

Clocks and Calendar

CLOCKS

The circumference of a dial of a clock (or watch) is divided into 60 equal parts called **minute spaces**. The clock has two hands—the hour hand and the minute hand. The hour hand (or short hand) indicates time in hours and the minute hand

(or long hand) indicates time in minutes. In an hour, the hour hand covers 5 minutes spaces while the minute hand covers 60 minutes spaces. Thus, in hour or 60 minutes, the minute hand gains 55 minutes spaces over the hour hand.

SOME BASIC FACTS

1. In every hour:

- (a) Both the hands coincide once.
- (b) The hands are straight (point in opposite directions) once. In this position, the hands are 30 minutes spaces apart.
- (c) The hands are twice at right angles. In this position, the hands are 15 minutes spaces apart.

2. The minute hand moves through 6° in each minute whereas the hour hand moves through $\frac{1^\circ}{2}$ in each minute. Thus, in one minute, the minute hand gains $5\frac{1}{2}$ than the hour hand.

3. (a) When the hands are coincident, the angle between them is 0° .

(b) When the hands point in opposite directions, the angle between them is 180° .

(c) The hands are in the same straight line, when they are coincident or opposite to each other. So, the angle between the two hands is 0° or 180° .

4. The minute hand moves 12 times as fast as the hour hand.

5. If a clock indicates 6.10, when the correct time is 6, it is said to be 10 minutes **too fast**. And if it indicates 5.50, when the correct time is 6, it is said to be 10 minutes **too slow**.

SOME USEFUL SHORT-CUT METHODS

1. The two hands of the clock will be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Explanation

At H O'clock the minute hand is $5H$ min spaces behind the hour hand.

\therefore the minute hand gains 55 minutes spaces in 60 minutes,

\therefore the minute hand will gain $5H$ minutes spaces in $\frac{60}{55}$

$\times 5H = \frac{60H}{11}$ minutes. Thus, the two hands of clock will

be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Illustration 1 At what time between 5 and 6 O'clock are the hands of a clock together?

Solution: Here, $H = 5$.

$$\therefore \frac{60H}{11} = \frac{60}{11} \times 5 = \frac{300}{11} = 27\frac{3}{11}$$

\therefore Hands of a clock are together at $22\frac{3}{11}$ minutes past 5 O'clock

2. The two hands of the clock will be at right angles between H and $(H + 1)$ O'clock at $(5H \pm 15)\frac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the minute hand will be $5H$ minutes spaces behind the hour hand. When the two hands are at right angle, they are 15 minutes spaces apart. So there are two cases:

Case I When the minute hand is 15 minutes spaces behind the hour hand, to be in this position, the minute hand will have to gain $(5H - 15)$ minutes spaces over the hour hand.

Case II When the minute hand is 15 minutes spaces ahead of the hour hand, to be in this position, the minute hand will have to gain $(5H + 15)$ minutes spaces over the hour hand. Combining the two cases, the minute hand will have to gain $(5H \pm 15)$ minutes spaces over the hour hand.

Now, 55 minutes spaces are gained in 60 minutes.

$\therefore (5H \mp 15)$ minutes spaces are gained in

$$\frac{60}{55}(5H \mp 15) = \frac{12}{11}(5H \mp 15) \text{ minutes}$$

So, they are at right angle at $(5H \mp 15)\frac{12}{11}$ minutes past H O'clock.

Illustration 2 At what time between 5 and 6 O'clock will the hands of a clock be at right angle?

Solution: Here $H = 5$

$$\therefore (5H \mp 15)\frac{12}{11} = (5 \times 5 \mp 15)\frac{12}{11} = 10\frac{10}{11} \text{ and } 43\frac{7}{11}$$

\therefore Hands of a clock are at right angle at $10\frac{10}{11}$ minutes past 5 and $43\frac{7}{11}$ minutes past 5.

3. The two hands of the clock will be in the same straight line but not together between H and $(H + 1)$ O'clock at

$$(5H - 30)\frac{12}{11} \text{ minutes past } H, \text{ when } H > 6$$

$$\text{and, } (5H + 30)\frac{12}{11} \text{ minutes past } H, \text{ when } H < 6$$

Illustration 3 Find at what time between 2 and 3 O'clock will the hands of a clock be in the same straight line but not together

Solution: Here, $H = 2 < 6$.

$$\begin{aligned} \therefore (5H + 30)\frac{12}{11} &= (5 \times 2 + 30)\frac{12}{11} \\ &= \frac{480}{11}, \text{ i.e., } 43\frac{7}{11} \end{aligned}$$

So, the hands will be in the same straight line but not together at $43\frac{7}{11}$ minutes past 2 O'clock

4. Between H and $(H + 1)$ O'clock, the two hands of a clock are M minutes apart at $(5H \mp M)\frac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the two hands are $5H$ minutes spaces apart

Case I Minute hand is M minute spaces behind the hour hand. In this case, the minute hand has to gain $(5H - M)$ minutes spaces over the hour hand.

Case II Minute hand is M minute spaces ahead of the hour hand. In this case, the minute hand has to gain $(5H + M)$ minute spaces over the hour hand.

Combining the two cases, the minute hand has to gain $(5H \pm M)$ minute spaces over the hour hand.

Now, 55 minutes spaces are gained in 60 minutes

$\therefore (5H \pm M)$ minutes spaces are gained in

$$\frac{60}{55}(5H \pm M) = \frac{12}{11}(5H \pm M) \text{ minutes.}$$

\therefore The hands will be M minutes apart at

$$\frac{12}{11}(5H \pm M) \text{ minutes past } H \text{ O'clock}$$

Illustration 4 Find the time between 4 and 5 O'clock when the two hands of a clock are 4 minutes apart

Solution: Here, $H = 4$ and $M = 4$.

$$\begin{aligned}\therefore \frac{12}{11}(5H \pm M) &= \frac{12}{11}(5 \times 4 \pm 4) \\ &= 26\frac{2}{11} \text{ and, } 17\frac{5}{11}\end{aligned}$$

\therefore The hands will be 4 minutes apart at $26\frac{2}{11}$ minutes past 4 and, $17\frac{5}{11}$ minutes past 4 O'clock

5. Angle between hands of a clock

(a) When the minute hand is behind the hour hand, the angle between the two hands at M

minutes past H O'clock = $30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees.

(b) When the minute hand is ahead of the hour hand, the angle between the two hands, at

M minutes past H O'clock = $30\left(\frac{M}{5} - H\right) - \frac{M}{2}$ degree.

Illustration 5 Find the angle between the two hands of a clock at 15 minutes past 4 O'clock

Solution: Here, $H = 4$ and $M = 15$

\therefore The required angle

$$\begin{aligned}&= 30\left(H - \frac{M}{5}\right) + \frac{M}{2} \text{ degrees} \\ &= 30\left(4 - \frac{15}{5}\right) + \frac{15}{2} \\ &= \frac{75}{2}, \text{ i.e., } 37.5^\circ\end{aligned}$$

6. The minute hand of a clock overtakes the hour hand at intervals of M minutes of correct time. The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right) \text{ minutes}$$

Illustration 6 The minute hand of a clock overtakes the hour hand at intervals of 65 minutes. How much a day does the clock gain or lose?

Solution: Here, $M = 65$

\therefore The clock gains or loses in a day by

$$\begin{aligned}&= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right) \\ &= \left(\frac{720}{11} - 65\right)\left(\frac{60 \times 24}{65}\right) \\ &= \frac{5}{11} \times \frac{12 \times 24}{13} = \frac{1440}{143} \\ &= 10\frac{10}{143} \text{ minutes}\end{aligned}$$

Since the sign is +ve, the clock gains by $10\frac{10}{143}$ minutes

CALENDAR

In this section, we will mainly deal with finding the day of the week on a particular given date. The process of finding it depends upon the number of odd days, which are quite different from the odd numbers. So, we should be familiar with **odd days**.

Odd Days

The days more than the complete number of weeks in a given period are called odd days.

Ordinary Year

An ordinary year has 365 days.

Leap Year

That year (except century) which is divisible by 4 is called a leap year, whereas century is a leap year by itself when it is divisible by 400.

For example, 1964, 1968, 1972, 1984, and so on, are all leap years whereas 1986, 1990, 1994, 1998, and so on, are not leap years.

Further, the centuries 1200, 1600, 2000 and so on, are all leap years as they are divisible by 400 whereas 900, 1300, 1500 and so on, are not leap years.

SOME BASIC FACTS

1. An ordinary year has 365 days, i.e., 52 weeks and 1 odd day.
2. A leap year has 366 days, i.e., 52 weeks and 2 odd days.
3. A century has 76 ordinary years and 24 leap years
 \therefore 100 years = 76 ordinary years + 24 leap years
 $= 76 \text{ odd days} + 24 \times 2 \text{ odd days}$
 $= 124 \text{ odd days} = 17 \text{ weeks} + 5 \text{ days}$
 \therefore 100 years contain 5 odd days.
4. 200 years contain 10 and therefore 3 odd days.
5. 300 years contain 15 and therefore 1 odd day.
6. 400 years contain (20 + 1) and therefore 1 odd day.
7. February in an ordinary year has no odd day, but in a leap year has one odd day.
8. Last day of a century cannot be either Tuesday, Thursday or Saturday.

9. The first day of a century must either be Monday, Tuesday, Thursday or Saturday.

Explanation

No. of odd days in first century = 5

\therefore Last day of first century is Friday

No. of odd days in two centuries = 3

\therefore Wednesday is the last day

No. of odd days in three centuries = 1

\therefore Monday is the last day

No. of odd days in four centuries = 0

\therefore Last day is Sunday

Since the order is continually kept in successive cycles, the last day of a century cannot be Tuesday, Thursday or Saturday. So, the last day of a century should be either Sunday, Monday, Wednesday or Friday. Therefore, the first day of a century must be either Monday, Tuesday, Thursday or Saturday.

SOME USEFUL SHORT-CUT METHODS

1. Working rule to find the day of the week on a particular date when reference day is given:

Step I Find the net number of odd days for the period between the reference date and the given date (Exclude the reference day but count the given date for counting the number of net odd days).

Step II The day of the week on the particular date is equal to the number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

Illustration 7 January 11, 1997 was a Sunday. What day of the week was on January 7, 2000?

Solution: Total number of days between January 11, 1997 and January 7, 2000

$$\begin{aligned}
 &= (365 - 11) \text{ in } 1997 + (365 \text{ days in } 1998) \\
 &\quad + (365 \text{ days in } 1999) + (7 \text{ days in } 2000) \\
 &= (50 \text{ weeks} + 4 \text{ odd days}) + (52 \text{ weeks} + 1 \text{ odd day}) \\
 &\quad + (52 \text{ weeks} + 1 \text{ odd day}) + (7 \text{ odd days})
 \end{aligned}$$

$$= 13 \text{ days} = 1 \text{ week} + 6 \text{ odd days}$$

Hence, January 7, 2000 would be 6 days ahead of Sunday, i.e., it was on Saturday

2. Working Rule to find the day of the week on a particular date when no reference day is given

Step I Count the net number of odd days on the given date

Step II Write:

Sunday for 0 odd day

Monday for 1 odd day

Tuesday for 2 odd days

$\vdots \quad \vdots \quad \vdots$

Saturday for 6 odd days.

Illustration 8 What day of the week was on June 5, 1999?

Solution: June 5, 1999 means 1998 years + first five months up to May of 1999 + 5 days of June

1600 years have 0 odd day

300 years have 1 odd day

98 years have 24 leap years + 74 ordinary years

$$= (24 \times 2) + (74 \times 1) \text{ days}$$

$$= 122 \text{ days} = 17 \text{ weeks} + 3 \text{ odd days}$$

Thus, 1998 years have 4 odd days.

January 1, 1999 to May 31, 1999 has

$$= (3 + 0 + 3 + 2 + 3 + 5) 2 \text{ odd days}$$

$$= 16 \text{ days} = 2 \text{ weeks} + \text{odd days}$$

Total number of odd days on June 5, 1999

$$= (4 + 2) \text{ odd days} = 6 \text{ odd days}$$

Hence, June, 5 1999 was Saturday.

Practice Exercises

DIFFICULTY LEVEL-1 (BASED ON MEMORY)

1. A watch is 1 minute slow at 1 p.m. on Tuesday and 2 minutes fast at 1 p.m. on Thursday. When did it show the correct time?
 - (a) 1.00 a.m. on Wednesday
 - (b) 5.00 a.m. on Wednesday
 - (c) 1.00 p.m. on Wednesday
 - (d) 5.00 p.m. on Wednesday

[Based on MAT, 2004]

2. The minute and the hour hand of a watch meet every 65 minutes. How much does the watch lose or gain time?
 - (a) 25 seconds
 - (b) 27 seconds
 - (c) 27.16 seconds
 - (d) 30 seconds
3. February 1, 1984 was a Wednesday just like the February 29, 1984. When will the calendar show another February with a similar situation?
 - (a) 2000
 - (b) 1996
 - (c) 2012
 - (d) 2004

[Based on MAT, 2003]

4. It was Wednesday on January 1, 2000. What would be the day on January 1, 2001?
 - (a) Monday
 - (b) Wednesday
 - (c) Thursday
 - (d) Friday
5. Out of the following four choices which does not show the coinciding of the hour hand and minute hand?
 - (a) 3.16.2
 - (b) 6.32.43
 - (c) 9.59.05
 - (d) 5.27.16
6. February 29, 2000 was a Tuesday. In this century, how many times February 29, will fall on a Tuesday?
 - (a) 1
 - (b) 2
 - (c) 3
 - (d) 4

[Based on IITM, Gwalior, 2003]

7. In a day how many times the minute-hand and hour-hand make right angle between them?
 - (a) 12
 - (b) 20
 - (c) 22
 - (d) 44
8. In a 366-day year, how many days occur 53 times?
 - (a) 2
 - (b) 1
 - (c) 0
 - (d) 3

9. In an ordinary year which months begin on the same day of the week:
 - (a) Feb; Nov
 - (b) Jan; Nov
 - (c) Feb; Oct
 - (d) Jan; Sept

10. If March 2, 1994 was on Wednesday, January 25, 1994 was on:
 - (a) Wednesday
 - (b) Thursday
 - (c) Tuesday
 - (d) Monday

11. Calendar for 2000 will also serve for:
 - (a) 2003
 - (b) 2006
 - (c) 2007
 - (d) 2005

12. January 7, 1992 was Tuesday. Find the day of the week on the same date after 5 years, i.e., on January 7, 1997?
 - (a) Tuesday
 - (b) Wednesday
 - (c) Saturday
 - (d) Friday

13. My watch was 3 minutes slow at 5 p.m. Tuesday and it was 5 minutes fast at 11 p.m. Wednesday. When did it give correct time?
 - (a) Wednesday 4.15 a.m.
 - (b) Wednesday 7.30 a.m.
 - (c) Tuesday 3.45 p.m.
 - (d) None of these

14. A clock gains 10 minutes in every 24 hrs. It is set right on Monday at 8 a.m. What will be the correct time on the following Wednesday, when the watch indicates 6 p.m.?
 - (a) 5.36 p.m.
 - (b) 5.40 p.m.
 - (c) 4.36 p.m.
 - (d) None of these

15. If a clock takes 22 seconds to strike 12, how much time will it take to strike 6?
 - (a) 10 seconds
 - (b) 12 seconds
 - (c) 14 seconds
 - (d) None of these

16. A clock strikes 4 taking 9 seconds. In order to strike 12 at the same rate, the time taken is:
 - (a) 36 seconds
 - (b) 27 seconds
 - (c) 30 seconds
 - (d) 33 seconds

[Based on MAT (Feb), 2008]

17. The hands of a clock are 10 cm and 7 cm, respectively. The difference between the distance traversed by their extremities in 3 days 5 h is:

(a) 4552.67 cm (b) 4557.67 cm
(c) 4555.67 cm (d) 4559.67 cm

[Based on MAT (May), 2006]

18. A clock gains 15 minutes per day. It is set right at 12 noon. What time will the clock show at 4.00 a.m., the next day?

(a) 4:10 a.m. (b) 4:15 a.m.
(c) 4:30 a.m. (d) 5:00 a.m.

[Based on MAT, 1998]

DIFFICULTY LEVEL-2 (BASED ON MEMORY)

1. An astronomical clock has its dial divided into 24 divisions instead of 12, and the small hand goes round in 24 hrs. The large hand goes round once every hour. If 24th hour is noon, then when are the hands at right angles between 24 and 1?

(a) $23\frac{2}{15}$ minutes past 24

(b) $2\frac{23}{15}$ minutes past 24

(c) $15\frac{15}{23}$ minutes past 24

(d) $15\frac{11}{23}$ minutes past 24

2. At what time are the hands of clock together between 7 p.m. and 8 p.m.?

(a) 7.45.54 (b) 7.36.27

(c) 7.37.49 (d) 7.38.11

3. A watch which gains uniformly, is 5 minutes slow at 8 o'clock in the morning on Sunday, and is 5 minutes 48 seconds fast at 8 p.m. on the following Sunday. When was it correct?

(a) 7:20 p.m. on Tuesday

(b) 9:20 p.m. on Wednesday

(c) 7:20 p.m. on Wednesday

(d) 9:20 p.m. on Tuesday

[Based on FMS (Delhi), 2004]

4. The relative speed of minute-hand with respect to hour-hand is:

(a) $\left(5\frac{1}{2}\right)^\circ$ per minutes (b) $\left(\frac{11}{12}\right)$ degree per minutes

(c) 6° per minutes (d) Both (a) and (b)

5. A clock loses 2 minutes in a hour and another clock gains 2 minutes in every 2 hrs. Both these clocks are set correctly at a certain time on Sunday and both the clocks stop simultaneously on the next day with the time shown being 9 a.m. and 10.06 a.m. What is the correct time at which they stopped?

(a) 9.54 a.m. (b) 9.44 p.m.

(c) 9.46 a.m. (d) 9.44 a.m.

6. My watch was 8 minutes behind at 8 p.m. on Sunday but within a week at 8 p.m. on Wednesday it was 7 minutes ahead of time. During this period at which time this watch has shown the correct time?

(a) Tuesday 10.24 a.m.

(b) Wednesday 9.16 p.m.

(c) It cannot show the correct time during this period

(d) None of the these

7. What are the possible times when a clock shows 35° angle between two hands between 3 p.m. and 4 p.m.?

(a) 20 min 25 seconds (b) 25 min 20 seconds

(c) 22 min 43 seconds (d) None of these

8. What is the area of the face of a clock described by its minute hand between 9 a.m. and 9.35 a.m., if the minute hand is 10 cm long?

(a) $157\frac{1}{7}$ cm² (b) $183\frac{1}{3}$ cm²

(c) $36\frac{2}{3}$ cm² (d) None of these

9. Between 5 and 6, a lady looked at her watch and mistaking the hour hand for the minute hand, she thought that the time was 57 minutes earlier than the correct time. The correct time was:

(a) 12 minutes past 5 (b) 24 minutes past 5

(c) 36 minutes past 5 (d) 48 minutes past 5

Directions (Q. 10 and 11): Read the following information carefully to answer these questions:

A person had left his home at the age of about 14 years. He remembers that the day was Monday. Since then, he has been fasting on every Tuesday. Today he is celebrating his 60th birth anniversary in a five-star hotel with his friends. As today is Tuesday, he is not taking anything except wine. At the end of the party, he discloses that it is his 2400th Tuesday of fasting.

10. Today is October 9, 2001. On which date had he left his home?

- (a) October 10, 1955 (b) October 9, 1955
(c) October 8, 1955 (d) None of these

11. He was born on:

- (a) Wednesday (b) Tuesday
(c) Monday (d) Thursday

12. A watch which gains uniformly, is 5 minutes slow at 8 O'clock in the morning on Sunday, and is 5 minutes 48 seconds fast at 8 p.m. on the following Sunday. When was it correct?

- (a) 7.20 p.m. on Tuesday
(b) 9.20 p.m. on Wednesday
(c) 7.20 p.m. on Wednesday
(d) 9.20 p.m. on Tuesday

13. A mechanical grandfather clock is at present showing 7 hrs 40 minutes 6 seconds. Assuming that it loses 4 seconds in every hour, what time will it show after exactly $6\frac{1}{2}$ hrs.

- (a) 2 hrs 9 minutes 40 seconds
(b) 2 hrs 10 minutes 6 seconds
(c) 14 hrs 9 minutes 34 seconds
(d) 14 hrs 10 minutes 32 seconds

14. Imagine that your watch was correct at noon, but then it began to lose 30 minutes each hour. It now shows 4 p.m., but it stopped 5 hrs ago. What is the correct time now?

- (a) 9.30 p.m. (b) 11 p.m.
(c) 1 a.m. (d) 1.30 a.m.

15. Two clocks are set right at 10 a.m. One gains 20 seconds and the other loses 40 seconds in 24 hrs. What will be the true time when the first clock indicates 4 p.m. on the following day?

- (a) $3.59\frac{2521}{4321}$ p.m. (b) $3.31\frac{1}{471}$ p.m.
(c) $3.59\frac{7}{12}$ p.m. (d) $3.57\frac{2521}{4321}$ p.m.

16. How much does a watch lose per day, if its hands coincide every 64 min?

- (a) 96 minutes (b) 90 minutes
(c) $36\frac{5}{11}$ minutes (d) $32\frac{8}{11}$ minutes

[Based on FMS (MS), 2006]

17. If March 1, 2006 was Wednesday, which day was it on March 1, 2002?

- (a) Wednesday (b) Thursday
(c) Friday (d) Saturday

[Based on FMS, 2009]

18. A watch loses $2\frac{1}{2}$ minutes per day. It is set right at 1 p.m.

on March 15. Let n be the positive correction, in min, to

be added to the time shown by the watch at a given time. When the watch shows 9 a.m. on March 21, n equals:

- (a) $14\frac{14}{23}$ (b) $14\frac{1}{14}$
(c) $13\frac{101}{115}$ (d) $13\frac{83}{115}$

[Based on FMS, 2010]

19. A man on his way to dinner shortly after 6:00 p.m. observes that the hands of his watch form an angle of 110° . Returning before 7:00 p.m. he notices that again the hands of his watch form an angle of 110° . The number of minutes that he has been away is:

- (a) $36\frac{2}{3}$ (b) 40
(c) 42 (d) 42.4

[Based on FMS, 2010]

20. The times between 7 and 8 o'clock, correct to the nearest minute, when the hands of a clock will form an angle of 84 degrees are:

- (a) 7:23 and 7:53 (b) 7:20 and 7:50
(c) 7:22 and 7:53 (d) 7:23 and 7:52

[Based on FMS, 2011]

21. Number of times the hands of a clock are in a straight line every day is:

- (a) 44 (b) 24
(c) 42 (d) 22

[Based on IIFT, 2005]

22. Three Vice Presidents (VPs) regularly visit the plant on different days. Due to labour unrest, VP (HR) regularly visits the plant after a gap of 2 days. VP (Operations) regularly visits the plant after a gap of 3 days. VP (Sales) regularly visits the plant after a gap of 5 days. The VPs do not deviate, from their individual schedules. CEO of the company meets the VPs when all the three VPs come to the plant together. CEO is on leave from January 5, to January 28, 2012. Last time, CEO met the VPs on January 3, 2012. When is the next time CEO will meet all the VPs?

- (a) February 6, 2012 (b) February 7, 2012
(c) February 8, 2012 (d) February 9, 2012

[Based on XAT, 2012]

23. At what time between 4 p.m. and 5 p.m. will the hands of a clock coincide?

- (a) $21\frac{9}{11}$ minutes past 4 p.m.
(b) $1\frac{1}{11}$ minutes past 4 p.m.
(c) $21\frac{9}{11}$ minutes before 5 p.m.
(d) $1\frac{1}{11}$ minutes before 5 p.m.

[Based on ATMA, 2008]

24. In a clock having a circular scale of twelve hrs, when time changes from 7.45 a.m. to 7.47 a.m. by how many degrees the angle formed by the hour hand and minute hand changes?

- (a) 10 (b) 11
(c) 12 (d) 15

[Based on XAT, 2010]

Answer Keys

DIFFICULTY LEVEL-1

1. (b) 2. (a) 3. (c) 4. (d) 5. (c) 6. (c) 7. (c) 8. (a) 9. (a) 10. (c) 11. (d) 12. (a) 13. (a)
14. (a) 15. (a) 16. (b) 17. (b) 18. (a)

DIFFICULTY LEVEL-2

1. (c) 2. (d) 3. (c) 4. (d) 5. (d) 6. (a) 7. (c) 8. (b) 9. (b) 10. (a) 11. (d) 12. (c) 13. (a)
14. (c) 15. (a) 16. (d) 17. (c) 18. (a) 19. (b) 20. (a) 21. (a) 22. (c) 23. (a) 24. (b)

Explanatory Answers

DIFFICULTY LEVEL-1

1. (b) Watch gains 3 minutes in 48 hrs. Therefore, after 16 hrs, it will show the correct time.

2. (a) In 60 minutes, the hour hand moves through an angle of 30°

\therefore In 65 minutes, the hour hand will move through an angle of $\frac{30}{60} \times 65 = 32.5^\circ$

In 60 minutes, the minute hand moves through an angle of 360°

\therefore In 65 minutes, the minute hand moves through an angle of $\frac{360}{60} \times 65 = 390^\circ = 360^\circ + 30^\circ$

\Rightarrow Difference of the angles made by the hour hand and the minute hand = 2.5° (In 65 minutes)

Now an angle of 30° is made by the minute hand in 5 minutes

\therefore An angle of 2.5° is made by the minute hand in $\frac{5}{30}$

$\times 2.5$ minutes, i.e., $\frac{2.5}{6}$ minutes, i.e., 25 seconds.

3. (c) The month of February 1984 had five Wednesdays. Only in a leap year this is possible. Seven leap years have to go by before this situation can occur again, because in each of the leap years the 29th would fall on a different weekday. Seven leap years means

$7 \times 4 = 28$ years have to pass after 1984. Therefore it will be the year 2012 when a February again has five Wednesdays.

4. (d) Year 2000 is a leap year, hence there are two odd days which will shift the whole calendar of the year 2001 by 2 days.

5. (c)

6. (c) In 2028, 2056 and 2084, i.e., after every 28 years.

7. (c) Since between 2 a.m. and 3 a.m. (2 p.m. and 3 p.m.) and 8 a.m. and 10 a.m. (8 p.m. and 10 p.m.) two hands of a clock make 90° angle only 3 times in rest of the each hour two hands make 90° angle 2 times.

8. (a)

9. (a) In an ordinary year, Feb. has no odd day

\therefore Feb. and March begin on same day of week

Also we know that, November and March begin on same day of the week.

10. (c) Number of days from January 25, 1994 to March 2, 1994 is

Jan	Feb	March	
6	+ 28	+ 2	= 36

\therefore Number of odd days = 1

∴ Day on January 25, 1994 is one day before the day on March 2, 1994

But March 2, 1994 was on Wednesday

∴ January 25, 1994 was on Tuesday.

11. (d) Starting with 2000, count for number of odd days in successive years till the sum is divisible by 7.

$$\begin{array}{ccccccccc} 2000 & 2001 & 2002 & 2003 & 2004 & & & & \\ 2 & +1 & +1 & +1 & +2 & = & 7 \end{array}$$

∴ No. of odd days up to 2004 = 0

∴ Calendar for 2000 will serve for 2005 also.

12. (a) During the interval we have two leap years as 1992 and 1996 and it contains February of both these years

∴ The interval has $(5 + 2) = 7$ odd days or 0 odd day

Hence, January 7, 1997 was also Tuesday.

13. (a) Time from 5 p.m. Tuesday to 11 p.m. Wednesday = 30 hrs

Clock gains 8 minutes in 30 hrs

$$\begin{aligned} \therefore \text{It gains 3 minutes in } \frac{30}{8} \times 3 \text{ hrs} \\ = 11 \text{ hrs } 15 \text{ minutes} \end{aligned}$$

∴ Correct time is 11 hrs 15 minutes after 5 p.m.

= 4.15 a.m. on Wednesday.

14. (a) Total number of hrs from Monday at 8 a.m. to the following Wednesday at 6 p.m.

$$24 \times 2 + 10 = 58 \text{ hrs}$$

24 hrs 10 minutes of this clock are the same as

24 hrs of a correct clock

$\frac{145}{6}$ hrs of the incorrect clock = 24 hrs of correct clock

$$58 \text{ hrs of the incorrect clock} = \frac{24 \times 6}{145} \times 58 \text{ hrs of}$$

$$\text{correct clock} = 57\frac{3}{5} \text{ hrs of correct clock}$$

Thus, the correct time on the following Wednesday will be 5.36 p.m.

15. (a) In order to hear 12 strikes, there are 11 intervals $(12 - 1)$ and time of each interval is uniform

$$\text{Hence, time to hear each strike is } \frac{22}{11} = 2 \text{ seconds}$$

Now, to hear six strikes, there are $6 - 1$, i.e., $5 \times 2 = 10$ seconds

Hence, it will take 10 seconds for a clock to strike 6.

16. (b) A clock strikes 4 taking 9 seconds.

$$\therefore \text{Time taken to strike 12} = \frac{9 \times 12}{4} = 27 \text{ seconds}$$

17. (b) Distance traversed by the extremity of the minute-

$$\text{hand in one hour} = 2 \times \frac{22}{7} \times 10$$

Distance traversed by the extremity of the minute-hand in 3 days and 5 hrs, i.e., in 77 hrs

$$= 2 \times \frac{22}{7} \times 10 \times 77 = 22 \times 220 = 4840 \text{ cm}$$

Distance traversed by the hour-hand in 12 hrs

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$

Distance traversed by the hour-hand in 77 hrs

$$= \frac{44}{12} \times 77 = \frac{11 \times 77}{3} = \frac{847}{3} = 282.33 \text{ cm}$$

∴ Required difference

$$= 4840 - 282.33 = 4557.67 \text{ cm}$$

18. (a) In 24 hrs a clock gains 15 minutes

∴ In 16 hrs (12 noon to 4 a.m.) a clock gains

$$= \frac{15}{24} \times 16 = 10 \text{ minutes}$$

∴ Required time = 4:10 a.m.

DIFFICULTY LEVEL-2

1. (c) Here angle covered by the large hand in 60 minutes = 360° and angle covered by the small hand in 60 minutes = $\frac{360^\circ}{24} = 15^\circ$. Required time when the hands are at right angles between 24 and 1

$$= \frac{60}{(360-15)} \times 90 \text{ minutes past 24}$$

$$= \frac{60}{345} \times 90 \text{ minutes past 24}$$

$$= \frac{360}{23} \text{ minutes past 24}$$

$$= 15 \frac{15}{23} \text{ minutes past 24}$$

2. (d) $\frac{210}{5.5} = \frac{210}{11} \times 2 = \frac{420}{11} = 38 \frac{2}{11}$ minutes = 38 minutes 11 seconds

Therefore, required time = 7:38:11.

3. (c) From 8 a.m. on Sunday to 8 p.m. on the following Sunday, the watch will be 348 seconds fast
 \therefore There will be difference of 648 seconds in the duration of 180 hrs

\therefore Difference of 300 seconds (5 minutes), so that the watch shows the correct time will be there after

$$\frac{180}{648} \times 300 \text{ hrs, i.e., } \frac{250}{3} \text{ hrs, i.e., } 83 \frac{1}{3} \text{ hrs, i.e., at 7.20 p.m. on Wednesday.}$$

4. (d) Both (a) and (b) are correct.

Relative speed = Speed of minute-hand – Speed of hour-hand

$$= 6^\circ - \left(\frac{1}{2}\right)^\circ = 5\left(\frac{1}{2}\right)^\circ$$

$$\text{and } 1 \text{ min} - \frac{1}{12} \text{ minutes} = \frac{11}{12} \text{ minutes}$$

5. (d) Actually they create a difference of 3 minute per hour and the two watches are showing a difference of 66 minutes. Thus, they must have been corrected 22 hrs earlier

Now, the correct time can be found by comparing any one of the watch

Since, second watch gains 1 min 1 hr so it must show 22 minutes extra than the correct time in 22 hrs

Hence, the correct time can be found by subtracting 22 minutes from 10:06.

6. (a) In 72 hrs my watch gains $(8 + 7) = 15$ minutes. To show the correct time watch must gain 8 minutes.

Since the watch gains 15 minutes in 72×60 minutes.

Therefore, the watch will gain 8 minutes in

$$\frac{72 \times 60 \times 8}{15} \text{ minutes} = \frac{72 \times 60 \times 8}{15} = 38 \text{ hrs } 24 \text{ minutes}$$

7. (c) $\frac{90-35}{5.5} = \frac{55}{11} \times 2 = 10$ minutes

So, the required time = 3:10:00

$$\text{Again, } \frac{90+35}{5.5} = \frac{125}{11} \times 2 = \frac{250}{11} = 22 \frac{8}{11} \text{ minutes} \\ = 22 \text{ minutes } 43 \text{ seconds}$$

8. (b) $\pi \times (10)^2 \times \frac{7}{12} = \frac{22}{7} \times 10 \times 10 \times \frac{7}{12} = 183 \frac{1}{3} \text{ cm}^2$

9. (b)

10. (a)

11. (d)

12. (c) From 8 a.m. on Sunday to 8 p.m. on the following Sunday, the watch will be 348 seconds fast.

\therefore There will be difference of 648 seconds in the duration of 180 hrs.

\therefore Difference of 300 seconds (5 minutes), so that the watch shows the correct time will be there after

$$\frac{180}{648} \times 300 \text{ hrs, i.e., } \frac{250}{3} \text{ hrs, i.e., } 83 \frac{1}{3} \text{ hrs, i.e., at}$$

7.20 p.m. on Wednesday.

13. (a)

14. (c)

15. (a) From 10 a.m. to 4 p.m. on the following day = 30 hrs

Now, 24 hrs 20 seconds of the first clock

= 24 hrs of the current clock

$$\therefore 1 \text{ hour of the first clock} = \frac{24 \times 180}{4321} \text{ hrs}$$

$$\therefore 30 \text{ hrs of the first clock} = \frac{24 \times 180 \times 30}{4321} \text{ hrs}$$

$$\text{Now, } \frac{24 \times 180 \times 30}{4321} \text{ hrs} = 29 \text{ hrs } 59 \frac{2521}{4321} \text{ minutes}$$

\therefore When the first clock indicates 4 p.m. on the following day the true time will be 3 hrs $59 \frac{2521}{4321}$ minutes.

16. (d) 60 minutes are gained in $\frac{60}{55} \times 60 = 65 \frac{5}{11}$ minutes

But they are together after 64 minutes

$$\text{Gain in 64 minutes} = 65 \frac{5}{11} - 64 = 1 \frac{5}{11} = \frac{16}{11} \text{ mins}$$

$$\text{Gain in 24 hrs} = \frac{16 \times 24 \times 60}{11 \times 64} = 32 \frac{8}{11} \text{ mins.}$$

17. (c) Since in between 2002 to 2006, this is a one leap year

$$\therefore \text{Total numbers} = (2006 - 2002) + 1 = 5$$

\therefore Required day = Wednesday - 5 = Friday.

18. (a) When the actual time elapsed in 24 hrs = $24 \times 60 = 1440$ min, the time elapsed on the faulty watch = 1437.5 min
From 1 p.m. on March 15 to 9 a.m. on March 21, the time elapsed on the faulty watch = 140 hrs = $140 \times 60 = 8400$ min.

\therefore The actual time elapsed

$$\begin{aligned} &= \frac{8400 \times 1440}{1437.5} \\ &= \frac{84 \times 144 \times 16}{23} = 8414 \frac{14}{23} \end{aligned}$$

$$\therefore \text{The correction } n = 8414 \frac{14}{23} - 8400 = 14 \frac{14}{23} \text{ mins}$$

19. (b) Let the time after 6 p.m. be x minutes.

The speed of the minute hand is 6° per minutes and the speed of the hour hand is 0.5° per minutes.

Initial distance between the hour and the minute hands at 6:00 p.m. is 180° .

$$\therefore (180 + 0.5x) - (6x) = 110^\circ$$

$$\Rightarrow 180 - 5.5x = 110^\circ$$

$$\therefore x \approx 12.72 \text{ minutes} \approx 12 \text{ minutes } 43 \text{ seconds}$$

Let the time before 7 p.m. be y minutes

$$\therefore 6y - 180 - 0.5y = 110^\circ$$

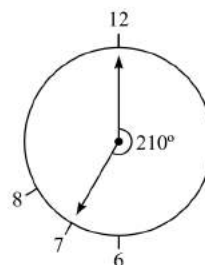
$$\therefore 5.5y - 180 = 110^\circ$$

$$\therefore y \approx 52.72 \text{ minutes} \approx 52 \text{ minutes } 43 \text{ seconds}$$

\therefore The man leaves at 06:12:43 p.m. and returns at 06:52:43 p.m.

\therefore He is away for 40 minutes.

20. (a) At 7 O'clock, the minute hand is 210° behind the hour hand.



The angle between the two hands will be 84° when the minute hand gains $(210^\circ - 84^\circ) = 126^\circ$ or when it gains $(210^\circ + 84^\circ) = 294^\circ$ with respect to hour hand. The relative speed of the minute hand with respect to hour hand = $5 \frac{1}{2}^\circ$ per minutes.

Thus, the time taken to gain 126° and 294° will be

$$= \frac{126}{\frac{11}{2}} = 22 \frac{10}{11} \text{ minutes and, } \frac{294}{\frac{11}{2}} = 53 \frac{6}{11} \text{ minutes}$$

Thus, the angle between the two hands will be 84° at $22 \frac{10}{11}$ minutes past 7 O'clock and $53 \frac{6}{11}$ minutes past

7 O'clock, i.e., approx 7:23 and, 7:53.

21. (a) The hands are in straight line twice in one hour

In 12 hrs the hands of a clock are in straight line $(11 \times 2) = 22$ times.

Hence, in one day it will be 44 times.

22. (c) After January 3, they will be together on January 15, January 27 and February 8, 2012

Since CEO is on leave up to January 28, he will meet the VPs on February 8, 2012

23. (a) At 4 O'clock, the hour hand is at 4 and the minute hand is at 12 i.e., they are 20 minutes space apart.

To be together, the minute hand have to gain 20 minutes over the hour hand

We know that 55 minutes are gained in 60 minutes

$$\therefore 20 \text{ m are gained in } \frac{60}{55} \times 20 = \frac{240}{11} = 21 \frac{9}{11} \text{ minutes}$$

i.e., $21 \frac{9}{11}$ minutes past 4 p.m.

24. (b) A minute hand in 60 minutes makes an angle of 360°

\therefore A minute hand in 45 minutes makes an angle of $(6 \times 45) = 270$.

An hour hand in 12 hrs make an angle of 360°

\therefore An hour hand in $\frac{31}{4}$ hrs make an angle of

$$\left(\frac{30 \times 31}{4} \right)^\circ = 232.5'$$

\therefore Angle between hour hand and minute hand at 7:45 a.m. is 37.5. Likewise calculate for 7:47 and calculate the difference.