POWERS AND ROOTS
SQUARE, CUBE, INDICES, SURDS SQUARING

Squaring of a number is largely used in mathematical calculations. There are so many rules for special cases. But we will discuss a general rule for squaring which is capable of universal application.

Squaring is multiplying the number by itself. For example,

$$25^2 = 25 \times 25 = 625$$

When the number is large, squaring by simple multiplication is obviously not very easy.

You should remember the following squares which will help you in taking square roots :

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

Short Methods in Squaring

Let *a, b* denote numbers

$$a^2 = a^2 - b^2 + b^2 = (a^2 - b^2) + b^2$$

or $$a^2 = [(a + b)(a - b)] + b^2 \quad ...(I)$$

$$(a + b)^2 = a^2 + b^2 + 2ab \quad ..(II)$$

$$(a - b)^2 = a^2 + b^2 - 2ab \quad ...(III)$$

These are very useful as we can write the given number as sum or difference of two convenient numbers.

Example : Find $(1213)^2 = ?$

$= [(1213 - 13)(1213 + 13)] + (13)^2$

$= (1200 \times 1226) + 169 = 1471200 + 169$

$= 1471369$

Squaring of a number ending in 5

Multiply the number formed after deleting 5 at the units place with the number, one higher than it. Annex 25 on the right side of the product and you will get the square of the given number.

Example : Find $(165)^2 = ?$

Solution : $16 \times 17 = 272$

So, $(165)^2 = 27225$

Properties of Squares

1. It cannot be a negative number.
2. It cannot have odd number of zeros at its end.
3. It cannot end with 2, 3, 7 or 8.
4. Square of an even number is always an even number.
5. Square of an odd number is always an odd number.
6. Every square number is either a multiple of 3 or exceeds multiple of 3 by unity.
7. Every square number is either a multiple of 4 or exceeds multiple of 4 by unity.
8. If a square number ends in 9, the digit preceding 9 must be either zero or even.
9. 1, 5, 6 and 0 at the end of a number reproduce themselves as the last digit in their squares.

Square of Decimal Number

Find the square of the number ignoring the decimal point. Put the decimal point leaving double the number of digits (from the right) as compared to that in the given number. In other words, the position of decimal place in the square is double of that in the original number. The square will lie between the square of integral part and the square of the number, one higher than the integral part.

Example : Find the square of 14.52 = ?

Solution : $(1452)^2 = 2108304$; $(14.52)^2 = 210.8304$

Square of Fraction :

$$\left(\frac{p}{q}\right)^2 = \frac{p^2}{q^2}$$

Square of $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}$ etc.

Multiply the integral part by one more than it. Add $\frac{1}{4}$ to the product and you will get the square of the given half fraction.

Example : Find the square of $4\frac{1}{2}$.

Solution : $\left(4\frac{1}{2}\right)^2 = 4\times5+\frac{1}{4} = 20+\frac{1}{4}$

$$= \frac{81}{4} = 20\frac{1}{4}$$

Square of Number Consisting of 9s only

Let the number consists of n 9s.

Write down $(n-1)$ 9*s*, followed by one 8, then $(n-1)$ zeros and finally annex 1 at the end.

Example : Find $(99999)^2$.

Solution : The given number consists of five 9s.

So, we will write four 9s followed by 8, then four zeros and finally 1.

i.e., $(99999)^2 = 9999800001$.

SQUARE ROOT

Square root is inverse of square. Square root of a given number may be defined as the number whose square is equal to the given number. In other words, square root of a given number is the number, which when multiplied by itself, gives the product equal to the given number.

Example : $\sqrt{4} = 2$ and $2 \times 2 = 4$,

$\sqrt{9} = 3$ and $3 \times 3 = 9$

There are two methods of finding square root of a number.

Method I : By Factorization

This method is generally used where the given number is a perfect square or when the number can be written as product of such factors whose square roots are known.

You should know following common square roots:

$\sqrt{0} = 0$	$\sqrt{15} = 3.873$
$\sqrt{1} = 1$	$\sqrt{16} = 4$
$\sqrt{2} = 1.414$	$\sqrt{17} = 4.123$
$\sqrt{3} = 1.732$	$\sqrt{19} = 4.359$
$\sqrt{4} = 2$	$\sqrt{21} = 4.583$
$\sqrt{5} = 2.236$	$\sqrt{22} = 4.690$
$\sqrt{6} = 2.449$	$\sqrt{23} = 4.796$
$\sqrt{7} = 2.646$	$\sqrt{25} = 5$
$\sqrt{9} = 3$	$\sqrt{36} = 6$
$\sqrt{10} = 3.162$	$\sqrt{49} = 7$
$\sqrt{11} = 3.317$	$\sqrt{64} = 8$
$\sqrt{13} = 3.606$	$\sqrt{81} = 9$
$\sqrt{14} = 3.742$	$\sqrt{100} = 10$

In factorization method, we write the given number as product of prime factors and take the product of prime factors, choosing one out of every pair.

Note :

1. Square root of a number greater than or equal to 1 but less than 100 consists of only one digit.
2. Square root of a number greater than or equal to 100 but less than 10000 consists of two digits.
3. In general, if the given number has *'n'* digits, its square root will have $n/2$ digits when n is even and $\frac{n+1}{2}$ digits when n is odd. This holds good for the case of pure decimal fractions too.

Some properties of exact square roots (*i.e.*, square roots are whole numbers)

1. A pure square number ending in 1 must have 1 or 9 as the last digit in its square root.
   For example, $\sqrt{81} = 9$; $\sqrt{121} = 11$.
2. If a square ends in 4, its square root must have 2 or 8 as the last digit.
   For example, $\sqrt{64} = 8$; $\sqrt{144} = 12$.
3. If a square ends in 5 or 00, its square root must have 5 or 0 respectively as the last digit.
   For example, $\sqrt{625} = 25$; $\sqrt{100} = 10$.
4. A square ending in 9 has 3 or 7 as the last digit in its square root.
   For example, $\sqrt{169} = 13$; $\sqrt{729} = 27$.

Method II : By Division

It is the most general method of finding square roots and is applicable to all cases.

Step I : Mark-off groups of two digits, starting from right. The extreme group may be either single digit or a pair.

Step II : Start division process from the extreme left group.

Step III : For the second stage, add the quotient to the divisor. The divisor of this stage will be equal to this sum with the quotient for this stage suffixed to it. The next dividend is always obtained by annexing the next pair of digits (of the dividend) to the remainder.

Step IV : For the next stage, again add the divisor and the quotient of the previous stage. The divisor for this stage will be formed in the same manner as explained for the second stage in step III.

Step V : Continue step IV till all the groups get exhausted, in case a remainder is left, annex two zeros to it and put a decimal point in the quotient.

At every stage after this we will annex two zeros to the remainder.

Continue to the number of decimal places required in the result. The quotient is equal to the square root of the given number.

CUBE ROOTS

If $a^3 = x$, then $a = \sqrt[3]{x}$; a is the cube root of x

Cube root of $8 = \sqrt[3]{2 \times 2 \times 2} = 2$

"Cube root of" $27 = \sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$

"Cube root of" $216 = \sqrt[3]{6 \times 6 \times 6} = 6$

"Cube root of" $0.000064 = \sqrt[3]{0.04 \times 0.04 \times 0.04} = 0.04$

Example : Evaluate $\sqrt[3]{1325 + \sqrt{20 + \sqrt{256}}}$

Solution : $\sqrt[3]{1325 + \sqrt{20 + 16}}$ $(\because \sqrt{256} = 16)$

$= \sqrt[3]{1325 + \sqrt{36}}$

$= \sqrt[3]{1325 + 6}$ $(\because \sqrt{36} = 6)$

$= \sqrt[3]{1331} = \sqrt[3]{11 \times 11 \times 11} = 11$

EXERCISE

1. The largest number of five digits which is a perfect square, is :

(*a*) 99999 (*b*) 99764
(*c*) 99976 (*d*) 99856
(*e*) None of these

2. The value of $\sqrt{2}$ up to three places of decimals is :

(*a*) 1.410 (*b*) 1.412
(*c*) 1.413 (*d*) 1.414
(*e*) None of these

3. $\dfrac{\left(\sqrt{7} + \sqrt{5}\right)}{\sqrt{7} - \sqrt{5}}$ is equal to :

(*a*) $6 + \sqrt{35}$ (*b*) $6 - \sqrt{35}$
(*c*) 2 (*d*) 1
(*e*) None of these

4. The least number by which 294 must be multiplied to make it a perfect square, is :

(*a*) 2 (*b*) 3
(*c*) 6 (*d*) 5
(*e*) None of these

5. The least number to be added to 269 to make it a perfect square , is :

(*a*) 31 (*b*) 16
(*c*) 7 (*d*) 20
(*e*) None of these

6. What is the smallest number by which 3600 be divided to make it a perfect cube?

(*a*) 9 (*b*) 50
(*c*) 300 (*d*) 450
(*e*) None of these

7. The smallest number of 4 digits, which is a perfect square is :

(*a*) 1000 (*b*) 1016
(*c*) 1024 (*d*) 1036
(*e*) None of these

8. $\sqrt{10} \times \sqrt{250} = ?$

(*a*) 46.95 (*b*) 43.75
(*c*) 50.25 (*d*) 50
(*e*) None of these

9. $\sqrt{?}/200 = 0.02$

(*a*) 0.4 (*b*) 4
(*c*) 16 (*d*) 1.6
(*e*) None of these

10. $\sqrt{.04} = ?$

(*a*) .02 (*b*) .2
(*c*) .002 (*d*) 1.2
(*e*) None of these

11. The greatest number of four digits which is a perfect square, is :

(*a*) 9996 (*b*) 9801
(*c*) 9900 (*d*) 9604
(*e*) None of these

12. $\sqrt[3]{?}\ / 200 = 0.02$

(*a*) 0.4 (*b*) 64
(*c*) 16 (*d*) 1/64
(*e*) None of these

13. If $\sqrt{256} \div \sqrt[3]{x} = 2$, then x is equal to :

(*a*) 64 (*b*) 128
(*c*) 512 (*d*) 1024
(*e*) None of these

14. $112/\sqrt{196} \times \sqrt{576}/12 \times \sqrt{256}/8 = ?$

(*a*) 8 (*b*) 12
(*c*) 16 (*d*) 32
(*e*) None of these

15. $(2\sqrt{27} - \sqrt{75} + \sqrt{12})$ is equal to :

(*a*) $\sqrt{3}$ (*b*) $2\sqrt{3}$
(*c*) $3\sqrt{3}$ (*d*) $4\sqrt{3}$
(*e*) None of these

16. $\sqrt{50} \times \sqrt{98}$ is equal to :

(*a*) 65.95 (*b*) 63.75
(*c*) 70.25 (*d*) 70
(*e*) None of these

17. The largest four-digit number which is a perfect cube, is :

(*a*) 9999 (*b*) 9261
(*c*) 8000 (*d*) 8467
(*e*) None of these

18. If $\sqrt{2} = 1.4142$, the square root of $\dfrac{(\sqrt{2}-1)}{\sqrt{2}+1}$ is equal to :

(*a*) 0.732 (*b*) 0.3652
(*c*) 1.3142 (*d*) 0.4142
(*e*) None of these

19. $\dfrac{\sqrt{121} \times 0.9}{1.1 \times 0.11} = ?$

(*a*) 2 (*b*) $\dfrac{900}{11}$
(*c*) 9 (*d*) 11
(*e*) None of these

20. $\sqrt{25}/15625 = \sqrt{?}/30625$

(*a*) 2 (*b*) 3.5
(*c*) 96.04 (*d*) 1225
(*e*) None of these

21. $\sqrt{3.61/10.24} = ?$

(*a*) 29/32 (*b*) 19/72
(*c*) 19/32 (*d*) 29/62
(*e*) None of these

22. $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = ?$

(*a*) $\sqrt{2}$ (*b*) 2
(*c*) 4 (*d*) 8
(*e*) None of these

23. $\dfrac{1}{\sqrt{9}-\sqrt{8}} = ?$

(*a*) $1/2\,(3-\sqrt{2})$ (*b*) $1/3+2\sqrt{2}$
(*c*) $(3-2\sqrt{2})$ (*d*) $(3+2\sqrt{2})$
(*e*) None of these

EXPLANATORY ANSWERS

1. (*d*) : Largest number of 5 digits is 99999.

```
3 ) 9 99 99 ( 316
  – 9
61 ) 99 (
   – 61
626 ) 3899 (
      3756
    – 143
```

So, required number $= (99999 - 143) = 99856$.

2. (*d*) :

```
1 ) 2.00 00 00 ( 1.414
  – 1
24 ) 100 (
    – 96
281 ) 400 (
     – 281
2824 ) 11900 (
      – 11296
```

So, $\sqrt{2} = 1.414$.

3. (*a*) : $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

$= \frac{(\sqrt{7}+\sqrt{5})^2}{7-5} = \frac{7+5+2\sqrt{7}\times\sqrt{5}}{2}$

$= \frac{12+2\sqrt{35}}{2} = 6+\sqrt{35}$.

4. (*c*) : 294 = 7 × 7 × 2 × 3. To make it a perfect square it must be multiplied by 2 × 3, *i.e.,* 6.

5. (*d*) :

```
   1 ) 269 ( 16
      -1
  26 ) 169 (
     - 156
        13
```

Required number to be added = $(17)^2 - 269$ = 20.

6. (*d*) : 3600 = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5.
To make it a perfect cube, we must divide it by 2 × 5 × 5 × 3 × 3 = 450

7. (*c*) : Smallest number of 4 digits = 1000

```
   3 ) 1000 ( 31
      - 9
  61 ) 100 (
      - 61
        39
```

So, required number = $(32)^2$ = 1024.

8. (*d*) : $\sqrt{10} \times \sqrt{250} = \sqrt{2500} = 50$.

9. (*c*) : Let $\sqrt{x}/200 = 0.02$
Then, $\sqrt{x} = 200 \times 0.02 = 4$
So, $x = 16$.

10. (*b*) : $\sqrt{.04} = \sqrt{4/100} = 2/10 = 0.2$

11. (*b*) : Greatest number of four digits = 9999
Now, 9999 = $(99)^2$ + 198
So, $(99)^2$ = 9999 – 198 = 9801,
So, required number = 9801.

12. (*b*) : Let $\sqrt[3]{x}/200 = 0.02$
Then, $\sqrt[3]{x} = 200 \times 0.02 = 4$,
So, $x = 4 \times 4 \times 4 = 64$.

13. (*c*) : $\sqrt{256}/\sqrt[3]{x} = 2 \Rightarrow 16 = 2\sqrt[3]{x}$
$\Rightarrow \sqrt[3]{x} = 8 \Rightarrow x = 512$

14. (*d*) : Given expression
= (112/14 × 24/12 × 16/8) = 32

15. (*c*) : $2\sqrt{27} - \sqrt{75} + \sqrt{12}$
$= 2\sqrt{9\times3} - \sqrt{25\times3} + \sqrt{4\times3}$
$= 6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$

16. (*d*) : $\sqrt{50} \times \sqrt{98} = \sqrt{4900} = 70$

17. (*b*) : Clearly, 9261 is a perfect cube.

18. (*d*) : $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{1}$

So $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{2} - 1 = 1.4142 - 1 = 0.4142$

19. (*b*) : Given expression $= \frac{\sqrt{121}\times0.9}{1.1\times0.11}$

$= \frac{11\times9\times1000}{11\times11\times10} = \frac{900}{11}$

20. (*c*) : $\frac{\sqrt{25}}{15625} = \frac{\sqrt{x}}{30625} \Rightarrow \sqrt{x} = \frac{30625\times5}{15625} = 9.8$
$\therefore\ x = 96.04$.

21. (*c*) : $\sqrt{3.61/10.24} = \sqrt{361/1024}$

$\frac{\sqrt{19\times19}}{\sqrt{32\times32}} = 19/32$

22. (*b*) : $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \frac{\sqrt{16\times2}+\sqrt{16\times3}}{\sqrt{4\times2}+\sqrt{4\times3}}$

$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = \frac{4}{2} = 2$

23. (*d*) : $\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}}$

$= \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$.