DETERMINANTS

EXERCISE – 1: Basic Subjective Questions

Section–A (1 Mark Questions)

1. If A is invertible matrix of order 3×3, then $|A^{-1}| =$ ……

2. If A is a matrix of order 3×3, then number of minors in determinant of A are……………

3. The sum of the products of elements of any row with the co-factors of corresponding elements is equal to……….

4. Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$.

5. Evaluate $\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$.

Section–B (2 Marks Questions)

6. Prove that $(A')^{-1} = (A^{-1})'$, where A is an invertible matrix.

7. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then find x.

8. The area of a triangle with vertices $(-3,0),(-3,0),(3,0)$ and $(0,k)$ is 9 sq. units. Find the value of k.

9. Find the value of the determinant

$$\Delta = \begin{vmatrix} \sin^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}.$$

10. Find the value of a if the value of the determinant $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & 1 \\ 0 & 4 & 2a \end{vmatrix}$ is 78.

11. Solve $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ using the properties of determinants.

12. If $\Delta = \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix}$, then show that Δ is equal to zero.

13. Evaluate: $\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ xz^2 & zy^2 & 0 \end{vmatrix}$.

Section–C (3 Marks Questions)

14. Prove that

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

15. Solve $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$ using the properties of determinants.

16. Find minors and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$.

17. Solve the system of equations

$2x + 5y = 1$

$3x + 2y = 7$

18. If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}, \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then prove that $\Delta + \Delta_1 = 0$.

19. If the co-ordinates of the vertices of an equilateral triangle with sides of length 'a' are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, then prove that

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}.$$

20. If *x, y, z* are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then find the value of $x^{-1} + y^{-1} + z^{-1}$.

21. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

22. If $A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$, then verify that $A(adjA)=|A|I$. Also find A^{-1}.

23. If $\begin{vmatrix}4-x&4+x&4+x\\4+x&4-x&4+x\\4+x&4+x&4-x\end{vmatrix}=0$, then find value of x.

24. Evaluate $\begin{vmatrix}x+\lambda&x&x\\x&x+\lambda&x\\x&x&x+\lambda\end{vmatrix}$.

Section–D (5 Marks Questions)

25. If $a,b,c,$ are positive and unequal, show that value of the determinant $\Delta=\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}$ is negative.

26. Evaluate the following determinant.

(i) $\begin{vmatrix}1&3&5\\2&6&10\\31&11&38\end{vmatrix}$ (ii) $\begin{vmatrix}67&19&21\\39&13&14\\81&24&26\end{vmatrix}$

27. Evaluate: $\begin{vmatrix}\cos(x+y)&-\sin(x+y)&\cos 2y\\\sin x&\cos x&\sin y\\-\cos x&\sin x&-\cos y\end{vmatrix}$

28. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

29. Show that the ΔABC is an isosceles triangle if the determinant

$$\begin{vmatrix}1&1&1\\1+\cos A&1+\cos B&1+\cos C\\\cos^2 A+\cos A&\cos^2 B+\cos B&\cos^2 C+\cos C\end{vmatrix}=0$$

30. Prove that $\begin{vmatrix}b+c&q+r&y+z\\c+a&r+p&z+x\\a+b&p+q&x+y\end{vmatrix}=2\begin{vmatrix}a&p&x\\b&q&y\\c&r&z\end{vmatrix}$.

EXERCISE – 2: Basic Objective Questions

Section–A (Single Choice Questions)

1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x is

(a) 3 (b) ± 3

(c) ± 6 (d) 6

2. Value of determinant $\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$ is

(a) 37 (b) – 37

(c) 74 (d) – 74

3. If $\begin{vmatrix} k & 1 & -2 \\ 3 & k & 1 \\ -2 & 3 & -3 \end{vmatrix} = -37$ then k is equal to

(a) 2 (b) -2

(c) $\frac{13}{3}$ (d) None of these

4. If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$, then the values of x are

(a) $\frac{1}{2}, -3$ (b) $\frac{1}{2}, 3$

(c) $-\frac{1}{2}, 3$ (d) $-\frac{1}{2}, -3$

5. If $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$, where x, y are natural numbers, then x and y can be respectively,

(a) 1, 8 (b) 2, 4

(c) a and b both (d) None of these

6. The value of determinant

$\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of

(a) n (b) a

(c) x (d) none of these

7. If $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$, then the value of λ is:

(a) -1 (b) -2

(c) -3 (d) 4

8. The value of determinant $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$ is

(a) 0 (b) 1

(c) $\sin A \sin B$ (d) $\cos A \cos B \cos C$

9. The value of $\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$, is

(a) 8 (b) -8

(c) 400 (d) 1

10. Cofactors of 2nd row elements of the determinant $\begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$ are, respectively

(a) 16, 8, 4 (b) $-16, -8, 4$

(c) $-16, 8, 4$ (d) None of these

11. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and C_{ij} is cofactor of a_{ij} in Δ, then the value of Δ is given by

(a) $a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$

(b) $a_{11}C_{11} + a_{12}C_{21} + a_{13}C_{31}$

(c) $a_{21}C_{11} + a_{22}C_{12} + a_{23}C_{13}$

(d) $a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

12. Area (in sq units) of the triangle with vertices $A(1,2), B(5,-2)$ and $C(-3,8)$ is.

(a) 0 (b) 8

(c) 4 (d) 7

13. If the points $(a_1,b_1),(a_2,b_2)$ and (a_1+a_2, b_1+b_2) are collinear, then

(a) $a_1b_2 = a_2b_1$ (b) $a_1b_2 = -a_2b_1$

(c) $a_1a_2 = b_2b_1$ (d) $a_1a_2 = -b_2b_1$

14. Find a, if $\begin{bmatrix} 1 & a & 2 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ is a singular matrix.

(a) 1 (b) – 1

(c) 2 (d) – 2

15. Adjoint of matrix $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 1 & -3\\ -1 & 2 & 3\end{bmatrix}$ is

(a) $\begin{bmatrix}9 & -1 & -4\\ -3 & -4 & 5\\ 5 & -3 & -1\end{bmatrix}$ (b) $\begin{bmatrix}9 & -1 & -4\\ -3 & 4 & -5\\ 5 & -3 & -1\end{bmatrix}$

(c) $\begin{bmatrix}9 & -1 & -4\\ -3 & 4 & 5\\ 5 & -3 & -1\end{bmatrix}$ (d) None of these

16. If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then $adj(AB)$ is

(a) $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ (b) $\begin{bmatrix}d & b\\ c & a\end{bmatrix}$

(c) $\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$ (d) None of these

17. If $A=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$, then the value of $|adj\ A|$ is

(a) a^{27} (b) a^9
(c) a^6 (d) a^2

18. For any 2×2 matrix, if $A(adj\ A)=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$, then $|A|=$

(a) 20 (b) 100
(c) 10 (d) 0

19. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and $A(\text{adj } A)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then find the value of k.

(a) 2 (b) 1
(c) 0 (d) –1

20. If $A=\begin{bmatrix}3 & 4\\ 2 & 4\end{bmatrix}, B=\begin{bmatrix}-2 & -2\\ 0 & -1\end{bmatrix}$, then $(A+B)^{-1}$

(a) is skew-symmetric matrix.
(b) $=A^{-1}+B^{-1}$
(c) Does not exist
(d) None of these

21. If A is a square matrix such that $A^2=I$, then A^{-1} is equal to

(a) $A+I$ (b) A
(c) O (d) $2A$

22. Solve the following equations by the matrix method $2x-y=-2$, $3x+4y=3$.

(a) $x=5/11, y=12/11$
(b) $x=-5/11, y=12/11$
(c) $x=5/11, y=-12/11$
(d) $x=-5/11, y=-12/11$

23. Solve the following equations by the matrix method. $5x-y+4z=5, 2x+3y+5z=2$ and $5x-2y+6z=-1$.

(a) $x=3, y=2, z=2$
(b) $x=-3, y=2, z=-2$
(c) $x=3, y=2, z=-2$
(d) $x=3, y=-2, z=-2$

24. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is Rs. 60. The cost of 2dozen pencils, 4 dozen pens & 6 dozen erasers is Rs. 90 & the cost of 6 dozen pencils, 2dozen pens & 3 dozen erasers is Rs. 70. Find the cost of each item per dozen by using matrices.

(a) Cost of pencils = Rs. 8 per dozen, Cost of pens = Rs. 5 per dozen, Cost of erasers = Rs. 8 per dozen.
(b) Cost of pencils = Rs. 5 per dozen, Cost of pens = Rs. 8 per dozen, Cost of erasers = Rs. 5 per dozen.
(c) Cost of pencils = Rs. 5 per dozen, Cost of pens = Rs. 5 per dozen, Cost of erasers = Rs. 8 per dozen.
(d) Cost of pencils = Rs. 5 per dozen, Cost of pens = Rs. 8 per dozen, Cost of erasers = Rs. 8 per dozen.

25. Find the area of the triangle with vertices: $(3, 8), (-4, 2), (5, 1)$.

(a) 30 (b) 30.5
(c) 32 (d) 32.5

26. The adjoint of the matrix $A=\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$ is

(a) $\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}$ (b) $\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$

(c) $\begin{bmatrix}3 & 1\\ -4 & 2\end{bmatrix}$ (d) None of these

27. Write A^{-1} for $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$.

(a) $\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$ (b) $\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}$

(c) $\begin{bmatrix}3 & -5\\ -1 & 2\end{bmatrix}$ (d) None of these

Section–C (Case Study Questions)

Case Study–1

28. Three shopkeepers Salim, Vijay and Venket are using polythene bags, handmade bags and newspaper's envelope as carry bags. It is found that Salim, Vijay and Venket are using (20, 30, 40), (30, 40, 20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Salim, Vijay and Venket spent ₹ 250, ₹ 270 and ₹ 200 on these carry bags respectively.

Using the concept of matrices and determinants, answer the following questions.

(i) What is the cost of one polythene bag?

(a) ₹ 1 (b) ₹ 2

(c) ₹ 3 (d) ₹ 5

(ii) What is the cost of one handmade bag?

(a) ₹1 (b) ₹2

(c) ₹3 (d) ₹5

(iii) What is the cost of one newspaper envelope?

(a) ₹ 1 (b) ₹ 2

(c) ₹ 3 (d) ₹ 5

(iv) Keeping in mind the environmental conditions, which shopkeeper is better.

(a) Salim (b) Vijay

(c) Venket (d) None of these

Case Study–2

29. Area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by the determinant. $\Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$ Since, area is a positive quantity, so we always take the absolute value of the determinant Δ. Also, the area of the triangle formed by three collinear points is zero. Based on the above information, answer the following questions.

(i) Find the area of the triangle whose vertices are (- 2, 6), (3, - 6) and (1, 5).

(a) 30 sq. units (b) 35 sq. units

(c) 40 sq. units (d) 15.5 sq. units

(ii) If the points $(2,-3), (k,-1)$ and $(0,4)$ are collinear, then find the value of 4k.

(a) 4 (b) $\frac{7}{140}$

(c) 47 (d) $\frac{40}{7}$

(iii) If the area of a triangle ABC, with vertices $A(1,3), B(0,0)$ and $C(k,0)$ is 3 sq. units, then a value of k is.

(a) 2 (b) 3

(c) 4 (d) 5

(iv) Using determinants, find the equation of the line joining the points $A(1,2)$ and $B(3,6)$.

(a) $y=2x$ (b) $x=3y$

(c) $y=x$ (d) $4x-y=5$

Case Study–3

30. Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies and is denoted by M_{ij}. Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by $A_{ij}=(-1)^{i+j}M_{ij}$, where M_{ij} is minor of a_{ij}. Also, the determinant of a square matrix A is the sum of the products of the elements of any row (or column) with their corresponding cofactors. For example, if $A=[a_{ij}]_{3\times3}, then |A|=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$.

Based on the above information, answer the following questions.

(i) Find the sum of the cofactors of all the elements of $\begin{vmatrix}1 & -2\\ 4 & 3\end{vmatrix}$.

(a) 2 (b) –2

(c) 4 (d) 1

(ii) Find the minor of a_{21} *of* $\begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$.

(a) 3 (b) –3
(c) 39 (d) –39

(iii) In the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find the value of $a_{32}.A_{32}$.

(a) 27 (b) –110
(c) 110 (d) –27

(iv) If $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then find the value of $|\Delta|$.

(a) 26 (b) 28
(c) 72 (d) 46

Section–C (Assertion & Reason Type Questions)

31. Let A be the matrix given by $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Assertion: A^{-1} does not exist.

Reason: $|A| \neq 0$.

(a) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation for Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

32. **Assertion:** The determinant of matrix $A = [a_{ij}]_{5\times5}$, where $a_{ij} + a_{ji} = 0$ for all i and j is zero.

Reason: The determinant of a skew-symmetric matrix of odd order is zero.

(a) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation for Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

33. **Assertion:** If a, b, c, d are real numbers and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^3 = O$, then $A^2 = O$

Reason: For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have $A^2 - (a+d)A + (ad - bc)I = O$.

(a) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation for Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

34. **Assertion:** The inverse of singular matrix $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0, i \geq j$ is $B = [a_{ij}^{-1}]_{n\times n}$

Reason: The inverse of singular square matrix does not exist.

(a) Both Assertion and Reason are correct; Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct; Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Reason is correct but Assertion is incorrect.

35. Let A be a 2×2 Matrix.

Assertion: $|adj\ (adj\ A)| = |A|$

Reason: $|(adj\ A)| = |A|^{n-1}$.

(a) Both Assertion and Reason are correct; Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct; Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Reason is correct but Assertion is incorrect.

EXERCISE – 3: Previous Year Questions

1. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ **(Delhi 2016)**

2. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8,$ then find the value of x. **(AI 2016)**

3. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$ write the value of x. **(AI 2015)**

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$, write the value of $|AB|$. **(Delhi 2015C)**

5. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find x. **(AI 2014)**

6. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$. **(Delhi 2014)**

7. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$. **(AI 2014 C)**

8. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x. **(AI 2013)**

9. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x. **(Delhi 2013C)**

10. Evaluate: $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$. **(AI 2011)**

11. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of $3|A|$. **(AI 2011C)**

12. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ x & x & y \\ -3 & -3 & -3 \end{vmatrix}$. **(AI 2015)**

13. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k. **(Foreign 2014)**

14. Let A be a square matrix of order 3×3. Write the value of $|2A|$, where $|A| = 4$. **(AI 2012, Delhi 2011 C)**

15. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x) - f(x)$. **(Delhi 2015)**

16. Using properties of determinants, prove that following:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

(AI 2015, Foreign 2014)

17. Using properties of determinants, prove that following: $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1-a^3)^2$ **(Foreign 2015)**

18. Using properties of determinants, prove that

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2.$$

(Delhi 2015C)

19. Using properties of determinants, sole for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

(AI 2015C, 2011)

20. Using properties of determinants, prove that

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

(Delhi 2011)

21. Prove the following using properties of determinants:

$$\begin{vmatrix} a+b+c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

(Delhi 2014, 2012C, 2008)

22. Using properties of determinants, prove the following:

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2++z^2$$

(Delhi 2014)

23. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab+bc+ca+abc$$

(AI 2014, Delhi 2012)

24. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

(AI 2014, 2010C)

25. Using properties of determinants, show that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$ **(AI 2014, 2009)**

26. Using properties of determinants, prove that:

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

(Foreign 2014)

27. Using properties of determinants, prove that:

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2$$

(Foreign 2014)

28. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(bc+ca+ab)$$

(Delhi 2014C)

29. Using properties of determinants, prove the following:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$ **(AI 2014C, 2012)**

30. Show that $\Delta = \Delta_1$, where

$$\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1, \\ Cz & z^2 & 1 \end{vmatrix}, \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$$ **(AI 2014C)**

31. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

(AI 2013)

32. Using properties of determinants, prove the following:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

(AI 2013)

33. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

(Delhi 2013C)

34. Using properties of determinants, prove the following:

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

$$=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

(Delhi 2012, 2010C)

35. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$$

(Delhi 2012C, 2009)

36. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

(AI 2012C)

37. Using properties of determinants, prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

(Delhi 2011, 2010C)

38. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$$ **(AI 2011)**

39. Using properties of determinants, solve the following for x:

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0,\ a \neq 0$$ **(AI 2011)**

40. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x+y+z)$, and hence find the quotient.

(Delhi 2016)

41. If a, b, and c are all non-zero and

$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then prove that

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1=0$ **(Foreign 2016)**

42. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$. Find the value of k if $D(k, 0)$ is a point such that area of ΔABD is 3 square units.

(AI 2013C)

43. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2^{nd} row. **(Foreign 2015)**

44. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32}.A_{32}$. **(AI 2013)**

45. Write the cofactor of the element a_{32} for

$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$. **(Delhi 2012)**

46. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$, write the minor of element a_{22}.

(Delhi 2012)

47. In the interval $\frac{\pi}{2} < x < \pi$, find the value of x for which the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular.

(AI 2015C)

48. Find $(adj\ A,)$ if $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$. **(Delhi 2014C)**

49. If A is a square matrix of order 3 such that $|adj\ A| = 64$, find $|A|$. **(Delhi 2013C)**

50. If A is an invertible square matrix of order 3 and $|A| = 5$, then find the value of $|adj\ A|$.

(AI 2013C, 2011C)

51. For what value of x, is the given matrix

$A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ singular **(AI 2013C, 2008)**

52. For what value of x, the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix? **(Delhi 2012C)**

53. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ singular? **(Delhi 2011)**

54. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1} in terms of A.

(AI 2011)

55. For what value of x is $A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ a singular matrix? **(AI 2011C)**

56. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$. **(Delhi 2015)**

57. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A. $(adj\ A) = |A| I_3$. **(AI 2015)**

58. If $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identify matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}. **(Foreign 2015)**

59. Find the inverse of $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. **(Delhi 2015C)**

60. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$. **(AI 2015 C)**

61. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find $adj\ A$ and verify that $A(adj\ A) = (adj\ A)A = |A| I_3$, **(Foreign 2016)**

62. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value? **(Delhi 2016)**

63. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question? **(AI 2016)**

64. A coaching institute of English (Subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society? **(Foreign 2016)**

65. Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of linear equations:
$8x + 4y + 3z = 19;\ 2x + y + z = 5;$
$x + 2y + 2z = 7$ **(Delhi 2016)**

66. A shopkeeper has 3 varieties of pens ‘A’ ‘B’ and “C. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of ‘A’ variety, 3 pens of ‘B’ variety and 2 pens of ‘C’ variety for ₹ 60. While Shikha purchased 6 pens of ‘A’ variety, 2 pens of “B’ variety and 3 pens of ‘C’ variety for ₹ 70. Using matrix method, find cost of each variety of pen. **(AI 2016)**

67. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students with a total award money of ₹ 1,000. School Q wants to spend ₹ 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹ 600, using matrices, find the award money for each value. Apart from the above three values, suggest one more value for awards. **(Delhi 2014)**

68. A total amount of ₹ 7,000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$ respectively. The total annual interest from these three accounts is ₹ 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. **(Delhi 2014C)**

69. Using matrices, solve the following system of equations:
$x + y - z = 3;\ 2x + 3y + z = 10;$
$3x - y - 7z = 1$ **(AI 2012)**

70. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations $x + 2y + z = 4, -x + y + z = 0,$
$x - 3y + z = 4$ **(Delhi 2012C)**

71. Determine the product
$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations:
$x - y + z = 4;\ x - 2y - 2z = 9;$
$2x + y + 3z = 1$ **(Delhi 2012C, 2010C)**

72. Find A^{-1}, where $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}$.

Hence solve the system of equations
$x+2y-3z=-4;\ 2x+3y+2z=2;$
$3x-3y-4z=11.$ **(Delhi 2012C)**

73. Using matrix method, solve the following system of equations :
$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,$
$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;\ x,\ y,\ z\neq 0$ **(Delhi 2011)**

74. If $A=\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}$, find A^{-1} and hence solve the following system of equations:
$3x-4y+2z=-1, 2x+3y+5x=7$ and $x+z=2$ **(Delhi 2011C)**

75. If A and B are square matrices of the same order 3, such that $|A|=2$ and $AB=2I$, write the value of $|B|$. **(Delhi 2019)**

76. Using properties of determinates, find the value of x for which $\begin{vmatrix}4-x & 4+x & 4+x\\ 4+x & 4-x & 4+x\\ 4+x & 4+x & 4-x\end{vmatrix}=0$. **(AI 2019)**

77. Using properties of determinates, prove that
$\begin{vmatrix}1 & 1 & 1+3x\\ 1+3y & 1 & 1\\ 1 & 1+3z & 1\end{vmatrix}$
$=9(3xyz+xy+yz+zx)$. **(2018)**

78. If A is a square matrix of order 3 with $|A|=9$, then write the value of $|2\cdot adj\ A|$. **(AI 2019)**

79. Given $A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$, compute A^{-1} and show that $2A^{-1}=9I-A$. **(2018)**

80. If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x+y+z=6, x+2z=7,$
$3x+y+z=12$. **(Delhi 2019)**

81. If A is square matrix of order 3, such that $A(adj\ A)=10I$, then $|adj\ A|$ is equal to. **(AI 2020, Delhi 2020)**

82. If A is a 3×3 matrix such that $|A|=8$, then $|3A|$ equals. **(AI 2020, Delhi 2020)**

83. For $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ write A^{-1}. **(AI 2020)**

84. If a,b,c are p^{th}, q^{th} and r^{th} terms respectively of a G.P, then prove that

$\begin{vmatrix}\log a & p & 1\\ \log b & q & 1\\ \log c & r & 1\end{vmatrix}=0$. **(AI 2020)**

85. Solve the following system of equations by matrix method:

$x-y+2z=7$
$2x-y+3z=12$
$3x+2y-z=5$ **(AI 2020)**

Answer Key

EXERCISE-1:

Basic Subjective Questions

1. $\frac{1}{|A|}$

2. 9

3. Value of determinant

4. 8

5. 0

7. ± 6

8. ± 3

9. 0

10. $-7, 3$

11. $x^3 - x^2 + 2$

12. 0

13. $2x^3y^3z^3$

15. $a^2(a+z+x+y)$

16. $M_{11} = 3, M_{12} = 4, M_{21} = -2, M_{22} = 1$

$A_{11} = 3, A_{12} = -4, A_{21} = 2, A_{22} = 1$

17. $x = 3, y = -1$

20. -1

23. $-12, 0$

24. $\lambda^2(3x+\lambda)$

26. (i) 0

(ii) -43

27. 0

28. The numbers are 1, 2, 3

EXERCISE-2:

Basic Objective Questions

1. (c) **2.** (b) **3.** (a) **4.** (a) **5.** (c)

6. (a) **7.** (c) **8.** (a) **9.** (b) **10.** (c)

11. (d) **12.** (c) **13.** (a) **14.** (b) **15.** (c)

16. (c) **17.** (c) **18.** (c) **19.** (b) **20.** (d)

21. (b) **22.** (b) **23.** (c) **24.** (d) **25.** (b)

26. (c) **27.** (c)

28. **(i)** (a) **(ii)** (d) **(iii)** (b) **(iv)** (a)

29. **(i)** (d) **(ii)** (d) **(iii)** (a) **(iv)** (a)

30. **(i)** (a) **(ii)** (b) **(iii)** (c) **(iv)** (b)

31. (d) **32.** (a) **33.** (a) **34.** (d) **35.** (a)

EXERCISE-3:

Previous Year Questions

1. $\frac{1}{2}$ **2.** 2 **3.** -2 **4.** -28

5. -2 **6.** 1 **7.** 0 **8.** 2

9. 1 **10.** 0 **11.** 6 **12.** 0

13. 27 **14.** 32 **15.** $ax(3x+2a)$

19. $x=0,\ 3a$ **38.** $x=4$

39. $x=\frac{-a}{3}$ **42.** $k=\mp 2$

43. 3 **44.** 110 **45.** 11 **46.** -7

47. $\frac{2\pi}{3}$ **48.** $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$ **49.** ± 8

50. 25 **51.** $x=1$ **52.** $x=\frac{13}{15}$

53. $x=3$ **54.** $A^{-1}=\frac{1}{19}A$ **55.** -2

56. $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$ **57.** $\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

58. $A^{-1}=\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ **59.** $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

62. Monthly income of Aryan = Rs. 90000
Monthly income of Babbon = Rs. 120000
We are encouraged to save a part of money every month.

63. Amount invested by trust = Rs. 25000
Helping and caring nature

64. Monthly fees paid by each poor child and Rich child are respectively Rs. 200 and Rs. 1000
The coaching institute offers an unbiased chance for the development and enhancement of the weaker section of our society.

65. $x=1,\ y=2,\ z=1$

66. Cost of pen of variety A, B, and C are respectively Rs. 5, Rs. 8, Rs. 8

67. Award for discipline, Politeness & Punctuality are respectively Rs. 100, Rs. 200, and Rs. 300

68. 1125, 1125, 4750 **69.** $x=3,\ y=1,\ z=1$

70. $A^{-1}=\frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix};x=z=2,y=0$

71. $x=3,\ y=-2,\ z=-1$ **72.** $x=3,\ y=2,\ z=1$

73. $x=2,\ y=3,\ z=5$ **74.** $x=3,\ y=2,\ z=-1$

75. 4 **76.** $0,\ -12$ **78.** 648

79. $A^{-1}=\begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ 2 & 1 \end{bmatrix}$ **80.** $A^{-1}=\frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

$x=3,\ y=1,\ z=2$

81. 100 **82.** 216 **83.** $\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$

85. $x=2,\ y=1,\ z=3$