Chapter 16

Playing with Numbers

Numbers in General Form

General Form of a Number

One-digit Number

The one-digit number is the number at the unit place.

For example,

$0b = 0 \times 10 + b \times 1 = 0 + \text{Ones digit.}$

So, 0b is a one-digit number.

Two-digit Number

Consider a 2 - digit number having the digit $'a'$ at its tens place and the digit $'b'$ at its unit's place.

The 2 - digit number ab is written as [ab does not mean a $\times$ b]

$a \times 10 + b \times 1 = 10a + b$

The 2 - digit number 52 is written as

$5 \times 10 + 2 \times 1 = 50 + 2$

Three-digit Number

Consider a 3 - digit number having the digit $'a'$ at its hundreds place,

digit 'b$'$ at its tens place and digit 'c$'$ at its unit's place.

The 3 - digit number abc is written as [ab does not mean a $\times$ b]

$a \times 100 + b \times 10 + c \times 1 = 100a + 10b + c$

For example: The 3 - digit number 357 is written as

$3 \times 100 + 5 \times 10 + 7 \times 1 = 300 + 50 + 7 = 357$

Let us take the number 62 and write it as

$$62 = 60 + 2 = 10 \times 6 + 2$$

Similarly, the number 37 can be written as

$$37 = 10 \times 3 + 7$$

In general, any two-digit number ab made of digits a and b can be written as

$$\mathrm{ab} = 10 \times \mathrm{a} + \mathrm{b} = 10\mathrm{a} + \mathrm{b}$$

What about ba?

$$\mathrm{ba} = 10 \times \mathrm{b} + \mathrm{a} = 10\mathrm{b} + \mathrm{a}$$

Let us now take number 351. This is a three-digit number. It can also **be written as**

$$351 = 300 + 50 + 1 = 100 \times 3 + 10 \times 5 + 1 \times 1$$

Similarly, $497 = 100 \times 4 + 10 \times 9 + 1 \times 7$

In general, a 3-digit number abc made up of digits a, b and c is written as

$$\begin{aligned} \mathrm{abc} &= 100 \times \mathrm{a} + 10 \times \mathrm{b} + 1 \times \mathrm{c} \\ &= 100\mathrm{a} + 10\mathrm{b} + \mathrm{c} \end{aligned}$$

In the same way,

$$\mathrm{cab} = 100\mathrm{c} + 10\mathrm{a} + \mathrm{b}$$
$$\mathrm{bca} = 100\mathrm{b} + 10\mathrm{c} + \mathrm{a} \text{ and so on}$$

In a 2-digit number, the unit's digit is two times the tens digit and the sum of the digits is 9. Find the number.

Let the tens digit be x. Then, the unit's digit = 2x

$x + 2x = 9 \Rightarrow 3x = 9 \Rightarrow x = 3$

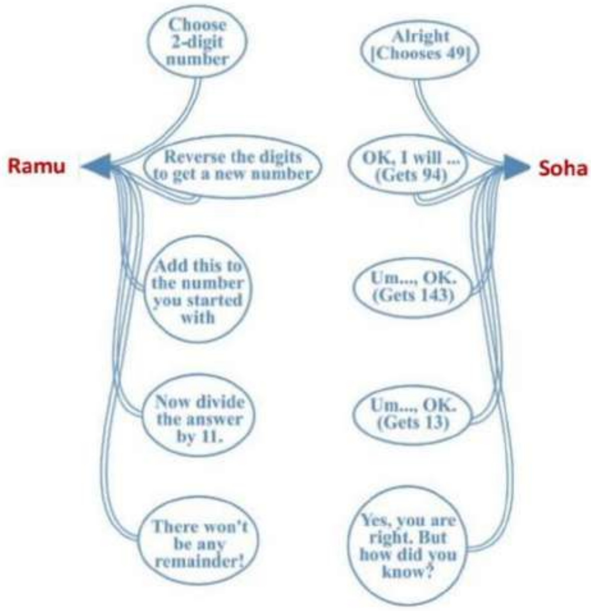
Tens digit = 2 and units' digit = $(2 \times 3) = 6$

Hence, the required number is $(10 \times 3 + 6) = 36$.

Games with numbers

Reversing the digits-2 digit Number

Ramu asks Sohan to think of a 2-digit number, and then to do whatever she asks him to do, to that number. Their conversation is shown in the following figure. Study the figure carefully before reading on.



Let ab be any two-digit number i.e., ab = 10a + b (i)

After reversing the digits, we get ba = 10b + a (ii)

Adding (i) and (ii), we get

$ab + ba = 10a + b + 10b + a = 11a + 11b$

Sum of two-digit number and the number obtained by reversing its digits is always divisible by 11 and quotient is $a + b$.

Subtracting (i) and (ii), we get

$ba - ab = 10b + a - 10a - b = 9b - 9a = 9(b - a)$ if $b>a$ and $9(a - b)$, if $a >b$

Difference of two-digit number and the number obtained by reversing its digits is always divisible by 9 and the quotient is $(a - b)$ if $a > b$ and $(b - a)$ if $b > a$.

Without performing actual addition and division, write the remainder when

(i) Sum of 29 and 92 is divided by 11

(ii) Difference of 29 and 92 is divided by 9.

(i) Since 29 obtained from 92 by reversing its digits.

Sum 29 and 92 is divisible by 11, hence remainder is zero (0).

(ii) Similarly, the difference of 29 and 92 is divisible by 9, hence the remainder is zero (0).

Reversing the digits-3 digit Number

Now let us consider a three-digit number $abc = 100a + 10b + c$

Reversing its digits i.e., $cba = 100c + 10b + a$

Case I: If $a < c$ then the difference between the numbers $abc - cba = 100a + 10b + c - 100c - 10b - a = 100a - a - 100c + c = 99a - 99c = 99(a - c)$

Hence, the difference is divisible by 99 and quotient $a - c$.

Case II: If $a < c$ then the difference will become

$cba - abc = 100c + 10b + a - 100a - 10b - c = 99c - 99a = 99(c - a)$

the difference is divisible by 99 and quotient $c - a$.

Case III: If $a = c$ then the difference is divisible $= 0$

In all cases, whether $a > c$ or $a < c$ or $a = c$, the difference is always divisible by 99 leaving remainder 0 and quotient $a - c$ or $c - a$.

Without performing actual division, find the remainder and quotient when

(i) Difference of 623 and 326 is divided by 99.

(ii) Difference of 239 and 932 is divided by 7.

(i) 326 is obtained from 623 by reversing its digits, where $3 > 6$. When the difference is divided by 99 gives remainder zero and quotient is $6 - 3 = 3$.

(ii) 932 is obtained from 239 by reversing its digits where $9 > 2$. Difference of numbers $= 99\ (9 - 2) = 99 \times 7$

When the difference is divided by 7, it will give the remainder zero and quotient 99.

Forming 3 - digit numbers with given three digits

Let the 3 - digit number be abc. On changing the order of the digits in a cyclic order, we obtain the numbers bca and cab.

On adding the three numbers,

$abc + bca + cab$

$= 100a + 10b + c + 100b + 10c + a + 100c + 10a + b$

$= 100a + 10a + a + 100b + 10b + b + 100c + 10c + c$

$= 111a + 111b + 111c$

$= 111(a + b + c) = 37 \times 3(a + b + c)$

The sum $abc + bca + cab$ is exactly divisible by 37, 3 and sum of its digits i.e. $(a + b + c)$.

For example, let us take the 3 - digit number 623.

Then, $623 + 236 + 362 = 1221 = 3 \times 37 \times (6 + 2 + 3) = 3 \times 37 \times 11$

The sum is exactly divisible by 37, 3 and 11.

Letters for Digits

Letters for digits or Cryptarithms

Here we will discuss some puzzles in which letters take the place of digits. Here, puzzles which involve only addition and multiplication.

There are two rules we follow while solving puzzles:

(a) Each letter in the puzzle must stand for just one digit. Each digit must be represented by just one letter. i.e., A→ 1 and 2 → B

(b) The first digit of a number cannot be zero. For example, we write the number "sixty-five" as 65 and not as 065 or 0065.

For example,

i) Find A in the addition.

$$\begin{array}{r} 3\ A\ 3 \\ +\ 1\ 1\ A \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

In one's column: $A+3 = 1$, i.e. one's digit is 1. $8 + 3 = 11$

Therefore, the value of A should be 8

The puzzle can be solved as

$$\begin{array}{rl} 3\ 8\ 3 & (3 + 8 = 11) \\ +\ 1\ 1\ 8 & (1 + 8 + 1 = 10) \\ \hline 5\ 0\ 1 & (1 + 3 + 1 = 5) \\ \hline \end{array}$$

ii) Find the digits A and B.

$$\begin{array}{cc} 1 & A \\ \times & A \\ \hline 9 & A \\ \hline \end{array}$$

In one's column: $A \times A = A$, therefore A is either 0, 5 or 6. A cannot be equal to zero.

If $A = 5$,

$$\begin{array}{cc} 1 & 5 \\ \times & 5 \\ \hline 7 & 5 \\ \hline \end{array} \qquad \begin{array}{l} (5 \times 5 = 25) \\ (5 \times 1) + 2 = 7 \end{array}$$

If $A = 6$,

$$\begin{array}{cc} 1 & 6 \\ \times & 6 \\ \hline 9 & 6 \\ \hline \end{array} \qquad \begin{array}{l} (6 \times 6 = 36) \\ (6 \times 1) + 3 = 9 \end{array}$$

iii) Find the digits P and Q,

$$\begin{array}{cccc} & & Q & P \\ & \times & Q & 5 \\ \hline 1 & 8 & 0 & P \\ \hline \end{array}$$

Here, we have to find the values of two letters P and Q.

Ones digit $P \times 5 = P$, so it must have $P = 0$ or 5. Firstly, we take $P = 0$.

Now, if $Q = 3$, then $QP \times Q5 = 30 \times 35 = 1050$, but here product is 180P, which is equal or more than 1800. So, we cannot take $Q = 3$.

If $Q = 4$, then $QP \times Q5 = 40 \times 45 = 1800$ i.e., 1800

P = 0 and Q = 4.

If P = 5 and Q = 3, then 35 × 35 = 1225, which is less than 180P.

And, if P = 5 and Q = 4, then 45 × 45 = 2025, which is greater than 180P. So, P =5 is not possible. Hence, P = 0 and Q = 4 is the correct answer.

Tests of Divisibility

Divisibility Rules

Number	Rule	Example
2	Last digit must be either 0, 2, 4, 6, 8	28, 570
3	Sum of digits must be divisible by 3	147; 1 + 4 + 7 = 12
4	Number formed by last 2 digits must be divisible by 4	144, 1096
5	Last digit must either be 0 or 5	20, 5275
6	The number must be divisible by both 2 and 3	1044;1 + 0 + 4 + 4 = 9
8	Number formed by last 3 digits must be divisible by 8	248, 1640
9	Sum of digits must be divisible by 9	675; 6+7+5 = 18
10	The last digit must be 0	30, 250
11	Difference of sum of digits at odd places and sum of digits at even places must either 0 or 11	4059; 4+5=9; 0+9=9 9 - 9 = 0

For example.

i) Test the divisibility of 458432 by 2.

The given number is 458432

As the digit in the unit place is an even number i.e. 2. Therefore, the number is divisible by 2.

$\frac{458432}{2} = 279216$

ii) Test the divisibility of 21465681 by 3.

The given number is 21465681

The sum of the digits = 2 + 1 + 4 + 6 + 5 + 6 + 8+1 = 33

$\frac{33}{3}$ = 11, therefore the number is divisible by 3.

$\frac{21465681}{3}$ = 7155227

iii) Test the divisibility of 24890 by 5.

The given number is 24890

As the digit in the unit place is 0. Therefore, the number is divisible by 5.

$\frac{24890}{5}$ = 4978

Find the value of x for which the number x805 is divisible by 9.

As x805 is divisible by 9, the sum of its digits should be divisible by 9.

Sum of its digits = (x + 8 + 0 + 5)

= (x + 13) must be divisible by 9

This is possible when x = 5

(x + 13) = (5 + 13)

= 18, which is divisible by 9

Therefore, the value of x is 5.

Check the divisibility of 3145587 by 3.

The sum of digits of 3145587 is 3 + 1 + 4 + 5 + 5 + 8 + 7 = 33 which is divisible by 3. Hence, 3145587 is divisible by 3.