

KBPE Class 10th Maths Question Paper With Solution 2019

QUESTION PAPER CODE S 1935

Answer any three questions from 1 to 4. Each question carries 2 scores. [3 * 2 = 6]

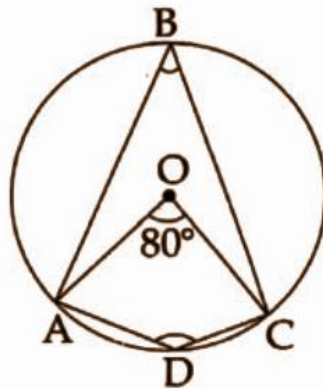
Question 1: In the figure, O is the centre of the circle.

$$\angle AOC = 80^\circ$$

[i] What is the measure of $\angle ABC$?

[ii] What is the measure of $\angle ADC$?

Solution:



Given $\angle AOC = 80^\circ$

[i] The measurement $\angle ABC = (1 / 2) * \angle AOC = 1 / 2 \times 80 = 40^\circ$.

[ii] $\angle ABC + \angle ADC = 180^\circ$

$$40^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 40^\circ$$

$$\angle ADC = 140^\circ$$

Question 2: [i] Write the first integer term of the arithmetic sequence $(1/7)$, $(2/7)$, $(3/7)$

[ii] What is the sum of the first 7 terms of the above sequence?

Solution:

[i] Given arithmetic sequence = $1/7 + 2/7 + 3/7$,.....

Common difference $d = 2/7 - 1/7 = 1/7$.

Hence the first integer term = $7/7 = 1$

[ii] $a = (1/7)$

$d = 2/7 - 1/7$

$= 1/7$

$n = 7$

$S_n = (n/2) (2a + [n - 1]d)$

$S_7 = (7/2) (2 * [1/7] + [7 - 1] * [1/7])$

$= (7/2) ([2/7] + 6 * (1/7))$

$= (7/2) ([2/7] + [6/7])$

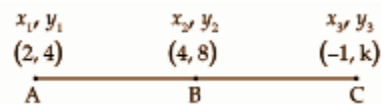
$= (7/2) (8/7)$

$= 4$

Question 3: [i] If C (-1, k) is a point on the line passing through the points A (2, 4) and B (4, 8) which number is k?

[ii] What is the relation between the x coordinate and the y coordinate of any point on this line?

Solution:



[i]

Points A, B and C are collinear.

Area of triangle ABC = 0

$(1/2) (x_1 [y_2 - y_3] + x_2 [y_3 - y_1] + x_3 [y_1 - y_2])$

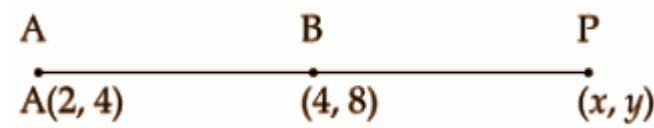
$|2 (8 - k) + 4 (k - 4) + (-1) (4 - 8)| = 0$

$16 - 2k + 4k - 16 - 4 + 8 = 0$

$2k = -4$

$$k = -2$$

[ii]



Area of triangle ABP = 0

$$(1 / 2) (x_1 [y_2 - y_3] + x_2 [y_3 - y_1] + x_3 [y_1 - y_2])$$

$$|2 (8 - y) + 4 (y - 4) + (x) (4 - 8)| = 0$$

$$16 - 2y + 4y - 16 - 4x = 0$$

$$2y - 4x = 0$$

$$2y = 4x$$

$$y = 2x$$

$$2x - y = 0$$

Question 4: [i] Find P(1) if $P(x) = x^2 + 2x + 5$

[ii] If $(x - 1)$ is a factor of $x^2 + 2x + k$, what is the value of k ?

Solution:

$$[i] P(x) = x^2 + 2x + 5$$

$$P(1) = 1^2 + 2 * 1 + 5$$

$$= 1 + 2 + 5$$

$$P(1) = 8$$

[ii] Since $(x - 1)$ is the factor of $x^2 + 2x + k$, then

$$x - 1 = 0$$

$$x = 1$$

$$(1)^2 + 2(1) + k = 0$$

$$1 + 2 + k = 0$$

$$k = -3$$

Answer any five questions from 5 to 11. Each question carries 3 scores. $[5 * 3 = 15]$

Question 5: [i] What is the remainder on dividing the terms of the arithmetic sequence 100, 107, 114 by 7?

[ii] Write the sequence of all three-digit numbers. Which leaves the remainder 3 on division by 7? Which is the last term of this sequence?

Solution:

[i] Given sequence be 100, 107, 114,

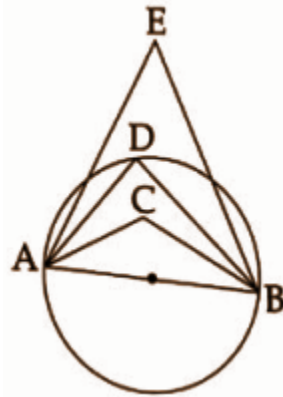
$$d = 7$$

$$\text{Remainder} = 100 / 7 = 2$$

[ii] 101, 108, 115

Hence the last three-digit term = 997.

Question 6: AB is the diameter of the circle. D is the point on the circle.



$\angle ACB + \angle ADB + \angle AEB = 270^\circ$. The measure of one among $\angle ACB$, $\angle ADB$ and $\angle AEB$ is 110° . Write the measures of $\angle ACB$, $\angle ADB$, $\angle AEB$.

Solution:

$$\angle ADB = 90^\circ \text{ (Measurement of semi circle angle)}$$

$$\angle ACB + \angle ADB + \angle AEB = 270^\circ \text{ (given)}$$

$$\angle ACB + 90^\circ + \angle AEB = 270^\circ$$

$$\angle ACB + \angle AEB = 270^\circ - 90^\circ = 180^\circ$$

The given condition is that any one of the angles $\angle ACB$, $\angle AEB$ be 110° .

$$\text{Take } \angle ACB = 110^\circ$$

$$\text{Hence } \angle AEB = 180^\circ - 110^\circ = 70^\circ$$

So the angles , $\angle ADB = 90^0$, $\angle ACB = 110^0$, $\angle AEB = 70^0$.

Question 7: If x is a natural number,

[a] What number is to be added to $x^2 + 6x$ to get a perfect square?

[b] If $x^2 + ax + 16$ is a perfect square number, then which number is a?

[c] If $x^2 + ax + b$ is a perfect square, prove that $a^2 = 4b$.

Solution:

Given $x^2 + 6x$

[a] $6x = 2ab$

$a = x$

$b = ?$

$b = 6x / 2x = 3$

Perfect square = $b^2 = 3^2 = 9$.

Hence 9 is to be added to them.

[b] Given, $x^2 + ax + 16$ is perfect square

This is the form of $a^2 + 2ab + b^2 = (a + b)^2$

$2ab = ax$

$a = x$

$b^2 = 16$

$b = \sqrt{16} = 4$

So, $(x + 4)^2 = x^2 + ax + 16$

Hence $a = 2ab = 2 \times 4 = 8$.

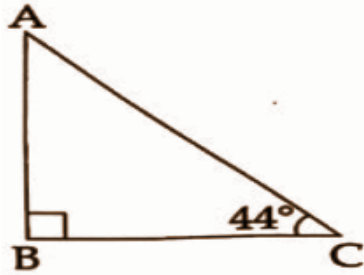
[c] Here b = the square of the half of a

$b = (a / 2)^2$

$b = a^2 / 4$

$a^2 = 4b$

Question 8: In the figure, $\angle B = 90^\circ$, $\angle C = 44^\circ$.



[a] What is the measure of A?

[b] Which among the following is $\tan 44^\circ$:

(AB / BC) , (AB / AC), (BC / AB), (BC / AC)

[c] Prove that $\tan 44^\circ * \tan 46^\circ = 1$

Solution:

[a] $\angle A + \angle B + \angle C = 180^\circ$

$$\angle A + 90^\circ + 44^\circ = 180^\circ$$

$$\angle A = 180^\circ - 90^\circ - 44^\circ$$

$$\angle A = 46^\circ$$

[b] In triangle ABC,

$$\tan 44^\circ = \text{opposite} / \text{adjacent}$$

$$= AB / BC \text{ \{from the figure\}}$$

[c] Take LHS = $\tan 44^\circ * \tan 46^\circ$

$$= \tan 44^\circ * \cot (90^\circ - 46^\circ) [\tan \theta = \cot (90 - \theta)]$$

$$= \tan 44^\circ * \cot 44^\circ$$

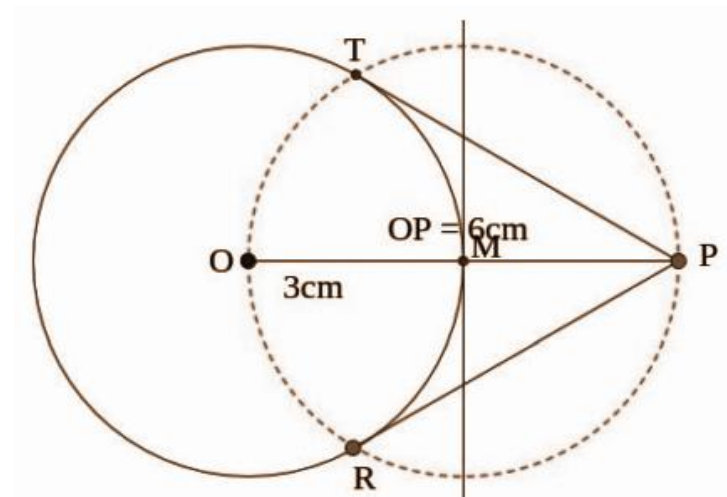
$$= \tan 44^\circ * [1 / \tan 44^\circ]$$

$$= 1$$

$$= \text{RHS}$$

Question 9: Draw a circle of radius 3 centimetres. Mark a point P at a distance of 6cm from the centre of the circle. Draw tangents from P to the circle.

Solution:



Steps of construction:

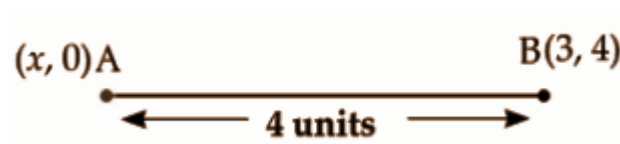
- Draw a circle of radius 3cm with O as the centre.
- From the centre O, draw $OP = 6\text{cm}$ and perpendicular to OP marking it as M.
- Draw another circle with centre M cutting T and R respectively.
- Join PT and PR which are the required tangents.

Question 10: [i] Find the coordinates of the point on the x-axis, which is at a distance of 4 units from (3, 4).

[ii] Find the coordinates of the point on the x-axis at a distance of 5 units from (3, 4).

Solution:

[i]



$$4 = \sqrt{(x - 3)^2 + (0 - 4)^2}$$

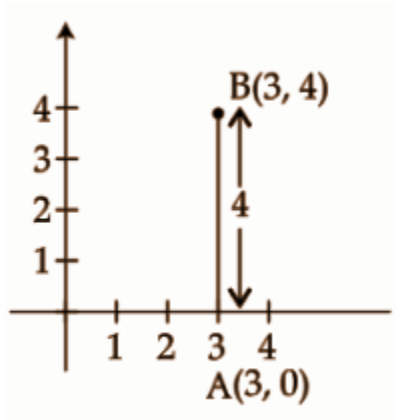
On squaring both sides,

$$4^2 = (x - 3)^2 + 16$$

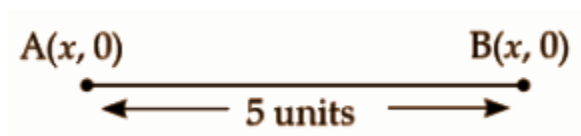
$$x - 3 = 0$$

$$x = 3$$

A (3, 0) is the required point.



[ii]



$$AB = 5$$

$$\sqrt{(x - 3)^2 + (0 - 4)^2} = 5$$

On squaring both sides,

$$(x - 3)^2 + 16 = 5^2$$

$$(x - 3)^2 + 16 = 25$$

$$(x - 3)^2 = 25 - 16$$

$$(x - 3)^2 = 9$$

$$(x - 3) = \pm 3$$

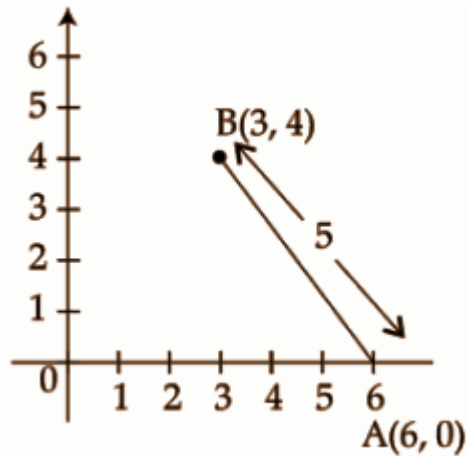
$$x - 3 = 6$$

$$x = 6$$

$$x - 3 = -3$$

$$x = 0$$

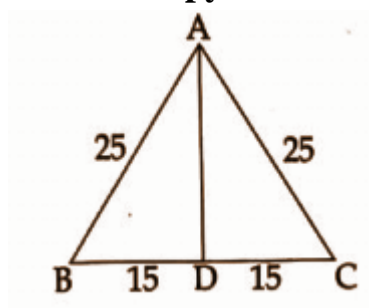
Hence, (6, 0) and (0, 0) is the required point.



Question 11: The given figure is the lateral face of a square pyramid. $AB = AC = 25$ centimeters and $BD = DC = 15$ centimeters.

[i] What is the length of its base edge?

[ii] Find the lateral surface area of the pyramid.



Solution:

Side of the base = diagonal / $\sqrt{2}$

$$= (30 / \sqrt{2}) * (\sqrt{2} / \sqrt{2})$$

$$= 15 \sqrt{2} \text{ cm}$$

$$= 15 * 1.414$$

$$= 17.210$$

Side of the base = 17.210 cm

Lateral surface area = $(1 / 2) * \text{perimeter of the base} * \text{slant height}$

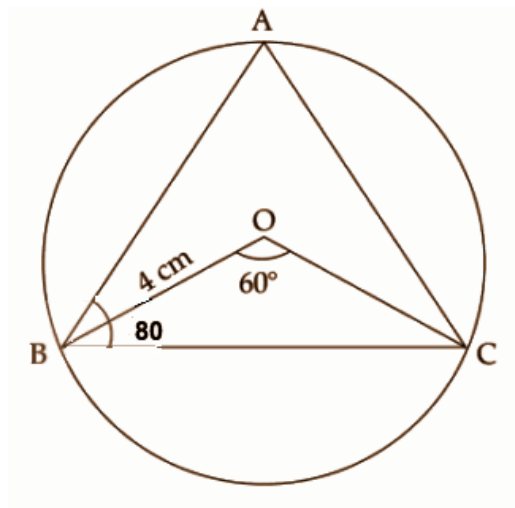
$$= (1 / 2) * (17.21) * 4 * 25$$

$$= 860.50 \text{ cm}^2$$

Answer any 7 questions from 12 to 21. Each question carries 4 scores. $[7 * 4 = 28]$

Question 12: In triangle ABC, $\angle A = 30^\circ$, $\angle B = 80^\circ$, the circumradius of the triangle is 4 centimetres. Draw the triangle. Measure the length of its smallest side.

Solution:



Steps of construction:

- Draw a circle of radius 4cm having a centre at O.
- Make an angle $\angle BOC = 60^\circ$.
- Construct an angle $\angle CBA = 80^\circ$.
- Join AC.
- $\triangle ABC$ is the required triangle.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$30^\circ + 80^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 110^\circ$$

$$\angle C = 70^\circ$$

$$30^\circ < 70^\circ < 80^\circ$$

$$\angle A < \angle C < \angle B$$

The smallest angle is $\angle A$.

BC is the smallest side of $\triangle ABC$.

Question 13: Find the following sums:

[i] $1 + 2 + 3 + \dots + 100$

[ii] $1 + 3 + 5 + \dots 99$

[iii] $2 + 4 + 6 + \dots 100$

[iv] $3 + 7 + 11 + \dots 199$

Solution:

[i] $1 + 2 + 3 + \dots 100$

$$a = 1$$

$$d = 2 - 1 = 1$$

$$\text{Last term} = 100 = l$$

$$l = a + (n - 1) d$$

$$100 = 1 + (n - 1) 1$$

$$100 = 1 + n - 1$$

$$n = 100$$

$$S_n = (n / 2) (n + 1)$$

$$S_{100} = (100 / 2) (100 + 1)$$

$$= (50) * (101)$$

$$= 5050$$

[ii] $1 + 3 + 5 + \dots 99$

$$a = 1$$

$$d = 3 - 1 = 2$$

$$\text{Last term} = 99 = l$$

$$l = a + (n - 1) d$$

$$99 = 1 + (n - 1) 2$$

$$99 = 1 + 2n - 2$$

$$99 = 2n - 1$$

$$100 = 2n$$

$$100 / 2 = n$$

$$50 = n$$

$$S_n = (n / 2) (a + a_n)$$

$$S_{50} = (50 / 2) (1 + 99)$$

$$= (25) * (100)$$

$$= 2500$$

[iii] $2 + 4 + 6 + \dots 100$

$$a = 2$$

$$d = 4 - 2 = 2$$

$$\text{Last term} = 100 = l$$

$$l = a + (n - 1) d$$

$$100 = 2 + (n - 1) 2$$

$$100 = 2 + 2n - 2$$

$$100 = 2n$$

$$n = 100 / 2$$

$$n = 50$$

$$S_n = (n / 2) (a + a_n)$$

$$S_{50} = (50 / 2) (2 + 100)$$

$$= (25) * (102)$$

$$= 2550$$

$$[\text{iv}] 3 + 7 + 11 + \dots\dots\dots 199$$

$$a = 3$$

$$d = 7 - 3 = 4$$

$$\text{Last term} = 199 = l$$

$$l = a + (n - 1) d$$

$$199 = 3 + (n - 1) 4$$

$$199 = 3 + 4n - 4$$

$$199 = 4n - 1$$

$$200 / 4 = n$$

$$n = 50$$

$$S_n = (n / 2) (a + a_n)$$

$$S_{50} = (50 / 2) (3 + 199)$$

$$= (25) * (202)$$

$$= 5050$$

Question 14: A box contains some green and blue balls. 7 red balls are put into it. Now the probability of getting a red ball from the box is $\frac{7}{24}$ and that of the blue ball is $\frac{1}{6}$.

[i] How many balls are there in the box?

[ii] How many of them are blue?

[iii] What is the probability of getting a green ball from the box?

Solution:

Let the number of green balls be x .

The number of blue balls is y .

Number of red balls = 7

Total number of balls = $x + y + 7$

$P(\text{red ball}) = 7 / 24$

$P(\text{blue ball}) = 1 / 3$

[i] Since $P(\text{red ball}) = 7 / 24$,

$$7 / [x + y + 7] = 7 / 24$$

$$24 = x + y + 7$$

$$24 - 7 = x + y$$

$$17 = x + y \text{ ---- (1)}$$

$P(\text{blue ball}) = 1 / 3$

$$y / [x + y + 7] = 1 / 3$$

$$3y = x + y + 7$$

$$2y = x + 7$$

$$-x + 2y = 7 \text{ ---- (2)}$$

On adding equation (1) and (2),

$$17 = x + y$$

$$-x + 2y = 7$$

$$3y = 24$$

$$y = 24 / 3$$

$$y = 8$$

Put $y = 8$ in equation (1),

$$17 = x + 8$$

$$17 - 8 = x$$

$$x = 9$$

Total number of balls = $8 + 9 + 7 = 24$

[ii] Number of blue balls

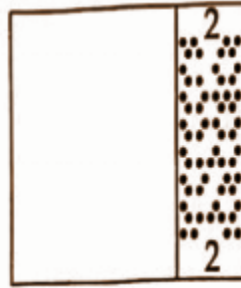
$$y / 24 = 1 / 3$$

$$3y = 24$$

$$y = 8$$

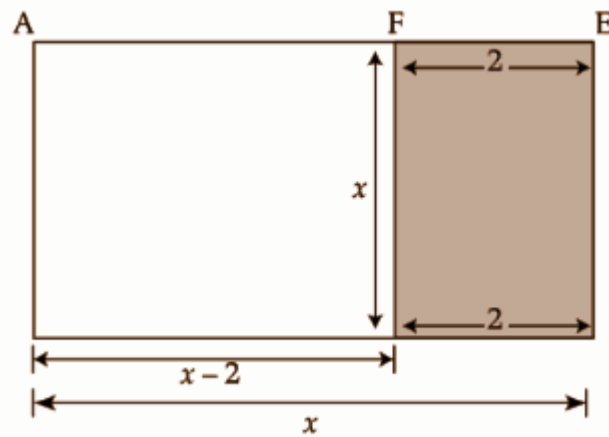
$$[iii] P(\text{green ball}) = x / 24 = 9 / 24 = 3 / 8$$

Question 15: Land is acquired for road widening from a square ground, as shown in the figure. The width of the acquired land is 2 meters. Area of the remaining ground is 440 square meters.



- [i] What is the shape of the remaining ground?
- [ii] What is the length of the remaining ground?

Solution:



- [i] The shape of the remaining ground is rectangular.
- [ii] Let the length be x and breadth be $x - 2$.

Given,

$$\text{Area} = 440 \text{ m}^2$$

$$L * B = 440$$

$$x * (x - 2) = 440$$

$$x^2 - 2x = 440$$

$$x^2 - 2x - 440 = 0$$

$$(x - 22)(x + 20) = 0$$

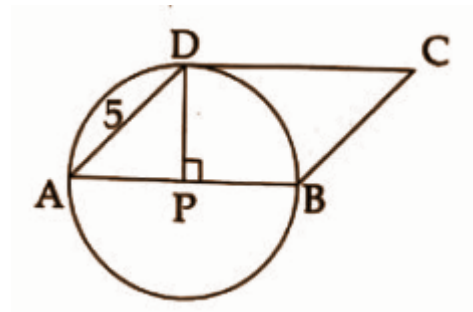
$$x = 22, -20$$

Since the values must be positive, $x = 22$ is taken.

$$\text{Length} = 22\text{m}$$

$$\text{Breadth} = 22 - 2 = 20\text{m}$$

Question 16: In the figure, P is the centre of the circle. A, B and D are points on the circle. $\angle P = 90^\circ$, $AD = 5\text{cm}$.



- (a) What is the measure of $\angle A$?
- (b) What is the area of the triangle APD?
- (c) Find the area of the parallelogram ABCD.

Solution:

[a] In triangle APD, $\angle P = 90^\circ$

$\angle A = \angle D$ [angle opposite to equal side are equal]

$\angle A + \angle ADP + \angle APD = 180^\circ$ [angle sum property of a triangle]

$$\angle A + \angle A + 90^\circ = 180^\circ$$

$$2\angle A = 90^\circ$$

$$\angle A = 45^\circ$$

[b] In triangle APD,

$$\sin 45^\circ = PD / AD$$

$$1 / \sqrt{2} = PD / 5$$

$$5 / \sqrt{2} = PD = AP$$

$$\text{Area of } \triangle ADP = (1 / 2) * AP * PD$$

$$= (1 / 2) * (5 / \sqrt{2}) * (5 / \sqrt{2})$$

$$= 25 / 4 \text{ cm}^2$$

$$\begin{aligned}
& [c] \text{ Area of a parallelogram} = \text{base} * \text{height} \\
& = AB * PD \\
& = 2AP * PD \\
& = 2 * (5 / \sqrt{2}) * (5 / \sqrt{2}) \\
& = 25 \text{ cm}^2
\end{aligned}$$

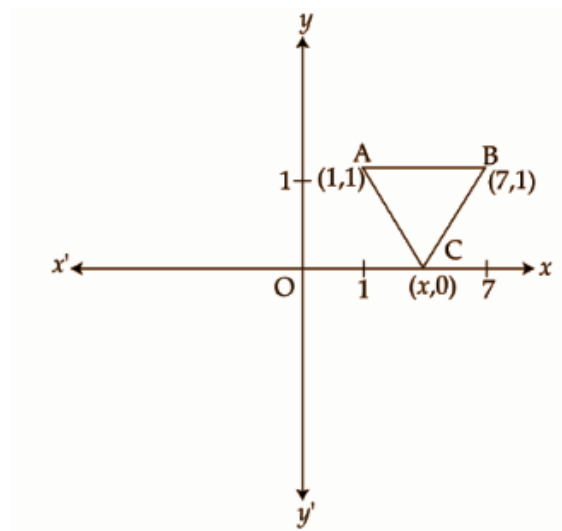
Question 17: [a] Draw the coordinates and mark the points A (1, 1), B (7, 1).

[b] Draw an isosceles triangle ABC with AB as the hypotenuse.

[c] Write the coordinates of C.

Solution:

[a]



$$AC = BC$$

$$\sqrt{(x - 1)^2 + 1} = \sqrt{(x - 7)^2 + 1}$$

On squaring both sides,

$$(x - 1)^2 + 1 = (x - 7)^2 + 1$$

$$x^2 + 1 - 2x + 1 = x^2 + 49 - 14x + 1$$

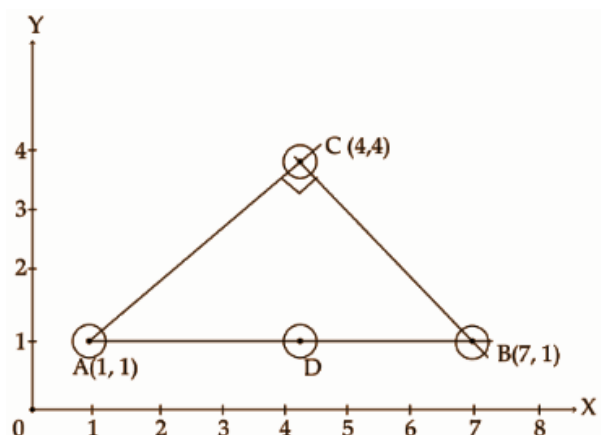
$$-2x + 14x = 49 - 1$$

$$12x = 48$$

$$x = 48 / 12$$

$$x = 4$$

[b]

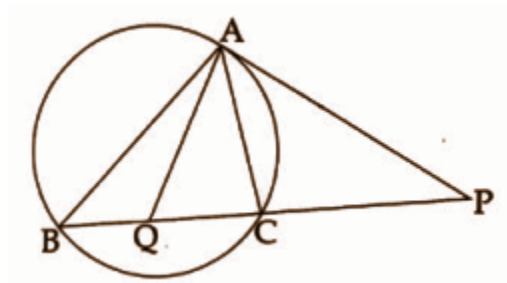


C (4, 0) is the required point.

Coordinate of C (4, 4) [AD = BC = CD]

The midpoint of the hypotenuse is equal distance from the vertex of the triangle.

Question 18: In the figure, chord BC is extended to P. Tangent from P to the circle is PA. AQ is the bisector of $\triangle BAC$.

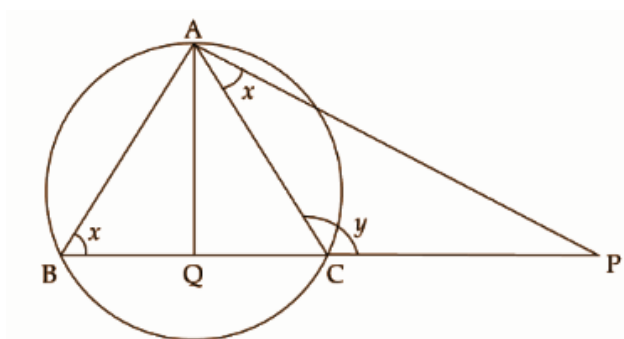


[a] Write one pair of equal angles from the figure.

[b] If $\angle PAC = x$ and $\angle PCA = y$, then prove that $\angle BAC = y - x$.

[c] Prove that $\angle PAQ = [y + x] / 2$

Solution:



$$[a] \angle BAC = \angle PAC$$

$$[b] \angle PAC = \angle ABC$$

$$\angle ACP = \angle BAC + \angle ABC \text{ [exterior angle property]}$$

$$y = \angle BAC + x$$

$$\angle BAC = y - x$$

$$[c] \angle PAQ = \angle PAC + \angle CAQ$$

$$= x + \left(\frac{1}{2} \right) * \angle BAC$$

$$= x + \left(\frac{1}{2} \right) * (y - x)$$

$$= x + \left(\frac{1}{2} \right) y - \left(\frac{1}{2} \right) x$$

$$\angle PAQ = \left(\frac{1}{2} \right) (x + y)$$

Question 19: If $(x - 1)$ is a factor of the second-degree polynomial $P(x) = ax^2 + bx + c$ and $P(0) = -5$.

[a] What is the value of c ?

[b] Prove that $a + b = 5$.

[c] Write a second-degree polynomial whose one factor is $x - 1$.

Solution:

[a] Given that $x - 1$ is a factor of the polynomial $ax^2 + bx + c$

$$x - 1 = 0$$

$$x = 1$$

$$P(1) = 0$$

$$a(1)^2 + b * 1 + c = 0$$

$$a + b + c = 0 \text{ ---- (1)}$$

Now, at $x = 0$, $P(0) = -5$

$$a * 0 + b * 0 + c = -5$$

$$c = -5$$

$$[b] a + b + c = 0$$

$$a + b - 5 = 0$$

$$a + b = 5$$

[c] Second-degree polynomial = $3x^2 + 2x - 5$ or $2x^2 - 3x + 5$ or $4x^2 + x - 5$ [any of them]

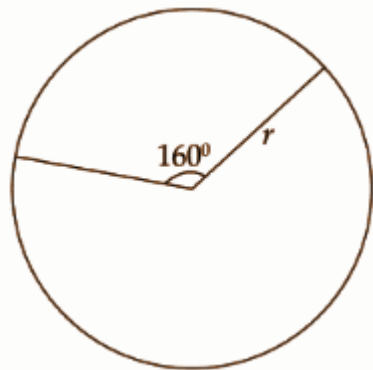
Question 20: A circular sheet of paper is divided into two sectors. The central angle of one of them is 160° .

[a] What is the central angle of the remaining sector?

[b] These sectors are bent into cones of maximum volume. If the radius of the small cone is 8 centimetres, what is the radius of the other?

[c] What is the slant height of the cone?

Solution:



[a] Central angle of the remaining sector = $360^\circ - 160^\circ = 200^\circ$

[b] R_1 is the radius of the small cone = 8cm

$$2\pi R_1 = 2\pi r (\theta_1) / 360^\circ$$

$$8 = r * (160^\circ / 360^\circ)$$

$$r = (360^\circ * 8) / 160^\circ$$

$$r = 18\text{cm}$$

$$2\pi R_2 = 2\pi r (\theta_2) / 360^\circ$$

$$R_2 = (18 * 200^\circ) / 360^\circ$$

$$R_2 = 10\text{cm}$$

[c] Slant height (l_1) = 18cm

Slant height (l_2) = 18cm

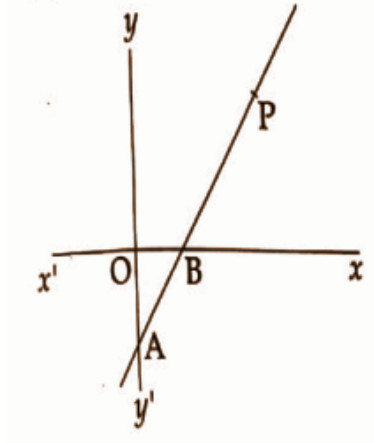
Question 21: Equation of the line AB is $3x - 2y = 6$. P is a point on the line. The line intersects the y-axis at A and the x-axis at B.

[a] What is the x coordinate of A?

[b] What is the length of OA?

[c] What is the length of OB?

[d] The x coordinate and the y coordinate of P are the same. Find the coordinates of P.



Solution:

Given, the equation of line AB is $3x - 2y = 6$

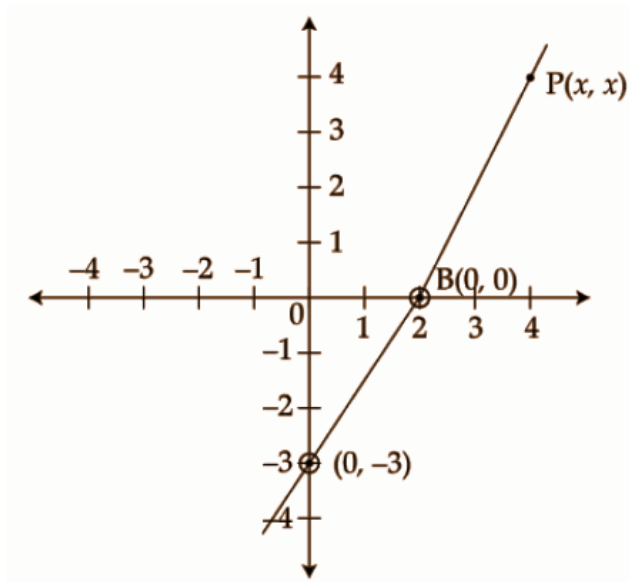
x	0	2
y	-3	0

[a] x coordinate of A = 0

[b] OA = 3 units

[c] OB = 2 units

[d]



A, B, P are collinear.

Area of $\triangle ABP = 0$

$$(1/2) [0(0 - x) + 2(x + 3) + x(-3 - 0)] = 0$$

$$2x + 6 - 3x = 0$$

$$x = 6$$

Hence, the coordinates of P are (6, 6).

Answer any 5 questions from 22 to 28. Each question carries 5 scores. [5 * 5 = 25]

Question 22: If the terms of the arithmetic sequence $(2/9), (3/9), (4/9), (5/9), \dots$ are represented as x_1, x_2, \dots then

[a] $x_1 + x_2 + x_3 =$

[b] $x_4 + x_5 + x_6 =$

[c] Find the sum of the first 9 terms.

[d] What is the sum of the first 300 terms?

Solution:

[a] $x_1 + x_2 + x_3$

$$= (2/9) + (3/9) + (4/9)$$

$$= 9/9$$

$$= 1$$

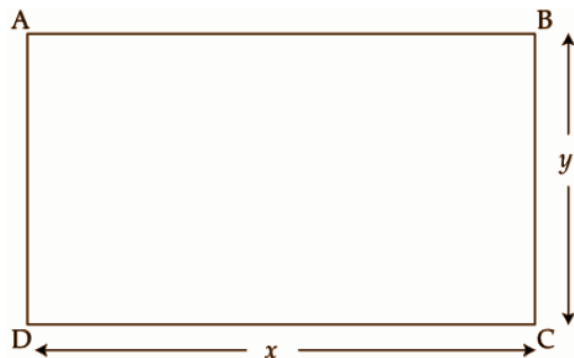
$$\begin{aligned}
 [\text{b}] \quad & x_4 + x_5 + x_6 \\
 &= (5/9) + (6/9) + (7/9) \\
 &= (18/9) \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 [\text{c}] \quad & n = 9 \\
 & a = 2/9 \\
 & d = (3/9) - (2/9) = (1/9) \\
 & S_n = (n/2) (2a + [n - 1]d) \\
 & S_9 = (9/2) (2 * [2/9] + [9 - 1] * (1/9)) \\
 &= (9/2) [(4/9) + (8/9)] \\
 &= (9/2) (12/9) \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 [\text{d}] \quad & n = 300 \\
 & a = 2/9 \\
 & d = (3/9) - (2/9) = (1/9) \\
 & S_n = (n/2) (2a + [n - 1]d) \\
 & S_{300} = (300/2) (2 * [2/9] + [300 - 1] * (1/9)) \\
 &= (300/2) [(4/9) + (299/9)] \\
 &= (150) (303/9) \\
 &= 5050
 \end{aligned}$$

Question 23: Draw a rectangle of area 12 square centimetres. Draw a square having the same area.

Solution:



For the given rectangle,

$$\text{Area} = 12\text{cm}^2$$

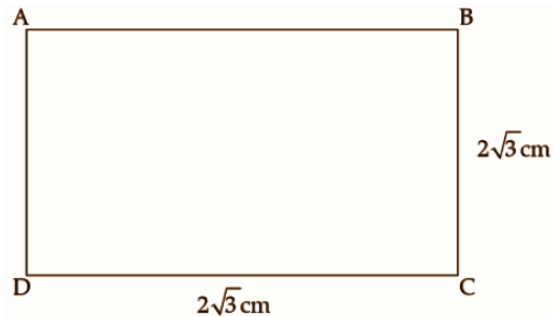
$$x * y = 12\text{cm}^2$$

For a square, $x = y$.

$$x * x = 12\text{cm}^2$$

$$x^2 = 12\text{cm}^2$$

$$x = 2\sqrt{3} \text{ cm}$$

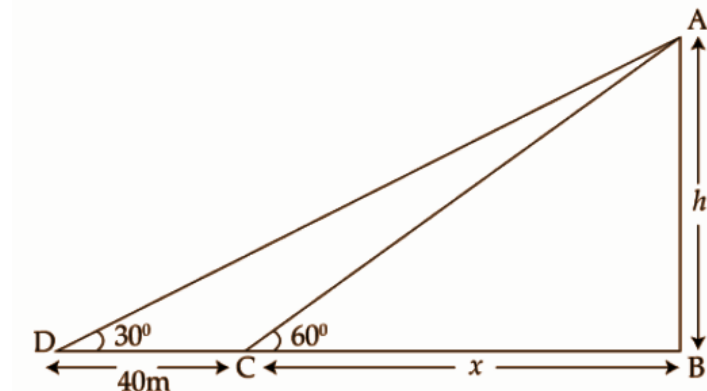


Question 24: A boy standing at one bank of a river sees the top of a tree on the other bank directly opposite to the boy at an elevation of 60° . Stepping 40 meters back, he sees the top of the elevation at 30° .

[a] Draw a rough sketch and find the height of the tree.

[b] What is the width of the river?

Solution:



Let AB be h and CB be x.

In $\triangle ABC$,

$$\tan 60^\circ = AB / BC$$

$$\sqrt{3} = h / x$$

$$h = \sqrt{3}x \text{ ---- (1)}$$

In $\triangle ABD$,

$$\tan 30^\circ = AB / BD$$

$$1 / \sqrt{3} = h / x + 40$$

$$x + 40 = \sqrt{3} (\sqrt{3}x) \text{ --- (2)}$$

$$x + 40 = 3x$$

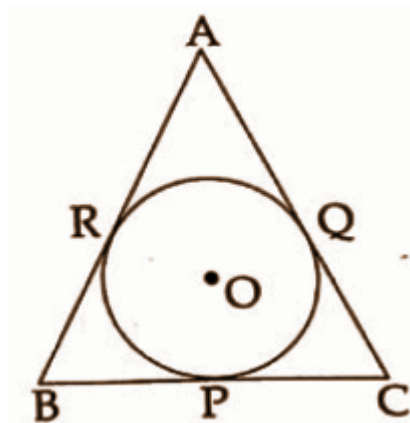
$$40 = 2x$$

$$x = 20$$

$$h = 20\sqrt{3} \text{ m}$$

Width of the river is 20m and the height of the tree is $20\sqrt{3}$ m.

Question 25: Circle with centre O touches the sides of a triangle at P, Q and R, $AB = AC$, $AQ = 4\text{cm}$ and $CQ = 6\text{cm}$.

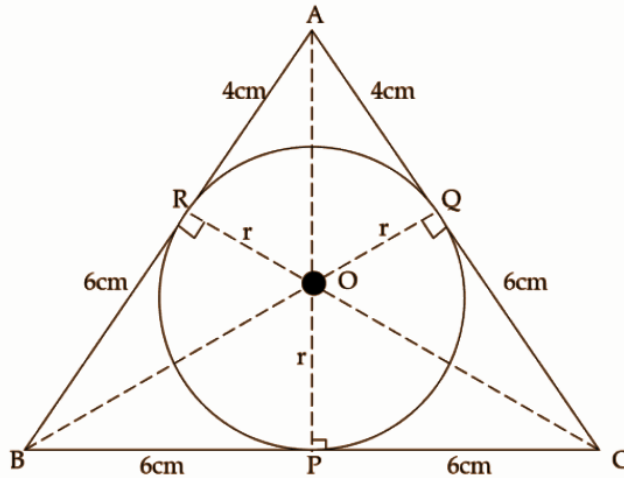


[a] What is the length of CP?

[b] Find the perimeter and the area of the triangle.

[c] What is the radius of the circle?

Solution:



[a] $CP = CQ$ [Length of external tangents are equal]

$CP = 6\text{cm}$

[b] Perimeter of triangle $= 4 + 6 + 6 + 6 + 4 + 6 = 32\text{ cm}$

For the area of $\triangle ABC$,

$$s = [AB + BC + CA] / 2$$

$$= [10 + 12 + 10] / 2$$

$$= 16\text{cm}$$

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(16)(16-10)(16-12)(16-10)}$$

$$= \sqrt{16 * 6 * 4 * 6}$$

$$= 48\text{cm}^2$$

[c] Area of $\triangle ABC = \text{area of } \triangle AOB + \text{area of } \triangle BOC + \text{area of } \triangle COA$

$$48 = (1/2) * 10 * r + (1/2) * 12 * r + (1/2) * 10 * r$$

$$48 * 2 = r(10 + 12 + 10)$$

$$48 * 2 = 32 * r$$

$$r = 3\text{cm}$$

Question 26: Radius of a cylinder is equal to its height. If the radius is taken as 'r', the volume of the cylinder is $\pi r^2 * r = \pi r^3$. Like this find the volumes of the solids, with the following measures.

Solids	Measures	Volume
Cone	radius = height = r	
Hemisphere	radius = r	
Sphere	radius = r	

[a] What is the ratio of the volumes of the cone, hemisphere, cylinder and the sphere?

[b] A solid metal sphere of radius 6cm is melted and recast into solid cones of radius 6cm and height 6cm. Find the number of cones.

Solution:

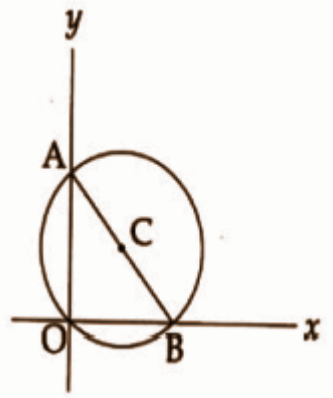
[a]

Solids	Measures	Volume
Cone	radius = height = r	$\frac{1}{3}\pi r^2 h \Rightarrow \frac{1}{3}\pi r^2 \times r \Rightarrow \frac{1}{3}\pi r^3$
Hemisphere	radius = r	$\frac{2}{3}\pi r^3$
Sphere	radius = r	$\frac{4}{3}\pi r^3$

$$\begin{aligned}
 [b] V_c : V_h : V_{cy} : V_s &= [\pi r^3 / 3] : [2 / 3] \pi r^3 : \pi r^3 : [4 / 3] \pi r^3 \\
 &= (1 / 3) : (2 / 3) : 1 : (4 / 3) \\
 &= 1 : 2 : 3 : 4
 \end{aligned}$$

$$\begin{aligned}
 \text{Number of cones} &= \text{Volume of the sphere} / \text{Volume of the cone} \\
 &= [(4 / 3) \pi r^3] / [\pi R^2 h / 3] \\
 &= \{[4 * \pi * 6^3] / (3)\} / \{[\pi * 6^2 * 6] / (3)\} \\
 &= 4
 \end{aligned}$$

Question 27: C is at the centre of the circle passing through the origin. Circle cuts the y-axis at A (0, 4) and the x-axis at B(4, 0).



[a] Write the coordinates of C.

[b] Write the equation of the circle.

[c] (0, 0) is a point on the circle. There is one more point on the circle with x and y coordinates equal. Which is that?

Solution:

[a] C is the midpoint of AB.

$$x = [4 + 0] / 2$$

$$= 4 / 2$$

$$x = 2$$

$$y = [4 + 0] / 2$$

$$= 4 / 2$$

$$y = 2$$

The coordinates of C are (2, 2).

[b] The equation of the circle is given by $(x - a)^2 + (y - b)^2 = r^2$

$$(x - 2)^2 + (y - 2)^2 = [\sqrt{(4 - 2)^2 + (0 - 2)^2}]^2$$

$$x^2 + 4 - 4x + y^2 + 4 - 4y = 8$$

$$x^2 + y^2 - 4x - 4y = 0$$

[c] Let P(x, x) be a point on the circle.

$$x^2 + x^2 - 4x - 4x = 0$$

$$2x^2 - 8x = 0$$

$$x = 0, 4$$

The required point is (4, 4).

Question 28: The table below shows the number of children in a class, sorted according to their heights.

Height (Centimetres)	Number of Children
130 -140	7
140 - 150	9
150 -160	10
160 -170	10
170 -180	9

If the students are directed to stand in a line according to the order of their heights starting from the smallest, then

- [a] The height of the child at what position is taken as the median?
- [b] What is the assumed height of the child in the 17th position?
- [c] Find the median height.

Solution:

Class interval	frequency	Cumulative frequency
130 - 140	7	7
140 - 150	9	16
150 - 160	10	26
160 - 170	10	36
170 - 180	9	45

[a] $N = 45$

Median is taken as $[N + 1] / 2$

$$= [45 + 1] / 2$$

$$= 46 / 2$$

$$= 23$$

The height of the child at the 23rd position is taken as the median.

[b] Height of the child in the 17th position between 150 - 160. Assumed height is 152cm.

$$\begin{aligned} \text{[c] Median} &= [l_1] + \{[(N / 2) - C] / cf\} * h \\ &= 150 + [22.5 - 16] / 10 * (10) \\ &= 150 + 6.5 \\ &= 156.5 \end{aligned}$$

Question 29: Read the following. Understand mathematical concepts in it and answer the questions that follow.

The remainders obtained on dividing the powers of two by 7 have an interesting property.

We can understand it from the table given below.

Number	2^1	2^2	2^3	2^4	2^5	2^6	2^7
Remainder	2	4	1	2	4	1	2

If the powers are 1, 4, 7 the remainder is 2.

If the powers are 3, 6, 9 the remainder is 1.

[a] What is the remainder on dividing 2^8 by 7?

[b] Write the sequence of powers of 2 leaving remainder 1 on division by 7.

[c] Check whether 2019 is a term of arithmetic sequence 3, 6, 9

[d] What is the remainder on dividing 2^{2019} by 7?

[e] Write the algebraic form of the arithmetic sequence 1, 4, 7

[f] Write the algebraic form of the sequence $2^1, 2^4, 2^7$ [powers of two leaving remainder 2 on division by 7].

Solution:

[a] If 2^8 is divided by 7, then the remainder is 7.

[b] $2^3, 2^6, 2^9$ when divided by 7 leaves a remainder 1.

[c] Yes

$$2019 = 3(n - 1) + 3$$

$$2019 = 3n - 9$$

$$2019 + 9 = 3n$$

$$2018 = 3n$$

$$2018 / 3 = n$$

$$n = 673 \text{ terms}$$

[d] 1 is the remainder on dividing 2^{2019} by 7.

$$[e] a_n = a + (n - 1)d$$

$$a_n = 1 + (n - 1)3$$

$$= 1 + 3n - 3$$

$$a_n = 3n - 2$$

[f] 1, 4, 7 n^{th} term is $3n - 2$.

So, the algebraic form is 2^{3n-2} .