Reflection at spherical mirror

Aperture : The edge of a spherical mirror is a circle. Part of the plane of circle, enclosed by the circle is called its aperture.

Paraxial Ray: A light ray incident on the mirror at very small angle then the ray is called paraxial ray.

Marginal Ray: A light ray incident on the mirror at finite angle then the ray is called marginal ray.

focus : Suppose a light ray AQ parallel to x axis become incident on a concave mirror at angle of incidence θ (fig).After reflection we have reflected ray QF at angle of reflection θ which intersects x axis at F. We want to calculate PF

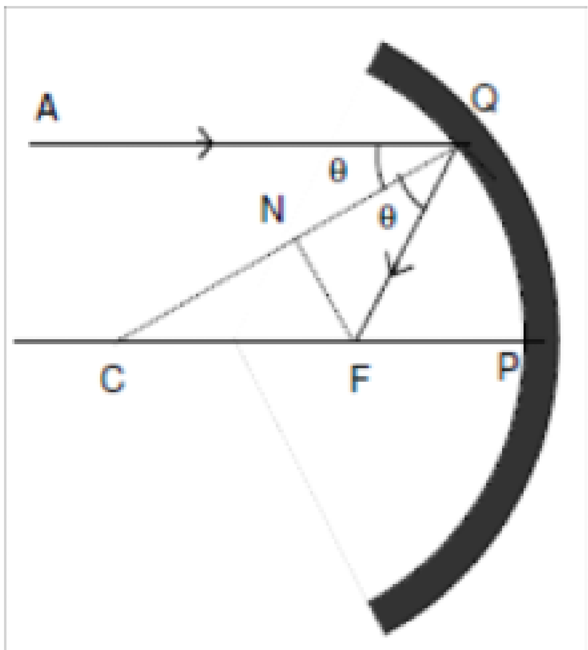
In triangle CFQ

$$\angle QCF = \angle AQC = \theta \qquad \text{(alternate angle)}$$

$\Rightarrow$ triangle CFQ is an isoscless triangle (CF=QF).

$\Rightarrow$ CN=QN=CQ/2=R/2

In triangle NFQ



$$\text{COS}\theta = \frac{\text{QN}}{\text{QF}} = \frac{\text{R}/2}{\text{QF}}$$

$$\Rightarrow \qquad \text{QF} = \frac{\text{R}}{2\text{COS}\theta}$$

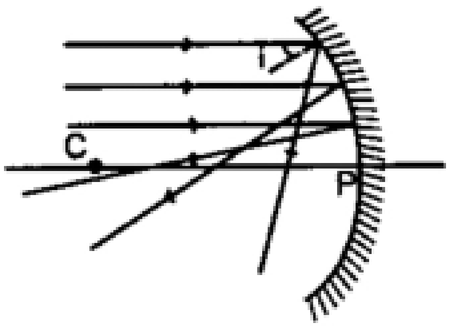
$$\Rightarrow \qquad \text{CF} = \text{QF} = \frac{\text{R}}{2\text{COS}\theta}$$

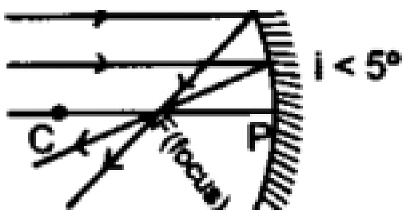
$$\therefore \qquad \text{PF}=\text{PC}-\text{CF}=\text{R}-\text{CF}= \text{R} - \frac{\text{R}}{2\text{COS}\theta}$$

For marginal rays θ is not small. Hence different light rays intersects x-axis at different points. But if we consider paraxial beam $(\theta \to 0)$

$$\Rightarrow \qquad \text{PF}=\frac{\text{R}}{2} \qquad (\text{As } \theta \to 0 \Rightarrow \cos\theta \to 1)$$

i.e all the light rays intersects x-axis at single point.This single point is called focus of the spherical mirror





As i increases cos i decreases.

Hence CQ increases

If i is a small angle cos i ≈ 1

$\therefore$ CQ = R/2

Principal axis : A line passing through focus and centre of curvature.

Pole : Point of intersection of principal axis and mirror.

Focal length (f) : The distance between focus and pole is called focal length .

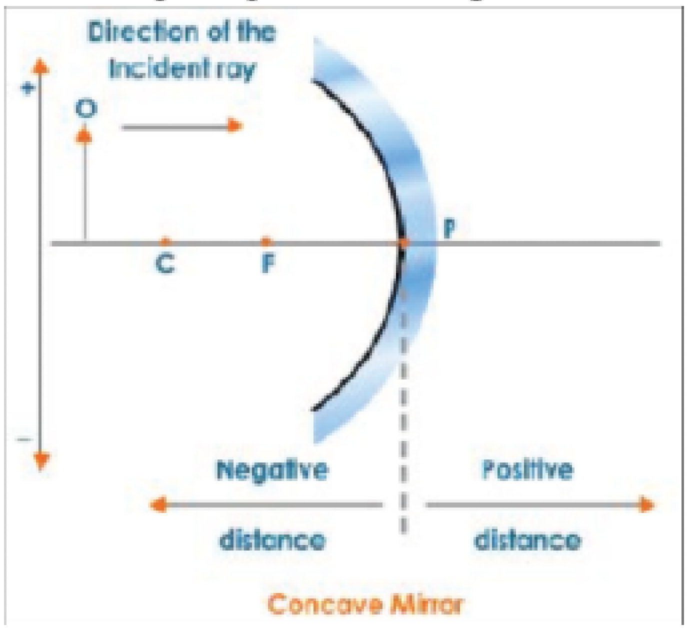
Sign Convention :

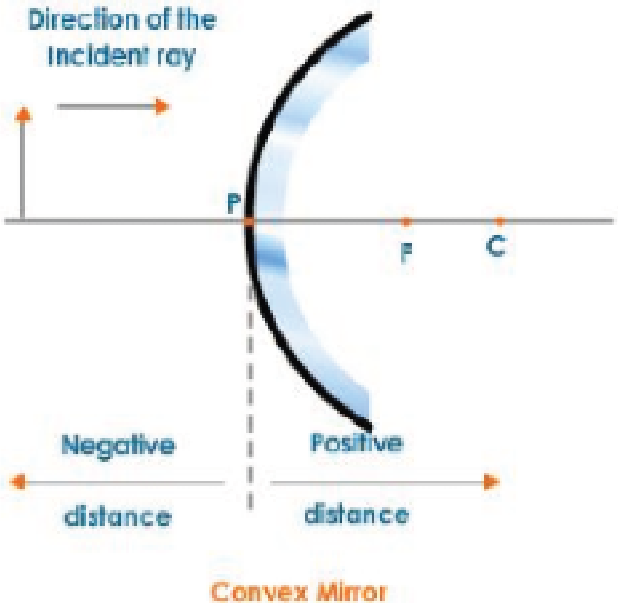
The following sign convention is used for measuring various distances in the ray diagrams of spherical mirrors:

All distances are measured from the pole of the mirror.

Distances measured in the direction of the incident ray are positive and the distances measured in the direction opposite to that of the incident rays are negative.

Distances measured along y-axis above the principal axis are positive and that measured along y-axis below the principal axis are negative.

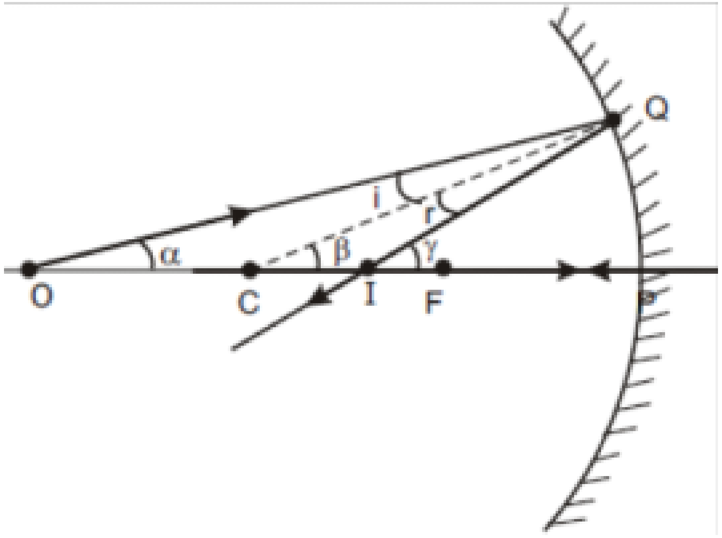




	Spherical Mirrors	Lenses
Focal Length	+ for concave mirrors	+ for a converging lens
	- for convex mirrors	- for a diverging lens
Object Distance	+ if object is in front of the mirror (real object)	+ if the object is to the left of the lens (real object)
	- if object is behind the mirror (virtual object)*	- if the object is to the right of the lens (virtual object)*
Image Distance	+ if the image is in front of the mirror (real image)	+ for an image (real) formed to the right of the lens by a real object
	- if the image is behind the mirror (virtual image)	- for an image (virtual) formed to the left of the lens by a real object
Magnification	+ for an image that is upright with respect to the object	+ for an image that is upright with respect to the object
	- for an image that is inverted with respect to the object	-for an image that is inverted with respect to the object.

Mirror formula

In this section we describe quantitatively where images are formed when light rays are reflected at spherical mirror. Consider two transparent media having indices a spherical mirror of radius R (Fig.). We assume that the object at O. Let us consider the paraxial rays leaving O. As we shall see, all such rays are reflected at the spherical surface and focus at a single point I, the image point.Figure shows a single ray leaving point O and reflecting to point I.



Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this rule to triangles OQC and CQI in Figure gives

$$i=\beta-\alpha \qquad r=\gamma-\beta$$

From law of reflection

$$i = r$$

$$\Rightarrow \quad \beta - \alpha = \gamma - \beta$$

$$\Rightarrow \quad \alpha + \gamma = 2\beta$$

$$\Rightarrow \quad \frac{QP}{OP}+\frac{QP}{IP}=\frac{QP}{CP} \qquad \text{(paraxial ray approximation)}$$

Taking sign convention

$$u = -OP \qquad v = -IP \qquad R = -CP$$

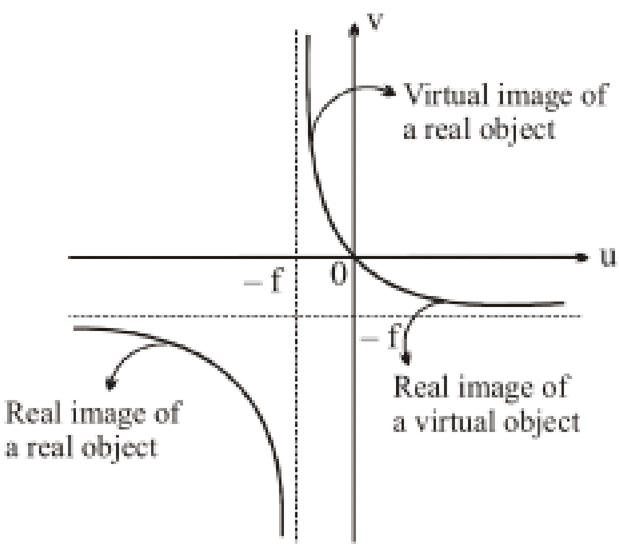
we get

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

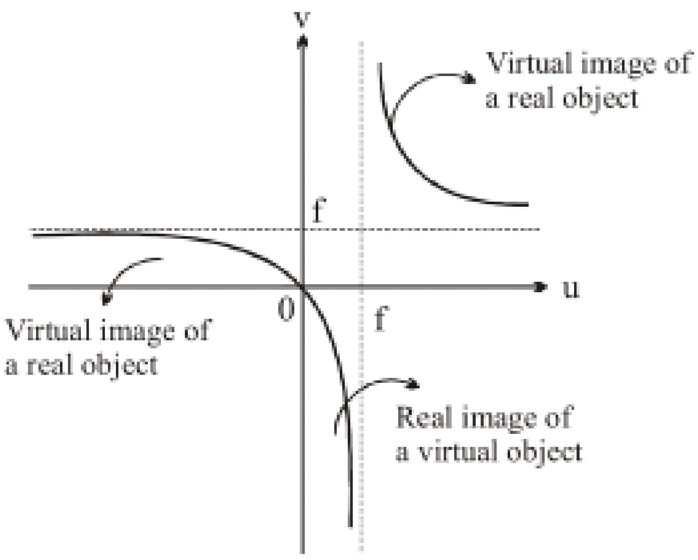
Note : This relation is same in all cases no matter object or image point is real or virtual, mirror is concave or convex.

Graph : v vs u :

(a) For concave mirror **(b) For convex mirror**



Concave mirror



Concave mirror

Image Tracing for Transverse Extended Object

In tracing image of a transverse extended object we should keep in mind following :

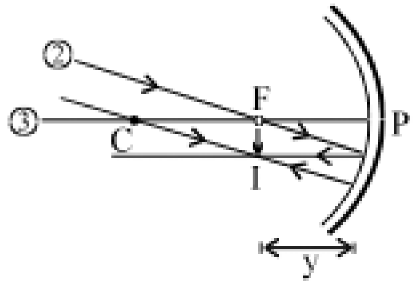
(1) A ray parallel to principal axis after reflection from the mirror passes or appears to pass through its focus

(2) A ray passing through or directed towards centre of curvature, after reflection from the mirror, retraces its path (as for it $\theta_1 = 0$ and so $\theta_2 = 0$).

(3) Ray drawn from the top of the object toward the focal point on the back side of the mirror and is reflected parallel to the principal axis.

Image Tracing in some cases

(i) When the object is placed at infinity, a real, inverted and very small image is formed at the focus.

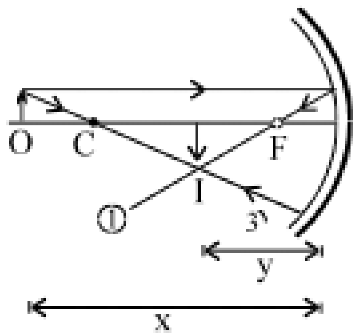


For a distance object image is formed at the focus

$x = \infty$

$v = -y$	where	$y = f_0$
$m = -\delta$	where	$\delta << 1$

(ii) When the object is placed beyond C ($2f_0 < x < \infty$), a real, inverted and diminished image is formed between F acid C.

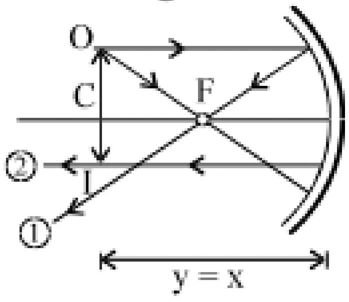


For an object placed beyond C, image is formed between C and F

$v = -y$	where	$f_0 < y < 2f_0$
$m = -\delta$	where	$0 < \delta < 1$

(iii) When the object is placed at C.

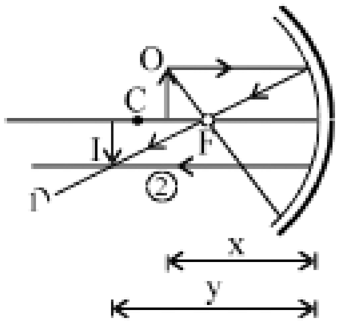
($x = 2f_0$), a real, inverted and equal size image is formed at C.



At C both object and image coincide

$v = -y$	where	$y = 2f_0$
$m = -\delta$	where	$\delta = 1$

(iv) When the object is placed between F and C ($f_0 < x < 2f_0$), a real, inverted and large image is formed beyond C.

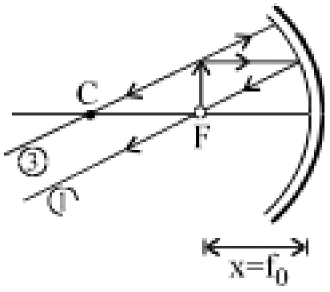


For an object placed between F and C image is formed beyond C.

$v = -y$ where $y < 2f_0$

$m = -\delta$ where $\delta > 1$

(v) When the object is placed at focus F, a real, inverted and very large image is formed at infinity.



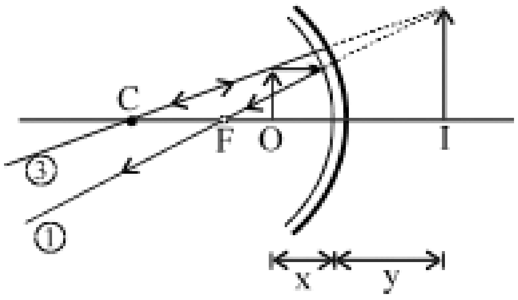
For an object placed at focus, image is formed at infinity

$v = -y$ where $y = \infty$

$m = -\delta$ where $\delta >> 1$

Note virtual object

When the object is placed between F and P, a virtual, erect and enlarged image is formed behind the mirror.



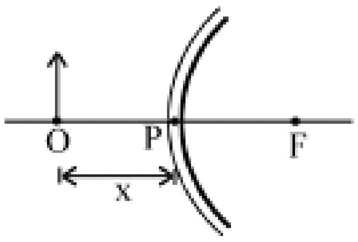
A virtual image is formed for an object placed within focus

$v = +y$

$m = +\delta$ where $\delta > 1$

Convex mirror

The fig. shows a convex mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P.



An object O placed in front of a convex mirror

According to Cartesian sign convention, the formulae may be modified as

$$u = -x \quad \text{and} \quad f = +f_0$$

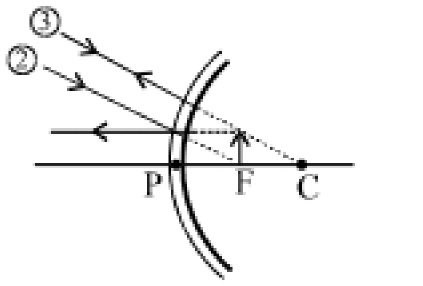
Thus $$v = \frac{xf_0}{f_0 + x}$$

The above expression shows that whatever may be the value of x, v is always positive and its value is always less than or equal to f_0.

The magnification formula may be modified as

$$m = \frac{f_0}{f_0 + x}$$

When the object is placed at infinity, a virtual, erect and very diminished image is formed at the focus.



For a distance object image is formed at the focus

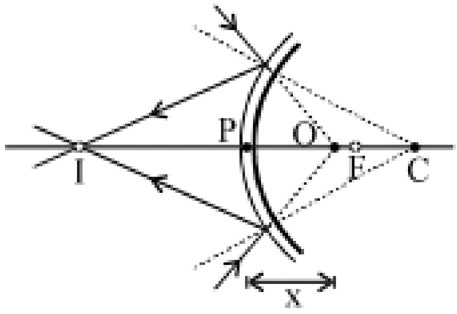
$x = \infty \qquad v = f_0 \qquad m << 1$

Illustration :

Can a convex mirror form real images ?

Sol. *Yes, only when the object is virtual and is placed between F and P.*

The fig. shows a convex mirror exposed to a converging beam which converges to a point that lies between F and P.



A real image formed by convex mirror

$$v = \frac{-xf_0}{f_0 - x}$$

v becomes negative (real image) only when $x < f_0$

FOR CONCAVE MIRROR

Position of object (real)	Position of image	Characteristics of image
At infinity	At. F	Real, inverted, highly diminished
Between infinity and C	Between C and F	Real, inverted, diminished
At C	At C	Real, inverted, same size as that of object
Between C and F	Between infinity and C	Real, inverted, magnified
At F	At infinity	Real, inverted, highly magnified
within F	Behind the mirror	Virtual, erect, magnified

FOR CONVEX MIRROR

Position of object (real)	Position of image	Characteristics of image
At infinity	At F	Virtual, erect, highly diminished
At a finite distance	Between P and F	Virtual, erect, diminished

Transverse or Lateral or linear magnification

It is defined as

$$m_T = \frac{\text{height of image}}{\text{height of object}} = \frac{h_I}{h_O}$$

After using geometry we get

$$m_T = -\frac{v}{u} = \frac{f}{f-u}$$

Note :

Sign of m_T states orientation of image w.r. to object and its magnitude compares size of image with size of object

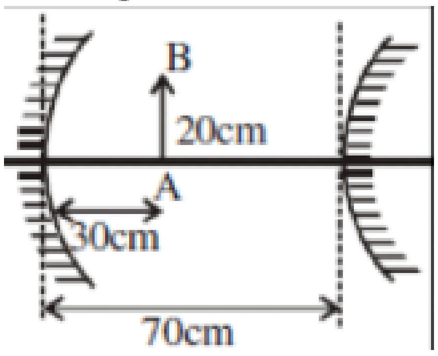
Newtonian formula for mirrors

here distances are measured from focus

$$uv = f^2 \quad \& \quad m_T = -\frac{f}{u}$$

Illustration :

A concave and convex mirror of focal length 10 cm and 15 cm are placed at distance 70 cm. An object *AB* of height 2 cm is placed at distance 30 cm from concave mirror. First ray is incident on concave mirror then on convex mirror. Find size position and nature of image.



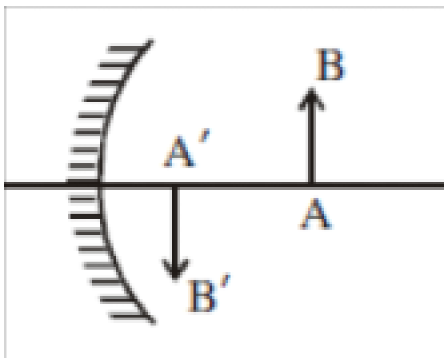
Sol. For concave mirror,

$$u = -30\text{cm}, \quad f = -10\text{cm}$$

Using

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{30} = \frac{-1}{10} \Rightarrow \quad v = -15\ cm$$



Now,

$$\frac{A'B'}{AB} = \frac{-V}{u} = \frac{(-15)}{(-30)} \quad \Rightarrow \quad A'B' = -1\ cm$$

Image formed by first reflection will be real inverted and diminished

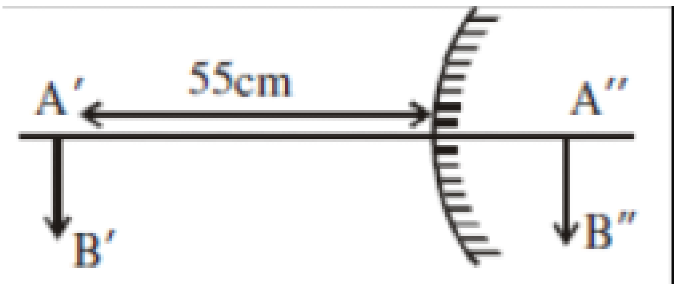
For convex mirror

$$u' = -55\text{cm}, \quad f' = +15\text{cm}$$

Using

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v'} - \frac{1}{55} = \frac{1}{15} \quad \Rightarrow \quad v' = \frac{165}{14}\ cm$$



Now,

$$\frac{A''B''}{A'B'} = -\frac{V'}{u'} = -\frac{\left(+\frac{165}{14}\right)}{(-55)} \Rightarrow \quad A''B'' = \left(-\frac{3}{14}\right)(-1) = -0.2\ cm$$

Image for Longitudinal Object

If an object is placed along principal axis then it is called longitudinal object.

If object is of very small size then longitudinal magnification is defined as.

$$m_L = \frac{length\ of\ image}{length\ of\ image} = \frac{dv}{du}$$

Now using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

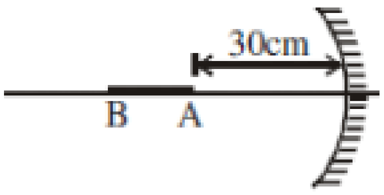
Differentiating w.r.to. u we get

$$\frac{dv}{du}=-\frac{v^2}{u^2}=-m_T^2$$

$$\Rightarrow \qquad m_L=-m_T^2$$

Illustration :

An object AB is placed on the axis of concave mirror of focal length 10 cm end A of the object is at 30 cm from mirror. Find the length of the image ***(a)*** *If length of object is 5 cm* ***(b)*** *If length of object is 1 mm*



Sol. *For point A,* $u=-30\text{cm}, \quad f=-10\text{cm}$

Using $\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \qquad \frac{1}{v'}-\frac{1}{30}=-\frac{1}{10} \quad \Rightarrow \quad v=-15\text{cm}$$

Similarly for point B $\quad u=-35\text{cm}$

Using $\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

we will get $\quad v'=-14\text{cm}$

Now size of image $|A'B'|=|v-v'|=|(-15)-(-14)|=1\text{cm}$

(b) *here* $u=-30\text{cm}, \ f=-10\text{cm}$

Using $\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \qquad \frac{1}{v'}-\frac{1}{30}=-\frac{1}{10} \quad \Rightarrow \quad v=-15\text{cm}$$

Now, $\quad \frac{dv}{du}=-\frac{(-15)^2}{(-30)^2} \quad \Rightarrow \quad |dv|=\frac{(15)^2}{(30)^2}|du|$

$$\Rightarrow \qquad |dv|=\left(\frac{225}{900}\right)(10^{-3})=2.5\times10^{-4}\ m$$

i.e.the length of the image is 2.5×10^{-4} *m.*

Relation between velocity of object and image if object is moving on principal axis:

we can write

velocity of object relative to mirror= $v_{O/M} = \frac{du}{dt}$

velocity of image relative to mirror= $v_{I/M} = \frac{dv}{dt}$

Now using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Differentiating w.r.to. t we get

$$\frac{dv}{dt}=-\frac{v^2}{u^2}\frac{du}{dt}$$

$$\Rightarrow \quad v_{I/M}=m_L v_{O/M}$$

Relation between velocity of object and image if object is moving perpendicular to principal axis:

we can write

velocity of object relative to mirror = $v_{O/Mirror} = \frac{dh_O}{dt}$

velocity of image relative to mirror = $v_{I/Mirror} = \frac{dh_I}{dt}$

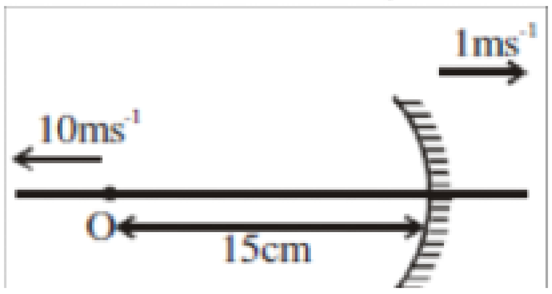
dividing we get

$$\frac{v_{I/Mirror}}{v_{O/Mirror}}=\frac{dh_I}{dh_O}=\frac{h_I}{h_O}=m_T$$

$$\Rightarrow \quad v_{I/Mirror}=m_T v_{O/Mirror}$$

Illustration :

A mirror of radius of curvature 20 cm and an object which is placed at distance 15 cm both are moving with velocity 1 ms^{-1} and 10 ms^{-1} as shown in figure. Find the velocity of image.



Sol. Using

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{15}=-\frac{1}{10} \quad \Rightarrow \quad v=-30\text{cm}$$

Now, using

$$V_{im}=-\frac{v^2}{u^2}V_{OM}$$

$$\Rightarrow \quad (V_i - V_m) = -\frac{v^2}{u^2}(V_0 - V_m)$$

$$\Rightarrow \quad V_i - (1) = -\frac{(-30)^2}{(-15)^2}\left[(-10) - (1)\right]$$

$$\Rightarrow \quad V_i = 45 \text{ cm/s}$$

Practice Exercise

Q.1 A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm. Find the nature and position of image if the point O is (a) 10 cm behind the mirror (b) 30 cm behind the mirror.

Q.2 A plane mirror is placed 22.5 cm in front of a concave mirror of focal length 10 cm. Find where an object can be placed between the two mirrors, so that the first image in both the mirror coincides.

Q.3 A concave mirror of focal length f produces a real image n times the size of the object. What is the distance of the object from the mirror?

Q.4 The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

Q.5 A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole. What is the size of the image?

Q.6 If a luminous point is moving at a speed V_0 towards a spherical mirror, along its axis, show that the speed at which the image of this object is moving will be given by :

$$V_i = \left[\frac{f}{u-f}\right]^2 V_0$$

Answers

Q.1 (a) –20 (b) +60 Q.2 15 cm from the concave mirror Q.3 (n+1)f/n

Q.4 Case I If the image is inverted (i.e., real) : 40 cm in front of the mirror.
Case II If the image is erect (i.e., virtual) : 20 cm in front of the mirror

Q.5 $b\left[\frac{f}{(u-f)}\right]^2$