MATHEMATICS

Relations and Functions

Speed TEST 61

Chapter-wise

No. of Questions	Maximum Marks	Time
30	**120**	**1 Hour**

GENERAL INSTRUCTIONS

- This test contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.
- You have to evaluate your Response Grids yourself with the help of solutions provided at the end of this book.
- Each correct answer will get you 4 marks and 1 mark shall be deduced for each incorrect answer. No mark will be given/ deducted if no bubble is filled. Keep a timer in front of you and stop immediately at the end of 60 min.
- The sheet follows a particular syllabus. Do not attempt the sheet before you have completed your preparation for that syllabus.
- After completing the sheet check your answers with the solution booklet and complete the Result Grid. Finally spend time to analyse your performance and revise the areas which emerge out as weak in your evaluation.

1. For the following relation
R = {(0, 0), (0, 1), (1, 1), (2, 1), (2, 2), (2, 0), (1, 0), (0, 2), (0, 1)}

(a) domain = {0, 1} (b) range = {0, 1, 2}
(c) both correct (d) None of these

2. The domain of the function $\sqrt{x^2-5x+6}+\sqrt{2x+8-x^2}$ is

(a) [2, 3] (b) [–2, 4]
(c) $[-2,2]\cup[3,4]$ (d) $[-2,1]\cup[2,4]$

3. If $3f(x)-f\left(\frac{1}{x}\right)=\log x^4$, then $f(e^{-x})$ is

(a) $1+x$ (b) $1/x$
(c) x (d) $-x$

4. The domain of the function $f(x)=\dfrac{1}{\sqrt{|x|-x}}$ is

(a) $(0,\infty)$ (b) $(-\infty,0)$
(c) $(-\infty,\infty)-\{0\}$ (d) $(-\infty,\infty)$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. $f(x) = \sqrt{|x|^2 - 5|x| + 6} + \sqrt{8 + 2|x| - |x|^2}$ is real for all x in

(a) $[-4, -3]$ (b) $[-3, -2]$

(c) $[-2, 2]$ (d) $[3, 4]$

6. $f(x) = \frac{x(x-p)}{q-p} + \frac{x(x-q)}{p-q}$, $p \neq q$. What is the value of $f(p) + f(q)$?

(a) $f(p-q)$ (b) $f(p+q)$

(c) $f(p(p+q))$ (d) $f(q(p-q))$

7. A real valued function $f(x)$ satisfies the functional equation

$$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$

where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to

(a) $-f(x)$ (b) $f(x)$

(c) $f(a) + f(a-x)$ (d) $f(-x)$

8. Domain of definition of the function

$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

(a) $(-1,0) \cup (1,2) \cup (2,\infty)$ (b) $(a, 2)$

(c) $(-1,0) \cup (a,2)$ (d) $(1,2) \cup (2,\infty)$.

9. Let $A = \{1, 2, 3, 4, 5\}$; $B = \{2, 3, 6, 7\}$. Then the number of elements in $(A \times B) \cap (B \times A)$ is

(a) 18 (b) 6

(c) 4 (d) 0

10. A relation R is defined in the set Z of integers as follows $(x, y) \in R$ iff $x^2 + y^2 = 9$. Which of the following is false?

(a) R = {(0, 3), (0, –3), (3, 0), (–3, 0)}

(b) Domain of R = {–3, 0, 3}

(c) Range of R = {–3, 0, 3}

(d) None of these

11. Let $f(x) = \sqrt{1 + x^2}$, then

(a) $f(xy) = f(x).f(y)$ (b) $f(xy) \geq f(x).f(y)$

(c) $f(xy) \leq f(x).f(y)$ (d) None of these

12. The domain of the function $f(x) = \sqrt{x - \sqrt{1 - x^2}}$ is

(a) $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$

(b) $[-1, 1]$

(c) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right)$

(d) $\left[\frac{1}{\sqrt{2}}, 1\right]$

13. Period of the function $\left|\sin^3 \frac{x}{2}\right| + \left|\cos^5 \frac{x}{5}\right|$ is :

(a) 2π (b) 10π

(c) 8π (d) 5π

14. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C) =$

(a) 288 (b) 1

(c) 12 (d) 2

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. If S = {1, 2, 3, 4, 5,} and R = {(x, y) : x + y < 6} then n (R) =

(a) 8 (b) 10

(c) 6 (d) 5

16. The function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, is

(a) neither an even nor an odd function

(b) an even function

(c) an odd function

(d) a periodic function

17. Let $f(x) = \frac{x}{1-x}$ and 'a' be a real number. If $x_0 = a$, $x_1 = f(x_0), x_2 = f(x_1), x_3 = f(x_2)$....... If $x_{2009} = 1$, then the value of a is

(a) 0 (b) $\frac{2009}{2010}$

(c) $\frac{1}{2009}$ (d) $\frac{1}{2010}$

18. The domain of the function $f(x) = \log_2\left(-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right)$ is

(a) (0, 1) (b) (0, 1]

(c) $[1, \infty)$ (d) $(1, \infty)$

19. The domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 3x + 2}}$ is

(a) $(-\infty, 1)$ (b) $(-\infty, 1) \cup (2, \infty)$

(c) $(-\infty, 1] \cup [2, \infty)$ (d) $(2, \infty)$

20. If (1, 3), (2, 5) and (3, 3) are three elements of A × B and the total number of elements in A × B is 6, then the remaining elements of A × B are

(a) (1, 5); (2, 3); (3, 5) (b) (5, 1); (3, 2); (5, 3)

(c) (1, 5); (2, 3); (5, 3) (d) None of these

21. If $f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right)$, then range of $f(x)$ is

(a) (0, 1) (b) (0, 1]

(c) [0, 1) (d) {0, 1}

22. The function $f(x) = \log\left(\frac{1+x}{1-x}\right)$ satisfies the equation

(a) $f(x+2) - 2f(x+1) + f(x) = 0$

(b) $f(x+1) + f(x) = f(x(x+1))$

(c) $f(x_1) \cdot f(x_2) = f(x_1 + x_2)$

(d) $f(x_1) + f(x_2) = f\left(\frac{x_1 + x_2}{1 + x_1 x_2}\right)$

23. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7n(n+1)}{2}$ (b) $\frac{7n}{2}$

(c) $\frac{7(n+1)}{2}$ (d) $7n + (n+1)$.

24. If { } denotes the fractional part of x, the range of the function $f(x) = \sqrt{\{x\}^2 - 2\{x\}}$ is

(a) ϕ (b) [0, 1/2]

(c) {0, 1/2} (d) {0}

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is equal to

(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{3f(x)+1}{f(x)+3}$

(c) $\frac{f(x)+3}{f(x)+1}$ (d) $\frac{f(x)+3}{3f(x)+1}$

26. The range of the function $f(x) = \frac{x^2-x+1}{x^2+x+1}$ where $x \in R$, is

(a) $(-\infty, 3]$ (b) $(-\infty, \infty)$

(c) $[3, \infty)$ (d) $\left[\frac{1}{3}, 3\right]$

27. The domain of the function $f(x) = \exp(\sqrt{5x-3-2x^2})$ is

(a) $[3/2, \infty)$ (b) $[1, 3/2]$

(c) $(-\infty, 1]$ (d) $(1, 3/2)$

28. If $f(x+y) = f(x) + 2y^2 + kxy$ and $f(a) = 2, f(b) = 8$, then $f(x)$ is of the form

(a) $2x^2$ (b) $2x^2 + 1$

(c) $2x^2 - 1$ (d) x^2

29. The relation R defined on the set A = {1, 2, 3, 4, 5} by R= $\{(x, y) : |x^2 - y^2| < 16\}$ is given by

(a) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}

(b) {(2, 2), (3, 2), (4, 2), (2, 4)}

(c) {(3, 3), (3, 4), (5, 4), (4, 3), (3, 1)}

(d) None of these

30. Which of the following relation is NOT a function

(a) $f = \{(x, x) \mid x \in R\}$ (b) $g = \{(x, 3) \mid x \in R\}$

(c) $h = \{(n, \frac{1}{n}) \mid n \in I\}$ (d) $t = \{(n, n^2) \mid n \in N\}$

RESPONSE GRID	
	25. ⓐⓑⓒⓓ 26. ⓐⓑⓒⓓ 27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

MATHEMATICS CHAPTERWISE SPEED TEST-61			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	38	Qualifying Score	50
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Speed Test-61

1. **(b)** We have domain of R = the set of first components of the ordered pairs in R = {0, 0, 1, 2, 2, 2, 1, 0, 0} = {0, 1, 2} range of R = the set of second components of the ordered pairs in R
= {0, 1, 1, 1, 2, 0, 0, 2, 1} = {0, 1, 2}

2. **(c)** $f(x)=\sqrt{(x-2)(x-3)}+\sqrt{-(x-4)(x+2)}$
The first part is real outside (2, 3) and the second is real in [–2, 4] so that the domain is [–2, 2] $\cup$ [3, 4].

3. **(d)** $3f(x)-f\left(\frac{1}{x}\right)=\log x^4$; $x\equiv\frac{1}{x}$
$3f\left(\frac{1}{x}\right)-f(x)=\log\left(\frac{1}{x}\right)^4$
After solving we get $f(x) = \log x$
$f(e^{-x})=\log_e e^{-x}=-x$

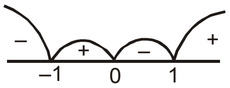
4. **(b)** $f(x)=\frac{1}{\sqrt{|x|-x}}$, define if $|x|-x>0$
$\Rightarrow |x|>x, \Rightarrow x<0$
Hence domain of $f(x)$ is $(-\infty, 0)$

5. **(d)** $\sqrt{|x^2|-5|x|+6}=\sqrt{(|x|-2)\,(|x|-3)}$
is real for $0\le|x|\le 4$
$\therefore$ $f(x)$ is real for all $0\le|x|\le 2$ or $3\le|x|\le 4$.

6. **(b)** In the definition of function
$$f(x)=\frac{x(x-p)}{q-p}+\frac{x(p-q)}{(p-q)}=p$$
Putting p and q in place of x, we get
$$f(p)=\frac{p(p-p)}{q-p}+\frac{p(p-q)}{(p-q)}=p$$
$\Rightarrow f(p)=p$
and $f(q)=\frac{q(q-p)}{q-p}+\frac{q(p-q)}{(p-q)}=q$
$\Rightarrow f(q)=q$
Putting $x=(p+q)$
$$f(p+q)=\frac{(p+q)(p+q-p)}{(q-p)}+\frac{(p+q)(p+q-q)}{(p-q)}$$
$$=\frac{(p+q)q}{(q-p)}+\frac{(p+q)(p)}{(p-q)}=\frac{pq+q^2-p^2-pq}{(q-p)}$$
$$=\frac{q^2-p^2}{q-p}=\frac{(q-p)(q+p)}{(q-p)}=p+q=f(q)+f(p)$$
So, $f(p)+f(q)=f(p+q)$

7. **(a)** $f(2a-x)=f(a-(x-a))=f(a)f(x-a)-f(0)f(x)$
$=f(a)f(x-a)-f(x)=-f(x)$
$[\because x=0, y=0,\ f(0)=f^2(0)-f^2(a)$
$\Rightarrow f^2(a)=0\ \Rightarrow f(a)=0]$
$\Rightarrow f(2a-x)=-f(x)$

8. **(a)** $f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$
$4-x^2\ne 0;\ x^3-x>0;$
$x\ne\pm\sqrt{4}$ and $-1<x<0$ or $1<x<\infty$
$\therefore D=(-1, 0)\cup(1,\infty)-\{\sqrt{4}\}$
$D=(-1, 0)\cup(1, 2)\cup(2,\infty)$.



9. **(c)** Here A and B sets having 2 elements in common, so A × B and B × A have 2^2 i.e., 4 elements in common. Hence, $n[(A\times B)\cap(B\times A)]=4$

10. **(d)** $x^2+y^2=9\Rightarrow y^2=9-x^2\Rightarrow y=\pm\sqrt{9-x^2}$
$x=0\Rightarrow y=\pm\sqrt{9-0}=\pm 3\in Z$
$x=\pm 1\Rightarrow y=\pm\sqrt{9-1}=\pm\sqrt{8}\notin Z$
$x=\pm 2\Rightarrow y=\pm\sqrt{9-4}=\pm\sqrt{5}\notin Z$
$x=\pm 3\Rightarrow y=\pm\sqrt{9-9}=0\in Z$
$x=\pm 4\Rightarrow y=\pm\sqrt{9-16}=\pm\sqrt{-7}\notin Z$ and so on.
$\therefore$ R = {(0, 3), (0, –3), (3, 0), (–3, 0)}
Domain of R = {x : (x, y) $\in$ R} = {0, 3, –3}
Range of R = {y : (x, y) $\in$ R} = {3, –3, 0}.

11. **(c)** $f(xy)=\sqrt{1+x^2y^2}$
$f(x)f(y)=\sqrt{1+x^2}\sqrt{1+y^2}=\sqrt{1+x^2y^2+x^2+y^2}$
$\ge\sqrt{1+x^2y^2}=f(xy)$
$\therefore f(xy)\le f(x)f(y)$

12. **(d)** For $f(x)$ to be defined, we must have
$x-\sqrt{1-x^2}\ge 0$ or $x\ge\sqrt{1-x^2}>0$
$\therefore x^2\ge 1-x^2$ or $x^2\ge\frac{1}{2}$.
Also, $1-x^2\ge 0$ or $x^2\le 1$.
Now, $x^2\ge\frac{1}{2}\Rightarrow\left(x-\frac{1}{\sqrt{2}}\right)\left(x+\frac{1}{\sqrt{2}}\right)\ge 0$
$\Rightarrow x\le-\frac{1}{\sqrt{2}}$ or $x\ge\frac{1}{\sqrt{2}}$
Also, $x^2\le 1\Rightarrow(x-1)(x+1)\le 0$
$\Rightarrow -1\le x\le 1$
Thus, $x>0,\ x^2\ge\frac{1}{2}$ and $x^2\le 1$
$\Rightarrow x\in\left[\frac{1}{\sqrt{2}},1\right]$

13. **(b)** Period of $\sin x = 2\pi \Rightarrow$ period of $\sin^3 x = 2\pi$

period of $|\sin^3 x| = \pi \Rightarrow$ period of $\left|\sin^3 \frac{x}{2}\right| = 2\pi$

period of $\cos^5 x = 2\pi \Rightarrow$ period of $|\cos^5 x| = \pi$

$\Rightarrow$ period of $\left|\cos^5 \frac{x}{5}\right| = 5\pi$

Thus required period = LCM of 2π & $5\pi = 10\pi$

14. **(d)** $n(A) = 4, n(B) = 3$

$n(A) \times n(B) \times n(C) = n(A \times B \times C)$

$4 \times 3 \times n(C) = 24 \Rightarrow n(C) = 24/12 = 2$

15. **(b)** We have $(x, y) \in R$ iff $x + y < 6$

Given the value $x = 1$, we get possible values of $y = 1, 2, 3, 4$. Thus 1R1, 1R2, 1R3, 1R4. Similarly we may find other values. The set of such ordered pairs is

$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$

$\therefore n(R) = 10$

16. **(c)** $f(x) = \log(x + \sqrt{x^2+1})$

$f(-x) = \log\left\{-x + \sqrt{x^2+1}\right\} = \log\left\{\frac{-x^2+x^2+1}{x+\sqrt{x^2+1}}\right\}$

$= -\log(x+\sqrt{x^2+1}) = -f(x)$

$\Rightarrow f(x)$ is an odd function.

17. **(d)** $x_0 = a,\ x_1 = f(x) = \frac{x_0}{1-x_0} = \frac{a}{1-a}$;

$x_2 = f(x_1) = \frac{x_1}{1-x_1} = \frac{\frac{a}{1-a}}{1-\frac{a}{1-a}} = \frac{a}{1-2a}$

$\therefore \quad x_{2009} = \frac{a}{1-2009a} = 1 \Rightarrow 1 - 2009a = a$

$\Rightarrow a = \frac{1}{2010}$

18. **(a)** $f(x)$ is defined if $-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) - 1 > 0$

$\Rightarrow \log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) < -1$

$\Rightarrow 1 + \frac{1}{x^{1/4}} > \left(\frac{1}{2}\right)^{-1}$

$\Rightarrow \frac{1}{x^{1/4}} > 1$

$\Rightarrow 0 < x < 1$

19. **(b)** For $f(x)$ to be defined, we must have

$x^2 - 3x + 2 = (x-1)(x-2) > 0 \Rightarrow x < 1$ or > 2

Domain of $f = (-\infty, 1) \cup (2, \infty)$.

20. **(a)** It is obvious.

21. **(b)** $f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right) = \ln\left(\frac{x^2+1-1+e}{2}\right) = \ln\left(1+\frac{e-1}{2}\right)$

22. **(b)** Let $f(x) = \log(g(x))$

$\therefore \quad f(x_1) + f(x_2) = \log(g(x_1)) + \log(g(x_2))$

$= \log(g(x_1) \cdot g(x_2))$

$\therefore$ Option (b) is correct

23. **(a)** $f(x+y) = f(x) + f(y)$.

Function should be $f(x) = mx$

$f(1) = 7;\ \therefore m = 7,\ f(x) = 7x$

$\sum_{r=1}^{n} f(r) = 7\sum_{1}^{n} r = \frac{7n(n+1)}{2}$

24. **(d)** $\{x^2\} - 2\{x\} \geq 0$

$\Rightarrow \{x\}(\{x\} - 2) \geq 0$

$\Rightarrow \{x\} \leq 0$ or $\{x\} \geq 2$

Second case is not possible.

Hence $\{x\} = 0$, as $\{x\} \leq [0, 1)$. Hence range of $f(x)$ contains only one element 0.

25. **(b)** Given $f(x) = \frac{x-1}{x+1}$

$\therefore f(2x) = \frac{2x-1}{2x+1}$

$= \frac{2(2x-1)}{2(2x+1)}$ (multiply and divide by 2)

$= \frac{4x-2}{4x+2} = \frac{3x+x-3+1}{3x+x+3-1} = \frac{3(x-1)+x+1}{3(x+1)+x-1}$

$= \frac{3\left[\frac{x-1}{x+1}\right]+1}{\frac{x-1}{x+1}+3} = \frac{3f(x)+1}{f(x)+3}$

26. **(d)** Let $y = \frac{x^2-x+1}{x^2+x+1}$

$\Rightarrow x^2(y-1) + x(y+1) + (y-1) = 0$

$\Rightarrow x = \frac{-(y+1) \pm \sqrt{(y+1)^2 - 4(y-1)^2}}{2(y-1)}$

$= \frac{-(y+1) \pm \sqrt{-3y^2+10y-3}}{2(y-1)}$ is real iff

$y - 1 \neq 0 \Rightarrow y \neq 1$

If $y = 1$ then original equation gives $x = 0$, so taking $y = 1$

Also $3y^2 - 10y + 3 \leq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow y \in \left[\frac{1}{3}, 3\right] \therefore$ Range is $\left[\frac{1}{3}, 3\right]$

27. **(b)** We have, $f(x) = \exp\left(\sqrt{5x-3-2x^2}\right)$

i.e., $f(x) = e^{\sqrt{5x-3-2x^2}}$

For Domain of $f(x)$, $\sqrt{5x-3-2x^2}$ should be +ve.

i.e., $\sqrt{5x-3-2x^2} \geq 0$

taking –ve sign common)

$\Rightarrow\ 2x(x-1)-3(x-1)\le 0$

$\Rightarrow\ (2x-3)(x-1)\le 0$

$\Rightarrow\ 2x-3\le 0$ or $x-1\ge 0$

$\Rightarrow\ x\le \frac{3}{2}$ or $x\ge 1$

$\therefore\ 1\le x\le \frac{3}{2}$ i.e., $x\in\left[1,\frac{3}{2}\right]$

Hence, domain of the given function is $[1, \frac{3}{2}]$.

28. (a) $f(x+y)=f(x)+2y^2+kxy$

$f(1+y)=2+2y^2+ky$, putting $x=1$

putting $y=1$,

$f(2)=8=2+2+k \Rightarrow k=4$

$\therefore f(1+y)=2+2y^2+4y=2(y+1)^2$

$\therefore f(x)=2x^2$

29. (d) Here $R=\{(x,y):|x^2-y^2|<16\}$

and given $A=\{1,2,3,4,5\}$

$\therefore$ R = {(1, 2) (1, 3) (1, 4); (2, 1) (2, 2) (2,3) (2, 4); (3, 1); (3, 2) (3,3) (3, 4); (4, 1) (4,2) (4, 3); (4, 4), (4, 5), (5, 4) (5, 5)}

30. (c) If $n=0$, then $h(n)$ is not defined, so, 'h' is not a function. All other are functions.