24 Height and Distance

Trigonometry is one of the most important branches of mathematics. Application of trigonometry includes:

- Measuring the heights of towers or big mountains
- Determining the distance of the shore from sea
- Finding the distance between two celestial bodies

The above examples are just few of the applications of **trigonometry**. In most of the applications, trigonometry is used to measure heights and distances. To measure heights and distances of different objects, we use **trigonometric ratios**.

Angle of Elevation: In order to see an object which is at a higher level compared to the ground level we are to look up.

The line joining the object and the eye of the observer is known as the line sight and the angle which this line of sight makes with the horizontal drawn through the eye of the observer is known as the angle of elevation.

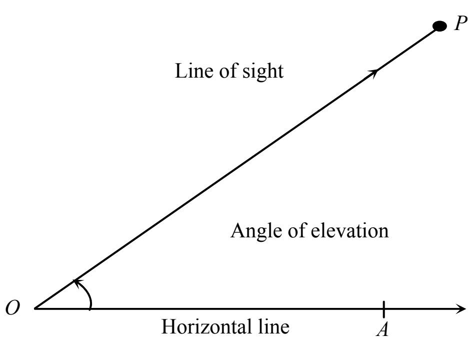


Figure: 24.1

Suppose a man from a point O looks up at an object P, placed above the level of his eye.

Then, the angle which the line of sight makes with the horizontal through O, is called the *angle of elevation* of P as seen from O.

$\therefore$ Angle of elevation of P from $O = \angle AOP$.

Angle of Depression: When the object is at a lower level tan the observer's eyes, he has to look downwards to have a view of the object.

It that case, the angle which the line of sight makes with the horizontal thought the observer's eye is known as the **angle of depression (Figure 24.2).**

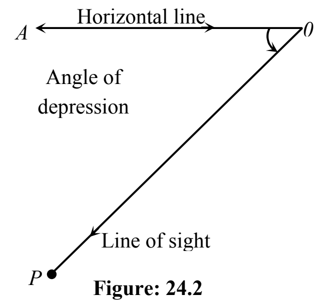
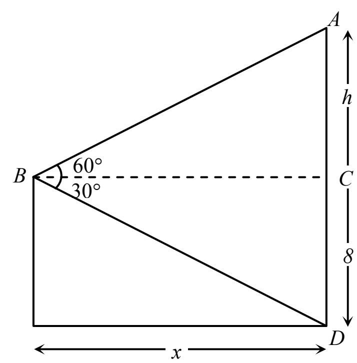


Figure: 24.2

Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the *angle of depression* of P as seen from O.

Example 1. A man is standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevations of the top of a hill as 60º and the angle of depression of the base of the hill as 30º. Calculation the distance of the hill from the ship and the height of the hill.

Solution: Let x be distance of hill from man and $h+8$ be height of hill which is required. Is right triangle ACB.



$\Rightarrow \quad \tan 60° = \dfrac{AC}{BC} = \dfrac{h}{x} \quad \Rightarrow \quad \sqrt{3} = \dfrac{h}{x}$

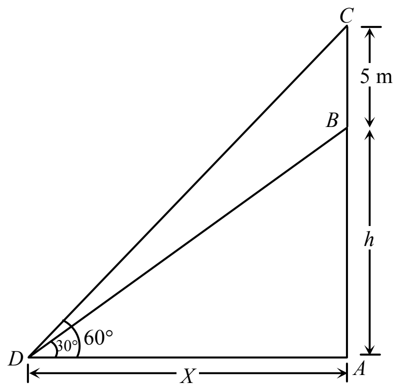
In right triangle BCD. $\quad \Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{8}{x}$

$\Rightarrow \quad x = 8\sqrt{3}$

$\therefore \quad \text{Height of hill} = h + 8 = \sqrt{3}.x + 8 = \left(\sqrt{3}\right)\left(8\sqrt{3}\right) + 8 = 32m.$

Distance of ship from hill $= x = 8\sqrt{3}\,m.$

Example 2. A vertical tower stands on a horizontal plane and is surmounted by vertical flag staff of height 5 meters. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30º and 60º find the height of tower.



Solution: Let AB be the tower of height h metre and BC be the height of flag staff surmounted on the tower, Let the point of the place be D at a distance x meter from the foot of the tower in ΔABD.

$\Rightarrow \quad \tan 30° = \frac{AB}{AD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \quad x = \sqrt{3}h \quad \quad \quad ...(i)$

In ΔABD

$\tan 60° = \frac{AC}{AD} \quad \Rightarrow \quad \sqrt{3} = \frac{5+h}{x}$

$\Rightarrow \quad x = \frac{5+h}{\sqrt{3}} \quad \quad \quad ...(ii)$

From (i) and (ii)

$\Rightarrow \quad \sqrt{3}h \frac{5+h}{\sqrt{3}}$

$\Rightarrow \quad 3h = 5 + h \quad \Rightarrow \quad 2h = 5$

$\Rightarrow \quad h = \frac{5}{2} = 2.5m$

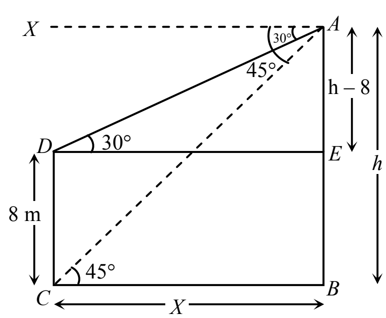
So, the height of tower = 2.5 m

Example 3. The angles of depressions of the top and bottom of 8m tall building from the top of a multistoried building are 30º and 45º respectively. Find the height of multistoried building and the distance between the two buildings.

Solution: Let AB be the multistoried building of height h and let the distance between two buildings be x metres.

$\angle XAC = \angle ACB = 45°$ [Alternate angles $\because$ AX || DE]

$XAD = ADE = 30°$ [Alternate angles $\because$ AX || BC]



In ΔADE

$\tan 30° = \frac{AE}{ED}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-8}{x}$

$(\because CB = DE = x)$

$\Rightarrow \quad x = \sqrt{3}(h-8) \quad \quad \quad ...(i)$

In ΔACB

$\tan 45° = \frac{h}{x} \quad \Rightarrow \quad 1 = \frac{h}{x}$

$\Rightarrow \quad x = h \quad \quad \quad ...(ii)$

Form (i) and (ii)

$\sqrt{3}(h-8) = h$

$\Rightarrow \quad \sqrt{3}h - 8\sqrt{3} = h \quad \Rightarrow \quad \sqrt{3}h - h = 8\sqrt{3}$

$\Rightarrow \quad h(\sqrt{3}-1) = 8\sqrt{3}$

$\Rightarrow \quad h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1} \quad \Rightarrow \quad h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow \quad h = 4\sqrt{3}(\sqrt{3}+1)$

$\Rightarrow \quad h = 4(3+\sqrt{3})$ metres

Form (ii) $x = h$

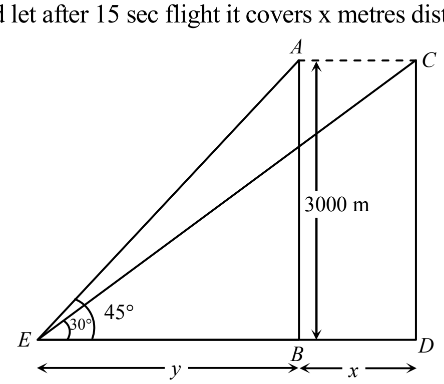
So, $x = 4(3+\sqrt{3})$ metres

Hence, height of multistoried building $= 4(3+\sqrt{3})$ metres

Distance between two building $= 4(3+\sqrt{3})$ metres

Example 4. The angle of elevation of an aeroplane from a point on the ground is 45º. After a flight of 15 sec, the elevation changes to 30º. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Solution: Let the point on the ground is E which is y metres from point B and let after 15 sec flight it covers x metres distance.



In ΔAEB.

$\Rightarrow \quad \tan 45° = \frac{AB}{EB}$

$$\Rightarrow \quad 1 = \frac{3000}{y}$$

$$\Rightarrow \quad y = 3000m \qquad \ldots (i)$$

In ΔCED

$$\Rightarrow \quad \tan 30° = \frac{CD}{ED}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3000}{x+y} \quad (\because AB = CD)$$

$$\Rightarrow \quad x + y = 3000\sqrt{3} \qquad \ldots (ii)$$

From equation (*i*) and (*ii*)

$$\Rightarrow \quad x + 3000 = 3000\sqrt{3}$$

$$\Rightarrow \quad x = 3000\sqrt{3} - 3000 \quad \Rightarrow \quad x = 3000\left(\sqrt{3}-1\right)$$

$$\Rightarrow \quad x = 3000 \times (1.732 - 1)$$

$$\Rightarrow \quad x = 2196$$

Speed of Aeroplane

$$= \frac{\text{Distance covered}}{\text{Time Taken}}$$

$$= \frac{2196}{15} \text{ m/sec.} = 146.4 \text{ m/sec.}$$

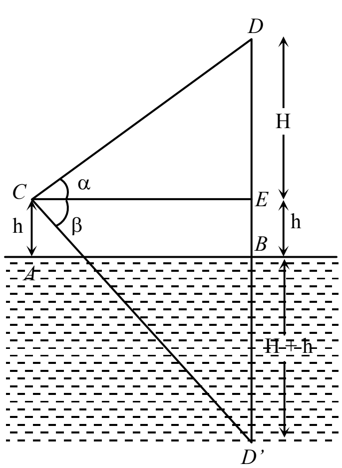
$$= \frac{2196}{15} \times \frac{18}{5} \text{ km/hr}$$

= 527.04 km/hr

Hence, the speed of aeroplane is 527.04 km/hr.

Example 5. If the angle of elevation of cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$.

Solution: Let AB be the surface of the lake and let C be a point of observation such that AC- h metres. Let D be the position of the cloud and D' be its reflection in the lake. Then BD = BD'.



In ΔDCE

$$\Rightarrow \quad \tan\alpha = \frac{DE}{CE}$$

$$\Rightarrow \quad CE = \frac{H}{\tan\alpha} \qquad \ldots (i)$$

$$\Rightarrow \quad \tan\beta = \frac{ED'}{EC}$$

In $\Delta CED'$

$$\Rightarrow \quad CE = \frac{h + H + h}{\tan\beta}$$

$$\Rightarrow \quad CE = \frac{2h + H}{\tan\beta} \qquad \ldots (ii)$$

From (*i*) and (*ii*)

$$\Rightarrow \quad \frac{H}{\tan\alpha} = \frac{2h+H}{\tan\beta}$$

$$\Rightarrow \quad H\tan\beta = 2h\tan\alpha + H\tan\alpha$$

$$\Rightarrow \quad H\tan\beta - H\tan\alpha + 2h\tan\alpha$$

$$\Rightarrow \quad H\left(\tan\beta - \tan\alpha\right) = 2h\tan\alpha$$

$$\Rightarrow \quad H = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} \qquad \ldots (iii)$$

In ΔDCE

$$\Rightarrow \quad \sin\alpha = \frac{DE}{CD}$$

$$\Rightarrow \quad CD = \frac{DE}{\sin\alpha}$$

$$\Rightarrow \quad CD = \frac{H}{\sin\alpha}$$

Substituting the value of H from (*iii*)

$$\Rightarrow \quad CD = \frac{2h\tan\alpha}{\left(\tan\beta - \tan\alpha\right)\sin\alpha}$$

$$\Rightarrow \quad CD = \frac{2h\frac{\sin\alpha}{\cos\alpha}}{\left(\tan\beta - \tan\alpha\right)\sin\alpha}$$

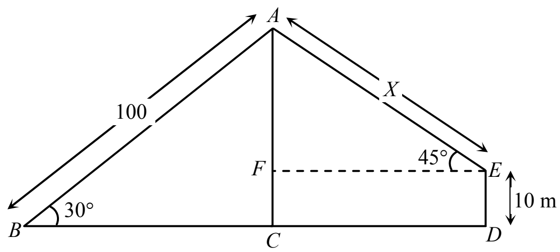
$$\Rightarrow \quad CD = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, the distance of the cloud from the point of observation is $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$

Hence Proved.

Example 6. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30º. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45º. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Solution: Let the length of second string b x m.



In ΔABC

$\sin 30° = \frac{AC}{AB}$

$\Rightarrow \frac{1}{2} = \frac{AC}{100} \Rightarrow AC = 50m$

In ΔAEF

$\sin 30° = \frac{AF}{AE}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AC - FC}{x}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{50 - 10}{x}$

[$\therefore$ AC = 50 m, FC = ED = 10 m]

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{40}{x}$

$\Rightarrow x = 40\sqrt{2}\, m$

(So, the length of string that the second boy must have so that the two kites meet $= 40\sqrt{2}$ m.)

Multiple Choice Questions

1. The angle of elevation of the top of a tower at point on the ground is 30°. If on walking 20 *metres* towards the tower, the angle of elevation become 60°, then the height of the tower is:

a. 10 *metre* **b.** $\frac{10}{\sqrt{3}}$ *metre*

c. $10\sqrt{3}$ *metre* **d.** None of these

2. The angle of elevation of a tower at a point distant *d* meters from its base is 30°. If the tower is 20 *m* high, then the value of *d* is

a. $10\sqrt{3}m$ **b.** $\frac{20}{\sqrt{3}}m$ **c.** $20\sqrt{3}m$ **d.** 10 *m*

3. The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground, forming a triangle is the same angle α. If R is the circum-radius of the triangle ABC, then the height of the tower is:

a. $R\sin\alpha$ **b.** $R\cos\alpha$ **c.** $R\cot\alpha$ **d.** $R\tan\alpha$

4. The angle of elevation of the top of a tower from a point *A* due south of the tower is α and from a point *B* due east of the tower is β. If *AB* =*d*, then the height of the tower is:

a. $\frac{d}{\sqrt{\tan^2\alpha - \tan^2\beta}}$ **b.** $\frac{d}{\sqrt{\tan^2\alpha + \tan^2\beta}}$

c. $\frac{d}{\sqrt{\cot^2\alpha + \cot^2\beta}}$ **d.** $\frac{d}{\sqrt{\cot^2\alpha - \cot^2\beta}}$

5. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 *meters* from the bank, he finds the angle to be 30°. The breadth of the river is:

a. 20 *m* **b.** 40 *m* **c.** 30 *m* **d.** 60 *m*

6. A vertical pole consists of two parts, the lower part being one third of the whole. At a point in the horizontal plane through the base of the pole and distance 20 *meters* from it, the upper part of the pole subtends an angle whose tangent is $\frac{1}{2}$. The possible heights of the pole are

a. 20 *m* and $20\sqrt{3}\,m$ **b.** 20 *m* and 60 *m*

c. 16 *m* and 48 *m* **d.** None of these

7. From a 60 metre high tower angles of depression of the top and bottom of a house are α and β respectively. If the height of the house is $\frac{60\sin(\beta - \alpha)}{x}$, then $x =$

a. sin α sin β **b.** cos α cos β

c. sin α cos β **d.** cos α sin β

8. An observer on the top of a tree, finds the angle of depression of a car moving towards the tree to be 30º. After 3 minutes this angle becomes 60º. After how much more time, the car will reach the tree

a. 4 *min* **b.** 4.5 *min* **c.** 1.5 *min* **d.** 2 *min*

9. A house of height 100 *m* subtends a right angle at the window of an opposite house. If the height of the window be 64 *m*, then the distance between the two houses is

a. 48 *m* **b.** 36 *m* **c.** 54 *m* **d.** 72 *m*

10. The length of the shadow of a pole inclined at 10^o to the vertical towards the sun is 2.05 *m*, when the elevation of the sun is 38^o. The length of the pole is:

a. $\frac{2.05 \sin 38^o}{\sin 42^o}$ **b.** $\frac{2.05 \sin 42^o}{\sin 38^o}$

c. $\frac{2.05 \cos 38^o}{\cos 42^o}$ **d.** None of these

11. The angle of elevation of the top of a tower from a point 20 *m* away from its base is 45^o. The height of the tower is:

a. 10 *m* **b.** 20 *m* **c.** 40 *m* **d.** $20\sqrt{3}\ m$

12. The horizontal distance between two towers is 60 *m* and the angular depression of the top of the first tower as seen from the top of the second, is 30^o. If the height of the second tower be 150 *m*, then the height of the first tower is

a. $150-60\sqrt{3}\,m$ **b.** 90 *m*

c. $150-20\sqrt{3}\,m$ **d.** None of these

13. From the top of a light house 60 *m* high with its base at the sea level, the angle of depression of a boat is 15^o. The distance of the boat from the foot of light house is:

a. $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60\ m$ **b.** $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60\ m$

c. $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\ m$ **d.** None of these

14. An observer in a boat finds that the angle of elevation of a tower standing on the top of a cliff is 60^o and that of the top of cliff is 30^o. If the height of the tower be 60 *m*, then the height of the cliff is

a. 30 *m* **b.** $60\sqrt{3}\ m$

c. $20\sqrt{3}\ m$ **d.** None of these

15. A tower subtends an angle α at a point *A* in the plane of its base and the angle of depression of the foot of the tower at a point *l* meters just above *A* is β. The height of the tower is:

a. $l\ \tan\beta\cot\alpha$ **b.** $l\ \tan\alpha\cot\beta$

c. $l\ \tan\ \alpha\ \tan\beta$ **d.** $l\ \cot\ \alpha\ \cot\beta$

16. The angle of elevation of a tower from a point *A* due south of it is 30^o and from a point *B* due west of it is 45^o. If the height of the tower be 100 *m*, then *AB* =

a. 150 *m* **b.** 200 *m* **c.** 173.2 *m* **d.** 141.4 *m*

17. An aeroplane flying horizontally 1 *km* above the ground is observed at an elevation of 60^o and after 10 *seconds* the elevation is observed to be 30^o. The uniform speed of the aeroplane in km/h is

a. 240 **b.** $240\sqrt{3}$

c. $60\sqrt{3}$ **d.** None of these

18. From a point *a metre* above a lake the angle of elevation of a cloud is α and the angle of depression of its reflection is β. The height of the cloud is:

a. $\frac{a\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ *metre* **b.** $\frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ *metre*

c. $\frac{a\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$ *metre* **d.** None of these

19. If the angle of depression of a point *A* on the ground from the top of a tower be 30^o, then the angle of elevation of the top of the tower from the point *A* will be:

a. 60^o **b.** 45^o

c. 30^o **d.** None of these

20. Two vertical poles of equal heights are 120 *m* apart. On the line joining their bottoms, *A* and *B* are two points. Angle of elevation of the top of one pole from *A* is 45^o and that of the other pole from *B* is also 45^o. If *AB* = 30 *m*, then the height of each pole is:

a. 40 *m* **b.** 45 *m* **c.** 50 *m* **d.** 42 *m*

21. At a distance 2*h* from the foot of a tower of height *h*, the tower and a pole at the top of the tower subtend equal angles. Height of the pole should be:

a. $\frac{5h}{3}$ **b.** $\frac{4h}{3}$ **c.** $\frac{7h}{5}$ **d.** $\frac{3h}{2}$

22. A house subtends a right angle at the window of an opposite house and the angle of elevation of the window from the bottom of the first house is 60^o. If the distance between the two houses be 6 *m*, then the height of the first house is

a. $6\sqrt{3}\ m$ **b.** $8\sqrt{3}\ m$

c. $4\sqrt{3}\ m$ **d.** None of these

23. The angle of elevation of the sun, when the shadow of the pole is $\sqrt{3}$ times the height of the pole, is

a. 60^o **b.** 30^o **c.** 45^o **d.** 15^o

24. A ladder rests against a wall so that its top touches the roof of the house. If the ladder makes an angle of 60^o with the horizontal and height of the house be $6\sqrt{3}$ meters, then the length of the ladder is

a. $12\sqrt{3}$ **b.** 12 *m*

c. $12/\sqrt{3}$ *m* **d.** None of these

25. If the angles of elevation of two towers from the middle point of the line joining their feet be 60^o and 30^orespectively, then the ratio of their heights is:

a. 2 : 1 **b.** $1:\sqrt{2}$ **c.** 3 : 1 **d.** $1:\sqrt{3}$

26. At a point on the ground the angle of elevation of a tower is such that its cotangent is 3/5. On walking 32 *m* towards the tower the cotangent of the angle of elevation is 2/5. The height of the tower is

a. 160 *m* **b.** 120 *m*

c. 64 *m* **d.** None of these

27. Some portion of a 20 *m* long tree is broken by the wind and the top struck the ground at an angle of 30^o. The height of the point where the tree is broken is:

a. 10 *m* **b.** $(2\sqrt{3}-3)\,20$ *m*

c. $\frac{20}{3}m$ **d.** None of these

28. The base of a cliff is circular. From the extremities of a diameter of the base the angles of elevation of the top of the cliff are 30^o and 60^o. If the height of the cliff be 500 *m*, then the diameter of the base of the cliff is:

a. $1000\sqrt{3}\,m$ **b.** $2000/\sqrt{3}\,m$

c. $1000/\sqrt{3}\,m$ **d.** $2000\sqrt{2}\,m$

29. The angle of elevation of the top of a tower from the top of a house is 60^o and the angle of depression of its base is 30^o. If the horizontal distance between the house and the tower be 12 *m*, then the height of the tower is:

a. $48\sqrt{3}$ *m* **b.** $16\sqrt{3}$ *m*

c. $24\sqrt{3}$ *m* **d.** $16/\sqrt{3}$ *m*

30. A man whose eye level is 1.5 *m* above the ground observes the angle of elevation of a tower to be 60^o. If the distance of the man from the tower be 10 *m*, the height of the tower is:

a. $(1.5+10\sqrt{3})m$ **b.** $10\sqrt{3}\,m$

c. $\left(1.5+\frac{10}{\sqrt{3}}\right)m$ **d.** None of these

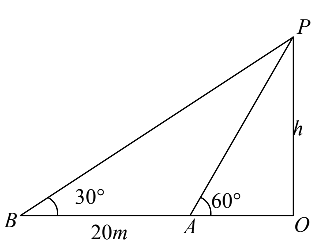
ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	c	d	c	a	b	d	c	a	a
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
b	c	b	a	b	b	b	b	c	b
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
a	b	b	b	c	a	c	b	b	a

SOLUTIONS

1. **(c)** $OA = h\cot 60^o, OB = h\cot 30^o$

$OB - OA = 20 = h(\cot 30° - \cot 60°)$

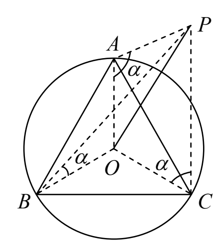


$\Rightarrow h = \frac{20}{\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$

2. **(c)** $20\cot 30^o = d$

$\Rightarrow d = 20\sqrt{3}$

3. **(d)** Since the tower makes equal angles at the vertices of the triangle, therefore foot of the tower is at the circumcentre.



From ΔOAP,

we have $\tan\alpha = \frac{OP}{OA}$

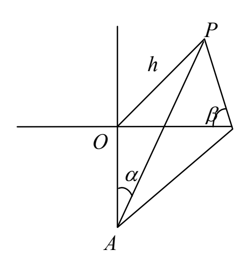
$\Rightarrow OP = OA\tan\alpha$

$\Rightarrow OP = R\tan\alpha$

4.

(c) $OB = h\cot\beta$,

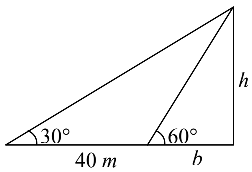
$OA = h\cot\alpha$



$h^2 = \frac{d^2}{\cot^2\beta + \cot^2\alpha}$

$\Rightarrow h = \frac{d}{\sqrt{\cot^2\beta + \cot^2\alpha}}$

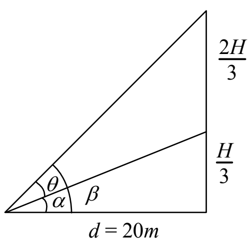
5. **(a)** $b = h\cot 60°,\ b+40 = h\cot 30^o$



$\Rightarrow \quad \dfrac{b}{b+40} = \dfrac{\cot 60^o}{\cot 30^o}$

$\Rightarrow b = 20m$

6. **(b)** $\dfrac{H}{3}\cot\alpha = d$ and $H\cot\beta = d$



or $\quad \dfrac{H}{3d} = \tan\alpha$ and $\dfrac{H}{d} = \tan\beta$

$$\tan(\beta-\alpha) = \frac{1}{2} = \frac{\frac{H}{d} - \frac{H}{3d}}{1+\frac{H^2}{3d^2}}$$

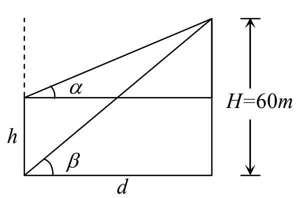
$\Rightarrow \quad 1 + \dfrac{H^2}{3d^2} = \dfrac{4H}{3d}$

$\Rightarrow \quad H^2 - 4dH + 3d^2 = 0$

$\Rightarrow \quad H^2 - 80H + 3(400) = 0$

$\Rightarrow \quad H = 20$ or $60m$

7. **(d)** $H = d\tan\beta$ and $H - h = d\tan\alpha$



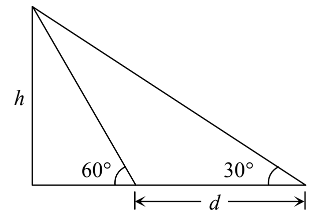
$\Rightarrow \quad \dfrac{60}{60-h} = \dfrac{\tan\beta}{\tan\alpha}$

$\Rightarrow \quad -h = \dfrac{60\tan\alpha - 60\tan\beta}{\tan\beta}$

$\Rightarrow \quad h = \dfrac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\dfrac{\sin\beta}{\cos\beta}}$

$\Rightarrow \quad x = \cos\alpha\sin\beta$

8. **(c)** $d = h\cot 30^o - h\cot 60^o$ and time $= 3\ min$

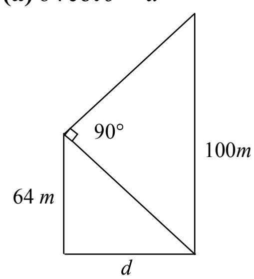


$\therefore \quad$ Speed $= \dfrac{h(\cot 30^o - \cot 60^o)}{3}$ *per minute*

It will travel distance $h\cot 60^o$ in

$\dfrac{h\cot 60^o \times 3}{h(\cot 30^o - \cot 60^o)} = 1.5$ *minute*

9. **(a)** $64\cot\theta = d$

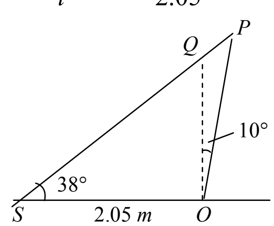


Also $(100-64)\tan\theta = d$

or $\quad (64)(36) = d^2$

$\therefore \quad d = 8\times 6 = 48\ m$

10. **(a)** $\dfrac{\sin 38^o}{l} = \dfrac{\sin(SPO)}{2.05}$

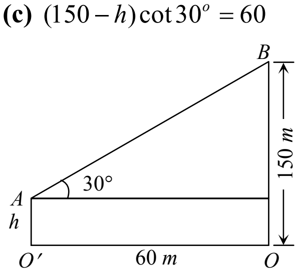


$= \dfrac{\sin(180^o - 38^o - 90^o - 10^o)}{2.05}$

$\Rightarrow \quad l = \dfrac{2.05\sin 38^o}{\sin 42^o}$

11. **(b)** $\tan 45^o = \dfrac{h}{20} \Rightarrow h = 20m$

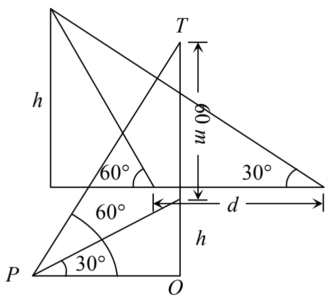
12. **(c)** $(150-h)\cot 30^o = 60$



$h = 150 - 20\sqrt{3}$

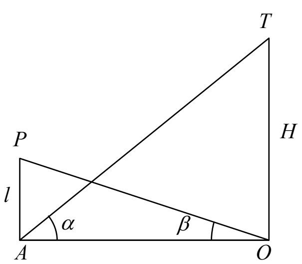
13. **(b)** Required distance $= 60\cot 15^o = 60\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

14. **(a)** $(60+h)\cot 60^o = h\cot 30^o \Rightarrow h = 30m$

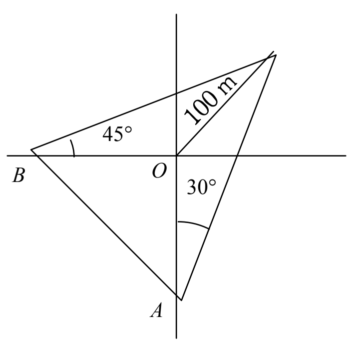


15. **(b)** From figure, we can deduce

$H = l\tan\alpha\cot\beta$



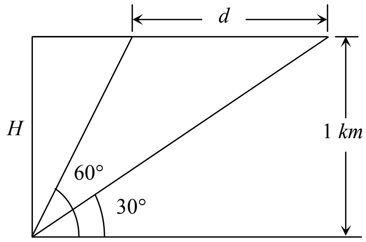
16. **(b)** $OB = 100\cot 45^o$



$OA = 100\cot 30^o$

$AB = \sqrt{(OA^2 + OB^2)} = 200m$

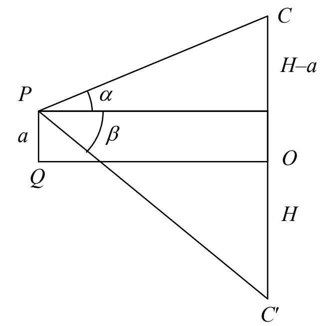
17. **(b)** $d = H\cot 30^o - H\cot 60^o$



Time taken = 10 *second*

Speed $= \dfrac{\cot 30^o - \cot 60^o}{10}\times 60\times 60 = 240\sqrt{3}$

18. **(b)** $(H+a)\cot\beta = (H-a)\cot\alpha$



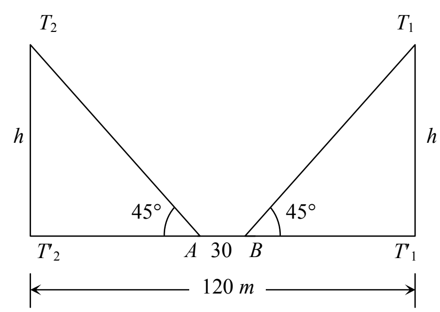
$$H = \frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$$

Using $\cot\alpha + \cot\beta = \dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$

and $\cot\alpha - \cot\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

19. **(c)** It is a fundamental concept.

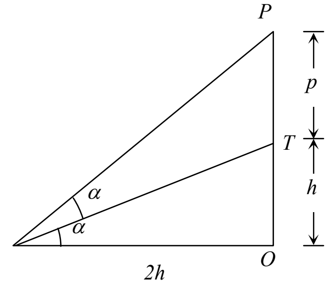
20. **(b)** $\tan 45^o = 1 = \dfrac{h}{T_2'A} \Rightarrow T_2'A = h$



Hence $120 = h + 30 + h \Rightarrow h = 45\ m$.

21. **(a)** $\tan\alpha = \dfrac{1}{2}$ and

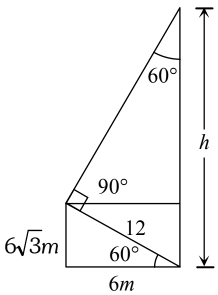
$\tan 2\alpha = \dfrac{P+h}{2h}$



$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \dfrac{p+h}{2h} = \dfrac{1}{1-\dfrac{1}{4}}$

$\Rightarrow \dfrac{p+h}{2h} = \dfrac{4}{3} \Rightarrow p = \dfrac{5h}{3}$

22. **(b)** $\frac{12}{h} = \sin 60^o$

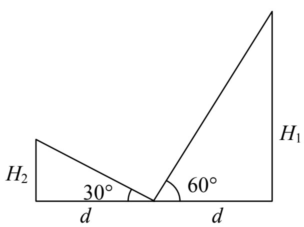


$\Rightarrow \quad h = 8\sqrt{3}\ m$

23. **(b)** $\tan\alpha = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^o$

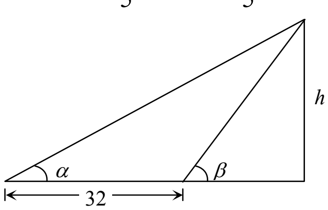
24. **(b)** Length of ladder = $\frac{6\sqrt{3}}{\sin 60^o} = 12\,m$

25. **(c)** $H_1 = d\tan 60^o,\ H_2 = d\tan 30^o$



$\frac{H_1}{H_2} = \frac{\tan 60^o}{\tan 30^o} = \frac{3}{1}$

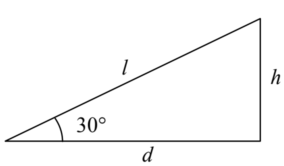
26. **(a)** $\cot\alpha = \frac{3}{5},\ \cot\beta = \frac{2}{5}$



$32 = h\cot\alpha - h\cot\beta$

$h = \left(\frac{32}{\cot\alpha - \cot\beta}\right) = \frac{32}{1/5} = 160m$

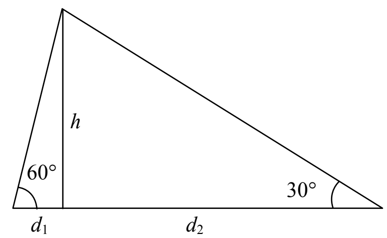
27. **(c)** $H = 20 = l + h, l = \frac{d}{\cos 30^o}, h = d\tan 30^o$



$\therefore \quad d = \frac{20}{(\sec 30^o + \tan 30^o)}$

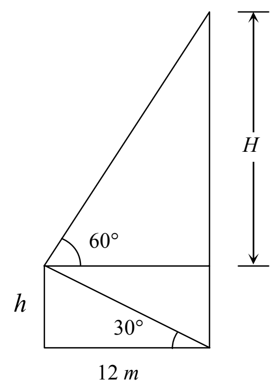
$= \frac{20}{\sqrt{3}}$ and hence, $h = d\tan 30^o = \frac{20}{3}m$

28. **(b)** $d_2 = h\cot 30^o = 500\sqrt{3}, d_1 = \frac{500}{\sqrt{3}}$



Diameter $D = 500\sqrt{3} + \frac{500}{3}\sqrt{3} = \frac{2000}{\sqrt{3}}m$

29. **(b)** $h = 12\tan 30^o = \frac{12}{\sqrt{3}}$



and $H = 12\tan 60^o + \frac{12}{\sqrt{3}}$

$= \ = 12\sqrt{3} + \frac{12}{\sqrt{3}} = 16\sqrt{3}m$

30. **(a)** $H = (10\tan 60^o + 1.5) = (10\sqrt{3} + 1.5)m$