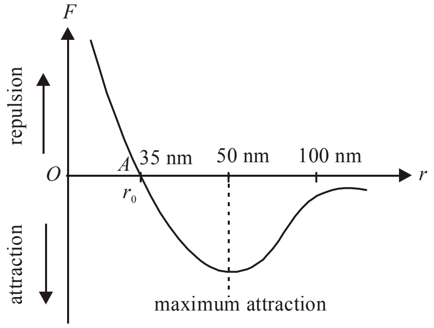
7

CHAPTER

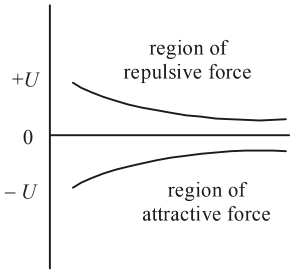
properties of matter

Intermolecular forces

- The force between two molecules is known as intermolecular forces.
- These forces are of electrical origin.
- They are also known as van der Waal forces. These forces are very weak forces.
- The potential energy is minimum and the kinetic energy is maximum when the two molecules are separated by $r = r_0$. This distance is known as normal distance.
  The molecules are in the state of equilibrium. No net force acts between the molecules.



- When $r > r_0$, the force is of attraction and potential energy is negative.
- When $r < r_0$, the force is of repulsion and potential energy is positive.
- At point A, the potential energy becomes minimum and the molecules cannot come closer. The distance OA denotes the minimum possible distance between molecules.
- Force of cohesion and adhesion are intermolecular forces.
- The potential energy is present in the molecules on account of intermolecular forces.
  $F = -\frac{dU}{dr}$, where U = potential energy.
- $F(r) = \frac{A}{r^{13}} - \frac{A}{r^7}$.
- **Positive and negative potential energy**
  (i) The value of potential energy U can be zero or positive or negative.
  (ii) The potential energy due to forces of attraction is negative.
  (iii) The potential energy due to forces of repulsion is positive.



- **States of equilibrium**

  (i) For a stable equilibrium, potential energy U is minimum. $\frac{d^2U}{dx^2}$ is positive.

  (ii) For unstable equilibrium, potential energy U is maximum. $\frac{d^2U}{dx^2}$ is negative.

  (iii) For neutral equilibrium, potential energy U is constant. $\frac{d^2U}{dx^2} = 0$.

- **Four states of matter**
  - (i) **Solid state :** The intermolecular forces are strongest, the kinetic energy of molecules is less than their potential energy and intermolecular distance remains constant.
  - (ii) **Liquid state :** The intermolecular forces are less strong than solids and more strong than gases, kinetic energy is more than potential energy and the intermolecular distance does not remain constant.
  - (iii) **Gas state :** The intermolecular forces are weakest, the kinetic energy of molecules is much larger than their potential energy and intermolecular distance does not remain constant.
  - (iv) **Plasma state :** It is a state of matter in which the medium is in the form of positive and negative ions.
- **Types of bonding :** There are mainly five types of bonding among the atoms or molecules.
  - (i) **Ionic bonding** - Cohesive energy is high and electrical conductivity is very low.

    e.g. NaCl, LiF. It is strongest bonding.
  - (ii) **Covalent bonding** - Cohesive energy is high. Semiconductors like Ge, Si, diamond have covalent bondings.
  - (iii) **Hydrogen bonding** - Cohesive energy is low. Insulators have this bonding.
  - (iv) **Hydrogen bonding** as DNA and H_2O.
  - (v) **Metallic bonding** as in metals.

ELASTICITY

- **For elastic constants : Inter relations**

  Y = Young's modulus, η = Rigidity modulus
  K = Bulk modulus, σ = Poisson's ratio

(a) $Y = 2\eta(1 + \sigma)$

(b) $Y = 3K(1 - 2\sigma)$

(c) $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$ or $Y = \frac{9K\eta}{\eta + 3K}$.

(d) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$

- **Range of σ**

  (a) $\sigma = -1$ to 0.5

  (b) When $\sigma = 0.5$, $K = \infty$ = matter is incompressible.This is not possible in practice.

  (c) When $\sigma = -1$, $\eta = \infty$ = not possible. σ in practice cannot be negative.

  (d) σ has no unit, no dimensions.

- **Adiabatic and isothermal elasticities**

  E_ϕ = adiabatic elasticity = γP where $\gamma = C_P/C_V$ for gas.

  E_θ = isothermal elasticity = P where P denotes pressure.

  $$\frac{E_\phi}{E_\theta} = \frac{\gamma P}{P} = \gamma = \frac{C_P}{C_V}.$$

- **Elastic potential energy**

  (a) Workdone per unit volume = $\frac{1}{2} \times \text{stress} \times \text{strain}$

  or Energy density = $\frac{1}{2} \times \text{stress} \times \text{strain}$

  (b) Workdone = $\frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

  (c) Energy density = $\frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \frac{\text{stress}}{\text{strain}} \times (\text{strain})^2 = \frac{1}{2} \times Y \times (\text{strain})^2$.

  (d) Workdone per unit volume = $\frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} = \frac{(\text{stress})^2}{2Y}$.

- **Force required to prevent a rod from increasing in length when the rod is heated**

  (a) Force = $YA\alpha\,(\theta)$ where α denotes coefficient of linear expansion, θ = change of temperature.

  (b) Thermal stress = $F/A = Y\alpha\theta$.

- **Interatomic force constant (k)**

  $$k = \frac{\text{Interatomic force}}{\text{Change in interatomic distance}}$$

  k = Young's modulus × interatomic distance

Cantilever and beam

- A beam, clamped at one end and loaded at free end is known as cantilever.
- Depression at the free end of cantilever = δ.

  $$\delta = \frac{Wl^3}{3YI_G}.$$

  where l denotes length of cantilever, I_G = geometrical moment of inertia of the cross-section of the beam.

- $I_G = \dfrac{bd^3}{12}$.

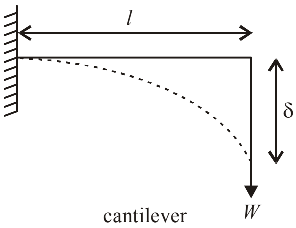
  For a beam with rectangular cross-section, b = breadth, d = thickness.
- $I_G = \dfrac{\pi r^4}{4}$.

  For a beam with circular cross-section having radius r.
- Depression produced at the centre of beam supported at two ends and loaded at the middle.

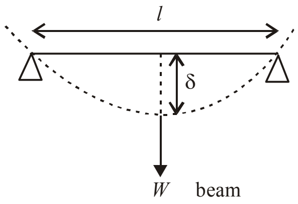
  $$\delta = \frac{Wl^3}{48YI_G}\text{, where } l \text{ denotes length of beam.}$$

  (i) $\delta = \dfrac{Wl^3}{4Ybd^3}$ for rectangular cross-section

  (ii) $\delta = \dfrac{Wl^3}{12\pi r^4 Y}$ for circular cross-section



cantilever



beam

Torsion of a cylinder and workdone in twisting

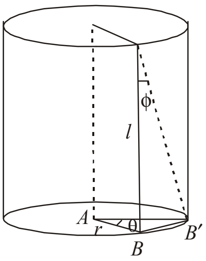
- Couple per unit twist is $C = \dfrac{\pi\eta r^4}{2l}$

  where l = length of cylinder, r = radius of cylinder

  η = modulus of rigidity of the material of cylinder.
- Workdone in twisting the cylinder through an angle θ

  $$= \int_0^\theta C\cdot\theta\cdot d\theta = \frac{1}{2}C\theta^2.$$
- Relation between angle of twist θ and angle of shear ϕ. Arc $BB' = r\theta = l\phi$

  $\therefore \quad \phi = \dfrac{r}{l}\cdot\theta$.



Variation in elasticity

- **Effect of temperature**
  - (i) In general elasticity decreases as the temperature increases.
  - (ii) INVAR is an exception. There is no effect of temperature on elasticity of invar. Invar is infact a short form of invariable.
- **Effect of impurities**
  - (i) If the impurity is more elastic, the elasticity of the material increases.
  - (ii) If the impurity is more plastic (less elastic) the elasticity of material decreases.
- On hammering or rolling elasticity increases.
- On annealing *i.e.* on alternate heating and cooling elasticity decreases.

Some salient points about elasticity

- Units and dimensions of coefficient of elasticity (Y, η and K) stress and pressure are same.

  $$\text{Unit} = \frac{\text{newton}}{(\text{metre})^2} = \frac{\text{N}}{\text{m}^2} = \text{Nm}^{-2}.$$

  $$\text{Dimensions} = [\text{ML}^{-1}\text{T}^{-2}]$$

- Coefficent of elasticity depends upon the material, its temperature and purity but not on stress or strain.
- For the same material, the three coefficients of elasticity *i.e.* Y, η and K have different magnitudes.
- **Compressibility :** The reciprocal of bulk modulus of elasticity is defined as compressibility.
- Young's modulus Y and rigidity modulus η are possessed by solid materials only.
- Every force produces extension along its own direction and simultaneous compression along perpendicular direction.
- Factor of safety $= \dfrac{\text{breaking stress}}{\text{normal working stress}} = \dfrac{\text{breaking stress} \times \text{area}}{\text{force}}$
- **Elastic relaxation time**
  - (i) The time delay in restoring the original form after removal of deforming force is called elastic relaxation time.
  - (ii) For phosphor bronze, silver and gold, the time is negligibly small.
  - (iii) For Quartz, it is minimum.
  - (iv) For glass, it is high.
- For liquids, modulus of rigidity η is zero.
- If real original length of wire = L, then $L = \dfrac{F_2L_1 - F_1L_2}{F_2 \sim F_1}$

  where F_1 and F_2 are the stretching forces and L_1 and L_2 are the respective final lengths of a wire.

  $$Y = \frac{F_1L}{A(L_1 - L)} = \frac{F_2L}{A(L_2 - L)}$$

$$F_1 (L_2 - L) = F_2 (L_1 - L)$$

$$\therefore \quad L = \frac{F_2L_1 - F_1L_2}{F_2 - F_1}.$$

- **Poisson's ratio (σ)**
  - (i) If $\sigma = 0.5$, there shall be no change in volume of wire on loading. This is not possible.
  - (ii) If σ is –ve, it means that if the length of a wire increases, its radius also increases. This is not possible.
  - (iii) $\sigma = 0.46$ for rubber, $\sigma = 0.25$ for steel, $\sigma = 0.20$ for glass.
- **Breaking stress** is fixed for a material. It depends upon the material.
- **Tensile strength** is the breaking stress for a wire of unit cross-section.
- **Quartz** is the best example of a perfectly elastic body and **putty** is the best example of a perfectly plastic body.
- (i) For a perfectly rigid body, strain produced by the applied force is zero.

  $$Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{0} = \infty.$$

  $Y = \infty =$ infinity.

  (ii) For a perfectly rigid body, strain produced is zero.

  $$K = \frac{\text{stress}}{\text{strain}} = \frac{\text{pressure}}{\text{zero}} = \infty = \text{infinity}$$

  (iii) Shear modulus of elasticity of a liquid is zero because a liquid does not oppose change in its shape.
- Potential energy of a stretched spring $U = \frac{1}{2}kx^2$ where k is spring constant and x denotes the amount of stretch.

  $F = kx$

  $dW = kx\, dx$

  $$W = \int_0^x kx\, dx = \frac{1}{2}kx^2$$

  $$W = \frac{k}{2}(l_2^2 - l_1^2)$$ when stretched from l_1 to l_2.
- **Ductile materials :** The materials which exhibit large plastic range beyond the elastic limit such that they can be drawn into wire springs and sheets are known as ductile materials.

  e.g. silver, steel, copper, aluminium.
- **Brittle materials** are those material which show a very small plastic range beyond the elastic limit. The breaking point lies very close to elastic limit e.g. rubber, glass.

HYDROSTATICS

- In fluid mechanics the following properties of fluid would be considered:
  - (i) when the fluid is at rest - **hydrostatics**
  - (ii) when the fluid is in motion - **hydrodynamics**.

Pressure due to a liquid

- The thrust exerted by a liquid at rest per unit area of the substance in contact with the liquid is called **pressure**. If F is the thrust exerted by a liquid on a surface of small area ΔA, then pressure is given by

$$P = \lim_{\Delta A \to 0} (F / \Delta A)$$

- The unit of pressure is dyne/cm^2 in CGS system and N/m^2 in SI. A pressure of one N/m^2 is also called pascal.
- **Pascal's law :** Pressure applied to enclosed liquid is transmitted equally in all directions, to every position of liquid and wall of container.
- **Hydrostatic pressure of liquid column:** Pressure = $\sigma g h$

  where σ = density of liquid, h = height of liquid column.
- **Brahma's hydraulic press** is bases upon Pascal's law of liquid pressure.
- **Unit of pressure** is pascal. Its symbol is Pa.

  Bar = 10^5 Pa, torr = 1 mm of Hg column.

Density and relative density

- Density = mass/volume.
  For water, density = 10^3 kg/m^3.
- One litre = 1000 cc = 1000 cm^3 = $1000 \times (10^{-2}\ \text{m})^3 = 10^{-3}\ \text{m}^3$.
- Relative density = $\dfrac{\text{Density of substance}}{\text{Density of water at } 4°\text{C}} = \dfrac{\text{Weight of substance in air}}{\text{Loss of weight in water}}$
- Density of mixture = $\dfrac{m_1 + m_2}{V_1 + V_2} = \dfrac{(m_1 + m_2)\rho_1\rho_2}{(m_1\rho_2 + m_2\rho_1)}$

  where m denotes mass and ρ denotes density of liquid.

  (i) If $m_1 = m_2 = m$, $\rho = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ or $\dfrac{2}{\rho} = \dfrac{1}{\rho_1} + \dfrac{1}{\rho_2}$

  $\therefore$ Density of mixture of two liquids is harmonic mean of the two densities.

  (ii) If $V_1 = V_2 = V$,

  $$\rho = \frac{m_1 + m_2}{V_1 + V_2} = \frac{\rho_1 V + \rho_2 V}{V + V} = \frac{\rho_1 + \rho_2}{2}.$$

  $\therefore$ Density of mixture of two liquids is arithmetic mean of two densities.
- Density of liquid varies with pressure.

  $\rho = \rho_0 \left[1 + \dfrac{\Delta P}{K}\right]$, where ΔP = change in pressure,

  K = bulk modulus of elasticity of liquid.
- Relative density, also known as specific gravity, has no unit, no dimension.

Archimede's principle

- When a solid body is immersed, partly or wholly, in a liquid at rest, it loses a weight which is equal to weight of the liquid displaced by the immersed portion of solid body.

- Observed weight = true weight – weight of liquid displaced
  or $T = Mg - mg = a\rho gh - a\sigma gh = ahg(\rho - \sigma)$
  or $T = ahg\rho\left(1-\frac{\sigma}{\rho}\right) = W\left(1-\frac{\sigma}{\rho}\right).$
  where ρ = density of solid, σ = density of liquid
  W = true weight of body
- Weight of liquid displaced = upthrust = loss in weight of body.

Laws of floatation

- Weight of floating body = weight of liquid displaced.
- Volume of body immersed = volume of liquid displaced.
- If density of solid body is greater than density of liquid, the body will sink in the liquid. Here $d_S > d_L$ or $\rho > \sigma$.
  Weight of body > weight of liquid displaced.
- If $d_S < d_L$, the solid body floats on the liquid surface. ($\rho < \sigma$).
  Here weight of body < weight of liquid displaced.
- If $d_S = d_L$ or $\rho = \sigma$, the body will stay at rest anywhere in the liquid.
- The floating body will be in stable equilibrium when the metacentre lies above centre of gravity of body.
- The floating body will be in unstable equilibrium when the metacentre lies below centre of gravity of body.
- The floating body will be in neutral equilibrium when the meta cente coincides with centre of gravity of body.
- The upward force or upthrust acting on the body immersed in a liquid is known as **Buoyant force** of buoyancy.
- The centre of gravity of displaced liquid is known as **centre of buoyancy**.
- If a person floats on his back on the surface of water, the apparent weight of the person is zero.

Surface tension

- **Surface tension :** The free surface of every liquid has always a tendency to contract to a minimum possible surface area and thus behaves like a stretched membrane having a tension in all directions parallel to the surface. Thus,
  Surface tension is the property of liquid by virtue of which its free surface behaves like a stretched membrane. The force which acts along the surface of a liquid, tending to contract its area to a minimum, is called the **surface tension** of the liquid.
  The dimension of surface tension is $[MT^{-2}]$.
- Surface tension = $\frac{\text{force}}{\text{length}} = \frac{\text{workdone}}{\text{change in area}}$
- Unit of surface tension = newton/metre = joule/(metre)^2.
- Dimensions of surface tension = $\frac{MLT^{-2}}{L} = [MT^{-2}]$

- **Cohesive force** is the force of attraction between molecules of same substance.
- **Adhesive force** is the force of attraction between molecules of different substances.
- **Molecular range** ≈ 10^{-9} m. It depends upon material. It is equal to the maximum distance upto which the molecules attract each other. Also it is equal to the radius of sphere of influence.
- Cohesive force (F_c) or adhesive force (F_a) is inversely proportional to eighth power of the distance between the molecules.
- $F_c \propto \frac{1}{r^8}$. Also $F_a \propto \frac{1}{r^8}$.
- Due to property of surface tension, a liquid tends to minimize its free surface area.
- **Surface tension** is the molecular phenomenon.

- **Surface energy** : To increase the surface area of liquid work has to be done against the force of surface tension. This additional potential energy stored per unit area of the surface is called surface energy. It can be shown that the surface energy per unit area is numerically equal to the surface tension of liquid.
- **Angle of contact** : A liquid is to be kept in a vessel and thus, liquid is in contact with some solid surface. For a pair of solid and liquid, the angle of contact θ, is defined as the angle between tangent to the liquid surface drawn at the point of contact and the solid surface inside the liquid.
  If θ < 90°, the liquid will have a meniscus concave upwards.
  If θ > 90°, the liquid surface will have a meniscus convex upwards.
  If θ = 90°, the surface of liquid at the point of contact is plane.
- **Angle of contact, meniscus, shape of liquid surface**

Property	**Angle of contact < 90°**	**Angle of contact = 90°**	**Angle of contact > 90°**
Substances	water and glass	water and silver	mercury and glass
Angle of contact	almost zero. acute angle	right angle = 90°	obtuse angle = 135°
Meniscus shape	concave	plane	convex
Capillary action	liquid rises	no effect	liquid falls
Sticking to solid	sticks/wets	does not wet	does not wet
Relation between F_a and F_c	$F_a > \frac{F_c}{\sqrt{2}}$ $F_a > F_c$	$F_a = \frac{F_c}{\sqrt{2}}$	$F_a < \frac{F_c}{\sqrt{2}}$ $F_c > F_a$
Shape of liquid surface	almost round	spreads on surface	flat

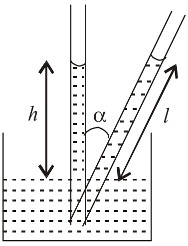
- **Capillarity** : A tube of very small radius is called a capillary. When such a clean tube of glass open at both ends is dipped vertically in water, the water rises in the tube upto a certain height above the water level outside the tube. On the other hand, if the tube is dipped in mercury, the mercury level falls below the outside level. The phenomenon of rise or fall of the liquids in a capillary tube is called **capillarity**.

- **Zurin's law :** Rh = constant $\Rightarrow$ $R_1h_1 = R_2h_2$
  R = radius of capillary tube, h = height of liquid in capillary tube.
- Liquid rises (water in glass capillary) or falls (mercury in glass capillary) due to property of surface tension.

  $T = \dfrac{R\rho gh}{2\cos\theta}$ where

  R = radius of capillary tube, h = height of liquid,
  ρ = density of liquid, θ = angle of contact,
  T = surface tension of liquid, g = acceleration due to gravity.
- **Tube of insufficient length**
  Liquid may rise to a height h but if the length of tube is less than this height, overflow of liquid does not occur. The liquid rises upto upper end and acquires a meniscus r' such that $r'h' = rh$.
- Tube kept in inclined position. The vertical height to which liquid rises remains the same.

  $\dfrac{h}{\cos\alpha} = l$ = inclined length



- **Excess pressure due to surface tension**:
  - (a) The pressure on concave side of a curved liquid surface is greater than the pressure on convex side.
  - (b) Excess pressure inside a liquid drop $P = 2T/R$, where R is the radius of drop. A drop has only one surface of contact of liquid and air.
  - (c) *Excess pressure inside a soap bubble* : A bubble has two surfaces of contact of soap and air.
    Excess pressure $P = 4T/R$.
  - (d) Excess pressure in air bubble inside a liquid $P = 2T/R$.
- **Combination/spraying of drops-work done**

(a) When a number of smaller drops are combined into a bigger drop, volume remains constant but area decreases. Energy is released and temperature of bigger drop increases.

(i) Equate volume, $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

$$n = \left(\frac{R}{r}\right)^3.$$

(ii) Decrease in surface area = $n \cdot 4\pi r^2 - 4\pi R^2 = 4\pi(nr^2 - R^2)$.

(iii) Work done = change in area × surface tension

(iv) Work done is converted into heat, $W = JH$.

Thereby temperature of bigger drop increases.

Specific heat of water = 1 cal g^{-1} $°C^{-1}$.

Density of water = 1 gcm^{-3}.

$$J\left(\frac{4}{3}\pi R^3\right)(1)\,(1) \times \Delta\theta = W$$

$$\therefore \quad \Delta\theta = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

(b) When a bigger drop of radius R is sprayed into a number of similar smaller drops, each of radius r, volume remains constant but surface area of drops increases. Energy is absorbed. Temperature of the system decreases.

(i) Equate volumes, $n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$$n = \left(\frac{R}{r}\right)^3.$$

(ii) Increase in surface area = $n \cdot 4\pi r^2 - 4\pi R^2 = 4\pi(nr^2 - R^2)$

$$= 4\pi\left(\frac{R^3}{r^3} \cdot r^2 - R^2\right) = 4\pi R^3\left(\frac{1}{r} - \frac{1}{R}\right)$$

(iii) Work done = T × change in area W = energy = $4\pi TR^3\left(\frac{1}{r} - \frac{1}{R}\right)$

(iv) Energy is absorbed. $W = JH$.

Thereby temperature of smaller drops falls.

$$J \times \left(\frac{4}{3}\pi R^3\right)(1)\,(1) \times \Delta\theta = W$$

$$\therefore \quad \Delta\theta = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right).$$

(c) Workdone in blowing a liquid drop

$W = T$ × change in surface area

or $W = T \times 4\pi(r_2^2 - r_1^2)$

If $r_2 = R$, $r_1 = 0$ or $W = 4\pi TR^2$.

(d) Workdone in blowing a soap bubble.

Soap bubble has two surfaces of contact.

$W = 2 \times T$ × change in surface area

or $W = 2T \times 4\pi(r_2^2 - r_1^2)$ or $W = 8\pi T(r_2^2 - r_1^2)$

If $r_2 = R$, $r_1 = 0$.

$W = 8\pi TR^2$.

- **Radius of new bubble when two bubbles coalesce**

  Let r_1 and r_2 be the radii of two bubbles. Let them coalesce into a bubble of radius r, under isothermal conditions. Let T denote the surface tension,

$$P_1 = \frac{4T}{r_1},\ P_2 = \frac{4T}{r_2},\ P = \frac{4T}{r}$$

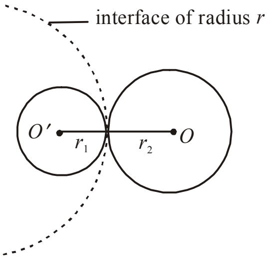
By Boyle's law, $P_1V_1 + P_2V_2 = PV$

$$\left(\frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(\frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right) = \frac{4T}{r}\left(\frac{4}{3}\pi r^3\right)$$

$r_1^2 + r_2^2 = r^2$.

- **Radius of interface**

  Consider two soap bubbles of radii r_1 and r_2 in contact with each other. Let r denote the radius of the common boundary/interface.



  Let P_1 and P_2 denote the excess pressure on the two sides of the interface and let P denote the resultant excess pressure.

$$P = P_1 - P_2 \quad \Rightarrow \quad \frac{4T}{r} = \frac{4T}{r_1} - \frac{4T}{r_2} \quad \Rightarrow \quad \frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}.$$

  Hence $r = \dfrac{r_1r_2}{r_2 - r_1}$.

- **Effect of temperature on surface tension**
  - (a) Surface tension of a liquid decreases with rise of temperature.
  - (b) An exception - Surface tension of molten cadmium or copper increases with increase in temperature.
  - (c) At critical temperature, surface tension of a liquid becomes zero.
  - (d) At boiling point, surface tension of a liquid becomes zero.
  - (e) At freezing point, surface tension becomes maximum.
- **Effect of impurity on surface tension**
  - (a) Soluble impurities cause increase in surface tension.
  - (b) Partially soluble impurities cause decrease in surface tension. Soaps, detergents, phenol reduce surface tension of water.

Some salient points about surface tension

- The angle of contact depends upon
  - (i) The nature of solid and liquid in contact.
  - (ii) The given pair of the solid and the liquid
  - (iii) The impurities and the temperature
- The angle of contact lies between 0° and 180°.

- The angle of contact does not depend upon the inclination of the solid in the liquid.
- The rise in temperature increases angle of contact.
- Addition of soluble impurities increases angle of contact.
- The water proofing material increases the angle of contact. Acute angle is converted into obtuse angle.
- Addition of detergent, soap etc. to water decreases angle of contact as well as surface tension.
- The surface tension of the liquid decreases due to electrification. Soap bubble expands when given a charge. Due to charge, a normal force acts in the outward direction on liquid surface.
- Due to contamination, surface tension decreases. Dust particles and lubricating materials reduce surface tension.
- Under weightlessness (as in a satellite) liquid does not rise in a capillary tube and splits into minute droplets. If liquid state is maintained, the liquid rises to the top of capillary tube and the radius of liquid meniscus adjusts itself for equilibrium.

  $Rdgh = 2T\cos\theta$ when Rh = constant.
- The force required to separate two glass plates, between which a liquid film is held is $\frac{2AT}{t}$ where t denotes thickness of the film. Obvioulsy greater force is needed for thinner films.
- Force due to surface tension on a hemispherical soap bubble.

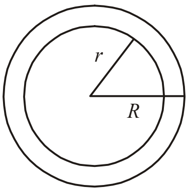
  Force = $2T \times$ circumference $= 2T \times 2\pi R = 4\pi RT$.
- For a liquid drop,

  $$P = \frac{2T}{R}, \quad \text{Volume of drop} = V = \frac{4}{3}\pi R^3.$$

  $$\therefore \quad PV = \frac{2T}{R} \times \frac{4}{3}\pi R^3 = \left(\frac{8\pi T}{3}\right) R^2$$

  $$\frac{PV}{R^2} = \frac{8\pi T}{3} = \text{constant}$$

  For two drops, $\frac{P_1V_1}{R_1^2} = \frac{P_2V_2}{R_2^2}$.
- Force on annular disc, having radii R and r, floating on a liquid sphere of surface tension is equal to T.

  $$T = \frac{\text{force}}{\text{length}} = \frac{F}{2\pi(R+r)}$$

  $F = 2\pi T(R + r)$.
- When two soap bubbles of equal radii coalesce, then the shape of resultant surface shall be plane.

$$r = \frac{r_1 r_2}{r_2 - r_1} = \frac{r^2}{\text{zero}} = \text{infinity} = \text{plane surface}$$

- When wax is coated on a glass capillary tube, it becomes water- proof. The angle of contact increases and becomes obtuse. Water does not rise in it. Rather it falls in the tube by virtue of obtuse angle of contact.

HYDRODYNAMICS

Viscosity

- When a layer of a liquid slips or tends to slip on another layer in contact, the two layers exert tangential force on each other. The directions are such that the relative motion between the layers is opposed. This property of a liquid to oppose relative motion between the layers is called viscosity.

Coefficient of viscosity

- Tangential force/viscous drag $F = -\eta \cdot A \cdot \frac{dv}{dx}$
  where dv/dx denotes velocity gradient between two layers of liquid each of area A.
  η = coefficient of viscosity of liquid.
- S.I. unit of η is decapoise = Nsm^{-2} or pascal-second.
  c.g.s. unit of η is poise = dyne sec cm^{-2}.
- Dimensions of $\eta = [ML^{-1}T^{-1}]$.
- Viscosity is due to transport of momentum.
- The value of viscosity (and compressibility) for ideal liquid is zero.

Poiseuille's formula and liquid resistance

- Volume of liquid flowing per second V through a horizontal capillary tube of length l, radius r, across a pressure difference P, under steamline motion, is given by

$$V = \frac{\pi P r^4}{8\eta l} = \frac{P}{R}.$$

- Liquid resistance $R = \frac{8\eta l}{\pi r^4}$.
- (i) Two capillary tubes are joined in series. $P = P_1 + P_2$ and V is same through the two tubes.
  (ii) Equivalent liquid resistance, $R_s = R_1 + R_2$.
- (i) Two capillary tubes are joined in parallel.
  Equivalent liquid resistance, $R_P = \frac{R_1 R_2}{R_1 + R_2}$
  or, $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$.
  (ii) In parallel, $V = V_1 + V_2$ but pressure difference P is same across both tubes.

Stoke's law and terminal velocity

- Stoke's law relates the backward dragging force F acting on a small sphere of radius r moving through a viscous medium of viscosity η with the velocity v.

$$F = 6\pi\eta r v.$$

- Terminal velocity is the maximum constant velocity of sphere of density ρ which falls freely in a viscous medium of density ρ_0.

$$v = \frac{2}{9}\cdot\frac{r^2(\rho - \rho_0)g}{\eta}.$$

  (i) If $\rho > \rho_0$, the body falls downwards.
  (ii) If $\rho < \rho_0$, the body moves upwards with the constant velocity.
  (iii) If $\rho_0 << \rho$, $v = \dfrac{2r^2\rho g}{9\eta}$.

Factors affecting viscosity

- **Effect of temperature**
  (i) If temperature increases, viscosity of liquid decreases. Viscosity of water, for example, at 80°C falls to one-third of its value at 10°C.
  (ii) The rate of diffusion of gases and also viscosity of all gases increases with increase in temperature *i.e.* $\eta \propto \sqrt{T}$.
- **Effect of pressure**
  (i) If pressure increases, viscosity of liquid increases but viscosity of water decreases at few hundred atmosphere.
  (ii) Viscosity of gases remains constant at high pressure but in low pressure region the viscosity of gases is directly proportional to pressure.

Streamlined and turbulent flow

- When a liquid flows in such a way that each liquid particle when passed through the same point follows exactly the same path as followed by the proceeding particles when passed through the same point then the flow is said to be streamlined and the path is called **streamline**.
- The fluid flows in streamline only when its velocity is less than certain value called **critical velocity**. While the motion of the particles of the fluid are disorderly, if velocity is more than critical value, the disorder motion of fluid is called **turbulent flow**.
- **Critical velocity**, $v_c = \dfrac{K\cdot\eta}{r\cdot\rho}$

  where K = Reynold's number
  η = coefficient of viscosity of liquid
  r = radius of capillary tube
  ρ = density of liquid.
- When $v \le v_c$, the flow of liquid is streamlined. v_c denotes the maximum velocity of a liquid under streamline flow. Viscosity dominates the flow.

- When $v > v_c$, the flow of liquid is turbulent. Such a flow is dominated by density of the fluid while viscosity has little effect on it. Lava, for example, is highly thick fluid emerging from a volcano with high speed.
- **Reynold's number** = $K = \dfrac{v_c \cdot \rho \cdot r}{\eta}$

(i) $K = \dfrac{\text{Inertial force}}{\text{Viscous force}}$
(ii) For $K < 3000$, the flow of liquid is streamlined.
(iii) For $K > 5000$, the flow becomes turbulent.
(iv) K has no unit, no dimension. It is a pure number.
(v) For narrow tubes and water, $K \approx 1000$.

- **Equation of continuity** : Equation of continuity is a special case of general law of conservation of matter stated as "for a steady state flow of an ideal fluid (incompressible and non-viscous) in a pipe, the rate of mass flow across any cross section is constant. If two sections A_1 and A_2 at right angles to a tube of flow be considered at two different positions, then velocities of flow v_1 and v_2 respectively at these positions are related to the sections as $A_1v_1 = A_2v_2$.

  Thus, when fluid flows through a smaller cross-section its velocity increases. This is known as equation of continuity.
- A fluid in steady or streamline flow may possess any or all of the three types of energy :

(i) **Kinetic energy** : Kinetic energy per unit volume = $\frac{1}{2}\rho v^2$
(ii) **Potential energy** : Potential energy per unit volume = ρgh
(iii) **Pressure energy** : Pressure energy of an incompressible fluid is because of its hydrostatic pressure P and pressure energy per unit volume = P.

Bernouilli's theorem

- It is the principle of conservation of energy for a flowing liquid. Under streamlined motion of a liquid, the sum total of pressure energy, kinetic energy and potential energy per unit volume at every point along its path remains constant.

  Mathematically, $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$.
- At greater depth, P is large and so v is small. Liquids flow slow at greater depths. Deeper waters run slow accordingly.
- The theorem is applicable to ideal liquid *i.e.* a liquid which is non-viscous, incompressible and irrotational.
- It is expressed as follows also: $\dfrac{P}{\rho g} + h + \dfrac{1}{2}\dfrac{v^2}{g} =$ a constant where

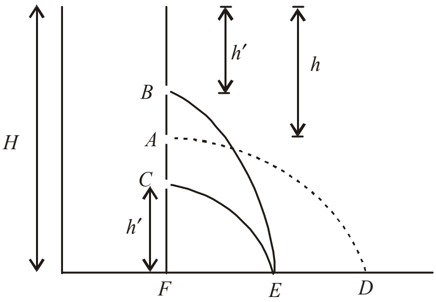
  $\dfrac{P}{\rho g}$ = pressure head, $\dfrac{1}{2}\dfrac{v^2}{g}$ = velocity head,

  h = potential head.

Torricelli's theorem and horizontal range

- $v = \sqrt{2gh}$ where $v =$ velocity of efflux.
- Velocity of efflux is the velocity acquired by a freely falling body in falling through a vertical distance h which is equal to depth of a hole, below free surface of liquid, from which liquid flows out.
- Horizontal range when water issues out of a hole at depth h below surface of water in a tank filled with water upto a height H. Water issues out from A.

  It covers a vertical distance $= (H - h)$ before striking the ground. Time for this vertical journey $= \sqrt{\dfrac{2(H-h)}{g}}$.



  Horizontal velocity of water is velocity of efflux $= \sqrt{2gh}$.

  Horizontal range $FD = \sqrt{2gh} \times \sqrt{\dfrac{2(H-h)}{g}}$

  Range $R = FD = 2\sqrt{h(H-h)}$

- **Special case**

  (i) For B, range $= 2\sqrt{h'(H-h')} = FE$

  For C, range $= 2\sqrt{(H-h')h'} = FE$

  Range is same whether the point is at distance h' below top or h' above bottom.

  (ii) Maximum range

  Range $R = 2\sqrt{h(H-h)}$

  For maximum horizontal range, $\dfrac{dR}{dH} = 0.$

  $R^2 = 4h(H - h)$

  $$2R \cdot \frac{dR}{dH} = 4(H-h) + 4h(-1)$$

$$= 4H - 4h - 4h = 4H - 8h$$

$$0 = 4H - 8h \quad \Rightarrow \quad h = H/2.$$

Range is maximum when liquid issues from a point at mid-height of tank.

$$R_{\max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$$

same as height of tank itself.

Some salient points about viscosity

- The cause of viscosity in liquids is the cohesive forces among molecules.
- The cause of viscosity in gases is diffusion.
- The viscosity of water is much higher than that of air. Hence it is more difficult to run through water than in air.
- In heavy machines lubricating oils of high viscosity are used.
- In light machines low viscosity oils are used for lubrication.
- 1 poise = 0.1 pascal second.
- Coefficient of viscosity is zero for ideal fluids.
  Incidentally, their compressibility is also zero.
- For water, $\eta = 10^{-3}$ pascal second = 10^{-2} poise.
  For glycerine, $\eta = 0.82$ pascal second = 8.2 poise.
- **Velocity gradient:** It is the relative velocity between two consecutive parallel liquid layers, unit distance apart, at right angles to the direction of flow of liquid.
  Its unit is $(\text{second})^{-1}$. Its dimension is $[T^{-1}]$.
- The path of liquid-stream issuing from a hole in a tall jar is parabolic.
- Two ping-pong balls freely suspended come closer when high speed air current is blown between them.
  Pressure between the balls falls due to high air velocity.
- After terminal velocity is acquired, the acceleration of a body falling through viscous fluid is zero. Gravitational force and viscous force balance each other.

End