CHAPTER

8

STATISTICS AND PROBABILITY

I. MEASURES OF DISPERSION

Key Points

✓ Measures of Variation (or) Dispersion of a data provide an idea of how observations spread out (or) scattered throughout the data.

✓ Different Measures of Dispersion are

1. Range 2. Mean deviation 3. Quartile deviation 4. Standard deviation 5. Variance
6. Coefficient of Variation

✓ Range R = L– S

✓ Coefficient of range = $\frac{L-S}{L+S}$ where L - Largest value; S - Smallest value.

✓ The mean of the squares of the deviations from the mean is called Variance. It is denoted by σ^2.

Variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

✓ The positive square root of Variance is called Standard deviation.

Standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$

✓ Formula Table for Standard Deviation (σ).

Data Type	Direct Method	Mean Method	Assumed mean method	Step Deviation method
Ungrouped Data	$\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}$	$\sqrt{\frac{\Sigma d^2}{n}}$	$\sqrt{\frac{\Sigma d^2}{n}-\left(\frac{\Sigma d}{n}\right)^2}$	$\sqrt{\frac{\Sigma d^2}{n}-\left(\frac{\Sigma d}{n}\right)^2}\times C$
Grouped Data	–	$\sqrt{\frac{\Sigma fd^2}{N}}$ $N=\Sigma f$	$\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}$ $N=\Sigma f$	$\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}\times C$ $N=\Sigma f$

- ✓ Standard deviation of first 'n' natural numbers

$$\sigma = \sqrt{\frac{n^2 - 1}{12}}$$

- ✓ The value of SD will not be changed if we add (or) subtract some fixed constant to all the values.
- ✓ When we multiply each value of a data by a constant, the value of SD is also multiplied by the same constant.

Example 8.1

Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution :

Largest value L = 67; Smallest value S =18

Range R= L − S = 67 −18 = 49

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$\text{Coefficient of range} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

Example 8.2

Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution :

Here Largest value L = 28

Smallest value S = 18

Range R = L −S

R = 28 −18 = 10Years.

Example 8.3

The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution :

Range R = 13.67

Largest value L = 70.08

Range R = L −S

13.67 = 70.08 −S

S = 70.08 −13.67 = 56.41

Therefore, the smallest value is 56.41.

Example 8.4

The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

Solution :

x_i	x_i^2
13	169
8	64
4	16
9	81
7	49
12	144
10	100
$\sum x_i = 63$	$\sum x_i^2 = 623$

Standard deviation

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{623}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{89-81} = \sqrt{8}$$

Hence, $\sigma \simeq 2.83$

Example 8.5

The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.

Solution :

Arranging the numbers in ascending order we get, 11.4, 12.5, 12.8, 16.3, 17.8, 19.2. |Number of observations n = 6

$$\text{Mean} = \frac{11.4+12.5+12.8+16.3+17.8+19.2}{6}$$

$$= \frac{90}{6} = 15$$

x_i	$d_i = x_i - \overline{x}$ $= x - 15$	d_i^2
11.4	–3.6	12.96
12.5	–2.5	6.25
12.8	–2.2	4.84
16.3	1.3	1.69
17.8	2.8	7.84
19.2	4.2	17.64
		$\sum d_i^2 = 51.22$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{51.22}{6}} = \sqrt{8.53}$$

Hence, $\sigma \simeq 2.9$

Example 8.6

The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Solution :

The mean of marks is 35.9 which is not an integer. Hence we take assumed mean, A = 35, n = 10.

x_i	$d_i = x_i - A$ $d_i = x_i - 35$	d_i^2
25	–10	100
29	–6	36
30	–5	25
33	–2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
	$\sum d_i = 9$	$\sum d_i^2 = 453$

Example 8.7

The amount that the children have spent for purchasing some eatables in one day trip of a school are 5, 10, 15, 20, 25, 30, 35, 40. Using step deviation method, find the standard deviation of the amount they have spent.

Solution :

We note that all the observations are divisible by 5. Hence we can use the step deviation method. Let the Assumed mean A = 20, n = 8.

x_i	$d_i = x_i - A$ $d_i = x_i - 20$	$d_i = \frac{x_i - A}{c}$ c = 5	d_i^2
5	–15	–3	9
10	–10	–2	4
15	–5	–1	1
20	0	0	0
25	5	1	1
30	10	2	4
35	15	3	9
40	20	4	16
		$\sum d_i = 4$	$\sum d_i^2 = 44$

Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} \times c$$

$$= \sqrt{\frac{44}{8} - \left(\frac{4}{8}\right)^2} \times 5 = \sqrt{\frac{11}{2} - \frac{1}{4}} \times 5$$

$$= \sqrt{5.5 - 0.25} \times 5 = 2.29 \times 5$$

$$\sigma \simeq 11.45$$

Example 8.8

Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

Solution :

Arranging the values in ascending order we get, 4, 7, 8, 10, 11 and n = 5

x_i	x_i^2
4	16
7	49
8	64
10	100
11	121
$\Sigma x_i = 40$	$\Sigma x_i^2 = 350$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{350}{5} - \left(\frac{40}{5}\right)^2}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

When we add 3 to all the values, we get the new values as 7,10,11,13,14.

x_i	x_i^2
7	9
10	100
11	121
13	169
14	196
$\Sigma x_i = 55$	$\Sigma x_i^2 = 635$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{635}{5} - \left(\frac{55}{5}\right)^2}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

From the above, we see that the standard deviation will not change when we add some fixed constant to all the values.

Example 8.9

Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.

Solution :

Given, n = 5

x_i	x_i^2
2	49
3	9
5	25
7	49
8	64
$\Sigma x_i = 25$	$\Sigma x_i^2 = 151$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{151}{5} - \left(\frac{25}{5}\right)^2}$$

$$= \sqrt{30.2 - 25}$$

$$= \sqrt{5.2} \simeq 2.28$$

When we multiply each data by 4, we get the new values as 8, 12, 20, 28, 32.

x_i	x_i^2
8	64
12	144
20	400
28	784
32	1024
$\Sigma x_i = 100$	$\Sigma x_i^2 = 2416$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{2416}{5} - \left(\frac{100}{5}\right)^2}$$

$$= \sqrt{483.2 - 400}$$

$$= \sqrt{83.2}$$

$$\sigma = \sqrt{16 \times 5.2}$$

$$= 4\sqrt{5.2} \simeq 9.12$$

From the above, we see that when we multiply each data by 4 the standard deviation also get multiplied by 4.

Example 8.10

Find the mean and variance of the first n natural numbers.

Solution :

$$\text{Mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+....+n}{n} = \frac{n(n+1)}{2\times n}$$

$$\text{Mean } \bar{x} = \frac{n+1}{2}$$

$$\text{Variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \begin{bmatrix} \Sigma x_i^2 = 1^2+2^2+3^2+....+n^2 \\ (\Sigma x_i)^2 = (1+2+3+....+n)^2 \end{bmatrix}$$

$$= \frac{n(n+1)(2n+1)}{6\times n} - \left[\frac{n(n+1)}{2\times n}\right]^2$$

$$= \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$$

$$\text{Variance } \sigma^2 = \frac{4n^2+6n+2-3n^2-6n-3}{12}$$

$$= \frac{n^2-1}{12}$$

Example 8.11

48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Solution :

x_i	f_i	$x_i f_i$	$d_i = x_i - \bar{x}$	d_i^2	$f_i d_i^2$
6	3	18	–3	9	27
7	6	42	–2	4	24
8	9	72	–1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	N=48	$\Sigma x_i f_i = 432$	$\Sigma d_i = 0$		$\Sigma f_i d_i^2 = 124$

$$\text{Mean } \bar{x} = \frac{\Sigma x_i f_i}{N} - \frac{432}{48} = 9 \text{ (Since } N = \Sigma f_i)$$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{124}{48}} = \sqrt{2.58}$$

$$\sigma \simeq 1.6$$

Example 8.12

The marks scored by the students in a slip test are given below.

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

Solution :

Let the assumed mean, A = 8

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
4	7	– 4	–28	112
6	3	– 2	– 6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
	N=29		$\Sigma f_i d_i = 4$	$\Sigma f_i d_i^2 = 240$

Standard deviation

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$$

$$\sigma = \sqrt{\frac{6944}{29 \times 29}} \; ; \sigma \simeq 2.87$$

Example 8.13

Marks of the students in a particular subject of a class are given below.

Marks	0-10	10-20	20-30	30-40
Number of students	8	12	17	14
Marks	40-50	50-60	60-70	-
Number of students	9	7	4	-

Find its standard deviation.

Solution :

Let the assumed mean, A = 35, c = 10

Marks	Mid value (x_i)	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
0-10	5	8	–30	–3	–24	72
10-20	15	12	–20	–2	–24	48
20-30	25	17	–10	–1	–17	17
30-40	35	14	0	0	0	0
40-50	45	9	10	1	9	9
50-60	55	7	20	2	14	28
60-70	65	4	30	3	12	36
		N=71			$\sum f_i d_i = -30$	$\sum f_i d_i^2 = 210$

$$\text{Standard deviation } \sigma = c \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$\sigma = 10 \times \sqrt{\frac{210}{71} - \left(-\frac{30}{71}\right)^2} = 10 \times \sqrt{\frac{210}{71} - \frac{900}{5041}}$$

$$= 10 \times \sqrt{2.779} \; ; \; \sigma \simeq 16.67$$

Example 8.14

The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23?

Solution :

$n = 15$, $\bar{x} = 10$, $\sigma = 5$;

$$\bar{x} = \frac{\sum x}{n}; \; \sum x = 15 \times 10 = 150$$

Wrong observation value = 8,

Correct observation value = 23.

Correct total = 150 − 8 + 23 = 165

$$\text{Correct mean } \bar{x} = \frac{165}{15} = 11$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\text{Incorrect value of } \sigma = 5 = \sqrt{\frac{\sum x^2}{15} - (10)^2}$$

$$25 = \frac{\sum x^2}{15} - 100 \text{ gives } \frac{\sum x^2}{15} = 125$$

Incorrect value of $\sum x^2 = 1875$

Correct value of $\sum x^2 = 1875 - 8^2 + 23^2 = 2340$

$$\text{Correct standard deviation } \sigma = \sqrt{\frac{2340}{15} - (11)^2}$$

$$\sigma = \sqrt{156 - 121} = \sqrt{35} \qquad \sigma \simeq 5.9$$

EXERCISE 8.1

1. **Find the range and coefficient of range of the following data.**
   (i) 63, 89, 98, 125, 79, 108, 117, 68
   (ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

i) Range $= L - S$

$= 125 - 63$

$= 62$

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{185} = 0.33$$

ii) Range $= L - S$

$= 61.4 - 13.6$

$= 47.8$

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{61.4-13.6}{61.4+13.6} = \frac{47.8}{75} = 0.64$$

2. **If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.**

Solution :

Given ; range = 36.8

Smallest value = 13.4

$\therefore$ R = L – S

$\therefore$ L = R + S

= 36.8 + 13.4

= 50.2

3. **Calculate the range of the following data.**

Income	400-450	450-500	500-550
Number of workers	8	12	30
Income	550-600	600-650	-
Number of workers	21	6	-

Solution :

Here, Largest value = L = 650

Smallest value = S = 450

$\therefore$ Range = L – S

= 650 – 450

= 200

4. **A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.**

Solution :

The pages yet to be completed by them are

60-32, 60-35, 60-37, 60-30, 60-33, 60-36, 60-35, 60-37, 60-37

= 28, 25, 23, 30, 27, 24, 25, 23

To find the SD of the data 28, 25, 23, 30, 27, 24, 25, 23

Arrange them in ascending order

x	$d = x - A$	d^2
23	– 2	4
23	– 2	4
24	– 1	1
25	0	0
25	0	0

27	2	4
28	3	9
30	5	25
	5	47
	$\sum d = 5$	$\sum d^2 = 47$

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{47}{8} - \left(\frac{5}{8}\right)^2}$$

$$= \sqrt{\frac{47}{8} - \frac{25}{64}}$$

$$= \sqrt{\frac{376 - 25}{64}}$$

$$= \frac{\sqrt{351}}{8}$$

$$= \frac{18.74}{8}$$

$$= 2.34$$

5. Find the variance and standard deviation of the wages of 9 workers given below:
₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.

Solution :

Given wages of a workers are ₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280

To find the variance and SD, arrange them in ascending order.

x	$d = \frac{x - 300}{10}$	d^2
280	–2	4
280	–2	4
290	–1	1
290	–1	1
300	0	0
310	1	1
310	1	1
320	2	4
320	2	4
	0	20
	$\sum d = 0$	$\sum d^2 = 20$

$$d = \frac{x - A}{c}$$

A - Assumed Mean   C - Common divisor

$$\sigma^2 = \frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2 \times c^2$$

$$= \frac{20}{9} - 0 \times 100$$

$$= \frac{2000}{9}$$

$$\sigma^2 = 222.2$$

$\therefore$ Variance = 222.2

$$\therefore \quad \text{S.D} = \sqrt{222.2}$$

$$= 14.906$$

$$\simeq 14.91$$

6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Solution :

A clock strikes bell at 1 o' clock once

twice at 2 o' clock,

3 times at 3 o' clock ....

$\therefore$ Number of times it strikes in a particular day

$= 2\,(1 + 2 + 3 + \ldots\ldots\ldots\, 12)$

$= 2\left(\frac{12 \times 13}{2}\right)$

$= 156$ times

To find the S.D of 2 (1, 2, 3, ...12)

$= 2\left[\sqrt{\frac{n^2-1}{12}}\right]$

$= 2\left[\sqrt{\frac{144-1}{12}}\right]$

$= 2\sqrt{\frac{143}{12}} = 2\sqrt{11.91}$

$= 2(3.45)$

$= 6.9$

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7. Find the standard deviation of first 21 natural numbers.

Solution :

SD of first 21 natural numbers

$= \sqrt{\frac{n^2-1}{12}}$

$= \sqrt{\frac{441-1}{12}}$

$= \sqrt{\frac{440}{12}}$

$= \sqrt{36.66}$

$= 6.0547$

$\simeq 6.05$

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8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution :

Given, S.D of a data = 4.5

Since each value is decreased by 5, then the new SD = 4.5

($\because$ S.D will not be changed when we add (or) subtract fixed constant to all the values of the data).

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9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution :

Given, S.D of a data = 3.6

Since each value is divided by 3 then the new S.D = $\frac{3.6}{3}$

$= 1.2$

New Variance = $(1.2)^2$

$= 1.44$

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10. The rainfall recorded in various places of five districts in a week are given below.

Rainfall (in mm)	**45**	**50**	**55**	**60**	**65**	**70**
Number of places	**5**	**13**	**4**	**9**	**5**	**4**

Find its standard deviation.

Solution :

x	f	$d = \frac{x-60}{5}$	d^2	$f \cdot d$	$f \cdot d^2$
45	5	– 3	9	–15	45
50	13	– 2	4	–26	52
55	4	–1	1	–4	4
60	9	0	0	0	0
65	5	1	1	5	5
70	4	2	4	8	16
				–32	122

$\sum f = N = 40, \quad \sum fd = -32 \quad \sum f.d^2 = 122$

$c = 5$

$$\therefore \sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \times c$$

$$= \sqrt{\frac{122}{40} - \left(\frac{-32}{40}\right)^2} \times 5$$

$$= \sqrt{\frac{122}{140} - \frac{1024}{1600}} \times 5$$

$$= \sqrt{\frac{4880 - 1024}{1600}} \times 5$$

$$= \frac{\sqrt{3856}}{40} \times 5$$

$$= \frac{62.096}{8} = 7.76$$

$\therefore$ S.D = 7.76

11. In a study about viral fever, the number of people affected in a town were noted as

Age in years	0-10	10-20	20-30	30-40
Number of people affected	3	5	16	18
Age in years	40-50	50-60	60-70	-
Number of people affected	12	7	4	

Solution :

C.I	mid value (x)	f	$d = \frac{x-35}{10}$	d^2	$f.d$	$f.d^2$
0-10	5	3	–3	9	–9	27
10-20	15	5	–2	4	–10	20
20-30	25	16	–1	1	–16	16
30-40	35-A	18	0	0	0	0
40-50	45	12	1	1	12	12
50-60	55	7	2	4	14	28
60-70	65	4	3	9	12	36
					3	139

$\therefore \sum f = 65$, $\sum fd = 3$, $\sum fd^2 = 139$ and $c = 10$

$$\therefore \sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \times c$$

$$= \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2} \times 10$$

$$= \sqrt{\frac{139}{65} - \frac{9}{65^2}} \times 10$$

$$= \sqrt{\frac{9035 - 9}{65^2}} \times 10$$

$$= \frac{\sqrt{9026}}{65} \times 10$$

$$= \frac{95.005}{65} \times 10$$

$$= 1.46 \times 10$$

$$= 14.6$$

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Solution :

C.I	mid value (x)	f	$d = \frac{x-34.5}{4}$	d^2	$f.d$	$f.d^2$
21-24	22.5	15	–3	9	–45	135
25-28	26.5	18	–2	4	–36	72
29-32	30.5	20	–1	1	–20	20
33-36	34.5	16	0	0	0	0
37-40	38.5	8	1	1	8	8
41-44	42.5	7	2	4	14	28
		84			–79	263

$\therefore \sum f = 94$, $\sum fd = -79$, $\sum fd^2 = 263$, $c = 4$

$$\therefore \sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \times c$$

$$= \sqrt{\frac{263}{84} - \left(\frac{-79}{84}\right)^2} \times 4$$

$$= \sqrt{\frac{263}{84} - \frac{6241}{84^2}} \times 4$$

$$= \sqrt{\frac{22092 - 6241}{84^2}} \times 4$$

$$= \frac{\sqrt{15851}}{84} \times 4$$

$$= \frac{125.9}{21}$$

$$= 5.995$$

$$\simeq 6$$

$$\therefore \sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \frac{64}{50^2}}$$

$$= \sqrt{\frac{3900 - 64}{50^2}}$$

$$= \frac{\sqrt{3836}}{50}$$

$$= \frac{61.935}{65}$$

$$= 1.238$$

$$\simeq 1.24$$

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation

Time taken (sec)	**8.5-9.5**	**9.5-10.5**	**10.5-11.5**	**11.5-12.5**	**12.5-13.5**
Number of students	**6**	**8**	**17**	**10**	**9**

Solution :

C.I	mid value (x)	f	$d = x - 11$	d^2	$f \cdot d$	$f \cdot d^2$
8.5-9.5	9	6	–2	4	–12	24
9.5-10.5	10	8	–1	1	–8	8
10.5-11.5	11	17	0	0	0	0
11.5-12.5	12	10	1	1	10	10
12.5-13.5	13	9	2	4	18	36
		50			8	78

$\therefore \sum f = 50, \sum fd = 8, \sum fd^2 = 78$ and $c = 1$

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

Solution :

Given $n = 100, \bar{x} = 60, \sigma = 15$

$$\therefore \frac{\sum x}{n} = 60$$

$$\Rightarrow \frac{\sum x}{100} = 60$$

$$\Rightarrow \sum x = 6000$$

$\therefore$ Corrected $\sum x = 6000 - (40 + 27) + (45 + 72)$

$= 6000 - 67 + 117$

$= 6050$

$\therefore$ Corrected mean $= \frac{6050}{100}$

$= 60.5$

$$\text{Variance} = \sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$225 = \frac{\sum x^2}{100} - 60^2$$

$$\therefore \frac{\sum x^2}{100} = 3825$$

$$\Rightarrow \sum x^2 = 382500$$

$\therefore$ The correct $\sum x^2 = 382500$

$\therefore$ Corrected $\sum x^2$

= Incorrect $\sum x^2 - 40^2 - 27^2 + 45^2 + 72^2$

= 382500 – 1600 – 729 + 2025 + 5184

= 387380

$$\therefore \text{Corrected } \sigma^2 = \frac{\text{Corrected } \sum x^2}{n} - (\text{Corr. mean})^2$$

$$= \frac{387380}{100} - (60.5)^2$$

$$= 3873.80 - 3660.25$$

$$= 213.55$$

$$\therefore \text{Corrected SD} = \sqrt{213.55}$$

$$= 14.6$$

---

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.

Solution :

Given n = 7, $\bar{x} = 8$, $\sigma^2 = 16$

5 of the observerations are 2, 4, 10, 12, 14

Let the emaining 2 observations be *a, b*.

$$\therefore \bar{x} = 8 \Rightarrow \frac{\sum x}{n} = 8$$

$$\Rightarrow \frac{42 + a + b}{7} = 8$$

$$\Rightarrow a + b = 56 - 42$$

$$\Rightarrow a + b = 14 \qquad \text{....(1)}$$

Also, $\sigma^2 = 16$

$$\Rightarrow \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = 16$$

$$\Rightarrow \frac{\sum x^2}{7} - 8^2 = 16$$

$$\Rightarrow \frac{\sum x^2}{7} = 80$$

$$\Rightarrow \sum x^2 = 560$$

$$\Rightarrow 2^2 + 4^2 + 10^2 + 12^2 + 10^2 + a^2 + b^2 = 560$$

$$\Rightarrow 460 + a^2 + b^2 = 560$$

$$\Rightarrow a^2 + b^2 = 100$$

$$\Rightarrow a^2 + (14 - a)^2 = 100 \qquad \text{(from (1))}$$

$$\Rightarrow a^2 + 196 + a^2 - 28a = 100$$

$$\Rightarrow 2a^2 - 28a + 96 = 0$$

$$\Rightarrow a^2 - 14a + 48 = 0$$

$$\Rightarrow (a - 8)(a - 6) = 0$$

$$a = 8, \qquad a = 6$$

$$\therefore b = 6, \qquad b = 8$$

II. COEFFICIENT OF VARIATION :

Key Points

- ✓ Coefficient of variation, CV = $\frac{\sigma}{\overline{x}} \times 100$.
- ✓ If the C.V value is less, then the observations of $\overline{x}$ corresponding data are consistent.
- ✓ If the C.V value is more, then the observations of corresponding data are inconsistent.

Example 8.15

The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution :

Mean $\overline{x} = 25.6$,

Coefficient of variation, C.V. = 18.75

$$\text{Coefficient of variation, C.V.} = \frac{\sigma}{\overline{x}} \times 100\%$$

$$18.75 = \frac{\sigma}{25.6} \times 100 \quad ; \qquad \sigma = 4.8$$

Example 8.16

The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	Weight
Mean	155 cm	46.50 kg^2
Variance	72.25 cm^2	28.09 kg^2

Which is more varying than the other?

Solution :

For comparing two data, first we have to find their coefficient of variations

Mean $\overline{x}_1 = 155\text{cm}$, variance $\sigma_1^2 = 72.25\text{cm}^2$

Therefore standard deviation $\sigma_1 = 8.5$

$$\text{Coefficient of variantion C.V}_1 = \frac{\sigma_1}{\overline{x}_1} \times 100\%$$

$$\text{C.V}_1 = \frac{8.5}{155} \times 100\% = 5.48\% \qquad \text{(for heights)}$$

Mean $\overline{x}_2 = 46.50\text{kg}$, variance $\sigma_2^2 = 28.09\,\text{kg}^2$

Standard deviation $\sigma_2 = 5.3\text{kg}$

$$\text{Coefficient of variantion C.V}_2 = \frac{\sigma_2}{\overline{x}_2} \times 100\%$$

$$\text{C.V}_2 = \frac{5.3}{46.50} \times 100\% = 11.40\% \quad \text{(for weights)}$$

$\text{C.V}_1 = 5.48\%$ and $\text{C.V}_2 = 11.40\%$

Since $\text{C.V}_2 > \text{C.V}_1$, the weight of the students is more varying than the height.

Example 8.17

The consumption of number of guava and orange on a particular week by a family are given below.

Number of Guavas	3	5	6	4	3	5	4
Number of Oranges	1	3	7	9	2	6	2

Which fruit is consistenly consumed by the family?

Solution :

First we find the coefficient of variation for guavas and oranges separately.

x_i	x_i^2
3	9
5	25
6	36
4	16
3	9
5	25
4	16
$\sum x_i = 30$	$\sum x_i^2 = 136$

Number of guavas, n = 7

$$\text{Mean } \bar{x}_1 = \frac{30}{7} = 4.29$$

$$\text{Standard deviation } \sigma_1 = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\sigma_1 = \sqrt{\frac{136}{7} - \left(\frac{30}{7}\right)^2} = \sqrt{19.43 - 18.40} \simeq 1.01$$

Coefficient of variation for guavas

$$C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\% = \frac{1.01}{4.29} \times 100\% = 23.54\%$$

x_i	x_i^2
1	1
3	9
7	49
9	81
2	4
6	36
2	4
$\sum x_i = 30$	$\sum x_i^2 = 184$

Number of oranges n = 7

$$\text{Mean } \bar{x}_2 = \frac{30}{7} = 4.29$$

$$\text{Standard deviation } \sigma_1 = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\sigma_2 = \sqrt{\frac{184}{7} - \left(\frac{30}{7}\right)^2} = \sqrt{26.29 - 18.40} = 2.81$$

Coefficient of variation for oranges :

$$C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100\% = \frac{2.81}{4.29} \times 100\% = 65.50\%$$

$C.V_1 = 23.54\%$ and $C.V_2 = 65.50\%$

Since $C.V_1 < C.V_2$, we can conclude that the consumption of guavas is more consistent than oranges.

EXERCISE 8.2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution :

Given $\sigma = 6.5, \bar{x} = 12.5$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{6.5}{12.5} \times 100$$

$$= \frac{13/2}{25/2} \times 100$$

$$= 13 \times 4$$

$$= 52\%$$

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution :

Given $\sigma = 1.2$, C.V = 25.6

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$25.6 = \frac{1.2}{\bar{x}} \times 100$$

$$\Rightarrow \bar{x} = \frac{120}{25.6}$$

$$\bar{x} = 4.69$$

3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution :

Given $\bar{x} = 15$, CV = 48, $\sigma = ?$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$\Rightarrow 48 = \frac{\sigma}{15} \times 100$$

$$\sigma = \frac{15 \times 48}{100} = \frac{720}{100} = 7.2$$

4. If n = 5 , $\bar{x}$ = 6 , $\sum x^2$ = 765 , then calculate the coefficient of variation.

Solution :

Given n = 5, $\bar{x} = 6$, $\sum x^2 = 765$, CV = ?

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{765}{5} - (6)^2}$$

$$= \sqrt{\frac{765 - 180}{5}}$$

$$= \sqrt{\frac{585}{5}}$$

$$= \sqrt{117}$$

$$= 10.82$$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{10.82}{6} \times 100$$

$$= \frac{1082}{6}$$

$$= 180.33\%$$

5. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Solution :

Given data is 24, 26, 33, 37, 29, 31.

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$\bar{x} = \frac{24 + 26 + 33 + 37 + 29 + 31}{6}$$

$$= \frac{180}{6}$$

$$= 30$$

To find σ_1 arrange them in ascending order.

x	$d = x - 31$	d^2
24	– 7	49
26	– 5	25
29	– 2	4
31	0	0
33	2	4
37	6	36
	– 6	118

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{118}{6} - \left(\frac{-6}{6}\right)^2}$$

$$= \sqrt{\frac{118}{6} - 1}$$

$$= \sqrt{\frac{112}{6}}$$

$$= \sqrt{18.6}$$

$$\sigma = 4.31$$

$$\therefore C.V = \frac{4.31}{30} \times 100$$

$$= \frac{43.1}{3}$$

$$= 14.36$$

$$\simeq 14.4\%$$

6. **The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.**

Solution :

Given data is 38, 40, 47, 44, 46, 43, 49, 53.

$$\bar{x} = \frac{38+40+47+44+46+43+49+53}{8}$$
$$= \frac{360}{8}$$
$$= 45$$

To find σ, arrange them in ascending order.

x	$d = x - 46$	d^2
38	– 8	64
40	– 6	36
43	– 3	9
44	– 2	4
46	0	0
47	1	1
49	3	9
53	7	49
	– 8	172

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$
$$= \sqrt{\frac{172}{8} - \left(\frac{-8}{8}\right)^2}$$
$$= \sqrt{\frac{172}{8} - 1} = \sqrt{\frac{164}{8}} = \sqrt{20.5} = 4.53$$

$$\therefore C.V = \frac{4.53}{45} \times 100$$
$$= \frac{453}{45}$$
$$= 10.07\%$$

7. **The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?**

Solution :

Sathya	**Vidhya**
$\sum x_1 = 460$	$\sum x_2 = 480$
$n = 5$	$n = 5$
$\therefore \bar{x}_1 = \frac{460}{5}$	$\therefore \bar{x}_2 = \frac{480}{5}$
$= 92$	$= 96$
$\sigma_1 = 4.6$	$\sigma_2 = 2.4$

$$\therefore C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$$
$$= \frac{4.6}{92} \times 100$$
$$= \frac{460}{92}$$
$$= 5$$
$$\therefore C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$$
$$= \frac{2.4}{96} \times 100$$
$$= \frac{240}{96}$$
$$= 2.5$$

∴ $C.V_2 < C.V_1$

∴ Vidhya is more consistent than Sathya.

8. **The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.**

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows highest variation and which shows lowest variation in marks?

Solution :

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

For Maths, C.V = $\frac{12}{56} \times 100 = 21.428$

For Science, C.V = $\frac{14}{65} \times 100 = 21.538$

For Social Science, C.V = $\frac{10}{60} \times 100 = 16.67$

Highest variation in Science.

Lowest variation in Social Science.

9. The temperature of two cities A and B in a winter season are given below.

Temperature of city A (in degree Celsius)	18	20	22	24	26
Temperature of city B (in degree Celsius)	11	14	15	17	18

Find which city is more consistent in temperature changes?

Solution :

Temperature of City 'A' :
18, 20, 22, 24, 26

$$\bar{x} = \frac{110}{5} = 22$$

x	$d = \frac{x-22}{2}$	d^2
18	– 2	4
20	– 1	1
22	0	0
24	1	1
26	2	4
	0	10

$$\therefore \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \times C$$

$$= \sqrt{\frac{10}{5} - 0} \times 2$$

$$= 2\sqrt{2}$$

$$\therefore \text{CV for city A} = \frac{\sigma}{x} \times 100$$

$$= \frac{2\sqrt{2}}{22} \times 100$$

$$= \frac{100 \times 1.414}{22}$$

$$= 6.427$$

Temperature of Ciy B 11, 14, 15, 17, 18

$$\bar{x} = \frac{75}{5} = 15$$

x	$d = x - 15$	d^2
11	– 4	16
14	– 1	1
15	0	0
17	2	4
18	3	9
	0	30

$$\therefore \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \times C$$

$$= \sqrt{\frac{30}{5} - 0}$$

$$= \sqrt{6}$$

$\therefore$ CV for city B $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{\sqrt{6}}{15} \times 100$

$= \frac{2.45}{15} \times 100$

$= 16.34$

$\therefore$ CV for City A < CV for City B.

$\therefore$ City A is more consistent in temperature changes.

III. PROBABILITY :

Key Points

- ✓ A **random experiment** is an experiment in which

  (i) The set of all possible outcomes are known (ii)Exact outcome is not known.
- ✓ The set of all possible outcomes in a random experiment is called a **sample space.** It is generally denoted by S.
- ✓ Each element of a sample space is called a **sample point.**
- ✓ In a random experiment, each possible outcome is called an **event.**
- ✓ An event will be a subset of the sample space.
- ✓ If an event E consists of only one outcome then it is called an **elementary event.**
- ✓ $P(E) = \frac{n(E)}{n(S)}$
- ✓ $P(S) = \frac{n(S)}{n(S)} = 1$. The probability of sure event is 1.
- ✓ $P(\phi) = \frac{n(\phi)}{n(s)} = \frac{0}{n(s)} = 0$. The probability of impossible event is 0.
- ✓ E is a subset of S and ϕ is a subset of any set.

  $\phi \subseteq E \subseteq S$ $\quad$ $P(\phi) \leq P(E) \leq P(S)$ $\quad$ $0 \leq P(E) \leq 1$
- ✓ The complement event of E is $\overline{E}$.
- ✓ $P(E) + P(\overline{E}) = 1$.

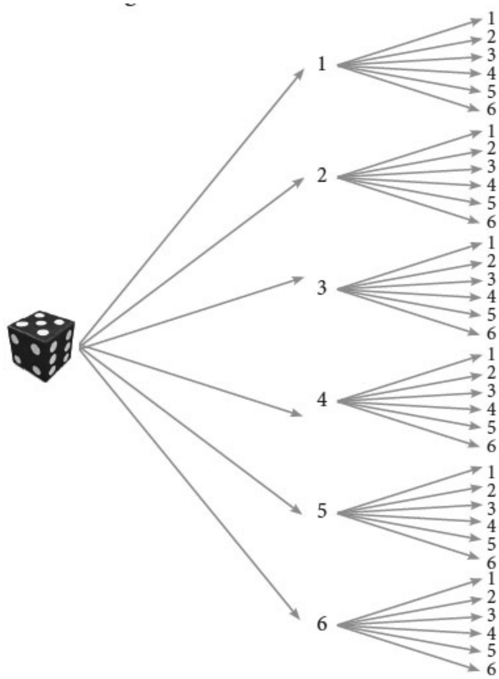
Example 8.18

Express the sample space for rolling two dice using tree diagram.

Solution :

When we roll two dice, since each die contain 6 faces marked with 1,2,3,4,5,6 the tree diagram will look like

Hence, the sample space can be written as



S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Example 8.19

A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution :

Total number of possible outcomes

n(S) = 5 + 4 = 9

(i) Let A be the event of getting a blue ball.

Number of favourable outcomes for the event A. Therefore, n(A) = 5

Hence we get, y = x +11 gives

$x - y + 11 = 0$.

Probability that the ball drawn is blue.

Therefore, P(A) = $\frac{n(A)}{n(S)} = \frac{5}{9}$

(ii) $\overline{A}$ will be the event of not getting a blue ball.

So $P(\overline{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$

Example 8.20

Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

Solution :

When we roll two dice, the sample space is given by

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) };

n (S) = 36

(i) Let A be the event of getting the sum of outcome values equal to 4.

Then A = {(1,3),(2,2),(3,1)}; n (A) = 3 .

Probability of getting the sum of outcomes equal to 4 is

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(ii) Let B be the event of getting the sum of outcome values greater than 10.

Then B = {(5,6),(6,5),(6,6)}; n (B) = 3

Probability of getting the sum of outcomes greater than 10 is

$$P(B)=\frac{n(B)}{n(S)}=\frac{3}{36}=\frac{1}{12}$$

(iii) Let C be the event of getting the sum of outcomes less than 13. Here all the outcomes have the sum value less than 13. Hence C = S .

Therefore, n(C) = n(S) = 36

Probability of getting the total value less than 13 is

$$P(C)=\frac{n(C)}{n(S)}=\frac{36}{36}=1$$

Example 8.21

Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution :

When two coins are tossed together, the sample space is

S = {HH,HT,TH,TT}; n(S) = 4

Let A be the event of getting different faces on the coins.

A = {HT,TH}; n(A) = 2

Probability of getting different faces on the coins is

$$P(A)=\frac{n(A)}{n(S)}=\frac{2}{4}=\frac{1}{2}$$

Example 8.22

From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Solution :

n (S) = 52

(i) Let A be the event of getting a red card.

n(A) = 26

Probability of getting a red card is

$$P(A)=\frac{26}{52}=\frac{1}{2}$$

(ii) Let B be the event of getting a heart card.

n (B) = 13

Probability of getting a heart card is

$$P(B)=\frac{n(B)}{n(S)}=\frac{13}{52}=\frac{1}{4}$$

(iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king.

n(C) = 2

Probability of getting a red king card is

$$P(C)=\frac{n(C)}{n(S)}=\frac{2}{52}=\frac{1}{26}$$

(iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen (Q), and King (K).

n(D) = 4 × 3 = 12

Probability of getting a face card is

$$P(D)=\frac{n(D)}{n(S)}=\frac{12}{52}=\frac{3}{13}$$

(v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10.

n(E) = 4 × 9 = 36

Probability of getting a number card is

$$P(E)=\frac{n(E)}{n(S)}=\frac{36}{52}=\frac{9}{13}$$

Example 8.23

What is the probability that a leap year selected at random will contain 53 saturdays.

(Hint: $366 = 52\times7 + 2$)

Solution :

A leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

The possible chances for the remaining two days will be the sample space.

S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)}

$$n(S) = 7$$

Let A be the event of getting 53^{rd} Saturday.

Then A = {Fri-Sat, Sat-Sun}; n(A) = 2

Probability of getting 53 Saturdays in a leap year is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

Example 8.24

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution :

Sample space

S = {1H,1T,2H,2T,3H,3T,4H,4T,5H,5T,6H,6T};

n (S) = 12

Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H}; n (A)= 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

Example 8.25

A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution :

Number of green balls is n (G) = 6

Let number of red balls is n (R) = x

Therefore, number of black balls is n (B) = $2x$

Total number of balls n(S) = $6 + x + 2x = 6 + 3x$

It is given that, P(G) = 3 × P(R)

$$\frac{6}{6+3x} = 3\times\frac{x}{6+3x}$$

$3x = 6$ gives, $x = 2$.

(i) Number of black balls = 2×2 = 4

(ii) Total number of balls = 6 +(3×2) = 12

Example 8.26

A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, …12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

Solution :

Sample space

S = {1,2,3,4,5,6,7,8,9,10,11,12}; n(S) = 12

(i) Let A be the event of resting in 7. n(A)=1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) Let B be the event that the arrow will come to rest in a prime number.

B = {2,3,5,7,11}; n (B) = 5

$$P(B)=\frac{n(B)}{n(S)}=\frac{5}{12}$$

(iii) Let C be the event that arrow will come to rest in a composite number.

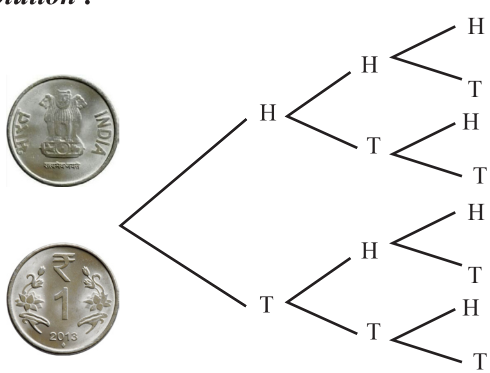
C = {4,6,8,9,10,12}; n (C)=6

$$P(C)=\frac{n(C)}{n(S)}=\frac{6}{12}=\frac{1}{2}$$

EXERCISE 8.3

1. Write the sample space for tossing three coins using tree diagram.

Solution :

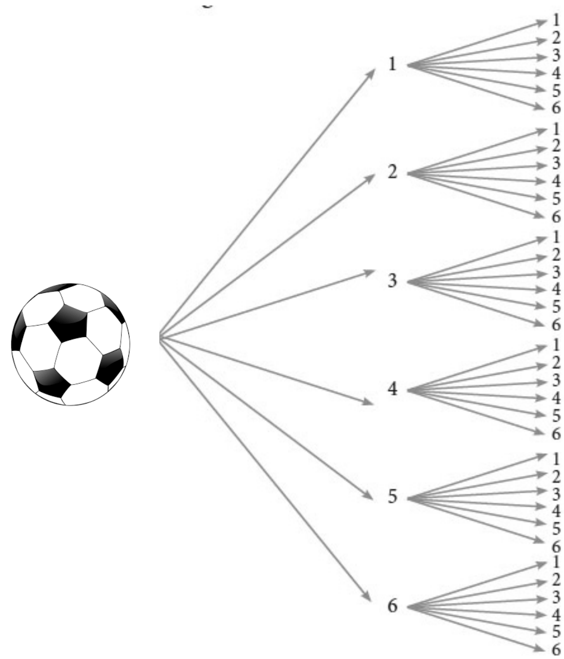


Sample space = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

2. Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).

Solution :

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}



3. If A is an event of a random experiment such that P(A) : P $(\overline{A})$=17:15 and n(S) = 640 then find (i) P $(\overline{A})$ (ii) n(A).

Solution :

Given P(A) : P$(\overline{A})$ = 17 : 15

$$\Rightarrow \quad \frac{1-P(\overline{A})}{P(A)}=\frac{17}{15}$$

$$\Rightarrow 15-15P(A)=17P(\overline{A})$$

$$\Rightarrow \quad 32P(\overline{A})=15$$

$$\Rightarrow \quad P(A)=\frac{17}{32}$$

$$\Rightarrow \quad \frac{n(A)}{n(S)}=\frac{17}{32}$$

$$\Rightarrow \quad n(A)=\frac{17}{32}\times 640=340$$

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution :

When a coin is tossed thrice,

S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

n(S) = 8

Let A bet the event of getting 2 tails continuously,

A = {(HTT), (TTH), (TTT)}

n(A) = 3

$$P(A)=\frac{n(A)}{n(S)}=\frac{3}{8}$$

---

5. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?

Solution :

n(S) = 1000

i) Let A be the event of getting perfect squares between 500 and 1000

A = {23^2, 24^2, 25^2, 26^2 ...... 31^2}

n(A) = 9

$$P(A)=\frac{9}{1000}$$

is the probablity for the 1st player to win a prize.

ii) When the card which was taken first is not replaced.

n(S) = 999

n(B) = 8

$$P(B)=\frac{8}{999}$$

---

6. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find *x*.

Solution :

Total number of balls in the bag

= x + 12. ($x \rightarrow$ red 12 $\rightarrow$ black)

i) Let A be the event of getting red balls

$$P(A)=\frac{n(A)}{n(S)}=\frac{x}{x+12}$$

ii) If 8 more red balls are added in the bag.

n(S) = x + 20

By the problem, $\frac{x+8}{x+20}=2\left(\frac{x}{x+12}\right)$

$$\Rightarrow (x+8)(x+12)=2x^2+40x$$

$$\Rightarrow x^2+20x+96=2x^2+40x$$

$$\Rightarrow x^2+20x-96=0$$

$$\Rightarrow (x+24)(x-4)=0$$

$$\therefore x=-24,\ 4$$

$$\therefore x=4$$

$$\therefore P(A)=\frac{4}{16}=\frac{1}{4}$$

---

7. Two unbiased dice are rolled once. Find the probability of getting

(i) a doublet (equal numbers on both dice)

(ii) the product as a prime number

(iii) the sum as a prime number

(iv) the sum as 1

Solution :

S ={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

i) Let A be the event of getting a doublet

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

n(A) = 6

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

ii) Let B the event of getting the product as a prime number.

B = {(1, 2), (1, 3), (1, 5), (2, 1), (3, 1), (5, 1)}

n(B) = 6

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

iii) Let C be the event of getting the sum of numbers on the dice is prime.

C = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

n(C) = 14

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{7}{36}$$

iv) Let D be the event of getting sum of numbers is 1.

n(D) = 0

P(D) = 0

8. Three fair coins are tossed together. Find the probability of getting

(i) all heads (ii) atleast one tail

(iii) atmost one head (iv) atmost two tails

Solution :

When 3 fair coins are tossed,

S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

n(S) = 8

i) Let A be the event of getting all heads.

A = {(HHH)}

n(A) = 1

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) Let B be the event of getting atleast one tail.

B = {(HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

n(B) = 7

$$P(B) = \frac{7}{8}$$

iii) Let C be the event of getting at most one head.

C = {(HTT), (THT), (TTH), (TTT)}

n(C) = 4

$$P(C) = \frac{4}{8} = \frac{1}{2}$$

iv) Let D - atmost 2 tails

D = {(HHH), (HHT), (HTT), (HTH), (THH), (THT), (TTH)}

n(D) = 7

$$P(D) = \frac{7}{8}$$

9. Two dice are numbered 1,2,3,4,5,6 and 1,1,2,2,3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Solution :

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

i) Let A - Sum of 2

n(A) = 2

$$\therefore P(A)=\frac{2}{36}$$

ii) Let B - Sum of 3

n(B) = 4

$$P(B)=\frac{4}{36}$$

iii) Let C - Sum of 4

n(C) = 6

$$P(C)=\frac{6}{36}$$

iv) Let D - Sum of 5

n(D) = 6

$$P(D)=\frac{6}{36}$$

v) Let E - Sum of 6

n(E) = 6

$$P(E)=\frac{6}{36}$$

vi) Let F - Sum of 7

n(F) = 6

$$P(F)=\frac{6}{36}$$

vii) Let G - Sum of 8

n(G) = 4

$$P(G)=\frac{4}{36}$$

viii) Let H - Sum of 9

n(H) = 2

$$P(H)=\frac{2}{36}$$

$$P(H)=\frac{2}{36}$$

---

10. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black

Solution :

S = {5R, 6W, 7G, 8B}

i) Let A - White ball

n(A) = 6

$$P(A)=\frac{6}{26}=\frac{3}{13}$$

ii) Let B - Black (or) red

n(B) = 5 + 8 = 13

$$P(B)=\frac{13}{26}=\frac{1}{2}$$

iii) Let C - not white

n(C) = 20

$$P(C)=\frac{20}{26}=\frac{10}{13}$$

iv) Let D - Neither white nor black

n(D) = 12

$$P(D)=\frac{12}{26}=\frac{6}{13}$$

11. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Solution :

Let x be the number of defective bulbs.

$\therefore$ n(S) = x + 20

Let A be the event of selecting defective balls

$\therefore$ n(A) = x

$$P(A) = \frac{x}{x+20}$$

$$\text{Given } \frac{x}{x+20} = \frac{3}{8}$$

$\Rightarrow$ $8x = 3x + 60$

$\Rightarrow$ $5x = 60$

$x = 12$

$\therefore$ Number of defective balls = 12.

12. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Solution :

Solution :

By the data given,

n(S) = 52 – 2 – 2 – 2 = 46

i) Let A be the event of selecting clubber card.

n(A) = 13

$$P(A) = \frac{13}{46}$$

ii) Let B - queen of red card.

n(B) = 0

P(B) = 0

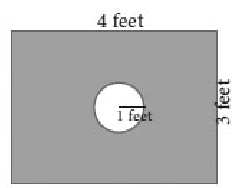
(queen diamond and heart are included in S)

iii) Let C - King of black cards

n(C) = 1 (encluding spade king)

$$\therefore P(C) = \frac{1}{46}$$

13. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



Solution :

Area of the rectangular region $= 4 \times 3$

$= 12\text{ft}^2$

Area of the circular region $= \pi r^2$

$= \pi \times 1^2$

$= \pi \text{ft}^2$

$$\therefore \text{Probability to win the game} = \frac{\pi}{12} = \frac{3.14}{12} = \frac{314}{1200} = \frac{157}{600}$$

14. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on

(i) the same day (ii) different days

(iii) consecutive days?

Solution :

Given n(S) = 6. (Monday - Saturday)

i) Prob. that both of them will visit the shop on the same day = $\frac{1}{6}$

ii) Prob. that both of them will visit the shop in different days = $\frac{5}{6}$.

($\because$ if one visits on Monday, other one visit the shop out of remaining 5 days).

iii) Prob. that both of them will visit the shop in consecutive days.

A = {(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat)}

n(A) = 5

$$P(A) = \frac{5}{6}$$

15. In a game, the entry fee is □150. Th e game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Solution :

S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (HTT), (TTT)}

n(S) = 8

i) P (gets double entry fee) = $\frac{1}{8}$ ($\because$ 3 heads)

ii) P (just gets for her entry fee) = $\frac{6}{8} = \frac{3}{4}$

($\because$ 1 (or) 2 heads)

iii) P (loses the entry fee) = $\frac{1}{8}$

($\because$ 3 no heads (TTT) only)

IV. ALGEBRA OF EVENTS:

Key Points

- ✓ $A \cap \overline{A} = \phi \quad A \cup \overline{A} = S$
- ✓ If A, B are mututally exclusive events, the $P(A \cup B) = P(A) + P(B)$.
- ✓ P (Union of mutually exclusive events) = $\sum$ (Probability of events)

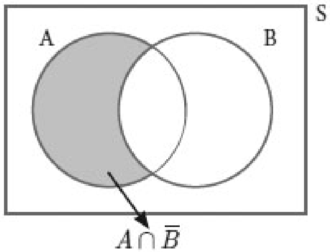
Theorem 1

If A and B are two events associated with a random experiment, then prove that

(i) $P(A \cap \overline{B})$ = P(only A) = $P(A) - P(A \cap B)$

(ii) $P(\overline{A} \cap B)$ = P(only B) = $P(B) - P(A \cap B)$

Proof



(i) By Distributive property of sets,

1. $(A\cap B)\cup(A\cap\overline{B}) = A\cap(B\cup\overline{B}) = A\cap S = A$

2. $(A\cap B)\cap(A\cap\overline{B}) = A\cap(B\cap\overline{B}) = A\cap\phi = \phi$

Therefore, $P(A) = P[(A\cap B)\cup(A\cap B)]$

$P(A) = P(A\cap B) + P(A\cap\overline{B})$

Therefore, $P(A\cap\overline{B}) = P(A) - P(A\cap B)$

That is, $P(A\cap\overline{B})$ = P (oly A) $= P(A) - P(A\cap B)$

(ii) By Distributive property of sets,

1. $(A\cap B) \cup (\overline{A}\cap B) = (A\cup\overline{A}) \cap B = S\cap B = B$

2. $(A\cap B) \cap (\overline{A}\cap B) = (A\cap\overline{A}) \cap B = \phi\cap B = \phi$

Therefore, the events $A\cap B$ and $\overline{A}\cap B$ are mutually exclusive whose union is B.

$P(B) = P[(A\cap B)\cup(\overline{A}\cap B)]$

$P(B) = P(A\cap B) + P(\overline{A}\cap B)$

Therefore, $P(\overline{A}\cap B) = P(B) - P(A\cap B)$

That is, $P(\overline{A}\cap B)$ = P (only B) $= P(B) - P(A\cap B)$

Theorem 2

(i) If A and B are any two events then

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

(ii) If A, B and C are any three events then

$P(A\cup B\cup C) = P(A) + P(B) + P(C)$

$- P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

Proof

(i) Let A and B be any two events of a random experiment with sample space S.

From the Venn diagram, we have the events only A, $A \cap B$ and only B are mutually exclusive and their union is $A \cup B$

Therefore,

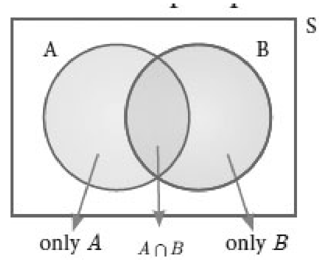
$P(A \cup B) = P$ [(only A) $\cup (A\cap B) \cup$ (only B)]

= P (only A) + $P(A\cap B)$ + P (only B)

$= [P(A)-P(A\cap B)]+P(A\cap B) + [P(B) - P(A\cap B)]$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

(ii) Let A, B, C are any three events of a random experiment with sample space S.



Let $D = B\cup C$

$P(A\cup B\cup C) = P(A\cup D)$

$= P(A) + P(D) - P(A\cap D)$

$= P(A) + P(B \cup C) - P[A\cap(B \cup C)]$

$=P(A) + P(B) + P(C) - P(B\cap C) -$

$P[(A\cap B)\cup(A\cap C)]$

$= P(A) + P(B) + P(C) - P(B\cap C) - P[(A\cap B)$

$- P(A\cap C) + P[(A\cap B)\cap(A\cap C)]$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

Example 8.27

If P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09 then find P(A∪B) .

Solution :

P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09

P(A∪B) = P(A)+P(B)−P(A∩B)

P(A∪B) = 0.37 + 0.42 − 0.09 = 0.7

Example 8.28

What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution :

Total number of cards = 52
Number of king cards = 4

Probability of drawing a king card = $\frac{4}{52}$
Number of queen cards = 4

Probability of drawing a queen card = $\frac{4}{52}$

Both the events of drawing a king and a queen are mutually exclusive

⇒ P(A∪B) = P(A)+P(B)

Therefore, probability of drawing either a king or a queen = $\frac{4}{52}+\frac{4}{52}=\frac{2}{13}$

Example 8.29

Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution :

When two dice are rolled together, there will be 6×6 = 36 outcomes. Let S be the sample space. Then n(S) = 36

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then A = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

B = {(1,3),(2,2),(3,1)}

Therefore, A ∩ B = {(2,2)}

Then, n(A) = 6 , n(B) = 3 , n(A∩B) = 1 .

$$P(A)=\frac{n(A)}{n(S)}=\frac{6}{36}$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{3}{36}$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{1}{36}$$

Therefore,

P (getting a doublet or a total of 4) = P(A∪B)

P(A∪B) = P(A) + P(B) – P(A∩B)

$$=\frac{6}{36}+\frac{3}{36}-\frac{1}{36}=\frac{8}{36}=\frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$

Example 8.30

If A and B are two events such that P(A)=$\frac{1}{4}$, P(B) = $\frac{1}{2}$ and P (A and B) = $\frac{1}{8}$, find

(i) P(A or B) (ii) P (not A and not B).

Solution :

(i) P(A or B) = P(A∪B)

= P(A) + P(B) – P(A∩B)

$$P(A \text{ or } B)=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$$

(ii) P (not A and not B) = $P(\overline{A}\cap\overline{B})$

= $P(\overline{A\cup B})$

= 1 – P(A∪B)

P (not A and not B) = 1 – $\frac{5}{8}=\frac{3}{8}$

Example 8.31

A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution :

Total number of cards = 52; n (S) = 52

Let A be the event of getting a king card.

n(A) = 4

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a heart card.

n(B) =13

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Let C be the event of getting a red card.

n (C) = 26

$$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$$

$P(A\cap B)$ = P (getting heart king) = $\frac{1}{52}$

$P(B\cap C)$ = P (getting red and heart) = $\frac{13}{52}$

$P(A\cap C)$ = P (getting red king) = $\frac{2}{52}$

$P(A\cap B\cap C)$=P (getting heart, king which is red)

$$= \frac{1}{52}$$

Therefore, required probability is

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$

Example 8.32

In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Solution:

Total number of students n(S)= 50.

Let A and B be the events of students opted for NCC and NSS respectively.

$n(A) = 28$, $n(B) = 30$, $n(A\cap B) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$$

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{18}{50}$$

(i) Probability of the students opted for NCC but not NSS

$P(A\cap\overline{B}) = P(A) - P(A\cap B) = \frac{28}{50} - \frac{18}{50} = \frac{1}{5}$

(ii) Probability of the students opted for NSS but not NCC.

$P(A\cap\overline{B}) = P(B) - P(A\cap B) = \frac{30}{50} - \frac{18}{50} = \frac{6}{25}$

(iii) Probability of the students opted for exactly one of them

$= P[(A\cap\overline{B})\cup(\overline{A}\cap B)]$

$= P(A\cap\overline{B}) + P(\overline{A}\cap B) = \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$

(Note that $(A\cap\overline{B})$, $(\overline{A}\cap B)$ are mutually exclusive events)

Example 8.33

A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

Solution:

$P(A) = 0.5$, $P(A \cap B) = 0.3$

We have $P(A \cup B) \le 1$

$$P(A) + P(B) - P(A \cap B) \le 1$$

$$0.5 + P(B) - 0.3 \le 1$$

$$P(B) \le 1 - 0.2$$

$$P(B) \le 0.8$$

Therefore, probability of B getting selected is atmost 0.8.

EXERCISE 8.4

1. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$ then find $P(A \cap B)$.

Solution :

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{10 + 6 - 5}{15}$$

$$= \frac{11}{15}$$

2. A and B are two events such that, P(A) = 0.42, P(B) = 0.48 , and $P(A \cap B) = 0.16$. Find (i) P,(not A) (ii) P,(not B) (iii) P,(A or B)

Solution :

a) $P(\text{not } A) = P(\overline{A}) = 1 - P(A)$

$= 1 - 0.42$

$= 0.58$

b) $P(\text{not } B) = P(\overline{B}) = 1 - P(B)$

$= 1 - 0.48$

$= 0.52$

c) $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) + P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.74$

3. If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, $P(A \cup B)$=0.65, then find P(B).

Solution :

Given A and B are mutually exclusive events

$P(A \cap B) = 0$

Also, $P(\text{not } A) = 0.45$

$\therefore P(\overline{A}) = 0.45$

$1 - P(A) = 0.45$

$P(A) = 0.55$

$P(A \cup B) = P(A) + P(B)$

$\therefore P(B) = P(A \cup B) - P(A)$

$= 0.65 - 0.55$

$= 0.10$

4. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\overline{A}) + P(\overline{B})$.

Solution :

Given $P(A \cup B) = 0.6$, $P(A \cap B) = 0.2$

$\therefore\ P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow 0.6 = P(A) + P(B) - 0.2$

$\therefore\ P(A) + P(B) = 0.8$

$\therefore\ P(\overline{A}) + P(\overline{B})$

$= 1 - P(A) + 1 - P(B)$

$= 2 - (P(A) + P(B))$

$= 2 - 0.8$

$= 1.2$

5. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

Solution :

Given $P(A) = 0.5$, $P(B) = 0.3$, $P(A\cap B) = 0$

P (neither A nor B)

$= P(\overline{A}\cap\overline{B})$

$= P\,(\overline{A\cup B})$

$= 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

$= 1 - (0.8)$

$= 0.2$

6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution :

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

$n(S) = 36$

Let A be the event of getting even number on the 1st die.

A = {(2, 1),(2, 2),(2, 3),(2, 4),(2, 5),(2, 6)
(4, 1),(4, 2),(4, 3),(4, 4),(4, 5),(4, 6)
(6, 1),(6, 2),(6, 3),(6, 4),(6, 5),(6, 6)}

$n(A) = 18$

$P(A) = \dfrac{18}{36}$

Let B - Total of face sum as 8.

B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

$n(B) = 5,\ P(B) = \dfrac{5}{36}$

$A\cap B$ = {(2, 6), (4, 4), (6, 2)}

$n(A\cap B) = 3$

$P(A\cap B) = \dfrac{3}{36}$

$\therefore\ P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \dfrac{18}{36} + \dfrac{5}{36} - \dfrac{3}{36}$

$= \dfrac{20}{36}$

$= \dfrac{5}{9}$

7. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Solution :

$n(S) = 52$

Let A - Red King

$n(A) = 2$

$P(A) = \dfrac{2}{52}$

Let B - Black Queen

$n(B) = 2$

$P(B) = \dfrac{2}{52}$

Here A and B are mutually enclusive

$\therefore P(A \cup B) = P(A) + P(B)$

$= \dfrac{4}{52}$

$= \dfrac{1}{13}$

---

8. A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution :

$S = \{3, 5, 7, 9, \ldots\ldots\ldots\ldots 35, 37\}$

$n(S) = 18$

Let A - multiple of 7.

$A = \{7, 14, 21, 28, 35\}$

$n(A) = 5$

$P(A) = \dfrac{5}{18}$

Let B - a prime number

$B = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

$n(B) = 11$

$P(B) = \dfrac{11}{18}$

Here $A \cap B = \{7\}$

$n(A \cap B) = 1$

$P(A \cap B) = \dfrac{1}{18}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$= \dfrac{5}{18} + \dfrac{11}{18} - \dfrac{1}{18}$

$= \dfrac{15}{18}$

$= \dfrac{5}{6}$

---

9. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution :

S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}

$n(S) = 8$

Let A - at most 2 tails

A = {(HHT), (HTH), (THH), (HTT), (THT), (TTH), (HHH)}

$n(A) = 7$

$P(A) = \dfrac{7}{8}$

Let B - atleast 2 heads

B = {(HHH), (HHT), (HTH), (THH)}

$n(B) = 4$

$P(B) = \dfrac{4}{8}$

$\therefore A \cap B$ = {(HHH), (HHT), (HTH), (THH)}

$n(A \cap B) = 4, P(A \cap B) = \dfrac{4}{8}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{7}{8} + \dfrac{4}{8} - \dfrac{4}{8}$

$= \dfrac{7}{8}$

10. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?

Solution :

Let A - electrification contract

B - not plumbing contract

Given

$$P(\mathrm{A}) = \frac{3}{5}, P(\overline{\mathrm{B}}) = \frac{5}{8}, P(A \cup B) = \frac{5}{7}$$

$$\Rightarrow P(B) = 1 - \frac{5}{8}$$

$$= \frac{3}{8}$$

$$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{3}{5} + \frac{3}{8} - \frac{5}{7}$$

$$= \frac{168 + 105 - 200}{280}$$

$$= \frac{73}{280}$$

11. In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?

Solution :

Let A - Female

B - Over 50 years

Given n(S) = 8000, n(A) = 3000,

n(B) = 1300 and $n(A \cap B) = \frac{30}{100} \times 3000 = 900$

$$\therefore P(A) = \frac{3000}{8000},\ P(B) = \frac{1300}{8000},\ P(A \cap B) = \frac{900}{8000}$$

$\therefore$ P (either a female (or) over 50 years)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3000 + 1300 - 900}{8000}$$

$$= \frac{3400}{8000}$$

$$= \frac{34}{80}$$

$$= \frac{17}{40}$$

---

12. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution :

S = {(HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)}

n(S) = 8

Let A - exactly 2 heads

A = {(HHT), (HTH), (THH)}

n(A) = 3

$P(A) = \frac{3}{8}$

Let B - atleast one tail

B = {(HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)}

n(B) = 7

$P(B) = \frac{7}{8}$

Let C - Consecutively 2 heads

C = {(HHH), (HHT), (THH)}

n(C) = 3

$P(C) = \frac{3}{8}$

$A \cap B = \{(HHT), (HTH), (THH)\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{8}$

$B \cap C = \{(HHT), (THH)\}$

$n(B \cap C) = 2$

$P(B \cap C) = \frac{2}{8}$

$C \cap A = \{(HHT), (THH)\}$

$n(C \cap A) = 2$

$P(C \cap A) = \frac{2}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$$

$$= \frac{8}{8} = 1$$

13. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$, $P(A\cup B\cup C) = \frac{9}{10}$, $P(A\cap B\cap C) = \frac{1}{15}$, then find P(A),P(B) and P(C) ?

Solution :

Given P(B) = 2. P(A), P(C) = 3 . P(A)

$P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$,

$P(A\cup B\cup C) = \frac{9}{10}$, $P(A\cap B\cap C) = \frac{1}{15}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$\Rightarrow \frac{9}{10} = P(A) + 2.P(A) + 3.P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$\Rightarrow 6\ .\ P(A) = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15}$$

$$\Rightarrow 6.P(A) = \frac{108 + 20 + 30 + 15 - 8}{120}$$

$$\Rightarrow 6.P(A) = \frac{165}{120}$$

$$\Rightarrow P(A) = \frac{165}{720} = \frac{11}{48}$$

$$\therefore P(A) = \frac{11}{48}$$

$$\therefore P(B) = 2\ .\ P(A) = 2 \times \frac{11}{48} = \frac{11}{24}$$

$$P(C) = 3\ .\ P(A) = 3 \times \frac{11}{48} = \frac{11}{16}$$

14. In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4:3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.

Solution :

Given n(S) = 35 and ratio of boys and girls=4:3

No. of boys = $\frac{4}{7} \times 35 = 20$

No. of boys = $\frac{3}{7} \times 35 = 15$

Let A - a boy with prime roll no

A = {2, 3, 5, 7, 11, 13, 19} ($\because$ only 20 boys)

n(A) = 7

$P(A) = \frac{7}{35}$

Let B - a girl with composite roll no.
B={21,22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35}
n(B) = 12

$\therefore$ P(B) = $\frac{12}{25}$

Let C - even roll no.
B={2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34}
n(C) = 17

$\therefore$ P(C) = $\frac{17}{35}$

$A\cap B = \{\quad\}$, $n(A\cap B) = 0$, $P(A\cap B) = 0$

$B\cap C = \{22, 24, 26, 28, 30, 32, 34\}$

$\therefore n(B\cap C) = 7 \Rightarrow P(B\cap C) = \frac{7}{35}$

$C\cap A = \{2\} \Rightarrow n(C\cap A) = 1$

$P(C\cap A) = \frac{1}{35}$

at $P(A\cap B\cap C) = 0$

$$\therefore P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$$= \frac{7}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0$$

$$= \frac{28}{35}$$

$$= \frac{4}{5}$$

EXERCISE 8.5

Multiple choice questions :

1. Which of the following is not a measure of dispersion?

(1) Range (2) Standard deviation
(3) Arithmetic mean (4) Variance

Ans : (3)

Hint :

A.M is not a measure of dispersion and it is a measure of central tendency.

2. The range of the data 8, 8, 8, 8, 8. . . 8 is

(1) 0 (2) 1 (3) 8 (4) 3

Ans : (1)

Hint :

Range = L – S
= 8 – 8 = 0

3. The sum of all deviations of the data from its mean is

(1) Always positive
(2) always negative
(3) zero
(4) non-zero integer

Ans : (3)

Hint :

Sum of all deviations of the data from the mean = 0

ie $\sum (x - \bar{x}) = 0$

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

(1) 40000 (2) 160900
(3) 160000 (4) 30000

Ans : (2)

Hint :

$$\bar{x} = 40,\ n = 100,\ \sigma = 3$$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$9 = \frac{\sum x^2}{100} - (40)^2$$

$$\frac{\sum x^2}{100} = 1609$$

$$\Rightarrow \sum x^2 = 160900$$

5. **Variance of first 20 natural numbers is**

(1) 32.25 (2) 44.25 (3) 33.25 (4) 30

Ans : (3)

Hint :

Variance for first 20 natural numbers

$$\sigma^2 = \frac{n^2 - 1}{12}$$

$$= \frac{400 - 1}{12}$$

$$= \frac{399}{12}$$

$$= 33.25$$

6. **The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is**

(1) 3 (2) 15 (3) 5 (4) 225

Ans : (4)

Hint :

$\sigma = 3$ of a data.

If each value is multiplied by 5,

then the new SD = 15

$\therefore$ Variance = $(\text{SD})^2$

$= 15^2$

$= 225$

7. **If the standard deviation of x, y, z is p then the standard deviation of $3x + 5$, $3y + 5$, $3z + 5$ is**

(1) 3p + 5 (2) 3p (3) p + 5 (4) 9p +15

Ans : (2)

Hint :

SD of x, y, z = p

$\Rightarrow$ SD of $3x, 3y, 3z$ = 3p

$\Rightarrow$ SD of $3x + 5, 3y + 5, 3z + 5$ = 3p.

8. **If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is**

(1) 3.5 (2) 3 (3) 4.5 (4) 2.5

Ans : (1)

Hint :

$\bar{x} = 4$, CV = 87.5, $\sigma = ?$

$$CV = \frac{\sigma}{x} \times 100$$

$$87.5 = \frac{\sigma}{4} \times 100$$

$$\therefore \sigma = \frac{87.5}{25}$$

$$= 3.5$$

9. **Which of the following is incorrect?**

(1) $P(A) > 1$ (2) $0 \le P(A) \le 1$

(3) $P(\phi) = 0$ (4) $P(A) + P(\bar{A}) = 1$

Ans : (1)

Hint :

$P(A) > 1$ is incorrect.

since $0 \le P(A) \le 1$

10. The probability a red marble selected at random from a jar containing *p* red, *q* blue and *r* green marbles is

(1) $\dfrac{q}{p+q+r}$ (2) $\dfrac{p}{p+q+r}$

(3) $\dfrac{p+q}{p+q+r}$ (4) $\dfrac{p+r}{p+q+r}$

Ans : (2)

Hint :

n (Red) = p, n(S) = $p + q + r$

Required probability = $\dfrac{p}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

(1) $\dfrac{3}{10}$ (2) $\dfrac{7}{10}$ (3) $\dfrac{3}{9}$ (4) $\dfrac{7}{9}$

Hint : **Ans : (2)**

P (digit at unit's place of the page is less than 7) = $\dfrac{7}{10}$

($\because$ n(S) = 10, A = {0, 1, 2, 3, 4, 5, 6}, n(A) = 7)

12. The probability of getting a job for a person is $\dfrac{x}{3}$ If the probability of not getting the job is $\dfrac{2}{3}$ then the value of *x* is

(1) 2 (2) 1 (3) 3 (4) 1.5

Hint : **Ans : (2)**

Given P(A) = $\dfrac{x}{3}$, P($\overline{A}$) = $\dfrac{2}{3}$

P(A) + P($\overline{A}$) = 1

$\Rightarrow \dfrac{x+2}{3} = 1$

$\Rightarrow x + 2 = 3$

$\Rightarrow x = 1$

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\dfrac{1}{9}$, then the number of tickets bought by Kamalam is

(1) 5 (2) 10 (3) 15 (4) 20

Hint : **Ans : (3)**

$n(S) = 135 \quad n(A) = x$

$\therefore P(A) = \dfrac{x}{135} = \dfrac{1}{9}$ (given)

$\Rightarrow x = \dfrac{135}{9} = 15$

14. If a letter is chosen at random from the English alphabets {a, b,..., z}, then the probability that the letter chosen precedes *x*

(1) $\dfrac{12}{13}$ (2) $\dfrac{1}{13}$ (3) $\dfrac{23}{26}$ (4) $\dfrac{3}{26}$

Hint : **Ans : (3)**

n(S) = 26 n(A) = 23 ($\because$ 26 – 3)

P(A) = $\dfrac{23}{26}$

15. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?

(1) $\dfrac{1}{5}$ (2) $\dfrac{3}{10}$ (3) $\dfrac{2}{3}$ (4) $\dfrac{4}{5}$

Hint : **Ans : (4)**

n(S) = 50, n(A) = 10, n(B) = 15, n(C) = 25

$P(B \cup C) = P(B) + P(C)$ ($\because$ B & C are mutually exclusive)

$= \dfrac{15}{50} + \dfrac{25}{50}$

$= \dfrac{40}{50}$

$= \dfrac{4}{5}$

UNIT EXERCISE - 8

1. **The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.**

Class Interval	0-20	20 - 40	40 - 60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Solution :

Given $\bar{x} = 62.8$, $\quad \Sigma f = 50$

$$\Rightarrow f_1 + f_2 + 30 = 50$$

$$\Rightarrow \quad f_1 + f_2 = 20$$

$$\Rightarrow \quad f_2 = 20 - f_1$$

C.I.	x	f	$d = \frac{x-70}{20}$	fd
0-20	10	5	–3	– 15
20-40	30	f_1	–2	$-2f_1$
40-60	50	10	–1	– 10
60-80	70	$20-f_1$	0	0
80-100	90	7	1	7
100-120	110	8	2	16
		50		$-2f_1-2$

$$\bar{x} = A + \left(\frac{\Sigma fd}{\Sigma f} \times c\right)$$

$$62.8 = 70 + \left(\frac{-2f_1 - 2}{50} \times 20\right)$$

$$\Rightarrow 62.8 = 70 + \left(\frac{-4f_1 - 4}{5}\right)$$

$$\Rightarrow \quad 314 = 350 - 4f_1 - 4 \quad \Rightarrow -4f_1 = -32$$

$$f_1 = \frac{32}{4} = 8$$

$$\therefore f_1 = 8, \quad f_2 = 20 - f_1$$

$$= 20 - 8 = 12.$$

2. **The diameter of circles (in mm) drawn in a design are given below.**

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

Calculate the standard deviation.

Solution :

C.I.	x	f	$d = \frac{x-42.5}{4}$	d^2	$f.d$	$f.d^2$
32.5-36.5	34.5	15	– 2	4	–30	60
36.5-40.5	38.5	17	– 1	1	–17	17
40.5-44.5	42.5	21	0	0	0	0
44.5-48.5	46.5	22	1	1	22	22
48.5-52.5	50.5	25	2	4	50	100
		100			25	199

$\therefore \Sigma f = 100, \Sigma fd = 25, \Sigma fd^2 = 199$

$$\therefore \sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times 4$$

$$= \sqrt{\frac{199}{100} - \left(\frac{25}{100}\right)^2} \times 4$$

$$= \sqrt{\frac{19900 - 625}{100^2}} \times 4$$

$$= \frac{\sqrt{19275}}{100} \times 4$$

$$= \frac{138.83}{25}$$

$$= 5.55$$

$\therefore$ S.D $= 5.55$

3. **The frequency distribution is given below.**

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a variance of 160. Determine the value of *k*.

Solution :

x	f	$d = \frac{x-A}{k}$	d^2	$f.d$	$f.d^2$
k	2	– 3	9	– 6	18
$2k$	1	– 2	4	– 2	4
$3k$	1	– 1	1	– 1	1
$4k$	1	0	0	0	0
$5k$	1	1	1	1	1
$6k$	1	2	4	2	4
	7			– 6	28

Given variance = 160

$$\therefore k^2\left(\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2\right) = 160$$

$$\Rightarrow \quad k^2\left[\frac{28}{7} - \left(\frac{-6}{7}\right)^2\right] = 160$$

$$\Rightarrow \quad k^2\left[4 - \frac{36}{49}\right] = 160$$

$$\Rightarrow \quad k^2\left[\frac{160}{49}\right] = 160$$

$$\Rightarrow \quad k^2 = \frac{16 \times 40}{16}$$

$$\Rightarrow \quad k^2 = 49$$

$\therefore k = 7 \quad (\because k$ is positive)

4. The standard deviation of some temperature data in degree celsius (°C) is 5. If the data were converted into degree Farenheit (°F) then what is the variance?

Solution : Given $\sigma_c = 5$

$$F = \frac{9c}{5} + 32$$

$$\Rightarrow \sigma_F = \frac{9}{5}\sigma_c$$

$$= \frac{9}{5} \times 5$$

$$= 9$$

($\because$ Add (or) subtract the value to a data won't effect the SD)

$$\therefore \sigma_F{}^2 = 9^2 = 81.$$

5. If for a distribution, $\Sigma(x - 5) = 3$, $\Sigma(x - 5)^2 = 43$, and total number of observations is 18, find the mean and standard deviation.

Solution :

Given $\Sigma(x - 5) = 3$,

$\Rightarrow \Sigma x - \Sigma 5 = 3$

$\Rightarrow \Sigma x - 5.\Sigma 1 = 3$

$\Rightarrow \Sigma x - 5(18) = 3$

$\Rightarrow \Sigma x = 93$

$\Sigma (x - 5)^2 = 43$, n = 18

$\Rightarrow \Sigma (x^2 - 10x + 25) = 43$

$\Rightarrow \Sigma x^2 - 10.\ \Sigma x + 25\ \Sigma 1 = 43$

$\Rightarrow \Sigma x^2 - 10(93) + 25(18) = 43$

$\Rightarrow \Sigma x^2 = 523$

i) Mean :

$$\bar{x} = \frac{\Sigma x}{n} = \frac{93}{18} = 5.17$$

ii) SD :

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2}$$

$$= \sqrt{\frac{523}{18} - \frac{8649}{324}}$$

$$= \sqrt{\frac{9414 - 8649}{324}}$$

$$= \frac{\sqrt{765}}{18} = \frac{27.65}{18} = 1.536$$

6. Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?

Prices in City A	**20**	**22**	**19**	**23**	**16**
Prices in city B	**10**	**20**	**18**	**12**	**15**

Solution :

CV for prices in City A

Given data is 20, 22, 19, 23, 16

$$\therefore \bar{x} = \frac{100}{5} = 20$$

To find σ_1 arrange them is ascending order.

x	$d = x - 20$	d^2
16	– 4	16
19	– 1	1
20	0	0
22	2	4
23	3	9
	0	30

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{30}{5}}$$

$$= \sqrt{6}$$

$$= 2.44$$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2.44}{20} \times 100$$

$$= 12.24$$

CV for prices in City B

Given data is 10, 20, 18, 12, 15

$$\therefore \bar{x} = \frac{75}{5} = 15$$

To find σ arrange them is ascending order.

x	$d = x - 15$	d^2
10	– 5	25
12	– 3	9
15	0	0
18	3	0
20	5	25
	0	68

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{68}{5}}$$

$$= \sqrt{13.6}$$

$$= 3.68$$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{3.68}{15} \times 100$$

$$= 24.53$$

∴ C.V for price in City A < City B

∴ Prices are very stable in City A.

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7. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution :

Given range = 20, Co.eff. of range = 0.2

$\Rightarrow$ L – S = 20 ...(1)

$$\frac{L-S}{L+S} = 0.2$$

$$\Rightarrow \frac{20}{L+S} = 0.2$$

$$\Rightarrow L + S = 100 \quad ...(2)$$

Solviong (1) and (2)

L = 60, S = 40

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8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Solution :

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S) = 36

Let A - Product of face value is 6.

A = {(1, 6), (2, 3), (3, 2), (6, 1)}
n(A) = 4

$P(A) = \frac{4}{36}$

Let B - Difference of face value is 5.

B = {(6, 1)}

n(B) = 1

$P(B) = \frac{1}{36}$

$A \cap B = \{(6, 1)\}$

$n(A \cap B) = 1$

$P(A \cap B) = \frac{1}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{36} + \frac{1}{36} - \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$

9. In a two children family, find the probability that there is at least one girl in a family.

Solution :

S = {(BB), (BG), (GB), (GG)}

n(S) = 4

Let A be the event of getting atleast one girl.

A = {(BG), (GB), (GG)}

$\therefore$ n(A) = 3

$\therefore P(A) = \frac{3}{4}$

10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.

Solution :

Given n(S) = 5 + x, 5 white balls
x black balls

By daa given,
P(B) = 2. P(W)

$\Rightarrow \frac{x}{5+x} = 2.\left(\frac{5}{5+x}\right)$

$\Rightarrow x = 10$

$\therefore$ No. of black balls = 10

11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?

Solution :

Given $P(E \cap T) = 0.5$; $P(\overline{E} \cap \overline{T}) = 0.1$

& P(E) = 0.75 $\Rightarrow P(\overline{E \cup T}) = 0.1$

$\Rightarrow P(E \cup T) = 1–01$

$= 0.9$

$P(E \cup T) = P(E) + P(T) - P(E \cap T)$

$0.9 = 0.75 + P(T) - 0.5$

$P(T) = 0.9 - 0.25$

$= 0.65$

$= \frac{65}{100}$

$= \frac{13}{20}$

12. The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.

Solution :

n(S) = 52 – 3 = 49

i) Let A - a diamond card

n(A) = 13

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{13}{49}$$

ii) Let B - a queen card

n(B) = 3 (except spade queen out of 4)

$$\therefore P(B) = \frac{3}{49}$$

iii) Let C - a spade card

n(C) = 10 (13 – 3 = 10)

$$\therefore P(C) = \frac{10}{49}$$

iv) Let D - 5 of heart

n(D) = 1

$$\therefore P(D) = \frac{1}{49}$$

PROBLEMS FOR PRACTICE

1. Find the SD of the data

   i) 45, 60, 62, 60, 50, 65, 58, 68, 44, 48

   ii) 8, 10, 15, 20, 22

   iii) 18, 11, 10, 13, 17, 20, 12, 19

(Ans: (i) 8.14, (ii) 5.44, (iii) 3.67)

2. Find the variance of the wages :

   Rs.210, Rs.190, Rs.220, Rs.180, Rs.200, Rs. 190, Rs.200, Rs.210, Rs.180

(Ans : 172.8)

3. Find the range of the heights of 12 girls in a class given in cm.

   120, 110, 150, 100, 130, 145, 150, 100, 140, 150, 135, 125

(Ans : 50)

4. The variance of 5 values is 36. If each value is doubled, find the SD of new values.

(Ans : 12)

5. For a group of 200 students, the mean and SD of scores were found to be 40 and 15 respectively. Later on, it was found that scores 43, 35 were misread as 34, 53 respectively. Find the correct mean, SD.

(Ans : 30.955, 14.995)

6. Mean of 100 items is 48 and their S.D is 10. Find the sum of all the items and the sum of the squares of all the items.

(Ans : 4800, 240400)

7. If the coefficient of variation of a collection of data is 57 and its SD is 6.84, find the mean.

(Ans : 12)

8. Calculate S.D from the data :

Marks :	10	20	30	40	50	60
No. of students :	8	12	20	10	7	3

(Ans : 13.45)

9. Find the SD for the data.

Age (in years) :	18	22	21	23	19
No. of students:	100	120	140	150	80

(Ans : 1.84)

10. The following table gives the distribution of income of 100 families in a village. Find the variance.

Income :	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of families :	18	26	30	12	10	4

(Ans : 1827600)

11. Find the coefficient of variation :

20, 18, 32, 24, 26

(Ans : 20.412)

12. Find the coefficient of variation of the data

Size (in cms) :	10-15	15-20	20-25	25-30	30-35	35-40
No. if items :	2	8	20	35	20	15

(Ans : 21.86)

13. Which of the following cricketers A or B is more consistent player, who scored runs in a cricket season.

A :	58	59	60	54	65	66	52	75	69	52
B :	84	56	92	65	86	78	44	54	78	68

(Ans : Player 'A')

14. Find the missing frequencies of the distribution whose mean is 28.2.

C.V	0-10	10-20	20-30	30-40	40-50
f :	5	f_1	15	f_2	6

(Ans : $f_1 = 8, f_2 = 16$)

15. A number is selected at random from 1 to 100. Find the probability that it is a perfect cube.

(Ans : 1/25)

16. A two digit number is formed of the digits 2, 5 and 9. Find the probability that it is divisible by 2 (or) 5, without repetition.

(Ans : 2/3)

17. From a set of whole numbers less than 40, find the probability of getting a number not divisible by 5 or 7.

(Ans : 12/41)

18. Two dices are thrown together. What is the probability that only odd numbers turn upon both the dices.

(Ans : 5/6)

19. What is the probability that a leap year to contain 53 sundays ?

(Ans : 2/7)

20. The probability that A, B and C can solve a problem are 4/5, 2/3, 3/7 respectively. The probability of the problem being solved by A and B is 8/15, B and C is 2/7, A and C is 12/35. The probability of the problem being solved by all the 3 is 8/35. Find the probability that the problem can be solved by atleast one of them.

(Ans : 101/105)

OBJECTIVE TYPE QUESTIONS

1. The range of first 20 whole numbers is

a) 19 b) 38 c) 20 d) 19.5

Ans : (c)

2. Variance of 1, 2, 3 is

a) 2/3 b) 2 c) 0 d) $\sqrt{2}/3$

Ans : (a

3. The sum of the squares deviations for 10 observations taken from their mean 50 is 250. The coefficient of variation is

a) 10% b) 40% c) 50% d) 15%

Ans : (a)

4. If A and B are mutually exclusive and S is the sample space such that P(A) = 1/3, P(B) and S = A∪B, their P(A) is

a) 1/4 b) 1/2 c) 4/3 d) 3/2

Ans : (a)

5. If the first 10 positive integers, in which we multiply each no. by – 1 and then add 1 to each, the variance of the numbers so obtained is

a) 8.25 b) 6.5 c) 3.87 d) 8.25

Ans : (c)

6 The variance of 15 observations is 4. If each observation is increased by 9, the variance of the new data is

a) 13 b) 36 c) 4 d) 16

Ans : (c)

7. Consider the numbers from 1 to 10. If 1 is added to each number, the variance of the numbers so obtained is

a) 6.5 b) 2.87 c) 3.87 d) 8.25

Ans : (c)

8. The probability of drawing neither an ace nor a king is

a) 2/13 b) 11/13 c) 4/13 d) 8/13

Ans : (b)

9. A number is chosen from 40 to 75. Find the prob. that it is divisible by 7 and 11.

a) 2/9 b) 1/9 c) 3/9 d) 4/9

Ans : (a)

10. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. The number of rotten apples is

a) 0.872 b) 1620 c) 162 d) 172

Ans : (c)

11. In a family of 3 children, probability of having atleast one boy is

a) 1/3 b) 7/8 c) 3/8 d) 1/2

Ans : (b)

12. If a card is drawn at random from 30 cards, the probability that the number on the card is not divisible by 3 is

a) 2/3 b) 1/3 c) 27/30 d) none

Ans : (a)

13. 3 digit numbers are made using the digits 4, 5, 9 without repetition. If a number is selected at random, the prob. that the number will be ended with 9 is

a) 1/3 b) 5/9 c) 1/2 d) none

(Ans : (a)

14. A letter of english alphabet is chosen at random. The prob. that the letter chosen is a consonant.

a) 5/26 b) 21/26 c) 7/13 d) 1

Ans : (b)

15. To prob. of a card to be a club card when it is taken from 52 cards where all red face cards are removed is

a) 3/23 b) 13/46 c) 10/23 d) 13/40

Ans : (b)