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CHAPTER

Theory of Errors

Q.1 The sides of a rectangle are (120 ± 0.05) m and (180 ± 0.06) m. The probable error in the area will be
(a) ± 16.8 sq m (b) ± 12.3 sq m
(c) ± 16.2 sq m (d) ± 11.53 sq m

Q.2 A plot of land 60 m × 20 m is measured by a steel tape. If the standard error of length and width measurements is taken as ±1 cm, then the standard error of the area of the plot would be
(a) ±0.1414 m^2 (b) ±0.566 m^2
(c) ±0.632 m^2 (d) ±0.8484 m^2

Q.3 A circle of radius 7 m has a standard error of 0.02 m on the radius. The standard error of its area is
(a) 0.04 m^2 (b) 0.14 m^2
(c) 0.28 m^2 (d) 0.88 m^2

Q.4 The difference between the most probable value of a quantity and its observed value is
(a) true error
(b) weighted observations
(c) conditional error
(d) residual error

Q.5 Theory of probability is applied to
(a) accidental errors only
(b) cumulative errors only
(c) both accidental and cumulative errors
(d) none of the above

Q.6 If the weight of an angle A is 3 and weight of angle B is 4, what will be the weight of $(3A - B + 90°)$?
(a) 1/7 (b) 1
(c) 4/13 (d) 91

Q.7 If a quantity A has a weight of 3, then the weight of $A/3$ will be
(a) 28 (b) 27
(c) 24 (d) 21

Q.8 The probable systematic error in precise levelling as recommended by IGA should not exceed
(a) $\pm 0.1\sqrt{K}$ mm (b) $\pm 0.2\sqrt{K}$ mm
(c) $\pm \sqrt{K}$ mm (d) $0.2\sqrt{K}$ mm
Where K is distance in kilometers

Q.9 If $A = 42° 10' 40''$ has weight of 3, then the weight of $A/4$ will be
(a) 48 (b) $\frac{3}{4}$
(c) $\frac{3}{16}$ (d) 12

Q.10 The residual error is the difference between
(a) true value and observed value of a quantity
(b) most probable value and observed value of a quantity
(c) most probable value and true value of a quantity
(d) none of the above

Q.11 The theory of least squares can be represented as
(a) $\Sigma e^2 = 0$ (b) $\Sigma W e \delta e = 0$
(c) Σe^2 = Minimum (d) $\Sigma 2W e \delta e = 0$
Where W = Weight of an observation and and e = Residual error

Q.12 If an equation $A + B = 55°$ has a weight of 3, then the weight of $180 - (A + B)$ is :
(a) 3 (b) $\frac{1}{3}$
(c) 9 (d) $\frac{1}{9}$

Q.13 The relationship between the probable error of single observation (E_s) and the probable error of the mean (E_m) is :
(a) $E_m = \frac{E_s}{n}$ (b) $E_m = \frac{E_s}{\sqrt{n}}$
(c) $E_m = \frac{E_s}{n^{2/3}}$ (d) $E_m = \frac{E_s}{2n^{1/2}}$
where, n = number of observations made

Q.14 Following are the observation equations :
$A = 25° 30'$
$2A = 51° 10'$
The normal equation in A will be
(a) $3A = 76° 40'$ (b) $5A = 127° 50'$
(c) Both (a) and (b) (d) None of the above

Q.15 Probable error of an observation of unit weight is given by
(a) ± standard error
(b) $\pm\frac{1}{\sqrt{5}}$ × standard error
(c) ± 0.5 × standard error
(d) ± 0.6745 × standard error

Q.16 The sides of a rectangular are (120 ± 0.05) m and (180 ± 0.06) m. The probable error in the area will be
(a) ± 16.80 m^2 (b) ± 12.35 m^2
(c) ± 16.70 m^2 (d) ± 16.20 m^2

Q.17 Pick the correct statement:
(a) The apparent error on reversal is twice the actual error.
(b) The correction may be made equal to half the observed discrepancy.
(c) The good results may be obtained from a defective instrument by reversing and taking the mean of two erroneous results
(d) All of the above

Q.18 If θ is the slope of the ground and l is the measured distance, the correction is
(a) $2l\sin^2\frac{\theta}{2}$ (b) $2l\cos^2\frac{\theta}{2}$
(c) $2l\tan^2\frac{\theta}{2}$ (d) $2l\cot^2\frac{\theta}{2}$

Q.19 The probable error of the adjustments bearing at the middle is
(a) $\frac{r\sqrt{n}}{2}$ (b) $\frac{r\sqrt{n}}{3}$
(c) $\frac{r\sqrt{n}}{4}$ (d) $\frac{r\sqrt{n}}{5}$

Q.20 Measuring with 30 m chain, 0.01 m too short, introduces
(a) +ve compensating error
(b) –ve compensating error
(c) +ve cumulative error
(d) –ve cumulative error

Q.21 The random errors tend to accumulate proportionally to
(a) number of operations involved
(b) reciprocal of operations involved
(c) square root of the number of operations involved
(d) cube root of the number of operation involved

Q.22 The slope correction may be ignored if
(a) slope of ground is less than 3°
(b) slope of ground is 1 in 19
(c) both (a) and (b)
(d) Neither (a) nor (b)

Q.23 The apparent error on reversal is
(a) equal to the actual error
(b) twice the actual error
(c) thrice the actual error
(d) None of these

Q.24 Correction per chain length of 100 links along a slope having a rise of 1 unit in n horizontal units, i.e.,
(a) $100/n^2$ (b) $100\,n^2$
(c) $100/n^3$ (d) $100/n$

Q.25 Accidental or compensating error of length L are proportional to
(a) L (b) $\sqrt{L}$
(c) $L^{1/3}$ (d) $\frac{1}{\sqrt{L}}$

Q.26 The correction to be applied to each 30 m chain length along 0° slope is

(a) 30 (sec θ – 1) (b) 30 (cos θ – 1)
(c) 30 (sin θ – 1) (d) 30 (tan θ – 1)

Q.27 A horizontal angle is measured five times and results are as follows

156° 45′ 34″ 156° 45′ 38″
156° 45′ 29″ 156° 46′ 30″
156° 45′ 32″

The standard error of the measurement is most nearly

(a) 3″ (b) 4″
(c) 8″ (d) 12″

Q.28 Which of following error obey probability law and is reduced in theory of errors and adjustments?

(a) Personal and accidental errors
(b) Systematic error
(c) Instrumental error
(d) All of the above

Q.29 The sides of a rectangular track were measured as 82.397 m and 66.132 m with a 30 m metallic tape too short by 25 mm. The error in the area of track is

(a) 0.24% (b) 0.17%
(c) 0.19% (d) 0.26%

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Answers Theory of Errors

1. (d)	2. (c)	3. (d)	4. (d)	5. (a)	6. (c)	7. (b)	8. (a)	9. (a)	10. (b)
11. (b)	12. (a)	13. (b)	14. (a)	15. (d)	16. (b)	17. (d)	18. (a)	19. (a)	20. (c)
21. (c)	22. (c)	23. (b)	24. (a)	25. (b)	26. (a)	27. (b)	28. (a)	29. (b)	

Explanations Theory of Errors

1. **(d)**
Error in area is given by

$$e_A = \sqrt{\left(e_l \frac{dA}{dl}\right)^2 + \left(e_b \frac{dA}{db}\right)^2}$$

$$\frac{dA}{dl} = b = 180$$

$$\frac{dA}{db} = l = 120$$

$$\therefore \quad e_A = \sqrt{(0.05 \times 180)^2 + (0.06 \times 120)^2}$$

$$= \sqrt{9^2 + 7.2^2}$$

$$\simeq 11.53 \text{ sq.m}$$

2. **(c)**
Standard error of area is given by

$$e_A = \sqrt{\left(e_l \frac{dA}{dl}\right)^2 + \left(e_b \frac{dA}{db}\right)^2}$$

$\because \quad A = l \times b$

$\because \quad \frac{dA}{dl} = b = 20$

and $\frac{dA}{db} = l = 60$

Now, $e_l = e_b = \pm 1\text{cm} = 0 = 0.01$ m

$$\therefore \quad e_A = \sqrt{(0.01 \times 20)^2 + (0.01 \times 60)^2}$$

$$= \pm 0.632 \text{ m}^2$$

3. **(d)**
Area of circle

$$A = \pi r^2$$

$$\therefore \quad \frac{dA}{dr} = 2\pi r$$

Let standard error in radius be e_r

∴Standard error in area,

$$e_A = e_r \frac{dA}{dr}$$

$$= 0.02 \times 2 \times \pi \times 7$$

$$= 0.88 \text{ m}^2$$

6. **(c)**
Given,
Weight of $A = 3$ (weight)
Weightof $B = 4$ (weight)

$$\therefore \text{ Weight of } 3A = \frac{3}{3^2} = \frac{1}{3}$$

∴ Weight of $3A - B$

$$= \frac{1}{\left(3 + \frac{1}{4}\right)} = \frac{4}{13}$$

$$\therefore \text{Weight of } 3A - B + 90 = \frac{4}{13}$$

Hence option (c) is correct.

7. **(b)**

$$A = 3 \text{ (weight)}$$

$$\text{Weight of } \frac{A}{3} = 3 \times (3)^2 = 27$$

Hence option (b) is correct.

9. **(a)**

$$A = 42^\circ 10' 40'' \qquad \text{Weight} = 3 \quad \ldots(i)$$

$$\text{Weight of } \frac{A}{4} = 3 \times (4)^2$$

$$= 3 \times 4^2 = 48$$

Hence option (a) is correct.

10. **(b)**
True error: It is the difference between true value and measured value.
Mean error: It is the average error of a quantity during different-different observations.
Conditional error: This type of error always occurs in same direction.
Residual error: It is the difference between the most probable value of a quantity and its observed value.
Hence option (b) is correct.

11. **(b)**
According to the theory of least squares, the most probable value of a quantity is the one for which the sum of squares of the residual errors is minimum.

Let $x_1, x_2, x_3 \ldots x_n$ are different measurement with corresponding weights, $w_1, w_2, w_3 \ldots w_n$.
Assuming x is most probable value of quantity.

Errors	Square of errors	Total error
$(x - x_1) = e_1$	e_1^2	$W_1 e_1^2$
$(x - x_2) = e_2$	e_2^2	$W_2 e_2^2$
$(x - x_3) = e_3$	e_3^2	$W_3 e_3^2$
—	—	—
—	—	—
$(x - x_n) = e$	e_n^2	$W_n e_n^2$

Sum of squares of error,

$$y = W_1 e_1^2 + W_2 e_2^2 + W_3 e_3^2 + \ldots W_n e_n^2$$

$$y = \Sigma W e^2$$

According to the principle of least squares

$$\frac{dy}{dx} = \Sigma 2We\,\delta e = 0$$

or $\quad \Sigma We\delta e = 0$

Hence option (b) is correct.

12. **(a)**

$$A + B = 55^\circ \qquad \ldots(Wt = 3)$$

If an equation is added to or subtracted from a constant, the weight remains unchanged
∴Weight of $180 - (A + B) = 3$
Hence option (a) is correct.

13. **(b)**
The relationship between the probable error of single observation (E_s) and the probable error of the mean (E_m) is equal to

$$E_m = \frac{E_s}{\sqrt{n}} = 0.6745 \sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

where, n = difference between any single measurement and mean of the series
n = no. of observations.

14. **(a)**

$$A = 25^\circ 30'$$

$$2A = 51^\circ 10'$$

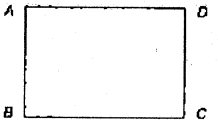
The normal equation in A,

$A + 2A = 25° 30' + 51° 10'$

$3A = 76° 40'$

Hence option (a) is correct.

16. (b)



Side, $AB = (120 \pm 0.05)$

Side, $BC = (180 \pm 0.06)$

Area, $S = 120 \times 180\ m^2$

Probable error in multiplication,

$$e_x = \pm S\sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

$$e_s = \pm 120 \times 180\sqrt{\left(\frac{0.05}{120}\right)^2 + \left(\frac{0.06}{180}\right)^2}$$

$e_s = \pm 11.53$

Hence no option is correct but most close option is (b).

27. (b)

Clearly movement number 4 has 1' blunder and should be discarded.

$$\therefore \quad \alpha_{mean} = \frac{\Sigma\alpha}{4}$$

$$= \frac{156°45'34'' + 156°45'29'' + 156°45'32'' + 156°45'38''}{4}$$

$$= 156° 45' 32.2''$$

Subtract the mean from each of the four angle, yielding four residuals.

$+ 0.8'', + 4.8'', - 4.2'', - 1.2''$

The standard error of measurement is, σ

$$= \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{4}}$$

$$= 3.8'' \simeq 4''$$

28. (a)

Both accidental and personal errors represents the limit of precision in determination of value. They obey the law of probability and therefore theory of errors and adjustment applies to them.

29. (b)

Let two sides of the track be x_1 and x_2, then area

$$y = x_1 x_2$$

If error in x_1 and x_2 are dx_1 and dx_2 respectively, then error in y

$$dy = \frac{\partial y}{\partial x_1} \times dx_1 + \frac{\partial y}{\partial x_2} \times dx_2$$

$$dx_1 = \frac{0.025}{30} \times 82.397 = 0.069\ m$$

$$dx_2 = \frac{0.025}{30} \times 66.132 = 0.055\ m$$

$$\therefore \quad dy = 66.132 \times 0.069 + 82.397 \times 0.055$$

$$= 9.095\ m^2$$

$$\% \text{ error} = \frac{9.095}{82.397 \times 66.132} \times 100 = 0.17\%$$

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