2

Number System

2.01 Review of Rational Numbers on the Number Line

Numbers have great impotance in our daily life. We have studied about different numbers begining from natural numbers to rational numbers. Now, we review them on the number line.

(i) Natural numbers

This line increases towards the right hand side from 1 to infinity.

Fig. 2.01

(ii) Whole numbers

This line increases towards the right hand side from 0 to infinity.

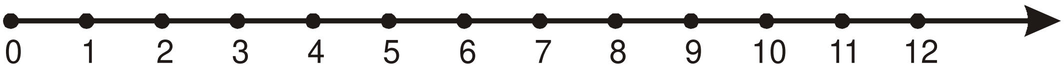

Fig. 2.02

(iii) Integers

This line increases towards both sides starting from 0 to infinity.

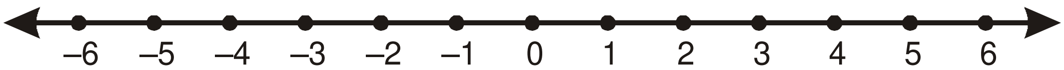

Fig. 2.03

(iv) Rational numbers

This line increases towards both sides starting from 0 to infinity.

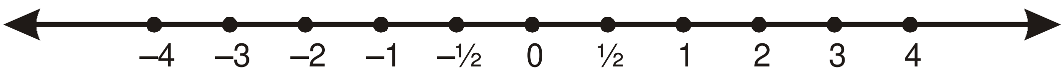

Fig. 2.04

But there are many numbers found between –1, 0 and 0, 1. We can use the concept of mean to find out the rational numbers between two rational numbers. There are infinitely rational numbers between two rational numbers.

With the course of changing time, numbers also developed. First of all, natural numbers were developed. Sum and multiplication of two natural numbers is also a natural number. The set of natural numbers is denoted by *N, i.e.,*

$$N = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$$

If we want to solve the question $x + 7 = 7$, then we get the value of x is 0. So we can not slove this equation with the help of natural numbers so the number zero is included to the set of natural numbers and this new set is known as set of whole numbers which is denoted by *W, i.e.,*

$$W = \{0, 1, 2, 3, 4, 5, 6, 7, \ldots\}$$

If we slove of the equation $x + 15 = 6$. and found the value of x, then we need a number $x = -9$ which is not a whole number. Here, again we need to develop an another set of numbers. If we include negative numbers to the set of whole numbers, we get another set of numbers which is known as set of integers. The set of integers is denoted by *I* or *Z, i.e.,*

$$Z = \{-3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Now, we notice that some more numbers also lie between two integers which come out when integer is divided by another integer for example $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \ldots$

A number which can be represented in the form of $\frac{p}{q}$ is called a rational number, where p and q both are integer but $q \neq 0$ either $\frac{p}{q}$ is turminating or it is recurring. In a set of rational number, natural numbers, whole numbers and integers are included.

There are infinitely rational numbers between two rational numbers.

2.02 Irrational Numbers

Let us see the number line again and think whether all the numbers have been included on this number line or some number have been still remaining. Let us discuss the remaining numbers that are not rational number. Such numbers that cannot be expressed in the form of $\frac{p}{q}$, where p and q are intergers and $q \neq 0$, is called an irrational number. As we know that there are infinitely rational numbers. In the same way there are infinitely irrational numbers, for which some examples are given below :

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi, 0.15150015000150000....$$

Recall that when we use the symbol "$\sqrt{\ }$", we assume that it is the square root of a positive number. So $\sqrt{25}=5$, though 5 and -5 both are square root of 25.

The set of all rational and irrational numbers is named as the set of real numbers which is denoted by R.

2.03. Real Numbers and their Decimal Expansions

Now, we will discuss decimal expansion to distinguish between rational and irrational numbers. We will also explain how real numbers can be represented on the number line using their decimal expansions. Let us start with the rational numbers.

Let us see three examples : $\frac{3}{8}, \frac{8}{9}, \frac{6}{7}$

$$\frac{3}{8}=0.375$$

$$\frac{8}{9}=0.88888...$$

$$\frac{6}{7}=0.857142857142...$$

In all of the above examples, $\frac{p}{q},\ (q \neq 0)$ is applied on rational number, then we get the various situation after dividing p by q which are following,

I Condition : Remainder becomes zero

In the example of $\frac{3}{8}$ the remainder becomes 0 after some steps. Decimal expansion of $\frac{3}{8}$ is 0.375. Let us consider some other example like $\frac{1}{4}=0.25,\ \frac{456}{125}=3.648$. In these examples, the decimal expansion terminates after a finite number of steps. The expansion of such numbers is called terminating decimal expansion.

II Condition : The remainder never becomes zero but we get a repeatition of digits in the quotient.

For example $\frac{8}{9}=0.888888...$ and $\frac{6}{7}=0.857142857142...$ The expansion of such numbers is non-terminating recurring.

In the decimal expansion of $\frac{8}{9}$, 8 repeats. So the usual way of showing repeatiton of 8 is to write it as $0.\overline{8}$. Similarly, in decimal expansion of $\frac{6}{7}$ the block 857142 repeats, So we write $\frac{6}{7}$ in the form $0.\overline{857142}$, where the bar above the digits indicates that block of digits that repeats. Similarly, 2.67474 can also be written as $2.6\overline{74}$. From above examples, we get non-terminating (repeating) decimal expansion. In this way we found that there are only two cases of decimal expansion in rational numbers, they can either be terminating or non terminating repeating (recurring) decimals.

Example 1. Show that 2.152786 is a rational number or express 2.152786 in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution : We have $2.152786 = \frac{2152786}{1000000}$, so it is a rational number.

Example 2. Show that 0.8888 . . . = $0.\overline{8}$ can be expressed as $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

Solution : Let

$$x = 0.\overline{8}$$

or $\qquad x = 0.8888\ ...$ $\qquad$ . . . (i)

Multiplying both the sides of equation 10, we get

$$10x = 10 \times (.8888....) = 8.888\ ...$$

$$10x = 8.888... \qquad \text{. . . (ii)}$$

Subtracting (i) from (ii), we have

$$9x = 8.000... = 8$$

$$\Rightarrow \qquad x = \frac{8}{9}$$

Example 3. Show that $0.\overline{47}$ = 0.474747 . . . can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution : Let $\qquad x = 0.4747...$ $\qquad$ . . . (i)

Since two digits are repeating, so multipling equation (i) by 100, we get

$$100x = 47.474747... \qquad \text{. . . (ii)}$$

Substracting (i) from equation (ii), we get

$$99x = 47.000... = 47$$

$\therefore \qquad x = \frac{47}{99}$

Example 4. Show that $0.12\overline{3} = 0.123333...$ **can be expressed as** $\frac{p}{q}$**, where p and q are integers and** $q \neq 0$

Solution : Let $\qquad x = 0.12333 \qquad \text{. . . (i)}$

Here, 1 and 2 do not repeat, but 3 repeats, since first two digits are non-repeating.

$$100x = 12.333 \qquad \text{. . . (ii)}$$

Again, multipling the equation (ii) by 10, we get

$$1000x = 123.333 \qquad \text{. . . (iii)}$$

Now subtracting equation (ii) form equation (iii), we get

$$900x = 111.000$$

$$x = \frac{111}{900} = \frac{37}{300}$$

So, every number with a non-terminating recurring decimal expansion can be expresses in the form of $\frac{p}{q}$, where p and q are integers and $p \neq 0$.

The decimal expansion of a rational number is either terminating or non-terminating recurring. Now, we think a number like $x = 0.150120015000150000...$ and we find that this number can not be changed in the form of $\frac{p}{q}$, (where p and q are integers and $q \neq 0$.)

So, due to specific property of number like this, it is called irrational number.

Hence, the number whose decimal expansion is non-terminating, non-recuring is called irrational number.

Infinitely many irrational numbers can be generated equivalent to x.

Decimal expansion of some irrational number $\sqrt{2}$, $\sqrt{3}$ are given here.

$$\sqrt{2} = 1.41421356237...$$

$$\sqrt{3} = 1.73205080756...$$

For over the years, mathematicians have developed various techniques to produce more and more digits in the decimal expansion of irrational numbers. For example, to find digits in the decimal expansion of $\sqrt{2}$ by the division method. From the sulbasutras (rules of chords), a mathematical book of vedic period (800 B.C.- 500 B.C.). We get the approximate value.

Similarly the history of finding the decimal expansion of π is also quite interesting

Greek's famous scientist Archimedes calculated the value of π in decimal expansion. He showed $3.140845 < \pi < 3.142857$. Aryabhatta (476-550AD), the great Indian mathematician and astronomer, found the value of π exact to four decimal places (3.1416). Using high speed computers and advanced algorithms, the value of π has been computed to many decimal places.

Example 5. Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Solution : We can easily calculate that $\frac{1}{7} = 0.142857142857... = 0.\overline{142857}$

and $\frac{2}{7} = 0.2857142857142... = 0.\overline{2857142}$

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$, we find a number which is non-terminating, non-recurring lying between them. Thus, we can find infinitely many numbers. An example of a number like this is 0.150150015000150000... .

Exercise 2.1

1. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3797 (iv) 7.4784478 . . .

(v) 1.101001000100001 . . .

2. Write three numbers whose decimal expansions are non-terminating non-recurring.

3. Write the following in decimal form and show the type of decimal expansion

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$

(v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

4. Express the following in the form of $\frac{p}{q}$, where p and q are the integers and $q \neq 0$:

(i) $0.\overline{3}$ (ii) $0.\overline{47}$ (iii) $1.\overline{27}$ (iv) $1.2\overline{35}$

5. Find three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Representation of Real Numbers on the Number Line

As we have studied in the previous section that a real number can be either a rational or an irrational number. So we can say that any real number can be represented on the number line in its unique point and every point on the number line represents a unique real number. Due to this, the number line is called the real number line. With the help of some examples we shall study the representation method of irrational numbers on the number line.

Example 6. Represent $\sqrt{2}$ on the real number line.

Solution : It can be done easily. Take a square *OABC* with the side of unit length (See the Fig. 2.05)

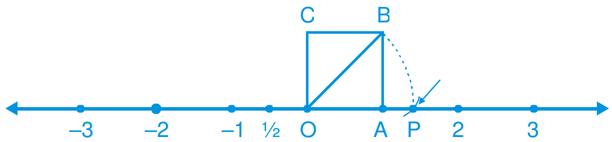

Fig. 2.05

Then using the Bodhyan theorem you can see that $OB = \sqrt{1^2 + 1^2} = \sqrt{2}$. Taking a distance of *OB* in *a* compass and with centre *O*, draw an arc on the number line that intersects it at the point *P*. Then point *P* on number line corresponds to $\sqrt{2}$.

Example 7. Represent $\sqrt{3}$ on the number line.

Solution : Let us see the Fig. 2.06.

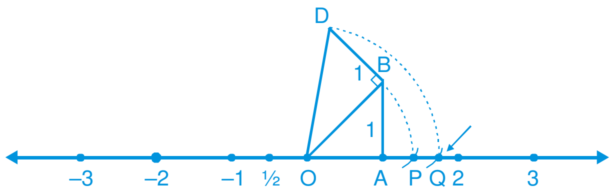

Fig. 2.06

Draw a perpendicular BD on the side OB with unit length. (As shown in the Fig. 2.06).

Then according to Bodhyan theorem $OD = \sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$. Taking *O* as centre and with the radius *OD*, draw an arc which intersect the number line at *Q* then *Q* corresponds to $\sqrt{3}$.

Similarly, after determining the place of $\sqrt{n-1}$ on the number line, we can easily determine the place of $\sqrt{n}$, where *n* is a positive integer.

Process of Successive Magnification

In the previous section, we have studied that a real number has its decimal expansion. We can represent the given number on a number line with the help of decimal expansion. Let us locate 2.05 on the number line as we see it on a sacle.

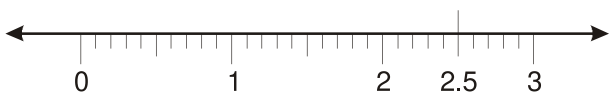

Fig. 2.07

For example, we want to determine the place of 2.775 on a number line. We have to consider the Fig 2.08 given below. We notice that 2.775 is located somewhere between 2 and 3. Divide this distance into ten equal parts. After that divide the distance between 2.7 and 2.8 in ten equal parts again. We divide the distance between 2.77 and 2.78 into ten equal parts. The point 2.775 is fifth point of this division. This process of visualisation of numbers in the number line through a magnifiying glass is called the process of successive magnification. So, we have seen that it is possible by sufficient successive magnifications to visualise the position of a real number with a terminating decimal expansion on the number line.

In the same way we can see the position of non-terminating non recurring real number on the number line with the help of this process.

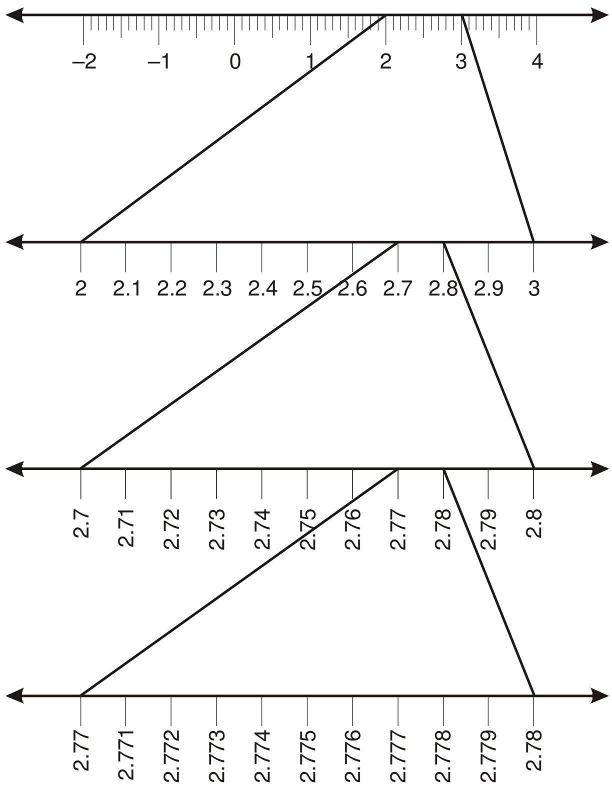

Fig. 2.08

Based on imagination of successive magnification form given in the above examples, we can again say that every real number is represented by a unique point on the number line. Further, every point on the number line represents one and only one real numer.

2.4. Geometrically Representation of a Real Number

If a is a natural number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$. The same definition can be extended for positive real numbers. Let $a > 0$ be a real number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$.

Now we shall show how the value of $\sqrt{x}$ can be found out on the number line geometrically, where x is a positive integer.

To find $\sqrt{x}$, for any positive real number x , we mark B so that $AB = x$ and as in Fig. 2.09 mark C so that $BC = 1$.

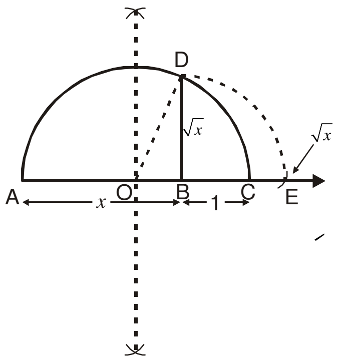

Fig. 2.09

In a right angled $\Delta\, OBD$ (Fig. 2.09), we have

So, $\quad OC = OD = OA = \dfrac{AB + BC}{2} = \dfrac{x+1}{2}$ unit

$$OB = AB - OA = x - \left(\frac{x+1}{2}\right) = \frac{x-1}{2} \text{ unit}$$

By bodhyan theorem, we get

$$BD^2 = OD^2 - OB^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 = \frac{4x}{4} = x$$

$\Rightarrow \qquad BD^2 = x$

$\Rightarrow \qquad BD = \sqrt{x}$

The representation of diagrametic and geometric method can be obtained by this construction which shows that $\sqrt{x}$ exists for all real numbers $x > 0$. If you want to know the position of $\sqrt{x}$ on the number line, then let us treat the line BC as the number line, with

B as zero, C as 1, and so on. Draw an arc with centre B and radius BD, which intersects the number line E (see the Fig. 2.09). Then E represents $\sqrt{x}$.

We can extend the idea of square roots to cube roots, fourth roots, fifth roots and in general n^{th} roots, where n is a positive integer. Recall your understanding of square roots and cube roots form earlier classes.

What is $\sqrt[3]{27}$? We know that $\sqrt[3]{27} = 3$. Find the value of $\sqrt[5]{243}$. If $b = \sqrt[5]{243}$ then $b^5 = 243$, $b^5 = (3)^5 \Rightarrow b = 3$. So, that value of $\sqrt[5]{243} = 3$. In the same way $\sqrt[n]{a}$ can be defined, where $a > 0$ and n is a positive integer.

Let $a > 0$ be real number and n is a positive integer, then $\sqrt[n]{a} = b$, if $b^n = a$ where $b > 0$.

Symbol "$\sqrt[n]{\ }$" is called the radical sign. $\sqrt[n]{a}$ can be expressed as $a^{1/n}$.

2.05. Operation on Real Numbers

We have studied in the previous classes, that rational numbers satisfy the commutative, associative and distributive laws for addition and multiplication. We have also studied that if we add, substract, multiply or divide (except zero) two rational numbers, we still get a rational number. That is rational numbers are closed with respect to addition, substration, multiplication and division. We also notice that irrational number also satisfy the commutative, associative, distributive laws for addition and multiplication. However the sum, difference, quotients and products of irrational are not always irrational. For example $\left(\sqrt{5}\right)-\left(\sqrt{5}\right)$, $\left(\sqrt{7}\right)\cdot\left(\sqrt{7}\right)$ and $\dfrac{\sqrt{13}}{\sqrt{13}}$ are rational numbers.

Let us see what happens when a rational number is added with an irrational number and a rational number is multiplied by an irrational number?

For an example, $\sqrt{5}$ is an irrational number, then the number $2+\sqrt{5}$ and $2\sqrt{5}$ are of what kind ? Undoubtfully these are irrational number because these numbers give the non-terminating non-recurring decimal expansion. Non-terminating non-recurring decimal expansion of irrational numbers can be understood more by the following examples.

Example 8. Check whether $7\sqrt{5}, \dfrac{7}{\sqrt{5}}, \sqrt{2}, \sqrt{2}+24,\ \pi-3$ are irrational or not.

Solution : We know that, $\sqrt{5} = 2.236...,\ \sqrt{2} = 1.4142...,\ \pi = 3.1415$

Then $\quad 7\sqrt{5} = 15.652...,\ \dfrac{7}{\sqrt{5}} = \dfrac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \dfrac{7\sqrt{5}}{5} = 3.1304...$,

$\sqrt{2}\quad 24\quad 25.4142....,\qquad 3\quad 0.1415$

All of these are non-terminating non-recurring decimals. Hence, all of these are irraitonal numbers.

Example 9. **Add :** $2\sqrt{3}+3\sqrt{5}$ and $\sqrt{3}-\sqrt{5}$.

Solution : $\left(2\sqrt{3}+3\sqrt{5}\right)+\left(\sqrt{3}-\sqrt{5}\right)$

$$=\left(2\sqrt{3}+\sqrt{3}\right)+\left(3\sqrt{5}-\sqrt{5}\right)$$

$$=(2+1)\sqrt{3}+(3-1)\sqrt{5}$$

$$=3\sqrt{3}+2\sqrt{5}$$

Example 10. **Multiply :** $6\sqrt{7}$ **by** $2\sqrt{7}$ **.**

Solution : $6\sqrt{7}\times 2\sqrt{7}=6\times 2\times\sqrt{7}\times\sqrt{7}$

$$=12\times 7=84$$

Example 11. **Divide :** $8\sqrt{15}$ **by** $2\sqrt{5}$ **.**

Solution : $8\sqrt{15}\div 2\sqrt{5}=\frac{8\sqrt{3\times 5}}{2\sqrt{5}}=\frac{8\sqrt{3}\sqrt{5}}{2\sqrt{5}}=4\sqrt{3}$

We can derive the following conclusions form these examples.

(i) The sum or difference of a rational number and an irrational number is irrational.

(ii) The product or quotient of a non-zero rational number with an irrational number is irrational.

(iii) If we add, substract, multiply or divide two irrational numbers, the result may be rational or irrational.

Here, some identities related to square root, which are useful for rationalizing have been given.

Let a and b are positive real numbers, then

(i) $\sqrt{ab}=\sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

(iii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b$

(iv) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}-\sqrt{d}\right)=\sqrt{ac}-\sqrt{ad}+\sqrt{bc}-\sqrt{bd}$

(v) $\left(\sqrt{a}+\sqrt{b}\right)^2=a+2\sqrt{ab}+b$

(vi) $\frac{1}{a+\sqrt{b}}=\frac{a-\sqrt{b}}{a^2-b}$

(vii) $\dfrac{1}{a+b\sqrt{x}}=\dfrac{a-b\sqrt{x}}{a^2-b^2x}$,where x is a natural number.

(viii) $\dfrac{1}{\sqrt{x}+\sqrt{y}}=\dfrac{\sqrt{x}-\sqrt{y}}{x-y}$, where x and y are natural numbers.

Above identities will be used to rationalise the denominator. When the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominatior is a rational number is called rationalising the denominator.

Example 12. Rationalise the denominator of $\dfrac{1}{\sqrt{2}}$.

Solution : We know that $\sqrt{2}\cdot\sqrt{2}=2$, which is rational number.

Multiply the numerator and denominator by $\sqrt{2}$ we have

$$\frac{1}{\sqrt{2}}=\frac{1}{2}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

In this form, it is easy to locate $\dfrac{1}{\sqrt{2}}$ on the number line. It is half way between 0 and $\sqrt{2}$.

Example 13. Rationalise the denominator of $\dfrac{1}{2+\sqrt{3}}$.

Solution : Here, the conjugate of the denominator $\left(2+\sqrt{3}\right)$ is $\left(2-\sqrt{3}\right)$. Multiply the numerator and denominator by $\left(2-\sqrt{3}\right)$, we have

$$\frac{1}{2+\sqrt{3}}=\frac{1}{\left(2+\sqrt{3}\right)}\times\frac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

Example 14. Rationalise the denominator $\dfrac{5}{\sqrt{3}-\sqrt{5}}$

Solution : By applying the identity (iii).

We have $\dfrac{5}{\sqrt{3}-\sqrt{5}}=\dfrac{5}{\sqrt{3}-\sqrt{5}}\times\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\dfrac{5\left(\sqrt{3}+\sqrt{5}\right)}{3-5}=\left(-\dfrac{5}{2}\right)\left(\sqrt{3}+\sqrt{5}\right)$

Example 15. Rationalise the denominator $\frac{1}{7+3\sqrt{2}}$

Solution : Here, conjugate of the denominator $\left(7+3\sqrt{2}\right)$ is $\left(7-3\sqrt{2}\right)$. Multiply the numerator and denominator by $\left(7-3\sqrt{2}\right)$, we have

$$\frac{1}{7+3\sqrt{2}}=\frac{1}{7+3\sqrt{2}}\times\left(\frac{7-3\sqrt{2}}{7-3\sqrt{2}}\right)=\frac{7-3\sqrt{2}}{49-18}=\frac{7-3\sqrt{2}}{31}$$

Exercise 2.2

1. Classify the following numbers as rational or irraitonal :

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

(iii) $\frac{2\sqrt{11}}{7\sqrt{11}}$ (iv) $\frac{1}{\sqrt{3}}$

(v) 5π

2. Rationalise the denominator of the following :

(i) $\frac{1}{5+3\sqrt{7}}$ (ii) $\frac{1}{\sqrt{2}+\sqrt{3}}$ (iii) $\frac{1}{\sqrt{7}-2}$

3. If $\frac{3+2\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$, then find the values of a and b. When a and b are rational numbers.

2.6. Laws of Exponent for Real Numbers

As we have studied in the previous classes about the laws of exponents. Here a, m and n are natural numbers a is called the base and m and n are the exponents.

Here a, m, n and b are natural numbers.

(i) $a^m a^n = a^{m+n}$ (ii) $\left(a^m\right)^n = a^{mn}$

(iii) $\frac{a^m}{a^n}=a^{m-n}, m>n$ (iv) $a^m b^m = \left(ab\right)^m$

What is $\left(a\right)^{o}$? Its value is 1. Therefore, $\left(a\right)^{o}=1$

The value of $\frac{1}{a^n} = a^{-n}$. Now we can use these laws on negative exponents. For example :

(i) $7^2 \cdot 7^{-5} = 7^{2-5} = 7^{-3} = \frac{1}{7^3}$

(ii) $\left(5^3\right)^{-7} = 5^{-21}$

(iii) $\frac{23^{-15}}{23^7} = 23^{-15-7} = 23^{-22}$

(iv) $(6)^{-3}(7)^{-3} = (42)^{-3}$

We can extend the laws of exponents that we have studied earlier, even when base is a positive real number and the exponents are rational numbers. In the previous section, we have defined $\sqrt[n]{a}$ for real number, where $a > 0$.

$$x^n = a \Rightarrow x = a^{1/n} = \sqrt[n]{a}$$

$$4^{3/2} = \left(4^{1/2}\right)^3 = 2^3 = 8$$

$$4^{3/2} = \left(4^3\right)^{1/2} = (64)^{1/2} = \left(8^2\right)^{1/2} = 8$$

Let $a > 0$ be a real number. Let m and n be integers such that m and n have no common factors other than 1, and $n > 0$, then :

(i) $a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ (ii) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$

Let $a > 0$ be a real number and p and q are rational numbers, then :

(i) $a^p \cdot a^q = a^{p+q}$ (ii) $\left(a^p\right)^q = a^{pq}$

(iii) $\frac{a^p}{a^q} = a^{p-q}$ (iv) $a^p b^p = (ab)^p$

Simplify :

(i) $4^{2/3} \cdot 4^{1/3} = 4^{2/3+1/3} = 4^{3/3} = 4^1 = 4$ (ii) $\left(3^{1/5}\right)^8 = 3^{8/5}$

(iii) $\frac{9^{1/5}}{9^{1/3}} = 9^{1/5-1/3} = 9^{(3-5)/15} = 9^{-2/15}$

Exercise 2.3

1. Find the value of :

(i) $81^{1/2}$ (ii) $64^{1/6}$ (iii) $(125)^{1/3}$

2. Find the value of :

(i) $(4)^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$

3. Simplify :

(i) $2^{2/3} \cdot 2^{1/7}$ (ii) $\left(\frac{1}{3^3}\right)^7$

4. Find the value of x :

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

Answer Sheet

Exercise 2.1

1. (i), (iv) and (v) irrational

(ii) and

(iii) rational

2. 0.01001000100001 . . .; 0.202002000200002 . . ., 0.00300030000 . . .

3. (i) 0.36 terminating ;

(ii) $0.\overline{09}$ non-terminating recuring;

(iii) 4.125 terminating ;

(iv) $0.\overline{230769}$ non-terminating recuring

(v) $0.\overline{18}$ non-terminating recuring ;

(vi) 0.8225 terminating

4. (i) $\frac{1}{3}$ (ii) $\frac{47}{99}$ (iii) $\frac{14}{11}$ (iv) $\frac{233}{990}$

5. 0.75075007500075000075 . . .

0.767076700767000767 . . .

0.808008000800008 . . .

Exercise 2.2

1. (i) irrational ; (ii) rational ; (iii) rational ; (iv) irrational ; (v) irrational

2. (i) $-\frac{1}{38}\left(5-3\sqrt{7}\right)$; (ii) $-\left(\sqrt{2}-\sqrt{3}\right)$; (iii) $\frac{1}{3}\left(\sqrt{7}+2\right)$

3. $a=\frac{13}{7},\ b=\frac{9}{7}$

Exercise 2.3

1. (i) 9 ; (ii) 2 ; (iii) 5

2. (i) 8 ; (ii) 4 ; (iii) 8

3. (i) $2^{\frac{2}{21}}$; (ii) 3^{-21}

4. $x=3$