_2(x) + f_3(x) = 0$

(D) $f_1(x) + f_3(x) = 0$

42. Let $A = [a_{ij}]$ is a square matrix of order 5. Then which of the following is/are correct?

(A) maximum number of distinct entries if A is an upper triangular matrix is 16
(B) maximum number of distinct entries if A is a non-null diagonal matrix is 6.
(C) minimum number of zeros if A is a lower triangular matrix is 10.
(D) maximum number of non-zero entries if A is skew-symmetric matrix is 20.

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

---

43. Let ΔABC be such that $2\overrightarrow{OA} + 3\overrightarrow{OB} + 6\overrightarrow{OC} = 0$, where O is origin. If area of ΔABC : area of $\Delta AOB = m : n$ where m, and n are relatively prime, then the value of $(m-n)$ is

44. Let circles C_1 and C_2 on Argand plane be given by $|z+1| = 3$ and $|z-2| = 7$ respectively. If a variable circle $|z - z_0| = r$ be inside circle C_2 such that it touches C_1 externally and C_2 internally then locus of Z_o describes a conic. If eccentricity of conic is $\frac{p}{q}$ where $p, q \in N$ and are relatively prime then the value of $|p - q|$ is

45. The function $f : R \to R$ satisfies $f(x^2) f''(x) = f'(x).f'(x^2) \forall x \in R$. Given that f(1) = 1 and $f'''(1) = 8$, then the value of $f'(1) + f''(1)$ is

46. If $f(x) = 2x + 1$, then the value of x, satisfying the equation $f(x) + f(f(x)) + f(f(f(x))) + f(f(f(f(x)))) = 116$ is equal to

47. In a right-angled triangle, the acute angles α, β are satisfying $\tan\alpha + \tan\beta + \tan^2\alpha + \tan^2\beta = 4$. If the hypotenuse is of length '4' then the area of triangle is

48. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be real numbers in arithmetic progression such that a_1 = 15 and a_2 is an integer. Given $\sum_{r=1}^{10} (a_r)^2 = 1185$ and $s(n) = \sum_{r=1}^{n} a_r$. If maximum value of n for which $s(n) \geq s(n-1)$ is a two-digit number pq, then value of (p+q) is

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

49. If $\lim\limits_{a\to\infty}\int_0^{\infty}\frac{x^2+ax+1}{1+x^4}.\tan^{-1}\left(\frac{1}{x}\right)dx$ is equal to $\frac{\pi^2}{k,}$, then value of k is

50. Let f(x) be a polynomial of degree 3. If the curve y = f(x) has relative extrema at $x=\pm\frac{2}{\sqrt{3}}$ and passes through (0, 0) and $(1,-2)$ dividing the circle $x^2+y^2=4$ in two parts, then the area bounded by $x^2+y^2=4$ and $y\ge f(x)$ is

51. Let $f(x)=x^2-2ax+a-2$ and $g(x)=\left[2+\sin^{-1}\frac{2x}{1-x^2}\right]$. If the set of real values of a for which $f(g(x))<0$ for all $x\in R$ is (K_1,K_2) then value of (K_1+K_2) is (where [.] is greatest integer function).

52. Ten identical balls are distributed in 5 different boxes kept in a row and labeled A, B, C, D and E. Find the number of ways in which the balls can be distributed in the boxes if no two adjacent boxes remain empty.

53. Let z = x + iy be a complex number, where x and y are real numbers. Let A and B be the sets defined by $A=\{Z:|Z|\le 2\}$ and $B=\{Z:(1-i)Z+(1+i)\overline{Z}\ge 4\}$. Find area of the region $A\cap B$.

54. Let P and Q be two circle externally tangent at point X. A straight line is tangent to P at point A and is tangent to Q at point B $(A\ne B)$. The line tangent to P and Q at X intersects line AB at a point Y. If AY = 10 and the radius of P is 9, then the radius of circle Q is

ALL INDIA TEST SERIES

TEST – 26

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

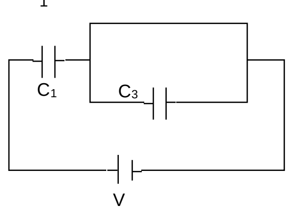
SECTION – A

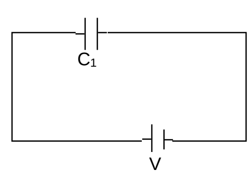
1. ABC

Sol. When $C_2 \to \infty \Rightarrow$ shorted

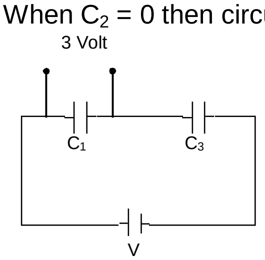
Then the circuit becomes

$\therefore V_1 = V = 12$ volt





When $C_2 = 0$ then circuit becomes



$\therefore V_{C_3} = 12 - 3 = 9V$

Since, C_1 and C_3 are in series, so charge will be same on both the capacitors.

$\therefore C_1 \times 3 = C_3 \times 9 \Rightarrow \frac{C_1}{C_3} = 3$

2. B

Sol. $0 = dU + PdV = d(a + bPV) + PdV$

After solving

$PV^{\frac{b+1}{b}}$ = constant

3. ABC

Sol. $V_{IP} = \left(\frac{f}{f-u}\right)^2 \times \frac{36}{\mu} = \left(\frac{-22.5}{-22.5+90}\right) \times \left(\frac{36}{4/3}\right) = 3\,cm/s$

4. AB

Sol. Distance of the target $x = (n)\frac{2\pi m}{qB} v\cos\theta$, where n is an integer

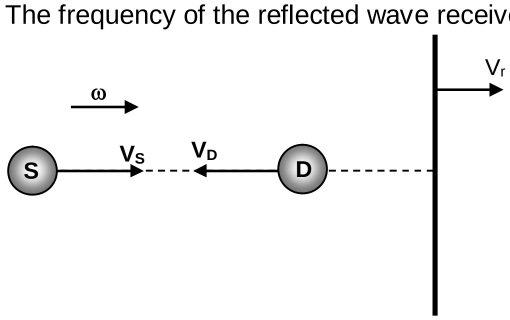
$\frac{nv}{B} = \frac{v_0}{B_0}$

5. BC

Sol. The frequency received by the reflector is

$f_1 = \left(\frac{V+W-V_r}{V+W-V_s}\right)f$

The frequency of the reflected wave received by the detector is



$f_2 = \left(\frac{V+W-V_D}{V+W-V_r}\right)\left(\frac{V+W-V_r}{V+W-V_s}\right)f$

$= \left(\frac{330-10-10}{330-10+20}\right)\left(\frac{330+10-20}{330+10-30}\right)527$

$f_2 = \left(\frac{310}{340}\right)\left(\frac{320}{310}\right) \times 527 = 496\,Hz$

The wavelength of the reflected wave received by the detector is

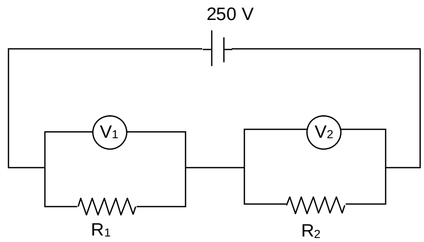
$\lambda_2 = \frac{V-W-V_D}{f_2} = \frac{330-10-10}{496} = \frac{5}{8}m$

6. BD

Sol. When switch S is open

$V_1 = \frac{250}{10000}(6000) = 150\,V$

$V_2 = \frac{250}{10000}(4000) = 100\,V$



When switch S is closed circuit reduces to shown in the figure.

So both the voltmeters show readings $V_1 = V_2 = \frac{250}{2} = 125\,V$

SECTION – B

7. 8

Sol. Initially $P_{in} = \frac{4T}{r}$, where P_{in} = inside pressure …(i)

$P_{outside}$ = 0 (always)

When charge is sprayed, for equilibrium $P'_{in} + \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{2r}$ ...(ii)

Since, isothermal process

$\therefore P'_{in} = \frac{P_{in}V_i}{V_f} = \frac{P_{in}}{8}$ ...(iii)

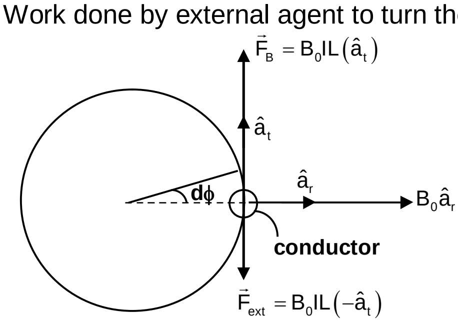
Using equations (i), (ii) and (iii) we have, $\frac{4T}{8r} + \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{2r} \Rightarrow \sigma = \sqrt{\frac{3T\varepsilon_0}{r}}$

8. 5

Sol. $\vec{F}_B = I(\vec{L} \times \vec{B}) = I(L\hat{K} \times B_0\hat{a}_r) = B_0IL\hat{a}_t$

$\vec{F}_{external} = B_0IL(-\hat{a}_t)$

Work done by external agent to turn the conductor by 2π radian



$$W = \int_0^{2\pi} B_0IL(-\hat{a}_t).rd\phi\hat{a}_t = -2\pi rB_0IL$$

$$P = 2\pi rB_0ILN = 0.5\,W$$

9. 5

Sol. When the wedge has moved through a distance y, let the point of contact of cylinder have moved down the plane through x (with respect to wedge). Let F be the friction between the cylinder and the plane.

Since there is no horizontal force on 9wedge + cylinder system)

$$M\frac{d^2y}{dt^2}+m\left(\frac{d^2y}{dt^2}-\cos\alpha\frac{d^2x}{dt^2}\right)=0 \qquad ...(i)$$

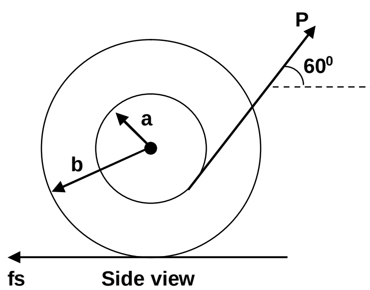
Also, $m\left(\frac{d^2y}{dt^2}\cos\alpha-\frac{d^2x}{dt^2}\right)=F-mg\sin\alpha$ and $\frac{ma}{2}\frac{d^2x}{dt^2}=\frac{ma^2}{2}\frac{d^2\theta}{dt^2}=Fa$, so that

$$2\frac{d^2y}{dt^2}\cos\alpha-3\left(\frac{d^2x}{dt^2}\right)=-2g\sin\alpha \qquad ...(ii)$$

Equation (i) and (ii) implies acceleration of the centre of the cylinder down the face, and relative to the wedge, is $\frac{2g\sin\alpha(M+m)}{3M+m+2m\sin^2\alpha}=5\ m/s^2$

10. 5

Sol. Assume that the drum rolls away from the force P, so that the friction F is away from P. Then:



$$Mk^2\frac{d^2\theta}{dt^2}=Pa-Fb \text{ and } Mb\frac{d^2\theta}{dt^2}=M\frac{d^2x}{dt^2}=F-P\cos 60^0$$

$$M\left(k^2+b^2\right)\frac{d^2\theta}{dt^2}=P\left[a-\frac{b}{2}\right]$$

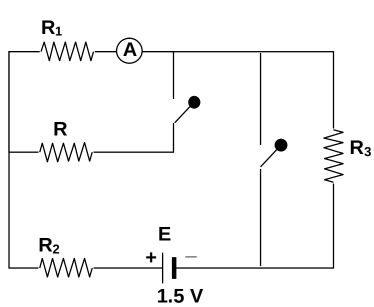
$$\Rightarrow 0.25\times 8\frac{d^2\theta}{dt^2}=10(1.5-1)$$

$$\Rightarrow \frac{d^2\theta}{dt^2}=2.5\,rad/s^2$$

and the required acceleration $=b\frac{d^2\theta}{dt^2}=2\times 2.5=5\ m/s^2$

11. 6

Sol. With both switches open: $I_A=\frac{E}{R_1+R_2+R_3}$



With both switches closed:
Current through E and R_2

$$I' = \frac{E}{R_2 + \frac{RR_1}{R+R_1}}$$

Current through the ammeter

$$I_A' = \frac{RI'}{R+R_1}$$

$$I_A = I_A'$$

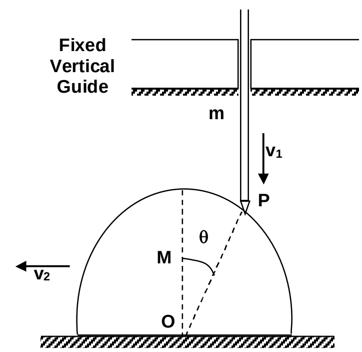
$$R = \frac{R_1R_2}{R_3} = 600\Omega$$

12. 2

Sol. $v_1\cos\theta = v_2\sin\theta$

$v_1 = v_2\tan\theta$

$v_1 = v_2\sqrt{3}$ (given $\theta = 60^0$)



Now using conservation of energy

$$mga(1-\cos\theta) = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$\Rightarrow \quad 2ga = 2v_1^2 + 4v_2^2$

$40 = 6v_2^2 + 4v_2^2$

$\Rightarrow \quad 10v_2^2 = 40$

$\Rightarrow \quad v_2 = 2\ m/s$

$\therefore \quad v_1 = 2\sqrt{3}\ \ m/s$

$$\omega = \frac{v_1\sin\theta + v_2\cos\theta}{a} = 2\ rad/s$$

SECTION – C

13. 00000.80

Sol. $I_{C.B.F} = K\left[A_1^2 + A_2^2 + 2A_1A_2\cos\phi\right] = I_0$

$KA^2 = \frac{I_0}{16}, \left(A_1 = A, A_2 = 3A \text{ and } \phi = 0^0\right)$

$I_0 = \frac{16}{25}K\left[A_1^2 + A_2^2 + 2A_1A_2\cos\phi\right] = 1$

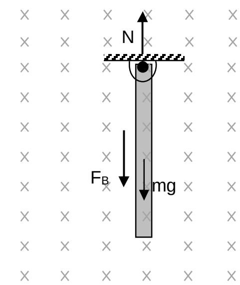
$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\times d\sin\theta = \frac{2\pi}{\lambda}\times d\frac{3}{5}$

So, $\phi = \pi$

Hence, $I = \frac{I_0}{4}\times\frac{16}{25} = \frac{5\times16}{100} = 0.80\ W/m^2$

14. 00032.50

Sol. Loss in gravitational potential energy = Gain in Kinetic energy



$$Mg\left[\frac{L}{2}-\frac{L}{2}\cos 60^0\right]=\frac{1}{2}\frac{ML^2}{3}\omega^2$$

$$\omega=\sqrt{\frac{3g}{2L}}=\sqrt{15}\text{ rad / s}$$

When rod is at its lowest position the forces acting on it are as shown in the diagram.

Magnetic force on the rod

$$F_B=\int dq\left(\vec{v}\times\vec{B}\right)=\int_{x=0}^{x=L}\left(\frac{q}{L}dx\right)\times(\omega X)\times B=\frac{1}{2}qBL\omega$$

$$\therefore N-mg-F_B=m\omega^2\frac{L}{2}$$

$$N=mg+F_B+M\omega^2\frac{L}{2}$$

$$=1\times10+\frac{1}{2}\times1\times\sqrt{15}\times2\sqrt{15}\times1+1\times\left(\sqrt{15}\right)^2\times\frac{1}{2}$$

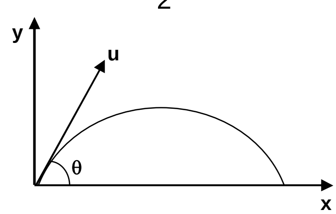
$$=10+15+7.5=32.5N$$

15. 00195.00

Sol. Coordinates of the ball at t = t is

$$x=u\cos\theta t$$

$$y=u\sin\theta t-\frac{1}{2}gt^2$$



and time of light is $\frac{2u\sin\theta}{g}$

Area under trajectory is

$$A=\int_0^x ydx=\int_0^{\frac{2\sin\theta}{g}}\left(u\sin\theta t-\frac{gt^2}{2}\right)u\cos\theta dt$$

$$\therefore A=\frac{2u^4}{3g^2}\sin^3\theta\cos\theta$$

For A to be maximum, $\frac{dA}{d\theta}=0$

$$\Rightarrow \theta=60^0$$

$$\therefore A_{max}=\frac{\sqrt{3}u^4}{8g^2}=194.85\approx195m^2$$

16. 00015.38

Sol. Total work done by gas per cycle, $\Delta W_{cycle}=P_0V_0$

During the process AB

$$\Delta Q_{AB} = nC_v\Delta T = n\frac{3R}{2}\Delta T = \frac{3}{2}V_0\left(2P_0 - P_0\right) = \frac{3}{2}P_0V_0$$

During the process BC

$$\Delta Q_{BC} = nC_P\Delta T = n\frac{5R}{2}\Delta T = \frac{5}{2}\times 2P_0\left(2V_0 - V_0\right) = 5P_0V_0$$

Efficiency of the cycle

$$\eta = \frac{\Delta W_{cycle}}{Q_{supplied}}\times 100 = \frac{P_0V_0}{\left(\frac{3P_0V_0}{2} + 5P_0V_0\right)}\times 100 = \frac{1}{13}\times 100 = 15.38\%$$

17. 00002.50

Sol. $v_0 = \omega h \sin\theta$

$$\Rightarrow \omega = \frac{v_0}{h\sin\theta} = \frac{2v_0}{h} = 2.50 \text{ rad/s}$$

18. 00004.50

Sol. When the ball A leaves the horizontal surface

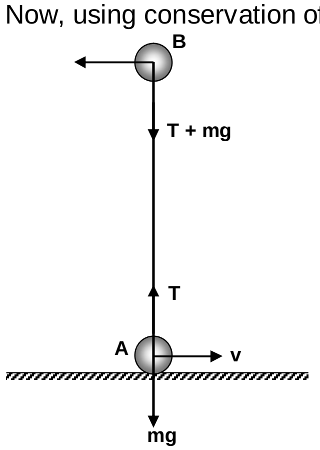
$T = mg$ … (i)

$$T + mg = \frac{m(2v)^2}{\ell}$$

$$2m = \frac{4mv^2}{\ell}$$

$$v = \sqrt{\frac{g\ell}{2}} \quad \text{...(ii)}$$

Now, using conservation of Energy



$$\frac{1}{2}mu^2 = 2\times\frac{1}{2}mv^2 + mg\ell$$

$$u^2 = 2v^2 + 2g\ell$$

$$u^2 = g\ell + 2g\ell$$

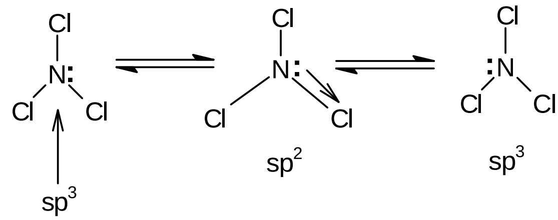
$$\Rightarrow u_{min} = \sqrt{3g\ell} = \sqrt{3\times 10\times 0.675} = 4.50 \text{ m / s}$$

Chemistry PART – II

SECTION – A

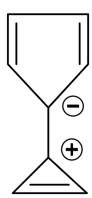
19. ACD

Sol.

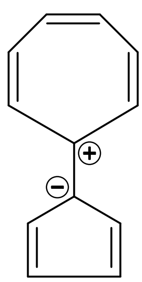
(A) 

(electronegative nitrogen become more electronegative because chlorine atoms pull electron density away from nitrogen than methyl group in $N(CH_3)_3$, creating electron defficient N, and destabililising the T.S)

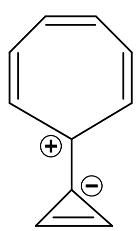
(B) After charge separation due to shifting of π electron

(1) 

2 aromatic ring stabilised by resonance.

(2) 

2 aromatic ring more stable than (1) due to extra resonance

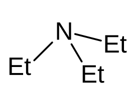
(3) 

1 aromatic ring

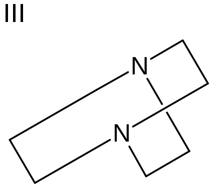
1 anti - aromatic ring required more energy for charge separation

Bond rotation energy is III > I > II

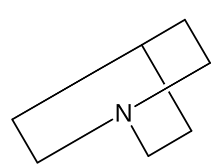
Bond rotation order II > I > III

(C) 

Least basic
due to amine inversion



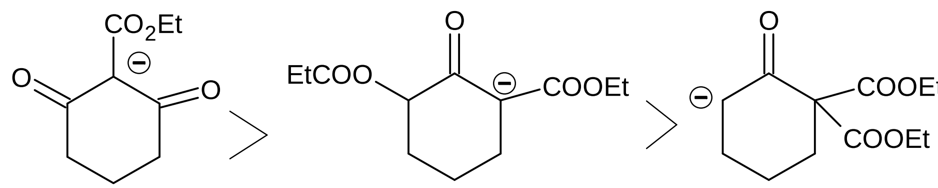
Less basic
due to - I of 2nd N



most basic

No amine inversion

(D) Stability of conjugate base α Acidic nature of conjugate acid



(stability order of C.B.)

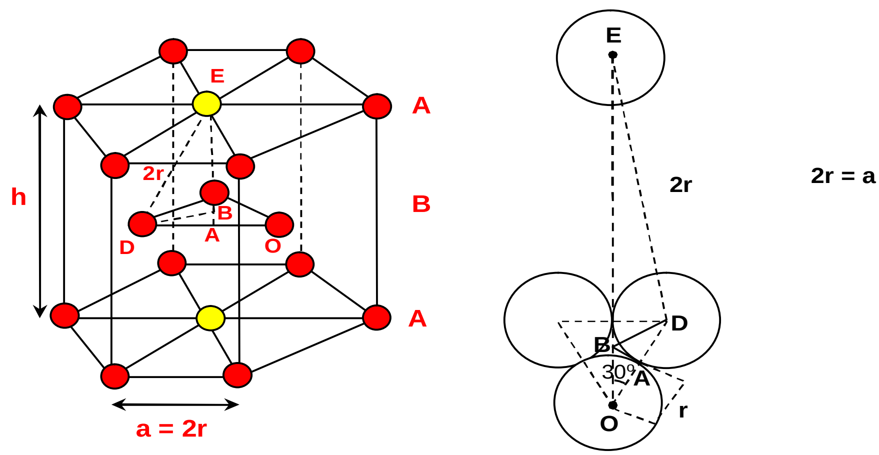
20. ABCD

Sol. (A) Co-ordination number in HCP = 12

(B) Volume $=$ Base area $\times$ height (h)

Base area = $6\times$ area of equilateral triangle

$= 6\times\frac{\sqrt{3}}{4}a^2$



$OA = r$

$\angle AOB = 30^o$

$OB = BD = \frac{2r}{\sqrt{3}}$

$EB = \sqrt{(2r)^2 - \left(\frac{2r}{\sqrt{3}}\right)^2}$

$= 2r\sqrt{\frac{2}{3}} = \frac{h}{2}$

$h = 4r\sqrt{\frac{2}{3}}$

Now, volume = $h\times$ area of base

$=4r\sqrt{\frac{2}{3}}\times 6\times\frac{\sqrt{3}}{4}a^2$

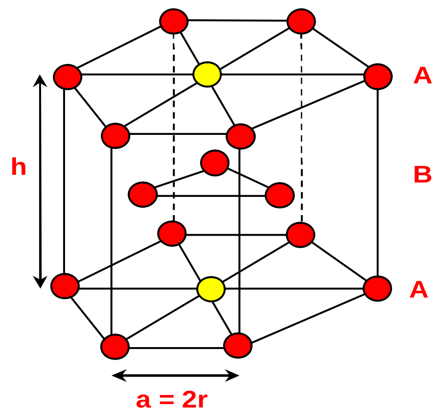
$= 4r\sqrt{\frac{2}{3}}\times 6\times\frac{\sqrt{3}}{4}(2r)^2$

$= 24\sqrt{2}\ r^3$

$(C) \Rightarrow$ Packing fraction $= \dfrac{6 \times \frac{4}{3}\pi r^3}{\text{volume of unit of unit cell}}$

$$= \frac{6 \times \frac{4}{3}\pi r^3}{24\sqrt{2}\ r^3} = \frac{\pi}{3\sqrt{2}}$$

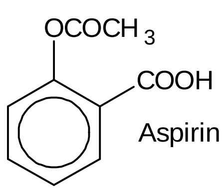
$(D) \Rightarrow Z = 12\ (\text{at corner}) \times \frac{1}{6} + 2(\text{face centres})\frac{1}{2} + 3(\text{in the body}) = 6$



Z = No. of atoms per unit cell.

21. ABCD

Sol. (A) Due to stabilization of intermediate
(B) Less steric hindrance
(C) Fact
(D) A is



Aspirin, an antipyretic.

22. ABC

Sol. Molten Zn and Pb are immiscible liquid and Ag is more soluble in molten Zn than molten Pb.

23. ABC

Sol. $ZnCl_2 + 2NaOH \rightarrow Zn(OH)_2 \xrightarrow{2NaOH} Na_2ZnO_2 + 2H_2O$

$Na_2ZnO_2 + H_2S \rightarrow ZnS \downarrow + 2NaOH$

$Na_2ZnO_2 + 4CH_3COOH \rightarrow (CH_3COO)_2\,Zn + 2CH_3COONa + 2H_2O$

$2(CH_3COO)_2\,Zn + K_4[Fe(CN)_6] \rightarrow Zn_2[Fe(CN)_6] + 4CH_3COOK$

$3Zn_2[Fe(CN)_6] + K_4[Fe(CN)_6] \rightarrow 2Zn_3K_2[Fe(CN)_6]_2 \downarrow$ White

24. ABCD

Sol. (A) Unacceptable because it is infinite over a finite region
(B) Unacceptable because it is not single valued
(C) Unacceptable because it is not continuous
(D) Unacceptable because its slope is discontinuous

SECTION – B

25. 4

Sol. **Statement – 1** Singlet carbene has zero unpaired electron hence spin multiplicity

$= 2s + 1 \qquad \Rightarrow s = \text{total spin}$

$= 1 \qquad \Rightarrow s = \frac{n}{2}, \qquad n = 0$

Statement – 2 $H^{\ominus}$,no. of electron = 2

According to (n + ℓ) rule

$1s^2 \qquad 2s^0 \qquad 2p^0$

$\downarrow$

ground state   1st excited state   2nd excited state

Hence

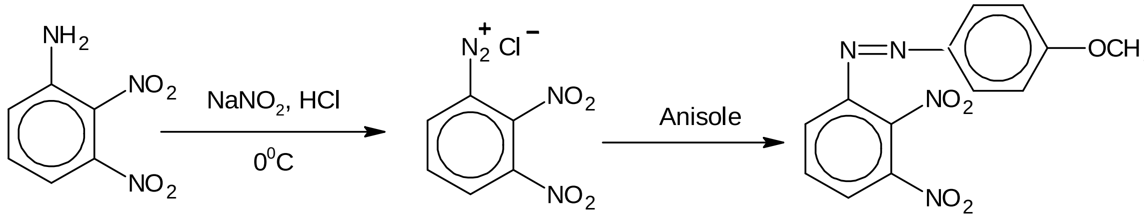
$$1s^1 \; 2s^o \; 2p^1 \Rightarrow 2p^1 = \underbrace{\boxed{\uparrow}\boxed{}\boxed{}}_{\text{Degeneracy} = 3}$$

Statement – 3 In presence of Mg vicinal dihalides involved in elimination reaction.

Statement – 4 $[AgF_4]^-$ and $[AuCl_4]^-$ both are dsp^2 hybridized.

26. 5

Sol.



A

27. 3

Sol. The lower IP indicates that X will be alkali metal hen X-OH should be a hydroxide of alkali metal and higher value of IP Q indicates that It should be a halogen so QOH be an oxyacid. Hence the statement that pH of QH is more than 7 is incorrect.

28. 3

Sol. $Xe \to Xe^+ + 1e^-, \; I.E = 1170 \text{ kJmol}^{-1}$

$O_2 \to O_2^+ + 1e^-, \; I.E = 1175 \text{ kJ mol}^{-1}$

29. 6

Sol.

change in oxidation state in Cr = 2 (+6 - 3) = 6

$$\overset{-3}{(NH_4)_2}\,\overset{+6}{Cr_2O_7} \longrightarrow \overset{O}{N_2} + \overset{+3}{Cr_2O_3} + 4H_2O$$

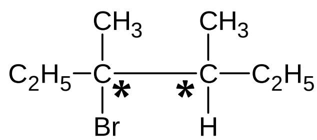
change in oxidation state = 2(-3-0)

In 'N" = - 6

Hence $n_{factor} = 6$

30. 4

Sol.



$= 2^2 = 4$

SECTION – C

31. 00009.65

Sol. $\Delta S^o = nF\left(\frac{dE^o}{dT}\right)_P$

$= 2 \times 96500C \times \left(5 \times 10^{-5} VK^{-1}\right)$

$= 9.65\ JK^{-1}$

32. 00001.69

Sol. $q = 0,\ w = -P\left(v_2 - v_1\right)$

$= -1\left(20-10\right)dm^3\ atm\ \ (\because\ 1\ atm\ dm^3 = 101.3\ J)$

$= -1013\ J$

$\Delta U = q + W \qquad \left(\text{from FLOT}\right)$

$dU = -1013\ J$

$\Rightarrow \Delta U = nC_V\Delta T$ and $\Delta H = nC_P\Delta T$

$\Rightarrow \frac{\Delta H}{\Delta U} = \frac{C_P}{C_V} = \frac{\left(C_V + R\right)}{C_V}$

$= \frac{5/2R}{3/2R}$

$\Delta H = \frac{5}{3}\Delta U \Rightarrow \Delta H = \frac{5}{3} \times \left(-1013\right)$

$= -1688\ J$

33. 00034.89

Sol. $V = \pi \times 1 \times 1 \times 10 \times 10^3$ Litre

$= 3.14 \times 10^4$ Litre

$P = 0.8 \times 26 = 20.8$ mm of Hg

PV=nRT

$\frac{20.8}{760} \times 3.14 \times 10^4 = n \times 0.0821 \times 300 \Rightarrow n = 34.89$ mol

34. 00116.42

Sol. $P = x_T P_T^o + x_B P_T^o$ $\qquad x_T, x_B$ mole fraction of Toluene and Benzene respectively.

= 67.525 mm of Hg = a

When last trace of liquid will disappear then $x_B\left(\text{vapour phase}\right) = \frac{1}{2},\ x_T\left(\text{vapour phase}\right) = \frac{1}{2}$

Then $\frac{1}{P} = \frac{x_T}{P_T^0} + \frac{x_B}{P_B^0}$

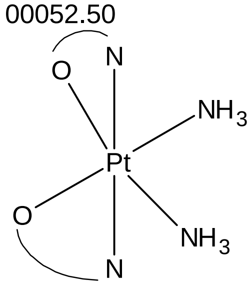
$$= \frac{0.5}{32.05} + \frac{0.5}{103}$$

P = 48.9 mm of Hg = b

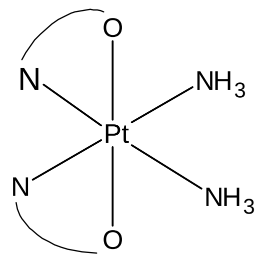
Then, $a + b = 116.42$ mm of Hg

35. 00052.50

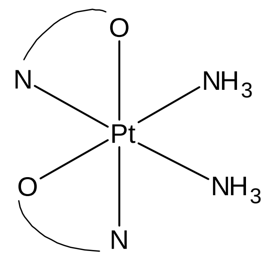
Sol.



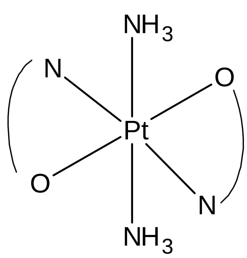
(i)



(ii)



(iii)



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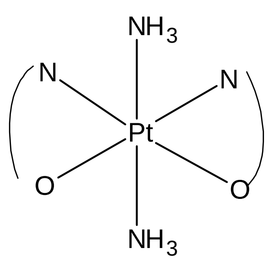
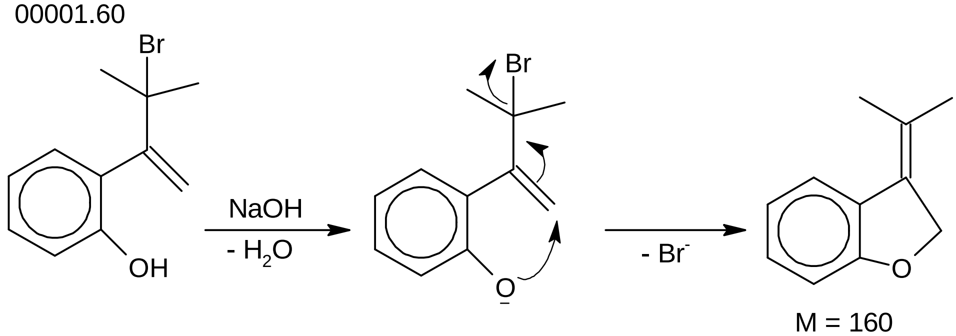


(v)

$\Rightarrow x = 5$ hence $\Rightarrow \frac{100+5}{2} = 52.5$

36. 00001.60

Sol.



Mathematics PART – III

SECTION – A

37. AD

Sol. Using family of lines $(a,b)=(1,1)$

$$\therefore \ell = \lim_{x\to 0^-}\frac{[\sin x]+\{\cos x\}}{\{x\}-1}=\frac{-2}{0}$$

38. AB

Sol. Using integration by parts

$$I_n = \left|x.\frac{1}{\left(1+x^2\right)^n}\right|_0^1 - \int_0^1 x.\frac{(-n).2x}{\left(1+x^2\right)^{n+1}}dx$$

$$=\frac{1}{2^n}+2n\int_0^1\frac{x^2+1-1}{\left(1+x^2\right)^{n+1}}dx$$

$$=\frac{1}{2^n}+2n\ I_n-2n\ I_{n+1}$$

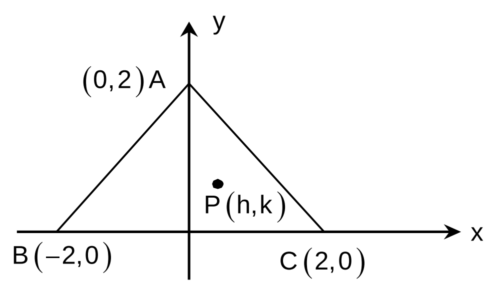
39. AC

Sol. $AB: y-x=2$

$AC: x+y=2$

$\therefore$ For P (h, k),

$$\left|\frac{h+k-2}{\sqrt{2}}\right|\left|\frac{h-k+2}{\sqrt{2}}\right|=2k^2$$



$$\frac{x^2}{16/3}+\frac{\left(y+2/3\right)^2}{(16/9)}=1 \qquad \therefore e=\sqrt{\frac{2}{3}}.$$

40. ABD

Sol. $\frac{dy}{dx}+y=f(x)\Rightarrow ye^x=\int e^x f(x)dx+c$

If $0\le x\le 2,\quad ye^x=\int e^x.e^{-x}dx+c\Rightarrow ye^x=x+c$

If $x>2,\qquad ye^x=\int e^{x-2}dx\Rightarrow ye^x=e^{x-2}+c$

41. AD

Sol. $f_1(x)+f_2(x)+f_3(x)=0\Rightarrow D=-4a^2+b^2-4ac<0$

$f_1(x)+f_2(x)=0$ $D=(2a+b)^2-4(2b+c)\Rightarrow$ Nothing can be said.
$f_1+f_3=0$ $D=b^2-4ac-8a^2<0$

42. ABC

Sol. (A) No. of entries $=1+\dfrac{n(n+1)}{2}$
(B) No. of entries $=5+1$
(C) Minimum no. of zeros $=\dfrac{n(n-1)}{2}$
(D) Maximum no. of non-zero entire =10.

SECTION – B

43. 5

Sol. $\text{Area}(\Delta ABC)=\dfrac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$
Given $2\vec{a}+3\vec{b}+6\vec{c}\Rightarrow\vec{a}\times\vec{b}=2(\vec{c}\times\vec{a})$
$\Rightarrow 2(\vec{b}\times\vec{a})+6(\vec{b}\times\vec{c})\Rightarrow 3(\vec{b}\times\vec{c})=\vec{a}\times\vec{b}$
$\therefore \dfrac{\text{Ar}(\Delta ABC)}{\text{Ar}(\Delta AOB)}=\dfrac{\frac{1}{2}|\vec{a}\times\vec{b}|\left(1+\frac{1}{2}+\frac{1}{3}\right)}{\frac{1}{2}|\vec{a}\times\vec{b}|}=\dfrac{11}{6}\therefore m=11,\ n=6$

44. 7

Sol. $C_1:(x+1)^2+y^2=9$ $\quad C_2:(x-2)^2+y^2=49$
$CC_1=r+r_1$ $\ CC_2=r_2-r\Rightarrow CC_1+CC_2=r_1+r_2=2a=10$
$C_1C_2=3=2ae\Rightarrow e=\dfrac{3}{10}$

45. 6

Sol. Put x =1 $f(1).f''(1)=(f'(1))^2$ Let $f'(1)=\alpha, f''(1)=\beta$
$\therefore \beta=\alpha^2$
Differentiate w.r.t x and put x = 1
$8+2\alpha\beta=2\alpha\beta+\alpha\beta\Rightarrow\alpha\beta=8$
$\therefore \alpha=2,\beta=4$

46. 3

Sol. $f(f(x))=4x+3$ and so on
$\therefore \text{LHS}=30x+26$ $\quad\therefore x=3.$

47. 4

Sol. $\because \alpha+\beta=90^o$ $\quad\therefore \tan\beta=\cot\alpha$
$\tan\alpha+\cot\alpha\ge 2, \tan^2\alpha+\cot^2\alpha\ge 2$ $\quad\therefore \alpha=\beta=\pi/4.$

48. 7

Sol. $15^2 + (15+d)^2 + (15+2d)^2 + ... + (15+9d)^2 = 1185$

On solving d = - 1

$s(n) \ge s(n-1) \Rightarrow n \le 16 \qquad \therefore n_{max} = 16$

SECTION – C

49. 00016.00

Sol. Put $x = 1/t \Rightarrow dx = -1/t^2 dt$

$$I = \int_{\infty}^{0} \frac{(t^2 + at + 1)}{1+t^4} \frac{t^4}{t^2} . \left(\frac{-1}{t^2}\right) \tan^{-1} t \, dt$$

$$I = \int_{0}^{\infty} \frac{x^2 + ax + 1}{1+x^4} \tan^{-1} x dx$$

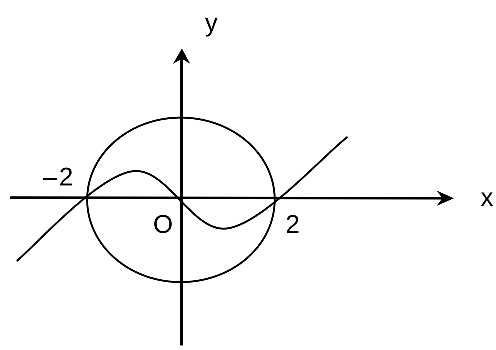
$$\therefore 2I = \int_{0}^{\infty} \frac{x^2 + ax + 1}{1+x^2} dx \quad \because \tan^{-1} x + \tan^{-1} \frac{1}{x} = \frac{\pi}{2}$$

$$\therefore I = \lim_{a \to \infty} \left(\frac{\pi^2}{8\sqrt{2a}} + \frac{\pi^2}{16} \right) \Rightarrow K = 16$$

50. 00006.28

Sol. $f'(x) = a\left(x^2 - \frac{4}{3}\right) \Rightarrow f(x) = \frac{2x}{3}(x^2 - 4)$

$\therefore$ Area bounded = 2π



51. 00003.40

Sol. $\because \sin^{-1}\left(\frac{2x}{1+x^2}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \therefore g(x)$ can be 0, 1, 2, 3

$f(g(x)) < 0 \Rightarrow f(0) < 0$ and $f(3) < 0$

$\therefore a \in \left(\frac{7}{5}, 2\right)$

52. 00771.00

Sol. Case 1 – No box empty,

$x_1 + x_2 + x_3 + x_4 + x_5 = 10, \; x_i \ge 1 \Rightarrow 126$

Case 2 – Exactly one empty, $= 420$

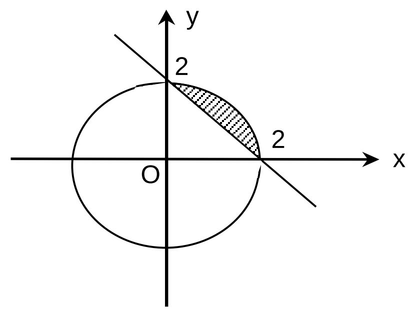
Case 3 – Exactly two empty = 216

Case 4 – Exactly three empty = 9

Total ways = 771

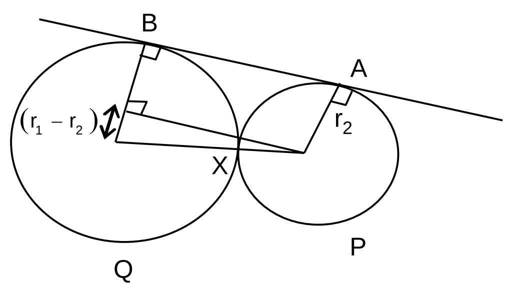
53. 00001.14

Sol. Area $= \frac{\pi.2^2}{4} - \frac{1}{2}.2.2 = \pi - 2$



54. 00011.11

Sol. $AB^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2$



$= 2\sqrt{r_1 r_2}$

$r_1 = \frac{100}{9} \because r_2 = 9$