Chapter 5

Resonance

LEARNING OBJECTIVES

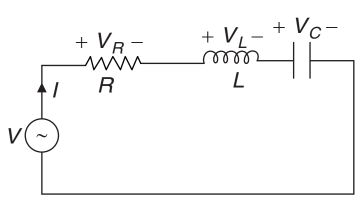
After reading this chapter, you will be able to understand:

- Series resonance
- Quality factor
- Frequency at which v_c is maximum
- Parallel resonance
- Filters
- Self-inductance
- Mutual inductance
- Coupling coefficient
- Magnetically coupled inductors in parallel
- Transformer coupling
- Laplace transform
- Network synthesis
- *L–C* emittance function
- Power relations in ac circuit

Introduction

Resonance in electrical circuits consisting of passive and active elements represents a particular state of the circuit when the current or voltage in the circuit is maximum or minimum with respect to the magnitude of excitation at a particular frequency. The circuit impedance being either minimum or maximum at the power factor unity.

Series Resonance



The circuit is said to be in resonance if the current is in phase with the applied voltage.

The impedance $Z = R + j\omega L - \frac{1}{j\omega C}$

$$Z = R + j\left(\omega L - \frac{1}{\omega c}\right)$$

At, resonance, the impedance is purely resistive

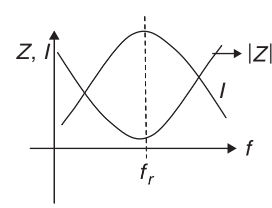
i.e., $\omega L - \frac{1}{\omega C} = 0$

$$\omega^2 = \frac{1}{LC}$$

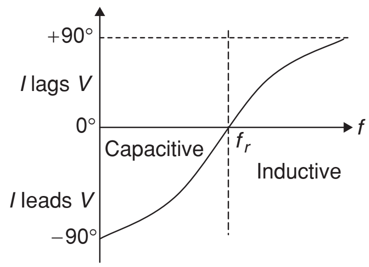
$$\omega = \frac{1}{\sqrt{LC}}$$

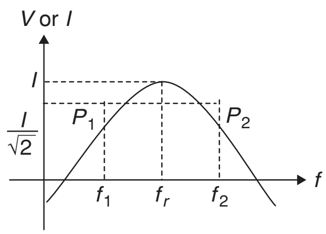
$f_r = \frac{1}{2\pi\sqrt{LC}}$ is the resonant frequency in Hz

At resonance the current, $I = \frac{V}{R}$, i.e., the current is maximum at resonance and the impedance is minimum.



- At zero frequency X_C and, Z are infinitely large, and X_L is zero because at zero frequency, the capacitor acts as an open circuit.
- As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C.
- At resonant frequency f_r, $X_C = X_L$ and $Z = R$.
- At frequencies above the resonant frequency f_r, X_L is larger than X_C, causing Z to increase.
- The phase angle as a function of frequency is shown in figure.





The bandwidth is the range of frequencies over which the current is equal to 70.7% of its value at the resonant frequency.

Bandwidth, $BW = f_2 - f_1$

Where f_2 and f_1 are cutoff frequencies or half power frequencies. At $f = f_1$ or f_2, the net reactance is R

i.e., $\frac{1}{\omega_1 C} - \omega_1 L = R$

$$\frac{1}{\omega_2 C} - \omega_2 L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \frac{1}{\omega_2 C} - \omega_2 L$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left[\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right]$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$\omega_r^2 = \omega_1 \omega_2$$

$$\omega_r = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

Bandwidth, $BW = \frac{R}{2\pi L}$

$$f_1 = f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \frac{R}{4\pi L}$$

The quality factor (Q-factor) is the ratio of the reactance of the coil to its resistance at resonant frequency.

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q}$$

$$Q = \frac{\omega L}{R} = \frac{1}{\omega C R}$$

$$\boxed{Q = \frac{f_r}{BW}}$$

Let V be the applied voltage across RLC circuit, at resonance, the current is I, the voltage across L is $V_L = IX_L = \frac{V}{R}\omega_r L$

$V_L = QV$

Similarly $V_C = VQ$

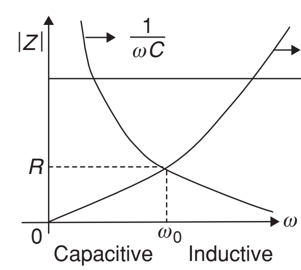
i.e., $Q = \frac{V_L}{V}$ or $\frac{V_C}{V}$

Quality Factor

A quality factor or figure of merit can be assigned to a component or to a complete circuit. It is defined as

$$Q = 2\pi\left(\frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}\right)$$

$$\boxed{Q = \frac{|X_L|}{R} = \frac{|X_C|}{R} = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}}$$



- For $\omega < \omega_0 \Rightarrow$ series RLC circuit behaves like (RC ckt) capacitive circuit
  $\Rightarrow$ Leading power factor
  $\Rightarrow V_C = QV\angle -90°$
- for $\omega = \omega_0 \Rightarrow$ Resistive nature; $pf = 1$
- for $\omega > \omega_0 \Rightarrow$ inductive nature
  $\Rightarrow$ Lagging power factor
  $V_L = I \cdot Z_L$

$V_L = QV \angle 90°$

Where $Q = \frac{\omega_0 L}{R}$

- **At resonance**:

$|V_L| = |V_C|$ and these are 180° out of phase

Solved Examples

Example 1: In a series RLC circuit, the Q factor at resonance is 80. If all the component values are doubled, then the new Q factor is

(A) $Q' = 160$ (B) $Q' = 40$
(C) $Q' = 80°$ (D) None of the above

Solution: (B)

For a series RLC circuits

$$Q = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}$$

Given $R' = 2R$

$L' = 2L$

$C' = 2C$

$$Q' = \frac{Q}{2} = 40.$$

Example 2: In a series RLC circuit, R = 8 Ω, X_L = 16, X_C = 16 and V_{in} = 80 V. The voltage across the capacitor is

(A) $V_C = 160 \angle 90°$ (B) $V_C = 40 \angle 90°$
(C) $V_C = 160 \angle -90°$ (D) $V_C = 80 \angle -90°$

Solution: (C)

$$V_C = QV \angle -90°$$

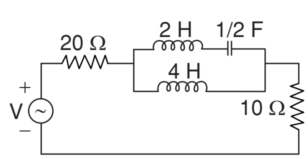
$$Q = \frac{|X_L|}{R} = \frac{16}{8} = 2$$

Given $V_{in} = V = 80$ V

$V_C = 2 \times 80 \angle -90°$

$= 160 \angle -90°.$

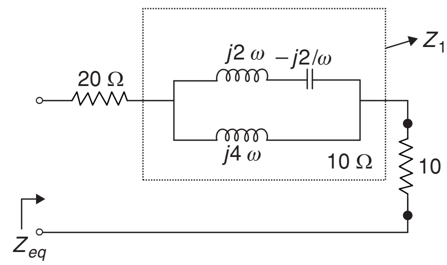
Example 3: For the circuit shown in the figure determine the resonant frequency (f_0).



(A) $f = 0$ Hz (B) $f = \frac{1}{2\pi}$ Hz

(C) $f = \frac{1}{\pi}$ Hz (D) None of the above

Solution: (B)



$$Z_{eq} = 20 + Z_1 + 10$$

$$Z_1 = (j4\omega) \parallel [j(2\omega - 2/\omega)]$$

$$= \frac{j4\omega j\left(2\omega - \frac{2}{\omega}\right)}{j4\omega + j2\omega - \frac{j2}{\omega}}$$

$$= \frac{j4\omega\left[2\omega - \frac{2}{\omega}\right]}{6\omega - \frac{2}{\omega}} \Rightarrow \frac{j4\omega[2\omega^2 - 2]}{6\omega^2 - 2}$$

$$= \frac{j4\omega[\omega^2 - 1]}{3\omega^2 - 1}$$

$$Z_{eq} = 20 + \frac{j\omega(\omega^2 - 1)\times 4}{3\omega^2 - 1} + 10$$

At resonance imaginary part equal to zero.

At $\omega = \omega_r$

$$\frac{\omega(\omega^2 - 1)\times 4}{3\omega^2 - 1} = 0$$

$$\Rightarrow \omega_r^2 - 1 = 0$$

$$\omega_r = 1$$

$$2\pi f_r = 1$$

$$f_r = \frac{1}{2\pi} \text{ Hz.}$$

Example 4: A series resonant circuit has L = 10 mH and C = 100 μF. The required R for the BW 16 Hz is

(A) R = 16 Ω (B) R = 0.16 Ω
(C) R = 160 Ω (D) R = 1.6 Ω

Solution: (B)

For a series RLC circuit characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$BW = \frac{R}{L}$$

$$\frac{R}{10\times 10^{-3}} = 16 \Rightarrow R = 0.16\ \Omega$$

Selectivity The selectivity of a resonating circuit is defined by the ratio of f_0 (the resonance frequency) to the bandwidth of the circuit.

$$\text{Selectivity} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

Frequency at which V_C is maximum V_C and V_L are not maximum at resonant frequency in case of series RLC resonance but at other frequency value. This can be obtained by

$$V_C = I \cdot X_C$$

$$= \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} \frac{1}{\omega c}$$

Where,

$$I = \frac{V}{|Z|}$$

The frequency at which V_C is maximum can be obtained by differentiate V_C wrt to ω. i.e.,

$$\frac{dV_c^2}{d\omega} = 0$$

$$\frac{dV_c^2}{d\omega} = 0 \quad \Rightarrow \quad \frac{-V\left[2\omega_0(RC)^2 + 2\left(\omega_0^2 LC - 1\right)\cdot 2\omega_0 LC\right]}{\left[(\omega_0 RC)^2 + \left(\omega_0^2 LC - 1\right)^2\right]^2}$$

But $V \neq 0$,

$2\omega_0^2 L^2 C + CR^2 - 2L = 0$

By simplification we get

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

Where $f_0 \Rightarrow$ at which voltage across capacitor is maximum.

The frequency at which V_L is maximum $V_L = I \cdot X_L$

$$= \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}\,\omega L$$

The frequency at which V_L is max is given by

$$\frac{dV_L^2}{d\omega} = 0$$

By the simplification we get

$2\omega^2 LC - \omega^2(RC)^2 - 2 = 0$

$\omega^2\,[2LC - (RC)^2] = 2$

$$\omega^2 = \frac{2}{2LC - (RC)^2}$$

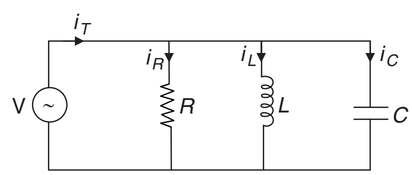
$$\boxed{f_{0L} = \frac{1}{2\pi} \times \frac{1}{\sqrt{LC - \frac{(RC)^2}{2}}}}$$

Where $f_{oL} \Rightarrow$ frequency at which the voltage across the inductor is max.

Parallel Resonance

A circuit consisting of a parallel connection of R, L and C is called a second order parallel resonant circuit, parallel resonance circuit is also called anti-resonance circuit. It acts as a band-reject filter.

Parallel Resonance



The admittance, $Y = G + \frac{1}{j\omega L} + j\omega C$

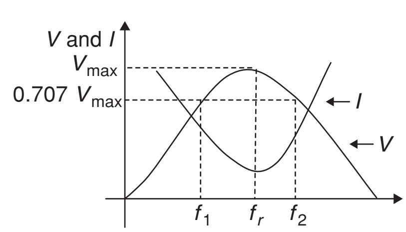
At resonance 'Y' is purely real, i.e.,

$$j\left(\omega C - \frac{1}{\omega L}\right) = 0$$

$$\Rightarrow \quad \omega C - \frac{1}{\omega L} = 0$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\ \text{Hz}$$



At resonance, the current is minimum and the impedance is maximum.

At lower half power frequency ω_1,

$$\omega_1 C - \frac{1}{\omega_1 L} = \frac{-1}{R}$$

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

At upper half power frequency ω_2,

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth, $BW = \omega_2 - \omega_1 = \dfrac{1}{RC}$

The quality factor $Q = \dfrac{\omega_r}{BW}$

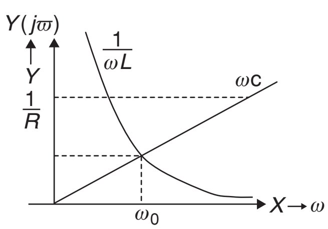
$$Q = \omega_r RC$$

$$Q = \frac{R}{\omega_r L}$$

At resonance $I_L = I_r Q\angle -90°$

$$I_C = I_r Q\angle 90°$$

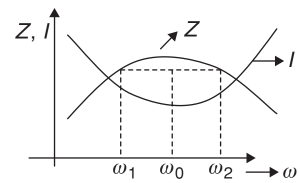
The resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ Hz



1. Below resonance the circuit acts like an RL circuit ($\omega < \omega_o$), i.e., lagging power factor.
2. Above resonance the circuit acts like an RC circuit ($\omega > \omega_o$), i.e., leading power factor.
3. At $\omega = \omega_0$ the circuit acts like an resistive nature.
4. $BW = \dfrac{1}{RC}$.
5. Quality factor $Q = R\sqrt{\dfrac{C}{L}}$
6. $\xi = \dfrac{1}{2Q}$

Note: Impedance of parallel resonant circuit is maximum at resonance. Current of parallel resonance circuit is minimum at resonance. i.e., $Z_{in} = R$ at $\omega = \omega_o$ is maximum.

$$\frac{I}{\omega} = \omega_o \frac{V_s}{R} \Rightarrow \text{minimum}$$



Examples 6: A series resonant circuit has an inductive reactance of 1 kΩ, a capacitive reactance of 1 kΩ and a resistance of 0.1 Ω. If the resonant frequency is 10 MHz, then the bandwidth of the circuit will be

(A) 1 kHz (B) 10 kHz
(C) 1 MHz (D) 0.1 kHz

Solution: (A)

$$BW = \frac{f_0}{Q} = \frac{f_0}{\left(\dfrac{\omega_0 L}{R}\right)}$$

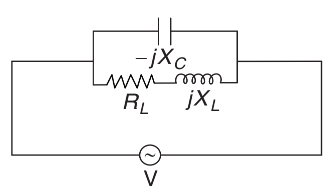
$$BW = \frac{0.1 \times 10 \times 10^6}{1000} = 1\text{ kHz}$$

Example 7:
Assertion(A): A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it.
Reason(R): At resonance, the power factor of the circuit is zero.

(A) Both A and R are true and 'R' is the correct explanation of A.
(B) Both 'A' and 'R' are true but 'R' is NOT the correct explanation of A.
(C) 'A' is true but 'R' is false.
(D) 'A' is false but 'R' is true.

Soltuion: (C)
At resonance the voltage across the circuit is in phase with the current through it and the power factor is '1'.
$\cos\phi = 1 \Rightarrow$ unity power factor

Example 8:



Find the resonant frequency for the tank circuit shown in figure

Solution:
The admittance is given by

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{-jX_C} = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R_L}{RL^2 + X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

At resonance Y is purely real

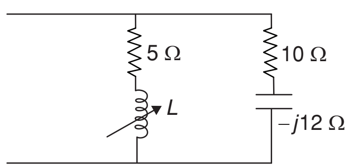
$$\therefore \quad \frac{1}{X_C} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\omega C = \frac{\omega L}{R_L{}^2 + \omega^2 L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

Example 9: Find the value of L at which the circuit resonates at a frequency of 1000 rad/sec in the circuit shown in figure.



Solution:

$$Y = \frac{1}{10 - j12} + \frac{1}{5 + jX_L}$$

$$Y = \frac{10 + j12}{10^2 + 12^2} + \frac{5 - jX_L}{25 + X_L^2}$$

$$Y = \frac{10}{10^2 + 12^2} + \frac{5}{25^2 + X_L^2} + j\left[\frac{12}{10^2 + 12^2} - \frac{X_L}{25 + X_L^2}\right]$$

At resonance Y is purely real, i.e.,

$$\frac{X_L}{25 + X_L^2} = \frac{12}{10^2 + 12^2}$$

$X_L{}^2 - 20.3X_L + 25 = 0$

$X_L = 18.98\ \Omega$ or $1.32\ \Omega$

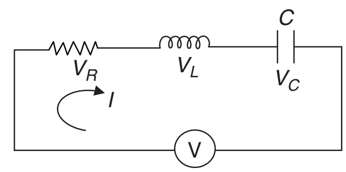
$X_L = \omega L = 18.98\ \Omega$ or $1.32\ \Omega$

$$L = \frac{18.98}{1000} \text{ or } \frac{1.32}{1000}$$

$L = 18.98$ mH or 1.32 mH

Example 10: A series RLC circuit is supplied by 220 V; 50 Hz; at resonance, the voltage across the capacitor = 550 V, $I = 1$ A; determine R, L and C.

Solution:



At resonance $X_L = X_C$

$$I = \frac{V}{R} = \frac{220}{R}$$

$$\Rightarrow R = 220\ \Omega$$

$$V_C = I_0 X_C$$

$$550 = 1 \cdot \frac{1}{\omega_0 C}$$

$$C = \frac{1}{550 \times 2\pi \times 50} = 5.75\ \mu\text{F}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{C}\left[\frac{1}{2\pi\, fr}\right]^2$$

$L = 1.75$ H

Example 11: A series RLC circuit consists of a 50 Ω resistance, 0.2 H inductance and 10 μF capacitor with an applied voltage of 20 V. Determine the resonant frequency. Find the Q-factor of the circuit. Compute the lower and upper frequency limits and also find the band width of the circuit

Solution:

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{0.2 \times 10 \times 10^{-6}}}$$

$= 112.5$ Hz

Quality factor $Q = \dfrac{\omega_r L}{R}$

$$= \frac{2\pi \times 112.5 \times 0.2}{50}$$

$= 2.83$

Lower frequency limit $f_1 = f_r - \dfrac{R}{4\pi L}$

$$= 112.5 - \frac{50}{4 \times \pi \times 0.2}$$

$= 92.6$ Hz

Upper frequency limit

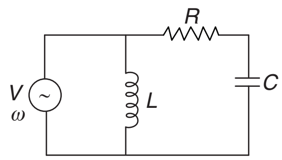
$$f_2 = f_r + \frac{R}{4\pi L} = 112.5 + \frac{50}{4\pi \times 0.2}$$

$$= 112.5 + 19.89$$

$$= 132.39 \text{ Hz}$$

Bandwidth, $BW = f_2 - f_1 = 39.79$ Hz

Example 12: Consider the following circuit



For what value of ω, the circuit shown above exhibits unity power factor?

(A) $\frac{1}{\sqrt{LC}}$

(B) $\frac{1}{\sqrt{LC + R^2C^2}}$

(C) $\frac{1}{\sqrt{LC - R^2C^2}}$

(D) $\frac{1}{RC}$

Solution: (C)
For unity power factor, imaginary part of impedance should be zero.

$$Z_{eq} = \frac{j\omega L\left(R - \frac{j}{\omega c}\right)}{R + j\omega L - \frac{j}{\omega c}}$$

$$Z_{eq} = \frac{j\omega l\left(R - \frac{j}{\omega c}\right)}{\left(R + j\omega L - \frac{j}{\omega c}\right)} \times \frac{\left(R - j\omega L + \frac{j}{\omega c}\right)}{\left(R - j\omega l + \frac{j}{\omega c}\right)}$$

At resonance imaginary part equal to zero

$$R^2 - \frac{L}{C} + \frac{1}{\omega^2 C^2} = 0$$

By simplification we get

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC - R^2C^2}}}$$

Filters The general transfer functions for second order filters:

	Filters		Transfer Function
1	LPF	$\Rightarrow$	$\frac{P}{s^2 + as + b}$
2	HPF	$\Rightarrow$	$\frac{Ps^2}{s^2 + as + b}$
3	BPF	$\Rightarrow$	$\frac{Ps}{s^2 + as + b}$
4	BSF	$\Rightarrow$	$\frac{Ps^2 + q}{s^2 + as + b}$
5	APF	$\Rightarrow$	$\frac{s^2 + Ps + q}{s^2 + as + b}$

Coupled Circuits

Self Inductance

When a current changes in a circuit, the magnetic flux linking the same circuit changes (and vice-versa) and an emf is induced in the circuit.

This induced emf is proportional to the rate of change of current.

$$v = L \cdot \frac{di}{dt} \tag{1}$$

Where

$v \rightarrow$ induced voltage

$\frac{di}{dt} \rightarrow$ rate of change of current

$L \rightarrow$ self inductance

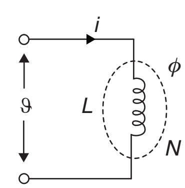
Inductance also expressed as

$$L = \frac{N\phi}{I}$$

$N \rightarrow$ no. of turns in the circuit.
$\phi \Rightarrow L$ flux linkage

$$v = L \cdot \frac{d\left(\frac{N\phi}{L}\right)}{dt}$$

$$= N \cdot \frac{d\phi}{dt} \tag{2}$$

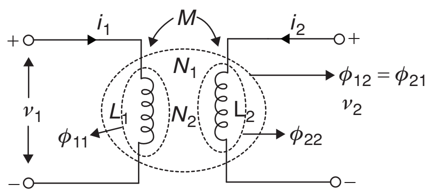


Equate the equation (1) and (2)

$$L \cdot \frac{di}{dt} = N \cdot \frac{d\phi}{di}$$

$$\boxed{L = N \cdot \frac{d\phi}{di}}$$

Mutual Inductance



The Induced voltage of coil-2 is given by

$$V_{L2} = N_2 \cdot \frac{d\phi_{12}}{dt} \qquad (3)$$

ϕ_{12} is related to the current of coil-I and the induced voltage is proportional to the rate of change of i_1.

$$V_{L_2} = M \cdot \frac{di_1}{dt} \qquad (4)$$

(3) = (4)
We get

$$\boxed{M = N_2 \cdot \frac{\phi_{12}}{i_1} = N_1 \cdot \frac{\phi_{21}}{i_2}}$$

Coupling Co-efficient

The amount of magnetic coupling is expressed by co-efficient of coupling (k)

$$K = \frac{\text{useful flux}}{\text{total flux}} = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$\boxed{M = K\sqrt{L_1 L_2}}$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

For ideal circuits $K = 1$ and practical circuits,range of K is $0 < K < 1$

Leakage factor

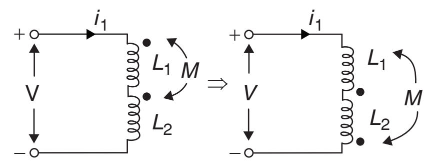
$$\text{Leakage factor} = \frac{\text{Total flux}}{\text{Useful flux}} = \frac{1}{K}$$

Dot conventions

1. When either both the currents are entering at dotted terminal or leaving the dotted terminal, mutual inductance will be added.
2. If one current is entering and other is leaving the dotted terminal or vice-versa then mutual inductance will be substracted.

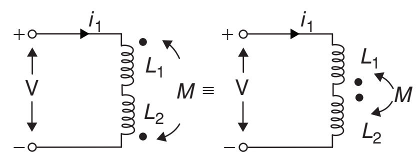
Series Connection of Coupled Coils

1.



$$\boxed{\text{Leq} = L = L_1 + L_2 + 2\,M} \Rightarrow \text{Aiding}$$

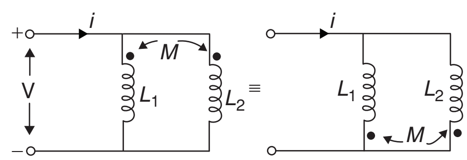
2.



$$\boxed{\text{Leq} = L = L_1 + L_2 - 2\,M} \Rightarrow \text{Opposition}$$

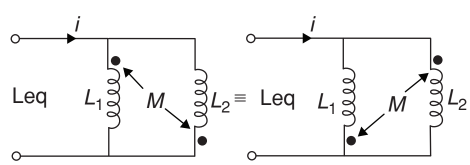
Magnetically Coupled Inductors in Parallel

1. Magnetically aiding



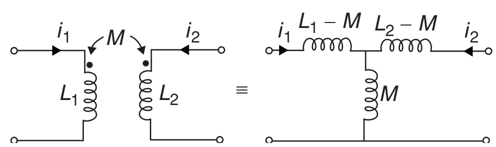
$$\boxed{\text{Leq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2\,M}}$$

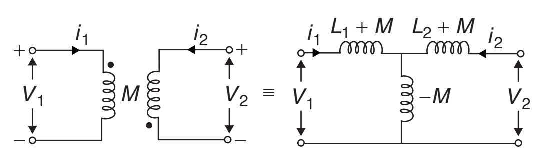
2. Magnetically opposition



$$\boxed{\text{Leq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2\,M}} \quad \Rightarrow$$

Equivalent circuit:

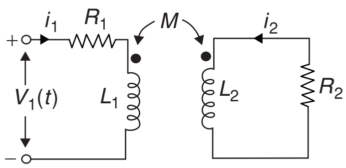




Transformer Coupling

Two inductors with self inductances L_1 and L_2, mutual inductance M, and coefficient of coupling K, are shown in figure, with dot convention

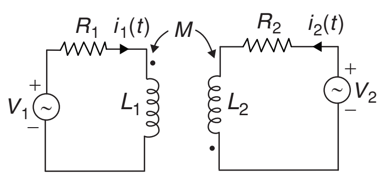
1.



$$V_1(t) = R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$0 = R_2 i_2(t) + L_2 \frac{di_2}{dt} + M \frac{di_1(t)}{dt}$$

2.

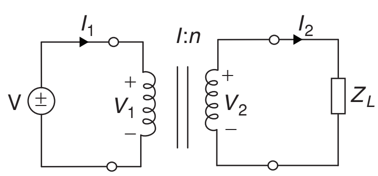


$$V_1 = R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$

$$V_2 = R_2 i_2(t) + L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$

Ideal Transformer

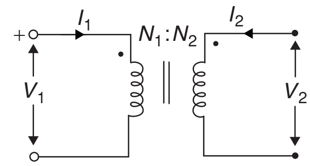
An ideal transformer is a unity coupled, lossless transformer in which the primary and secondary coils have infinite self inductances.



$$V_1 = N_1 \frac{d\phi}{dt} = v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{n}{1} = \frac{I_1}{I_2}$$

1. When $n = 1$, generally called the transformer an isolation transformer.
2. $n > 1 \Rightarrow$ step up transformer ($V_2 > V_1$)
3. $n < 1 \Rightarrow$ step-down transformer ($V_2 < V_1$)



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{-I_1}{I_2}$$

The input impedance as seen by the source is

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{n^2} \cdot \frac{V_2}{I_2}$$

$$\boxed{Z_{in} = \frac{Z_L}{n^2}}$$

$$\boxed{Z_{in} = Z_L \left(\frac{N_1}{N_2}\right)^2}$$

Examples 13: Two couples coils have self inductances L_1 = 15 mH and L_2 = 18 mH. The coefficient of coupling (k) being 0.75 in the air, find voltage in the second coil and flux of first coil provided the second coil has 500 turns and the circuit current is given by $i_1 = 5.\ \sin 314t$ A.

Solution: $M = K\sqrt{L_1 K_2}$

$$= 0.75\sqrt{15 \times 18} \times 10^{-3}$$

$$= 12.32 \times 10^{-3} \text{ H}$$

The voltage induced in the second coil is

$$V_{L_2} = M \cdot \frac{di_1}{dt} = 12.32 \times 10^{-3} \cdot \frac{d}{dt}(5 \sin 314t)$$

$$= 12.32 \times 10^{-3} \times 5 \times 314 \times \cos 314t$$

$$V_{L2} = 19.34 \cos 314t \text{ V}$$

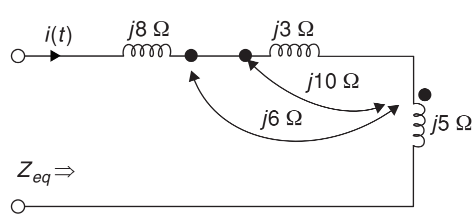
$$M = \frac{N_2 \phi_{12}}{i_1}$$

$$M = \frac{500 \times k \times \phi_1}{i_1}$$

$$\phi = \frac{12.32 \times 10^{-3} \times 5 \sin 314t \times 1}{0.75 \times 500}$$

$$\phi_1 = 16.42 \times 10^{-5} \sin 314t$$

Example 14: Impedance z as shown in figure is



(A) $j29\ \Omega$ (B) $j9\ \Omega$
(C) $j19\ \Omega$ (D) $j24\ \Omega$

Solution:

$Z_{eq} = L_1 + L_2 + L_3 \pm 2M_{12} \pm 2M_{13} \pm 2M_{23} \ldots$

$\therefore Z_{eq} = j8 + j3 + j5 + 2 \times j10 - 2 \times j6$

$= j16 + j8 = j24\ \Omega$

Example 15: Two coils have self-inductances of 0.09 H and 0.01 H and a mutual inductance of 0.015 H. The coefficient of coupling between the coil is

(A) 0.06 (B) 0.5
(C) 1.0 (D) 0.05

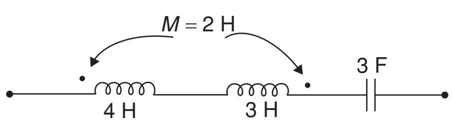
Solution:

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.015}{\sqrt{0.09 \times 0.01}}$$

$$= \frac{0.015}{3 \times 10^{-2}}$$

$$= 0.5$$

Example 16: The resonant frequency of the given series circuit is



(A) $f = \frac{1}{4\pi}$ Hz (B) $f = \frac{1}{3\pi}$ Hz

(C) $f = \frac{1}{6\pi}$ Hz (D) $f = \frac{1}{2\sqrt{3}\pi}$ Hz

Solution:

$$f = \frac{1}{2\pi\sqrt{L_{eq}c}}$$

$$\text{Leq} = L_1 + L_2 \pm 2\,M$$

$$\text{Leq} = 4 + 3 - 2 \times 2 = 3\ \text{H}.$$

$$f = \frac{1}{2\pi\sqrt{3 \times 3}}$$

$$f = \frac{1}{6\pi}\ \text{Hz}$$

Network Functions and Laplace Transform

Laplace Transform

Let $f(t)$ be a time function which is zero for $t \le 0$ and which is arbitrarily defined for $t > 0$.

Then the Laplace transform of $f(t)$, is defined by

$$L\{f(t)\} = F(s) = \int_{o^+}^{\infty} f(t)e^{-st} \cdot dt$$

In general

$$\boxed{F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \cdot dt}$$

The inverse Laplace transform can also be expressed as an integral

$$L^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma_o - j\infty}^{\sigma_o + j\infty} F(s) \cdot e^{st} ds.$$

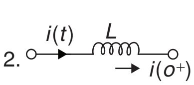
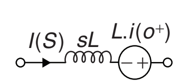
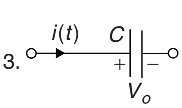
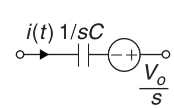
Table 1 *Laplace transform pairs*

$f(t)$	$F(s)$
1. 1	$\frac{1}{s}$
2. $Au(t)$	$\frac{A}{s}$
3. At	$\frac{A}{s^2}$
4. e^{-at}	$\frac{1}{s+a}$
5. $t \cdot e^{-at}$	$\frac{1}{(s+a)^2}$
6. sin at	$\frac{a}{s^2+a^2}$
7. cos at	$\frac{s}{s^2+a^2}$
8. $\frac{df(t)}{dt}$	$sF(s) - f(o^+)$
9. $\int_o^t f(t) \cdot dt$	$\frac{F(s)}{s}$
10. $f(t-a)$	$e^{-aS} \cdot F(s)$
11. $\int f_1(\tau) \cdot f_2(t-\tau) \cdot d\tau$	$F_1(s) \cdot F_2(s)$

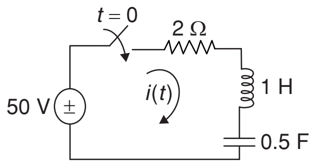
Initial-value and find-value theorems

1. $\underset{t \to o^+}{\text{Lt}}\ f(t) = f(o^+) = \underset{S \to \infty}{\text{Lim}}\{s \cdot F(s)\}$
2. $\underset{t \to \infty}{\text{Lt}}\ f(t) = f(\infty) = \underset{S \to o}{\text{Lim}}\{s \cdot F(s)\}$

Table 2 *Circuits in the S-domain*

	Time domain	S-domain	S-domain voltage
1.			R.I(s)
2.			sLI(s)–L.i(o^+)
3.			$\frac{I(s)}{sC} + \frac{V_o}{s}$

Examples 17: In the RLC circuit shown in figure, there is no initial charge on the capacitor. If the switch is closed at $t = 0$, the resultant current is



(A) $i(t) = j25 \cdot e^{-t}.\cos t$ A (B) $i(t) = 50 \cdot e^{-t} \sin t$ A
(C) $i(t) = 25 \cdot e^{-t} \sin 2t$ A (D) None of the above

Solution: (B)
Given $i(o^+) = 0$ and $V_c(o^+) = 0$
For $t > 0$ apply KVL
The time domain equation of the given circuit is

$$R \cdot i(t) + L \cdot \frac{di(t)}{dt} + \frac{1}{c}\int_0^t i(t)dt = V. \tag{5}$$

Convert Eqn (5) into LPF

$$R \cdot I(s) + sL \cdot I(s) + \frac{1}{sC} \cdot I(s) = \frac{V}{s}$$

$$\left(2 + S + \frac{2}{s}\right) \cdot I(s) = \frac{50}{s}$$

$$I(s) = \frac{50}{s^2 + 2s + 2}$$

Apply inverse Laplace transform to $I(s)$

$$i(t) = j25\{e^{-1-jt} - e^{-1+jt}\}$$

$$= 50 \cdot e^{-t} \cdot \sin t \text{ A}$$

Network Synthesis

Consider a network as shown in figure

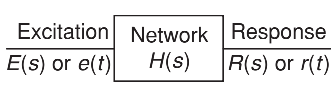


Figure 1 General Network

The system function is defines as

$$\Rightarrow \quad H(s) = \frac{R(s)}{E(s)}$$

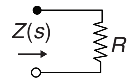
Example:

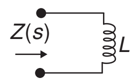
1. Driving point impedance $Z(s) = \dfrac{V(s)}{I(s)}$

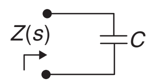
2. Driving point admittance $V(S) = \dfrac{I(s)}{V(s)}$

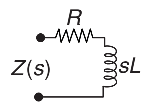
Note: Immitance = Impedance (or) admittance

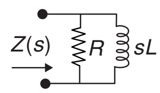
Driving point immitance of simple networks

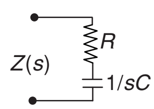
1.  $\Rightarrow Z(s) = R;\ Y(s) = \dfrac{1}{R}$

2.  $\Rightarrow Z(s) = Ls;\ Y(s) = \dfrac{1}{sL}$

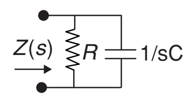
3.  $\Rightarrow Z(s) = \dfrac{1}{sC};\ Y(s) = sC$

4.  $\Rightarrow Z(s) = R + Ls;\ Y(s)\dfrac{1}{R + sL}$

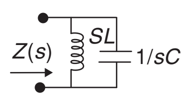
5.  $\Rightarrow Z(S) = \dfrac{RLs}{R + Ls};\ y(s) = \dfrac{\frac{1}{L} + \frac{1}{R}s}{s}$

6.  $\Rightarrow Z(s) = R + 1/sC$

$$Y(s) = \frac{sC}{1 + RCs}$$

7.  $\Rightarrow Z(S) = \dfrac{1}{Cs + \frac{1}{R}}$

$$Y(s) = \frac{1}{R} + sC$$

8.  $\Rightarrow Z(s) = \dfrac{sL}{1 + s^2LC}$

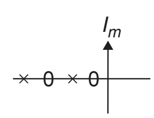
$$Y(s) = \frac{1 + s^2LC}{sL}$$

R-L Driving point impedance The impedance function is of the form

$$Z(s) = K_o + K_v s + \sum_{i=1}^{n} \frac{K_i}{s + \sigma_i}$$

Properties:

1. Poles and zeros of an R-L impedance lie on σ axis and alternate

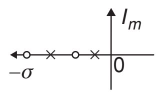


2. The residues are real and negative.
3. The singularity closest to the origin is a zero.

R-C driving point impedance The R-C impedance function has the general form $Z(s) = \dfrac{K_o}{s} + K_v + \sum_{i=1}^{n} \dfrac{K_i}{s + \sigma_i}$

Properties:

1. The poles and zeroes lie on the negative real zxis and alternate.



2. The residues of the poles, K_i, are real and +ve.
3. R-C impedance ⇔ R-L admittance.
4. The singularity nearest the origin being a pole.

L-C immitance function The general expansion is given by

$$Z(S) = \frac{K_o}{s} + \sum_{i=1}^{n} \frac{2K_i s}{s^2 + w_i^2} + \cdots + K_v s$$

Properties:

1. $Z(s)$ or $Y(s)$ is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the $j\omega$ axis and alternate.
3.

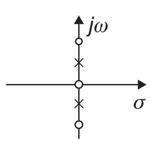
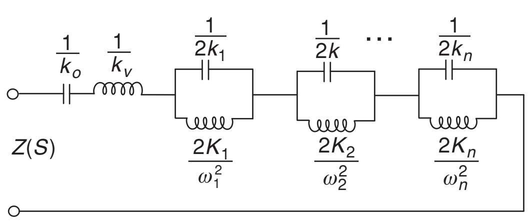


Figure 2 L-C pole-zero realization

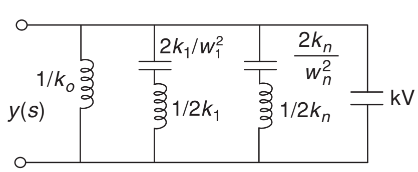
4. The highest power of the numerator and denominator differ by unity.
5. The lowest power of the numerator and denominator differ by unity.
6. There is either a pole or a zero at the origin and at infinity.

Network Function Have Four Canonical Forms

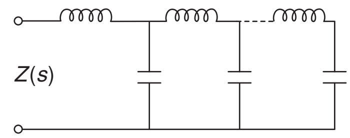
Foster Form I



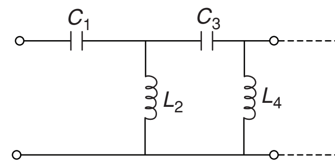
Foster form II



Cauer form I



Cauer from II

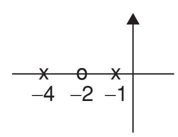


Examples 15: The network function

$F(s) = \dfrac{(s+2)}{(s+1)(s+4)}$ represents an

(A) R-L impedance
(B) R-C impedance
(C) R-C impedance and an R-L admittance
(D) R-C admittance and R-L impedance

Solution: (C)
Pole zero realization:

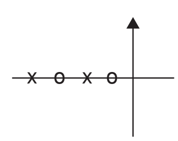


⇒ pole nearby origin so R-C impedance
⇒ R-C impedance ⇔ R-L admittance and vice-versa.

Example 18: The first critical frequency nearest the origin of the complex frequency plane for an R-L driving point impedance function will be

(A) A zero in the left-half plane
(B) A zero in the right-half plane
(C) A pole in the LH-plane
(D) Either a pole or zero in the LH-plane

Solution: (A)
For R-L impedance function



∴ Critical frequency is a zero nearby origin in the LH-plane.

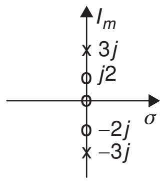
Example 19: Driving point impedance $Z(S) = \dfrac{s(s^2+4)}{(s^2+9)}$ is not realizable because the

(A) Number of zeros is more than the no. of poles.
(B) Poles and zeros lie on the imaginary axis.
(C) Poles and zeros are not located on the real axis.
(D) Poles and zeros do not alternated on imaginary axis.

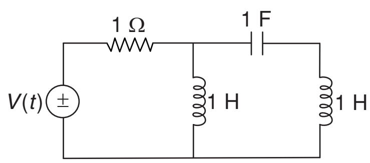
Solution: (D)

The pole-zero realization of $Z(S)$ is shown below zeros $\Rightarrow Z = 0$ and $Z = \pm j2 = S$

Poles $S^2 + 9 = 0 \Rightarrow S = \pm j3$



Example 20: The driving point impedance of the network shown below is



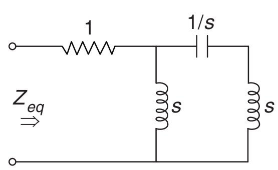
(A) $Z(S) = s + 1$

(B) $Z(S) = \dfrac{2s^2+1}{s^3+2s^2+s+1}$

(C) $Z(S) = \dfrac{1}{s+1}$

(D) $Z(S) = \dfrac{s^3+2s^2+s+1}{2s^2+1}$

Solution: (D)



$$Z_{eq} = 1 + s||(s + 1/s)$$

$$= 1 + \frac{s\left(s+\frac{1}{s}\right)}{s+s+\frac{1}{s}}$$

$$= \frac{s^3+2s^2+s+1}{2s^2+1}$$

Power Relations in AC Circuit

In a passive AC circuit, let the instantaneous voltage be $V(t) = V_m \sin \omega t$. The current is given by $i(t) = I_m \sin(\omega t - \phi)$, ϕ being the phase difference between voltage and current at any instant.

The instantaneous power p is given by $P = vi = V_m I_m \cdot \sin\omega t \cdot \sin(\omega t - \phi)$

$$= \frac{1}{2} \cdot V_m \cdot I_m \cdot \cos\phi \cdot - \frac{1}{2} V_m \cdot I_m \cos(2\omega t - \phi) \qquad (6)$$

In Eqn (6) contains a double frequency term and the magnitude of the average value of this term is zero.

∴ The avrage power in the passive circuit is given by

$$P_{avg} = \frac{1}{2} \cdot V_m \cdot I_m \cdot \cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi$$

$$P = V_{rms} \cdot I_{rms} \cdot \cos\phi = V \cdot I \cdot \cos\phi (W)$$

Loads	Power Factor(cosϕ)
R	Unity
L	zero lag
C	zero lead
R – L	lagging
R – C	leading

Apparent power

S = voltage × current

$= V \cdot I = V_{rms} I_{rms}$

Relative power $Q = V \cdot I \sin\phi$ (VAR)

$$S = \sqrt{P^2 + Q^2} VA.$$

Average value

$$X_{avg} = \frac{1}{T} \int_o^T x(t) \cdot dt$$

Where T is the time period of periodic function $x(t)$

RMS or effective value

$$X_{rms} = \sqrt{\frac{1}{T} \int_o^T [x(t)]^2 \cdot dt}$$

1. RMS value of $A \sin\omega t$ and $A \cos\omega t$ is $\dfrac{A}{\sqrt{2}}$
2. If $x(t) = a_o + (a_1 \cos \omega t + a_2 \cos 2\,\omega t + \cdots) + (b_1 \sin \omega t + b_2 \sin 2\,\omega t + \cdots)$

$$X_{rms} = \sqrt{a_o^2 + \frac{1}{2}\left(a_1^2 + a_2^2 + \cdots\right) + \frac{1}{2}\left(b_1^2 + b_2^2 + \cdots\right)}$$

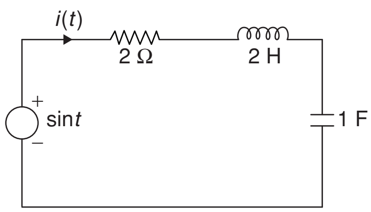
Exercises

Practice Problems I

Directions for questions 1 to 30: Select correct alternative from the given choices.

1. A series RLC circuit has a resonance frequency of 1 kHz and a quality factor $Q = 100$. If each of R, L and C is doubled from its original value, the new Q of the circuit is
   (A) 25 (B) 50
   (C) 100 (D) 200

2. The differential equation for the current $i(t)$ in the circuit of the figure is



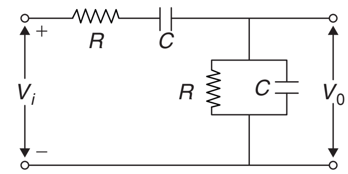
   (A) $2\dfrac{d^2i}{dt^2}+2\dfrac{di}{dt}+i(t)=\sin t$
   (B) $2\dfrac{d^2i}{dt^2}+2\dfrac{di}{dt}+2i(t)=\cos t$
   (C) $2\dfrac{d^2i}{dt^2}+2\dfrac{di}{dt}+i(t)=\cos t$
   (D) $\dfrac{d^2i}{dt^2}+2\dfrac{di}{dt}+2i(t)=\sin t$

3. In a series RLC circuit, $R = 2$ kΩ, $L = 1$ H and $C = \dfrac{1}{400}$ μF. The resonant frequency is
   (A) 2×10^4 Hz (B) $\dfrac{1}{\pi}\times 10^4$ Hz
   (C) 10^4 Hz (D) $2\pi \times 10^4$ Hz

4. The current $i(t)$ through a 10 Ω resistor in series with an inductance, is given by $i(t) = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$ Amperes
   The RMS value of the current and the power dissipated in the circuit are
   (A) $\sqrt{41}$ A, 410 W, respectively
   (B) $\sqrt{35}$ A, 350 W, respectively
   (C) 5 A, 250 W, respectively
   (D) 11 A, 1210 W, respectively

5. Two 2 H inductance coils are connected in series and are also magnetically coupled to each other, the co-efficient of coupling being 0.1. The total inductance of the combination can be
   (A) 0.4 H (B) 3.2 H
   (C) 4.0 H (D) 3.6 H

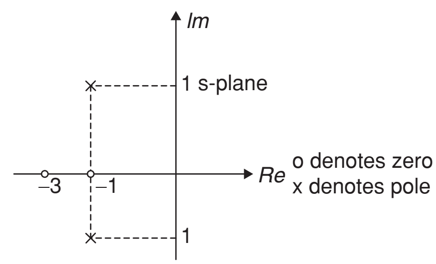
6. The R-C circuit shown in the figure is



   (A) A low-pass filter (B) A high-pass filter
   (C) A band-pass filter (D) A band-reject filter

7. A certain series resonant circuit has a band width of 1000 Hz. If the existing coil is replaced by a coil with lower Q, what happens to the bandwidth
   (A) It increases (B) It is zero
   (C) It decreases (D) It remains the same

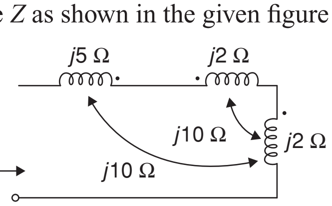
8. The driving point impedance $Z(s)$ of a network has the pole-zero locations as shown in the figure If $Z(0) = 3$, then $Z(s)$ is



   (A) $\dfrac{3(s+3)}{s^2+2s+3}$ (B) $\dfrac{2(s+3)}{s^2+2s+2}$
   (C) $\dfrac{3(s-3)}{s^2-2s-2}$ (D) $\dfrac{2(s-3)}{s^2-2s-3}$

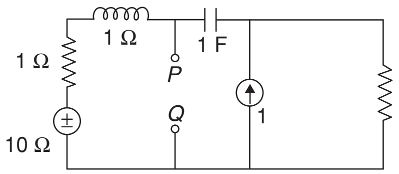
9. Consider the following statements S_1 and S_2
   S_1: At the resonant frequency the impedance of series R-L-C circuit is zero.
   S_2: In a parallel G-L-C circuit, increasing the conductance G results in increase in its Q-factor. Which one of the following is correct?
   (A) S_1 is FALSE and S_2 is TRUE
   (B) Both S_1 and S_2 are TRUE
   (C) S_1 is TRUE and S_2 is FALSE
   (D) Both S_1 and S_2 are FALSE

10. Impedance Z as shown in the given figure is



   (A) $j29$ Ω (B) $j9$ Ω
   (C) $j19$ Ω (D) $j39$ Ω

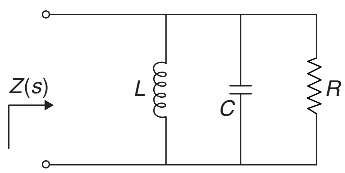
11. The Thevenin equivalent impedance Z_{th} between the nodes P and Q in the following circuit is



(A) 1 (B) $1+s+\frac{1}{s}$

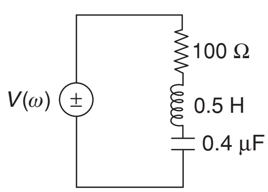
(C) $2+s+\frac{1}{s}$ (D) $\frac{s^2+s+1}{s^2+2s+1}$

12. The driving point impedance of the following network is given by $Z(s)=\frac{0.2s}{s^2+0.1s+2}$. The component values are



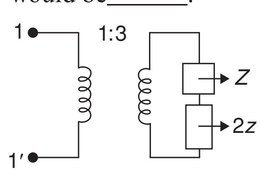
(A) $L = 5$ H, $R = 0.5$ Ω, $C = 0.1$ F
(B) $L = 0.1$ H, $R = 0.5$ Ω, $C = 5$ F
(C) $L = 5$ H, $R = 2$ Ω, $C = 0.1$ F
(D) $L = 0.1$ H, $R = 2$ Ω, $C = 5$ F

13. The resonant frequency for the series R-L-C circuit shown in the figure is



(A) 550 Hz (B) 670 Hz
(C) 1100 Hz (D) 355 Hz

14. If an ideal centre tapped 1:3 transformer is loaded as shown in the figure, the impedance measured across the terminals 11′ would be_______.



(A) $\frac{2}{3}Z$ (B) Z

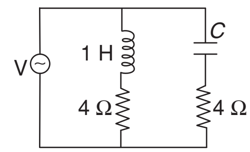
(C) $\frac{Z}{9}$ (D) $\frac{Z}{3}$

15. What is the ratio of the heating effects of two current waves of equal peak value, one being sinusoidal and the other rectangular in wave form?

(A) 1 : 2 (B) $1:\sqrt{2}$
(C) $\sqrt{2}:1$ (D) 2 : 1

16. The form factor of a half wave and a full wave rectified sine wave are _______.
(A) 1.11, 1.57 (B) 1.57, 1.11
(C) 1.414, 1.11 (D) 1.11, 1.414

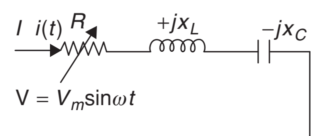
17. The value of capacitance C in the given AC circuit, to make it a pure resistive circuit and for the supply current to be independent of its frequency is _______.



(A) $\frac{1}{12}$ F (B) $\frac{1}{8}$ F

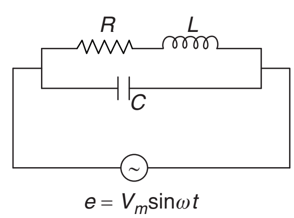
(C) $\frac{1}{4}$ F (D) $\frac{1}{16}$ F

18.



In the circuit shown in the figure. if R is varied from 0 to ∞ the locus of the tip of the current phasor is
(A) Circle (B) Semi circle
(C) Exponential curve (D) Sine curve

19.

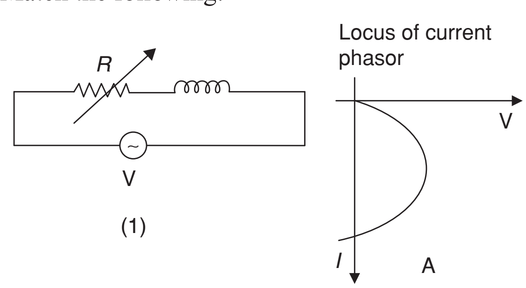


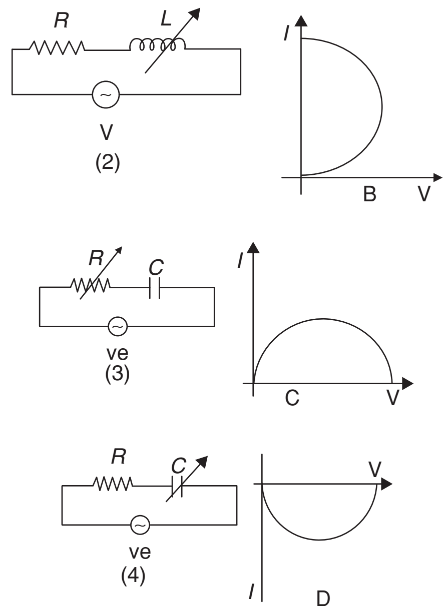
The dynamic impedance of the above circuit at resonance is

(A) $\frac{R}{LC}$ (B) $\frac{1}{R}\sqrt{\frac{L}{C}}$

(C) $\frac{L}{RC}$ (D) $\frac{C}{RL}$

20. Match the following:





(A) 1 – D, 2 – A, 3 – B, 4 – C
(B) 1 – A, 2 – D, 3 – B, 4 – C
(C) 1 – B, 2 – C, 3 – A, 4 – D
(D) 1 – C, 2 – B, 3 – D, 4 – A

Common Data for Questions 21 and 22: A low pass π section filter consists of an inductance of 25 mH in the series arm and two capacitors 0.2 uF in the shunt arms.

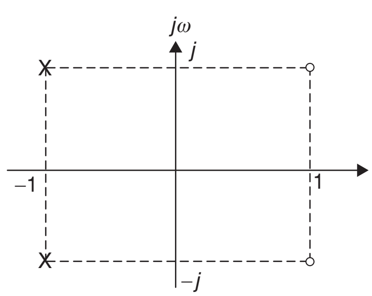
21. The cut-off frequency is ______.

(A) $\frac{10^4}{\pi}$ (B) $\frac{1}{10^4\pi}$
(C) $\frac{5000}{\pi}$ (D) $\frac{1}{5000\pi}$

22. Design impedance is ______.

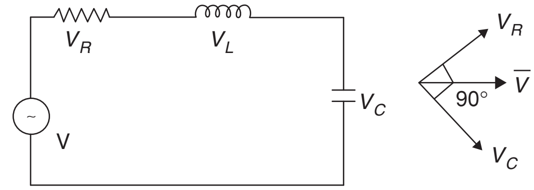
(A) 25 Ω (B) 250 Ω
(C) 15 Ω (D) 2.5 Ω

23. An impedance has the pole-zero patterns shown in figure. It must be composed of



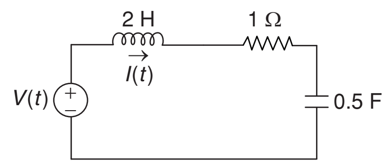
(A) RLC elements (B) RL elements only
(C) RC elements only (D) LC elements only

24. For the series R-L-C circuit of Figure I shown below, phasor diagram (involving some phasors) is shown in Figure II. The operating frequency of the circuit is



(A) Equal to resonant frequency
(B) Less than resonant frequency
(C) Twice the resonant frequency
(D) Greater than the resonant frequency

25. Which one of the following represents the state equation of the given R-L-C series circuit?



(A) $\begin{bmatrix}\dot{\phi}\\ q'c\end{bmatrix}=\begin{bmatrix}-2 & -0.5\\ 0 & 0.5\end{bmatrix}\begin{bmatrix}\phi\\ qc\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}v(t)$

(B) $\begin{bmatrix}\dot{\phi}\\ \dot{q}_c\end{bmatrix}=\begin{bmatrix}-2 & 2\\ -05 & 0.5\end{bmatrix}\begin{bmatrix}\phi\\ qc\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}v(t)$

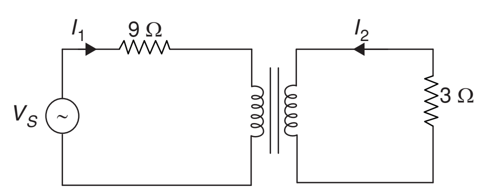
(C) $\begin{bmatrix}\dot{\phi}\\ \dot{q}_c\end{bmatrix}=\begin{bmatrix}-0.5 & -2\\ 0.5 & 0\end{bmatrix}\begin{bmatrix}\phi\\ qc\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}v(t)$

(D) $\begin{bmatrix}\dot{\phi}\\ \dot{q}_c\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0.5 & 2\end{bmatrix}\begin{bmatrix}\phi\\ qc\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}v(t)$

26. When two inductances are connected in series aiding the equivalent inductance of 14 H and in opposition is 6 H. Find out co-efficient of coupling '*K*'

(A) 0.408 (B) 0.534
(C) 0.628 (D) 0.707

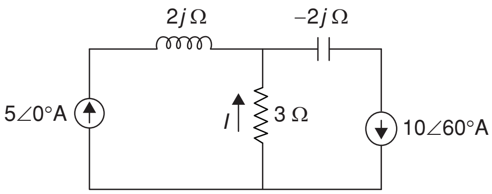
27. If the secondary winding of the ideal transformer shown in the circuit of figure has 50 turns, the number of turns in the primary winding for maximum power transfer to the 3 Ω resister will be



1:50
Ideal transformer

(A) 80 (B) 87
(C) 90 (D) 100

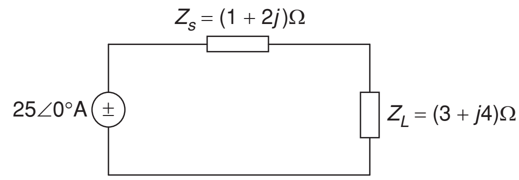
28. For the circuit in figure the instantaneous current $i(t)$ is



(A) $\frac{10\sqrt{3}}{2}\angle 90°$A (B) $5\angle -60°$A

(C) $5\angle 60°$A (D) $\frac{10\sqrt{3}}{2}\angle -90°$A

29. An AC sources of RMS voltage 25 V with internal impedance $Z_s = (1 + 2j)\Omega$ feeds a load of impedance $Z_L = (3 + j4)\Omega$, in the figure below. The reactive power consumed by the load is



(A) 60 VAR (B) 70 VAR
(C) 62.5 VAR (D) 78 VAR

30. The *n/w* function $\frac{(s+1)(s+4)}{s(s+2)(s+6)}$ is a

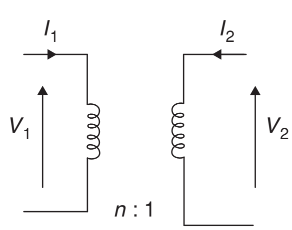
(A) RL impedance function
(B) RC impedance function
(C) LC impedance function
(D) All the above

Practice Problems 2

Directions for questions 1 to 28: Select correct alternative from the given choices.

1. For parallel RLC circuit, which one of the following statements is not correct?
(A) The bandwidth of the circuit decreases if R is increased
(B) The bandwidth of the circuit remains same if 'L' is increased
(C) At resonance, input impedance is a real quantity
(D) At resonance, the magnitude of the input impedance attains its minimum value

2. The ABCD parameters of an ideal n : 1 transformer shown in the figure are $\begin{bmatrix} n & o \\ o & x \end{bmatrix}$. The value of x will be

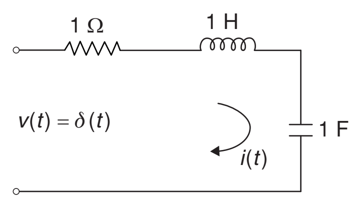


(A) n (B) $\frac{1}{n}$

(C) n^2 (D) $\frac{1}{n^2}$

3. What is the total reactance of a series RLC circuit at resonance
(A) Equal to X_L (B) Equal to X_C
(C) Equal to R (D) Zero

4.

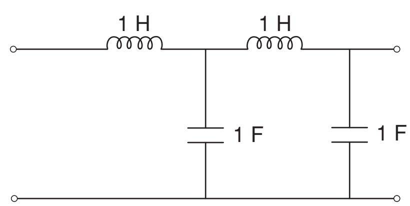


The circuit shown in the figure is initially relaxed. The Laplace transform of the current $i(t)$ is

(A) $\frac{s+1}{s^2+2s+1}$ (B) $\frac{s+1}{s^2+s+1}$

(C) $\frac{s}{s^2+s+1}$ (D) $\frac{s}{s^2+2s+1}$

5. Driving point impedance of the network shown in the figure is



(A) $\frac{s^4+3s^2+1}{s^3+2s}$ (B) $\frac{s^2+1}{s(s^2+2)}$

(C) $\frac{s^4+3s^3+2s^2+1}{s^3+2s}$ (D) $\frac{s^2+1}{s+1}$

6. The two windings of a transformer have an inductance of 3 H each. If the mutual inductance between them is also 3 H, then
(A) Transformation ratio is 3.
(B) It is an ideal transformer.
(C) It is a perfect transformer as the coefficient of coupling is 1.
(D) None of the above

7. The complex power in a single phase AC circuit is given by _______.

(A) VI (B) V^*I
(C) VI^* (D) V^*I^*

8. In a series resonance circuit which of the following statements are true.

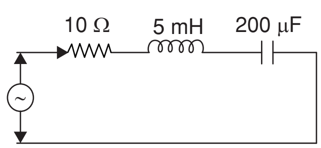
(i) *PF* is unity.
(ii) Voltage magnification takes place.
(iii) Current magnification takes place.
(iv) Resonant frequency depends on resistance.

(A) (i) and (ii) (B) (ii) and (iv)
(C) (i), (ii), and (iv) (D) (i) and (iii)

9. The condition for parallel resonance is

(A) Net reactance is zero
(B) Net susceptance is zero
(C) Net reactive power is zero
(D) Reactive component of net current is zero

10. In the circuit of the given figure, the magnitudes of V_L and V_c are equal at what value of supply frequency _______.



(A) $\frac{1}{15\pi}$ (B) $\frac{2}{15}\pi$
(C) $\frac{10^3}{2\pi}$ (D) 2π

11. If the voltage and current in an AC circuit are given by $v = 200 \sin(\omega t + 30)$ and $i = 10 \sin(\omega t - 60)$ then the p.f of circuit is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$
(C) 0 (D) $\frac{1}{\sqrt{2}}$

12. If $v = 200 \sin(2\,\omega t + 30)$, $i = 10 \sin(\omega t + 30)$ then the phase difference between v and i is

(A) 0 (B) 30
(C) 60° (D) None

13. In a series R-L-C circuit for frequencies less than the resonant frequency the circuit is ______.

(A) Inductive (B) Capacitive
(C) Resistive (D) None

14. In a parallel resonance circuit for frequencies greater than the resonant frequency the circuit is ______.

(A) Inductive (B) Capacitive
(C) Resistance (D) None

15. The Q of a circuit can be increased by _______.

(A) Increasing the *BW*
(B) Decreasing the *BW*
(C) Increasing the *R*
(D) None

16. In a R-L-C series circuit the bandwidth is increased by _______.

(A) Decreasing *L* (B) Decreasing *C*
(C) Increasing *R* (D) Decreasing *R*

17. In a R-L-C. series, resonant circuit at the half power points.

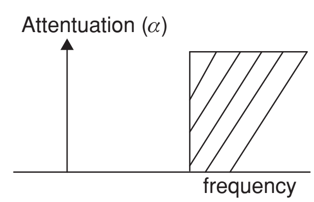
(A) The current is half of the current at resonance.
(B) The impedance is half of the impedance at resonance.
(C) The resistance is equal to the resultant reactance.
(D) None of the above

18. Which of the following statements are true

(i) The higher the value of Q, the more selective will be the circuit and lesser will be the Bandwidth.
(ii) Impedance of parallel resonant circuit is maximum at resonance.

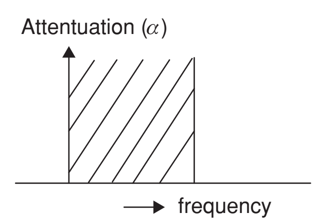
(A) (i) and (ii) (B) (i) only
(C) (ii) only (D) Both are false

19. Match the following:

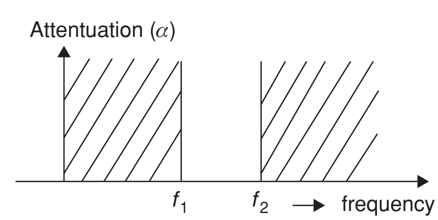


(1)

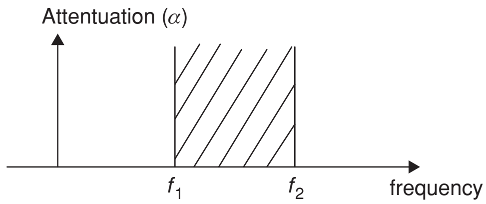
(A) Band pass (B) Low pass
(C) High pass (D) Band elimination



(2)



(3)



(4)

(A) 2 – B, 1 – C, 3 – D, 4 – A
(B) 1 – B, 2 – C, 3 – A, 4 – D
(C) 1 – A, 2 – D, 3 – B, 4 – C
(D) 1 – A, 2 – D, 3 – C, 4 – B

20. Given that

$f_1 \rightarrow$ lower cut-off frequency

$f_2 \rightarrow$ higher cut-off frequency

Then resonant frequency 'f_r', quality factor at resonance and selectivity are given by

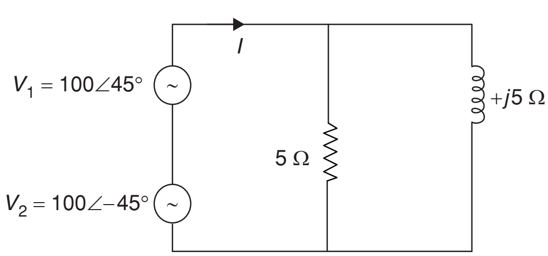
(A) $\sqrt{f_1 f_2}, \dfrac{f_2 - f_1}{f_r}, \dfrac{f_r}{f_2 - f_1}$

(B) $f_1 f_2, \dfrac{f_r}{f_2 - f_1}, \dfrac{f_2 - f_1}{f_r},$

(C) $\sqrt{f_1 f_2}, \dfrac{f_r}{f_2 - f_1}, \dfrac{f_2 - f_1}{f_r}$

(D) $\dfrac{1}{\sqrt{f_1 f_2}}, \dfrac{f_r}{f_2 - f_1}, \dfrac{f_2 + f_1}{f_2 - f_1}$

21. The phase angle of the current I with respect to the voltage V_2 in the circuit shown in the figure is



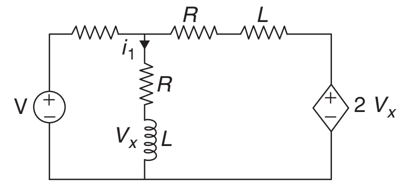
(A) 0°
(B) –45°
(C) +45°
(D) +90°

22. The conditions for defining driving point functions are.

(i) Response and excitation are applied to different terminals.
(ii) The network should not contain independent sources.
(iii) The network should be initially relaxed

(A) (i), (ii) and (iii)
(B) (i) and (iii)
(C) (ii) and (iii)
(D) (i) and (ii)

23. Find the state equation for the circuit given.



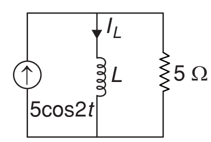
(A) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5\text{ V}$

(B) $L\dfrac{di_2}{dt} = -0.7V_x + 1.5i_2R - .5\text{ V}$

(C) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5\text{ V}$

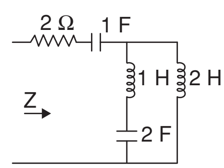
(D) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R - .5\text{ V}$

24. What is the value of the inductance if current through it is given as $i(t) = 0.707 < -45°$. The input current is $5\cos 2t$.



(A) 2 H
(B) 1 H
(C) 5 H
(D) 3 H

25. Find the driving point admittance of the network given below.



(A) $\dfrac{3s^3 + 2s}{2s^4 + 6s^3 + 7s^2 + 4s + 2}$

(B) $\dfrac{3s^2 + 2}{2s^4 + 5s^2 + 4s + 2}$

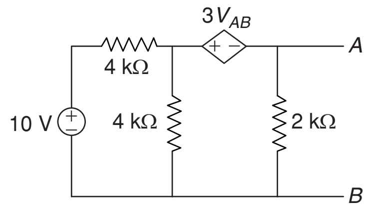
(C) $\dfrac{(3s^2 + 2)s}{2s^4 + 7s^3 + 6s^2 + 4s}$

(D) $\dfrac{5s^3 + 2s}{s^4 + 7s^3 + 6s^2 + 4s + 2}$

26. An unknown impedance Z is connected across a voltage source $V = \sqrt{50}\cos(\omega t + 50)$. The current flowing through the circuit is $\sqrt{18}\cos(10t + 75)$. What is the value of Z.

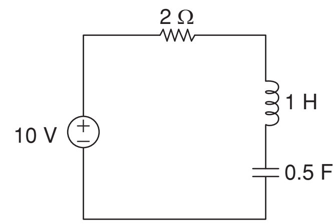
(A) 1.18 Ω Resistor in series with 84.7 mH inductor.
(B) 1.18 Ω Resistor in parallel with 84.7 mH inductor.
(C) 1.18 Ω Resistor in series with 84.7 mF Capacitor.
(D) 1.18 Ω resistor in parallel with 84.7 mF capacitor.

27. Find the Thevenins resistance associated with the circuit.



(A) 1 kΩ (B) 0.45 kΩ
(C) 2 kΩ (D) 0.22 kΩ

28. Find the current $i(t)$ through the circuit given



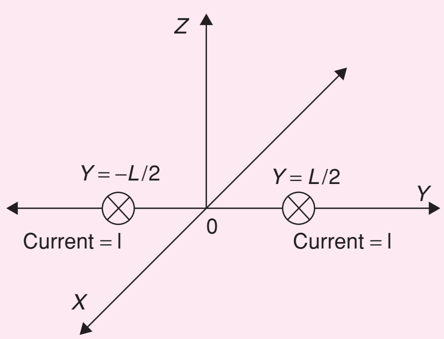
(A) $10e^{-t}\cos t$ (B) $\frac{10}{\sqrt{2}}e^{t}\cos\sqrt{2}t$

(C) $10e^{-t}\sin t$ (D) $\frac{10}{\sqrt{2}}e^{t}\sin\sqrt{2}t$

Previous Years' Questions

1. Two identical coupled inductors are connected in series. The measured inductances for the two possible series connections are 380 μH and 240 μH. Their mutual inductance in μH is _______. **[2014]**

2. A steady current I is flowing in the $-x$ direction through each of two infinitely long wires at $y = \pm\frac{L}{2}$ as shown in the figure. The permeability of the medium is μ_0. The $\vec{B}$-field at (0, L, 0) is **[2015]**



(A) $-\frac{4\mu_0 t}{3\pi L}\hat{Z}$ (B) $+\frac{4\mu_0 I}{3\pi L}\hat{Z}$

(C) 0 (D) $-\frac{3\mu_0 I}{4\pi L}\hat{Z}$

3. The self inductance of the primary winding of a single phase, 50 Hz, transformer is 800 mH, and that of the secondary winding is 600 mH. The mutual inductance between these two windings is 480 mH. The secondary winding of this transformer is short circuited and the primary winding is connected to a 50 Hz, single phase, sinusoidal voltage source. The current flowing in both the windings is less than their respective rated currents. The resistance of both windings can be neglected. In this condition, what is the effective inductance (in mH) seen by the source? **[2015]**

(A) 416 (B) 440
(C) 200 (D) 920

4. A symmetrical square wave of 50% duty cycle has amplitude of ±15 V and time period of 0.4π ms. This square wave is applied across a series RLC circuit with $R = 5$ Ω, $L = 10$ mH, and $C = 4$ μF. The amplitude of the 5000 rad/s component of the capacitor voltage (in Volt) is ________. **[2015]**

5. Two identical coils each having inductance L are placed together on the same core. If an overall inductance of αL is obtained by interconnecting these two coils, the minimum value of α is ______. **[2015]**

6. The flux linkage (λ) and current (i) relation for an electromagnetic system is $\lambda = \left(\sqrt{i}\right)/g$. when i = 2A and g(air - gap length) = 10 cm, the magnitude of mechanical force on the moving part, in N, is ________. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** C	**3.** B	**4.** C	**5.** D	**6.** C	**7.** A	**8.** B	**9.** D	**10.** B
11. A	**12.** D	**13.** D	**14.** D	**15.** A	**16.** B	**17.** D	**18.** B	**19.** A	**20.** B
21. A	**22.** B	**23.** D	**24.** B	**25.** C	**26.** C	**27.** B	**28.** A	**29.** C	**30.** B

Practice Problems 2

1. D	**2.** B	**3.** D	**4.** C	**5.** A	**6.** C	**7.** C	**8.** A	**9.** B	**10.** C
11. C	**12.** D	**13.** B	**14.** B	**15.** B	**16.** A	**17.** C	**18.** A	**19.** B	**20.** C
21. A	**22.** C	**23.** A	**24.** C	**25.** A	**26.** C	**27.** D	**28.** C		

Previous Years' Questions

1. 35 **2.** A **3.** A **4.** 190 to 192 V **5.** 0 **6.** 188