2 Motion

Section-A : JEE Advanced/ IIT-JEE

A	**1.**	2R, πR	**2.**	d/v	**3.**	0.6 m/s				
B	**1.**	T	**2.**	T	**3.**	F				
C	**1.**	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(a)
	6.	(b)	**7.**	(a)	**8.**	(a)	**9.**	(c)		
D	**1.**	(b)	**2.**	(a, c, d)	**3.**	(a, b, c)				
E	**1.**	$\frac{\alpha\beta}{\alpha+\beta}t;\ \frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}t^2$	**2.**	**(i)** 0 ; **(ii)** 0	**3.**	No	**4.**	mid point of AB, 3.53 sec.		
	5.	17.32, 11.547 m from B	**6.**	1 sec, $(5\sqrt{3}, 5)$ in metres						
	7.	**(a)** $\frac{u^2 \sin 2\alpha}{g\cos\theta}$ **(b)** $\frac{u\cos(\alpha+\theta)}{\cos\theta}$	**8.**	$\vec{u} = (3.75\hat{i}+6.25\hat{j})$ m/s, t = 1 sec.	**9.**	45°, 2 m/sec.				
H	**1.**	**(b)**								
I	**1.**	**5**	**2.**	**5**	**3.**	**8**				

Section-B : JEE Main/ AIEEE

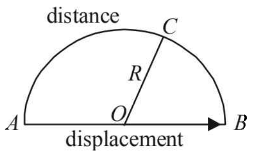
1.	(c)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(b)	**6.**	(a)
7.	(c)	**8.**	(a)	**9.**	(b)	**10.**	(d)	**11.**	(c)	**12.**	(d)
13.	(c)	**14.**	(d)	**15.**	(a)	**16.**	(d)	**17.**	(c)	**18.**	(b)
19.	(b)	**20.**	(a)	**21.**	(d)	**22.**	(d)	**23.**	(c)	**24.**	(c)
25.	(a)	**26.**	(a)	**27.**	(d)	**28.**	(c)	**29.**	(b)	**30.**	(b)
31.	(c)	**32.**	(b)								

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Displacement $= AOB = 2R$

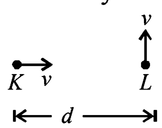
Distance $= ACB = \pi R$



2. The relative velocity of K w.r.t L along the line KL is

$$\vec{v}_{KL} = \vec{v}_K - \vec{v}_L = \vec{v}_K + (-\vec{v}_L)$$
$$= v$$

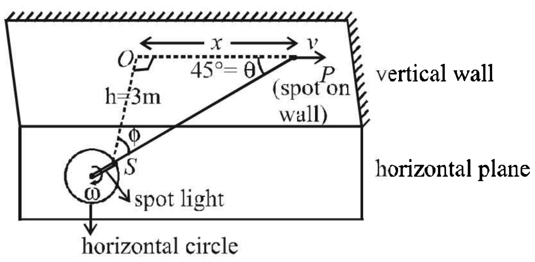
($\because$ the component of velocity of L along KL is zero)



The displacement of K till K and L meet is d.

$\therefore$ Time taken for K and L to meet will be $= \frac{d}{v}$

3.



The velocity (v) of spot $= dx/dt$

and the angular speed (ω) of spot light $= \frac{d\phi}{dt}$

From ΔSOP,

$\tan\phi = \frac{x}{h}$ $\quad\therefore\quad x = h\tan\phi$

$\therefore\quad \frac{dx}{dt} = h\sec^2\phi\,\frac{d\phi}{dt}$ $\quad\therefore\quad v = (h\sec^2\phi)\,\omega$

$\therefore\quad v = 3\sec^2 45° \times 0.1$ $\quad[\because \theta+\phi = 90°]$

$\therefore\quad v = 3\times 2\times 0.1 = 0.6$ m/s

B. True/False

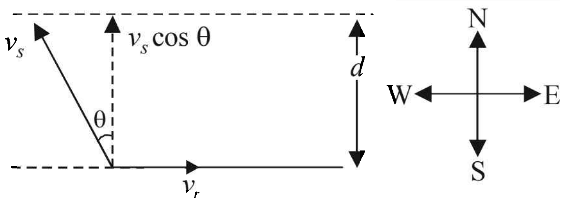
1. **KEY CONCEPT**
When the two balls are thrown vertically upwards with the same speed u then their final speed v at the point of projection is $v^2 - u^2 = 2 \times g \times s$
Here, $s = 0$
$\therefore\quad v = u$ for both the cases

2. T.E. = P.E. + K.E.
T.E. = Constant
At P, K.E. is minimum and P.E. is maximum. Since K.E. is minimum speed is also minimum.

3. The pressure exerted will be different because one train is moving in the direction of earth's rotation and other in the opposite direction.

C. MCQs with ONE Correct Answer

1. (a)

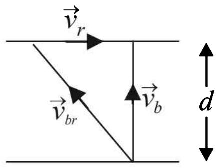


Time taken to cross the river $t = \frac{d}{v_s \cos\theta}$

NOTE : For time to be minimum, $\cos\theta$ = maximum
$\Rightarrow \theta = 0°$
The swimmer should swim due north.

2. (b) Shortest route corresponds to $\vec{v}_b$ perpendicular to river flow

$\therefore\ t = \frac{d}{v_b} = \frac{d}{\sqrt{v_{br}^2 - v_r^2}}$



or $\frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$

$\Rightarrow v_r = 3$ km/h

3. (b) $|\text{Average velocity}| = \frac{|\text{displacement}|}{\text{time}}$

$= \frac{2r}{t} = 2 \times \frac{1}{1} = 2$ m/s.

4. (a) KEY CONCEPT
Before hitting the ground, the velocity v is given by $v^2 = 2gd$ (quadratic equation and hence parabolic path) Downwards direction means **negative** velocity. After collision, the direction become positive and velocity decreases.

Further, $v'^2 = 2g \times \left(\frac{d}{2}\right) = gd$;

$\therefore\ \left(\frac{v}{v'}\right) = \sqrt{2}$ or $v = v'\sqrt{2} \Rightarrow v' = \frac{v}{\sqrt{2}}$

As the direction is reversed and speed is decreased graph (a) represents these conditions correctly.

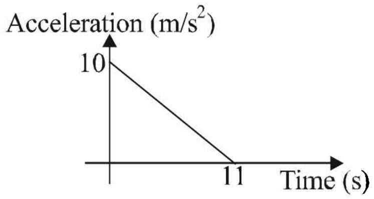
5. (a) $s_n = \frac{a}{2}(2n-1)$;

$s_{n+1} = \frac{a}{2}[2(n+1)-1] = \frac{a}{2}(2n+1)$

$\frac{s_n}{s_{n+1}} = \frac{2n-1}{2n+1}$

6. (b) Change in velocity = area under the graph

$= \frac{1}{2} \times 10 \times 11 = 55\ \text{m/s}$



Since, initial velocity is zero, final velocity is 55 m/s.

7. (a) The equation for the given v-x graph is

$v = -\frac{v_0}{x_0}x + v_0$ ... (i)

$\frac{dv}{dx} = -\frac{v_0}{x_0}$

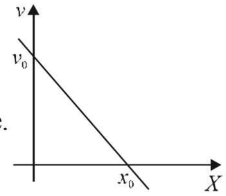
$\therefore\ v\frac{dv}{dx} = -\frac{v}{x_0} \times v = -\frac{v_0}{x_0}\left[-\frac{v_0}{x_0}x + v_0\right]$ from (1)

$\therefore\ a = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}$ .... (ii) $\left[\because a = v\frac{dv}{dx}\right]$

On comparing the equation (ii) with equation of a straight line

$y = mx + c$

we get $m = \frac{v_0^2}{x_0^2} = +\text{ve}$,



i.e. $\tan\theta = +$ ve, i.e., θ is acute.

Also $c = -\frac{v_0^2}{x_0^2}$,

i.e., the y-intercept is negative
The above conditions are satisfied in graph (a).

8. (a) At $t = 0$, the relative velocity will be zero.

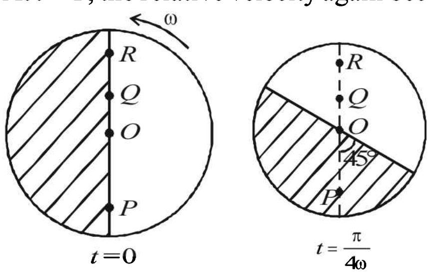
At $t = \frac{T}{4}$, the relative velocity will be maximum in magnitude.

At $t = \frac{T}{2}$, the relative velocity will be zero.

At $t = \frac{3T}{4}$, the relative velocity will be maximum in magnitude

At $t = T$, the relative velocity again becomes zero.

9. (c)



The x-coordinate of $P = v_x \times t = \omega R \times \frac{\pi}{4\omega} = \frac{\pi R}{4}$

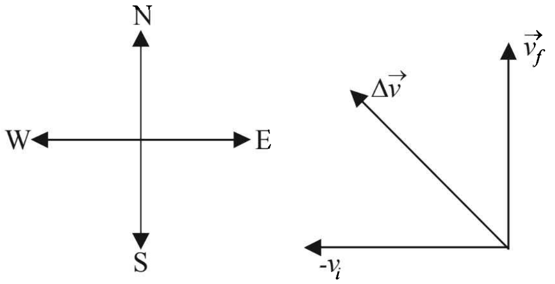
This horizontal distance travelled will be greater than any point on the disc between O and P. Therefore the landing will be in unshaded area. In the same way, the horizontal distance travelled by Q is always less than that of any point between O and R. Therefore the landing will be in unshaded area.

D. MCQs with ONE or MORE THAN ONE Correct

1. (b) Average acceleration

$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t} = \frac{\vec{v_f} + (-\vec{v_i})}{t} = \frac{\Delta\vec{v}}{t}$

To find the resultant of $\vec{v_f}$ and $-\vec{v_i}$, we draw a diagram

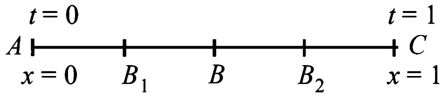


$|\Delta\vec{v}| = \sqrt{v_f^2 + v_i^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ m/s

$|\vec{a}| = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$

Since, $|\vec{v_f}| = |\vec{-v_i}|$, $\therefore\ \vec{v}$ is directed towards N – W.

2. **(a, c, d)** **Note :** α cannot remain positive for all t in the interval $0 \le t \le 1$. This is because since the body starts from rest, it will first accelerate. Finally it stops therefore α will become negative. Therefore α will change its direction. Options (a) and (d) are correct.



Let the particle accelerate uniformly till half the distance (A to B) and then retard uniformly in the remaining half distance (B to C).

The total time is 1 sec. Therefore the time taken from A to B is 0.5 sec.

For A to B

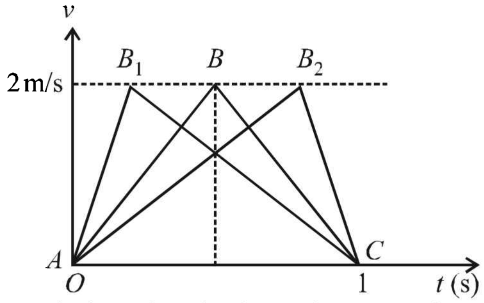
$S = ut + \frac{1}{2}at^2 \qquad 0.5 = 0 + \frac{1}{2} \times a \times (0.5)^2$

$\therefore\ a = 4$ m/s^2

$\therefore\ V_B = 0 + 4 \times 0.5 = 2$ m/s^2

Note : Now, if the particle accelerates till B_2 then for covering the same total distance in same time, acceleration should be less than 4 m/s^2 but |deceleration| should be greater than 4 m/s^2. And if the particle accelerates till B_1, then for covering the same total distance in the same time, the acceleration should be greater than 4 m/s^2 and | deceleration | < 4 m/s^2.

The same is depicted by the graph.



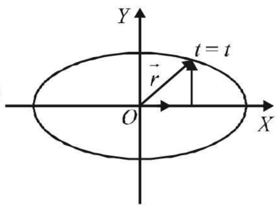
So, the | acceleration | must be greater than or equal to 4 m/s^2 at some point or points in the path.

3. **(a, b, c)** $x = a\cos pt \Rightarrow \cos(pt) = \frac{x}{a}$ ... (1)

$y = b\sin pt \Rightarrow \sin(pt) = \frac{y}{b}$ ... (2)

Squaring and adding (1) and (2), we get,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



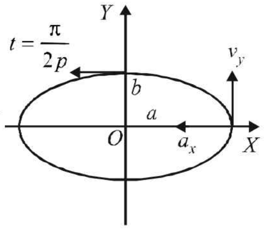
$\therefore$ The path of the particle is an ellipse.

From the given equations we can find,

$\frac{dx}{dt} = v_x = -ap\sin pt;\ \frac{d^2x}{dt^2} = a_x = -ap^2\cos pt$

$\frac{dy}{dt} = v_y = \text{pb}\cos pt$

and $\frac{d^2y}{dt^2} = a_y = -bp^2\sin pt$



At time $t = \frac{\pi}{2\text{p}}$ or $pt = \frac{\pi}{2}$

a_x and v_y become zero (because $\cos\frac{\pi}{2} = 0$). Only v_x and a_y are left,

or we can say that velocity is along negative x-axis and acceleration along negative y-axis.

Hence, at $t = \frac{\pi}{2p}$, velocity and acceleration of the particle are normal to each other.

At $t = t$, position of the particle $\vec{r}(t) = x\hat{i} + y\hat{j}$ $= a\cos pt\,\hat{i} + b\sin pt\,\hat{j}$ and acceleration of the particle is

$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} = -p^2[a\cos pt\,\hat{i} + b\sin pt\,\hat{j}]$

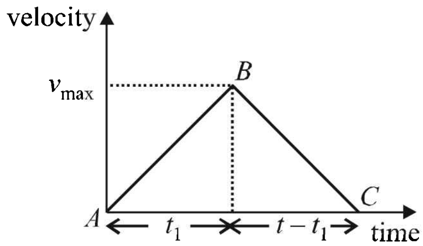
$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$

Therefore, acceleration of the particle is always directed towards origin.

At $t = 0$, particle is at $(a, 0)$ and at $t = \frac{\pi}{2p}$, particle is at $(0, b)$. Therefore, the distance covered is one fourth of the elliptical path and not a.

E. Subjective Problems

1.



Distance travelled = area of ΔABC

$= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times t \times v_{max}$

$= \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha + \beta}t = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2$

2. $\sqrt{x} = t - 3 \Rightarrow x = t^2 + 9 - 6t\ \therefore\ v = \frac{dx}{dt} = 2t - 6$

(i) For velocity to be zero, $2t - 6 = 0 \Rightarrow t = 3$ sec.

The displacement is $x = 9 + 9 - 6 \times 3 = 0$

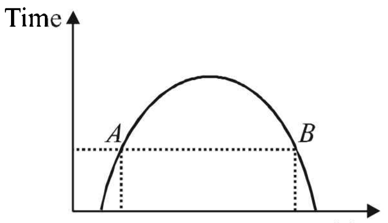
(ii) $a = \frac{dv}{dt} = 2$ $\therefore$ At $t = 0, v = -6\ ms^{-1}$

At $t = 6$ sec, $v = 6\ ms^{-1}$

$\therefore$ Work done = Change in K.E. = $[K.E_f - K.E_i]$

$$= \frac{1}{2}m(6)^2 - \frac{1}{2}m(6)^2 = 0$$

3.



As shown, at a given instant of time, the body is at two different positions A and B which is not possible.

4. If a body drops from a height H above the ground then the time taken by it to reach the ground

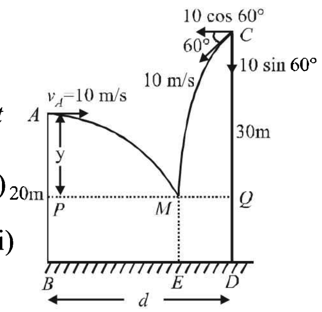
$$t = \sqrt{\frac{2H}{g}} \quad \therefore \quad t = \sqrt{\frac{2 \times 61.25}{9.8}} = 3.53\,s$$

5. (i) Let t be the time taken for collision.

For mass *m* thrown horizontally from *A*.

For horizontal motion

$PM = 10\,t$ ... (i)



For vertical motion

$u_y = 0; s_y = y; a_y = g; t_y = t$

$\therefore\ y = \frac{1}{2}gt^2$ ... (ii)

$v_y = u_y + a_y t = gt$ ... (iii)

For mass 2m thrown from *C*

For horizontal motion $QM = [10 \cos 60°]\,t$

$QM = 5\,t$ ... (iv)

For vertical motion $v_y = 10 \sin 60° = 5\sqrt{3}$; $a_y = g$

$s_y = y + 10; t_y = t$

Now, $v_y = 5\sqrt{3} + gt$ ... (v)

and $(s_y) = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow\ y + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2$ ... (vi)

From (ii) and (vi)

$$\frac{1}{2}gt^2 + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2 \Rightarrow t = \frac{2}{\sqrt{3}}\text{sec}$$

$$\therefore\ BD = PM + MQ = 10\,t + 5\,t = 15\,t = 15 \times \frac{2}{\sqrt{3}}$$

$$= 10\sqrt{3} = 17.32\text{ m}$$

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

$m \times 10 - 2\,m\,10 \cos 60° = 3\,m \times v_x$

$\Rightarrow\ 10\,m - 10\,m = 3\,m \times v_x \Rightarrow v_x = 0$

Since, the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E.

$$BE = PM = 10\,t = 10 \times \frac{2}{\sqrt{3}} = 11.547\text{ m}$$

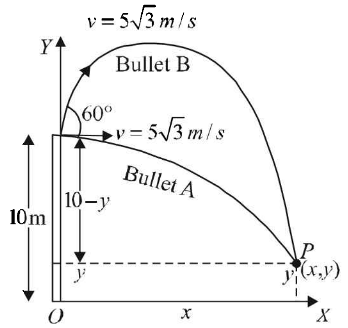
6. **For Bullet A.** Let t be the time taken by bullet A to reach P.

Vertical motion

$u_y = 0; s_y = 10 - y; a_y = 10\text{ m/s}^2; t_y = t$

$s_y = u_y t + \frac{1}{2}a_y t^2$

$10 - y = 5\,t^2$ ... (i)



Horizontal motion

$x = 5\sqrt{3}\,t$ ... (ii)

For bullet B.

Let $(t + t')$ be the time taken by bullet B to reach P

Vertical Motion

Let us consider upward direction negative and downward positive. Then

$u_y = -5\sqrt{3}\sin 60° = -7.5\text{ m/s}, a_y = +10\text{ m/s}^2$

$s_y = +(10 - y); t_y = t + t',\ s_y = u_y t + \frac{1}{2}a_y t^2$

$10 - y = -7.5\,(t + t') + 5\,(t + t')^2$ ... (iii)

Horizontal motion

$x = (5\sqrt{3}\cos 60°)\,(t + t')$

$\Rightarrow\ 5\sqrt{3}\,t + 5\sqrt{3}\,t' = 2x$ ... (iv)

Substituting the value of x from (ii) in (iv), we get

$5\sqrt{3}\,t + 5\sqrt{3}\,t' = 10\sqrt{3}\,t$

$\Rightarrow\ t = t'$

Putting $t = t'$ in eq. (iii)

$y - 10 = 15\,t - 20\,t^2$ ... (v)

Adding (i) and (v)

$0 = 15\,t - 15\,t^2 \Rightarrow t = 1$ sec.

(ii) Putting $t = 1$ in eq. (ii), we get $x = 5\sqrt{3}$

Putting $t = 1$ in eq. (i), we get $y = 5$

Therefore, the coordinates of point P are $(5\sqrt{3}, 5)$ in metres.

7. **(a)** u is the relative velocity of the particle with respect to the box. Resolve u.

u_x is the relative velocity of particle with respect to the box in x-direction.

u_y is the relative velocity with respect to the box in y-direction.

Since, there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground also.

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Y-direction motion (Taking relative terms w.r.t. box)

$u_y = +u \sin\alpha$

$a_y = -g\cos\theta$

$s_y = 0$ (activity is taken till the time the particle comes back to the box.)

$t_y = t$

$s_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = (u \sin\alpha)\, t - \frac{1}{2} g\cos\theta \times t^2$

$\Rightarrow \quad t = 0$ or $t = \dfrac{2u\sin\alpha}{g\cos\theta}$

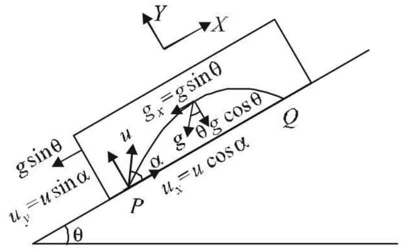
X - direction motion (Taking relative terms w.r.t. box)

$u_x = +u\cos\alpha\,;\ a_x = 0,\ t_x = t,\ s_x = s_x$

$s_x = u_x t + \frac{1}{2} a_x t^2 \Rightarrow s_x = u\cos\alpha \times \dfrac{2u\sin\alpha}{g\cos\theta} = \dfrac{u^2\sin 2\alpha}{g\cos\theta}$

(b) For the observer (on ground) to see the horizontal displacement to be zero, the distance travelled by the box in time $\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)$ should be equal to the range of the particle.

Let the speed of the box at the time of projection of particle be U. Then for the motion of box with respect to ground.



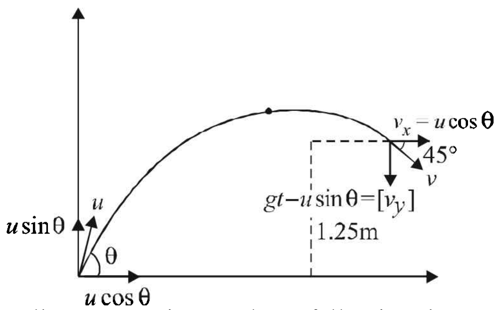
$u_x = -U;\ a_x = -g\sin\theta\,;\ t_y = \dfrac{2u\sin\alpha}{g\cos\theta}\,;\ s_x = \dfrac{-u^2\sin 2\alpha}{g\cos\theta}$

$s_x = u_x t + \frac{1}{2} a_x t^2$

$\dfrac{-u^2\sin 2\alpha}{g\cos\theta} = -U\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right) - \dfrac{1}{2} g\sin\theta \left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)^2$

On solving we get $U = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

8. Let 't' be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal.



According to question, we have following three conditions.

(i) Vertical displacement of stone is 1.25 m.

Therefore, $1.25 = (u\sin\theta)\, t - \frac{1}{2} g t^2$

where g = 10 m/s^2

or $(u\sin\theta)\, t = 1.25 + 5t^2$ ... (i)

(ii) Horizontal displacement of stone = 3 + displacement of object A.

Therefore, $(u\cos\theta)t = 3 + \frac{1}{2} a t^2$

where $a = 1.5$ m/s^2

or $(u\cos\theta)t = 3 + 0.75\, t^2$ ... (ii)

(iii) Horizontal component of velocity of stone = vertical component (because velocity vector is inclined at 45° with horizontal.)

Therefore $(u\cos\theta) = gt - (u\sin\theta)$ ... (iii)

(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion. Therefore, $gt > u\sin\theta$. In upward motion $u\sin\theta > gt$). Multiplying equation (iii) with t we can write,

$(u\cos\theta)\, t + (u\sin\theta)\, t = 10\, t^2$ ... (iv)

Now, (iv) – (ii) – (i) gives $4.25\, t^2 - 4.25 = 0$ or $t = 1$s

Substituting $t = 1$s in (i) and (ii), we get,

$u\sin\theta = 6.25$ m/s or $u_y = 6.25$ m/s

and $u\cos\theta = 3.75$ m/s.

or $u_x = 3.75$ m/s therefore $\vec{u} = u_x\hat{i} + u_y\hat{j}$

or $\vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s

9. **(a)** Let the ball strike the trolley at B. Let

$\vec{v}_{BG}$ = velocity of ball w.r.t. ground

$\vec{v}_{TG}$ = velocity of trolley w.r.t. ground

$\therefore$ Velocity of ball w.r.t. trolley

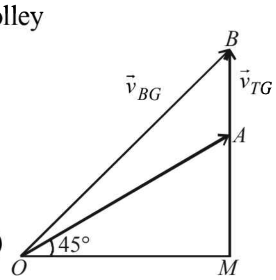
$\vec{v}_{BT} = \vec{v}_{BG} - \vec{v}_{TG}$ ... (i)

From triangle OAB

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\therefore \quad \overrightarrow{OA} + \vec{v}_{TG} = \vec{v}_{BG}$

$\therefore \quad \overrightarrow{OA} = \vec{v}_{BG} - \vec{v}_{TG}$ ... (ii)



From (i) and (ii) $\overrightarrow{OA} = \vec{v}_{BT}$

$\Rightarrow$ velocity of ball w.r.t. trolley makes an angle of 45° with the X-axis

(b) Here $\theta = 45°$

$\therefore \quad \phi = \dfrac{4\theta}{3} = \dfrac{4\times 45}{3} = 60°$

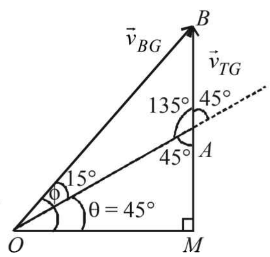
In ΔOMA,

$\theta = 45° \Rightarrow \angle OAM = 45°$

$\therefore \quad \angle OAB = 135°$

Also $\angle BOA = 60° - 45° = 15°$

Using sine law in ΔOBA



$\dfrac{v_{BG}}{\sin 135°} = \dfrac{v_{TG}}{\sin 15°} \Rightarrow v_{BG} = 2$ m/s

H. Assertion & Reason Type Questions

1. **(b)** Statement-1 is true. For a moving observer, the near by objects appear to move in the opposite direction at a large speed. This is because the angular speed of the near by object w.r.t observer is large. As the object moves away the angular velocity decreases and therefore its speed seems to be less. The distant object almost remains stationary.

Statement-2 is the concept of relative velocity which states that

$\vec{v}_{21} = \vec{v}_{2G} - \vec{v}_{1G}$

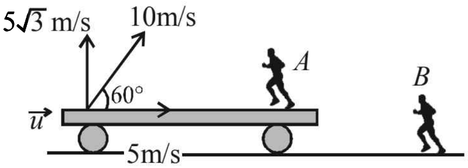
where G is the laboratory frame.

Thus both the statement are true but statement-2 is not the correct explanation of statement-1.

I. Integer Value Correct Type

1. **5** From the perspective of observer A, considering vertical motion of the ball from the point of throw till it reaches back at the initial height.

$U_y = +5\sqrt{3}$ m/s, $S_y = 0$, $a_y = -10\text{m/s}^2$, $t = ?$



Applying $S = ut + \frac{1}{2}at^2 \Rightarrow 0 = 5\sqrt{3}t - 5t^2$

$\therefore \quad t = \sqrt{3}$ sec

Considering horizontal motion from the perspective of observer B. Let u be the speed of train at the time of throw.

The horizontal distance travelled by the ball = $(u + 5)\sqrt{3}$.

The horizonal distance travelled by the boy

$= \left[u\sqrt{3} + \frac{1}{2}a(\sqrt{3})^2\right] + 1.15$

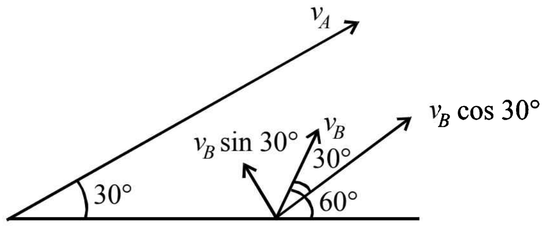
As the boy catches the ball therefore

$(u+5)\sqrt{3} = u\sqrt{3} + \frac{3}{2}a + 1.15$

$\therefore 5\sqrt{3} = 1.5a + 1.15 \quad \therefore 7.51 = 1.5a$

$\therefore a \approx 5 \text{ m/s}^2$

2. **5**



Here

$v_A = v_B \cos 30°$

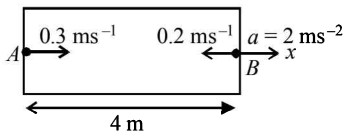
$\therefore 100\sqrt{3} = v_B \times \frac{\sqrt{3}}{2}$

$\therefore v_B = 200 \text{ ms}^{-1}$

$\text{Time} = \frac{\text{displacement}}{\text{velocity}}$

$\therefore t_0 = \frac{500}{v_B \sin 30°} = \frac{500}{200 \times \sin 30°} = 5 \text{ sec}$

3. **8**



For ball A

$u_1 = 0.3 \text{ ms}^{-1}$, $a_1 = -2\text{ms}^{-2}$, $s_1 = x$, $t_1 = t$

$\therefore \quad s_1 = u_1 t_1 + \frac{1}{2}a_1 t_1^2$

$x = 0.3t - t^2$ ...(1)

For ball B

$u_2 = 0.2 \text{ ms}^{-1}$, $a_2 = 2\text{ms}^{-2}$, $s_2 = 4 - x$, $t_2 = t$

$\therefore \quad s_2 = u_2 t_2 + \frac{1}{2}a_2 t_2^2$

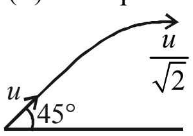
$4 - x = 0.2t + t^2$ ...(2)

From (1) and (2) $t = 8$ sec

Section-B JEE Main/ AIEEE

1. **(c)** Let u be the speed with which the ball of mass m is projected. Then the kinetic energy (E) at the point of projection is

$E = \frac{1}{2}mu^2$ ...(i)

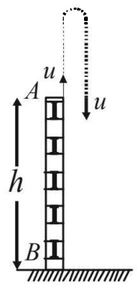


When the ball is at the highest point of its flight, the speed of the ball is $\frac{u}{\sqrt{2}}$ (Remember that the horizontal component of velocity does not change during a projectile motion).

$\therefore$ The kinetic energy at the highest point

$= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\frac{mu^2}{2} = \frac{E}{2}$ [From (i)]

2. **(b)** Ball A is thrown upwards from the building. During its downward journey when it comes back to the point of throw, its speed is equal to the speed of throw. So, for the journey of both the balls from point A to B.



We can apply $v^2 - u^2 = 2gh$.

As u, g, h are same for both the balls, $v_A = v_B$

3. **(c)** **Case-1 :** $u = 50 \times \frac{5}{18}$ m/s, $v = 0$, $s = 6\text{m}$, $a = a$

$v^2 - u^2 = 2as \Rightarrow 0^2 - \left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$

$\Rightarrow -\left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$ ....(i)

Case-2 : $u = 100 \times \frac{5}{18}$ m/sec , $v = 0, s = s, a = a$

$\therefore v^2 - u^2 = 2as$

$\Rightarrow 0^2 - \left(100 \times \frac{5}{18}\right)^2 = 2as$

$\Rightarrow -\left(100 \times \frac{5}{18}\right)^2 = 2as \quad \ldots \text{(ii)}$

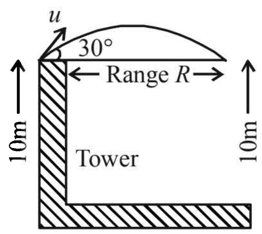
Dividing (i) and (ii) we get

$\frac{100 \times 100}{50 \times 50} = \frac{2 \times a \times s}{2 \times a \times 6} \Rightarrow s = 24\text{m}$

4. **(d)** From the figure it is clear that range is required

$R = \frac{u^2 \sin 2\theta}{g}$

$= \frac{(10)^2 \sin(2 \times 30°)}{10} = 5\sqrt{3}$



5. **(b)** $x = \alpha t^3$ and $y = \beta t^3$

$v_x = \frac{dx}{dt} = 3\alpha t^2$ and $v_y = \frac{dy}{dt} = 3\beta t^2$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4} = 3t^2\sqrt{\alpha^2 + \beta^2}$

6. **(a)** We know that $s = ut + \frac{1}{2}gt^2$, or $h = \frac{1}{2}gT^2 (\because u=0)$

now for $T/3$ second, vertical distance moved is given by

$h' = \frac{1}{2}g\left(\frac{T}{3}\right)^2 \Rightarrow h' = \frac{1}{2} \times \frac{gT^2}{9} = \frac{h}{9}$

$\therefore$ position of ball from ground $= h - \frac{h}{9} = \frac{8h}{9}$

7. **(c)** $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

$\therefore \vec{A} \times \vec{B} = 0$

Angle between them is 0, π, or 2π

from the given options, $\theta = \pi$

8. **(a)** The angle for which the ranges are same is complementary.

Let one angle be θ, then other is $90° - \theta$

$T_1 = \frac{2u \sin\theta}{g}, T_2 = \frac{2u \cos\theta}{g}$

$T_1 T_2 = \frac{4u^2 \sin\theta \cos\theta}{g} = 2R \left(\because R = \frac{u^2 \sin^2\theta}{g}\right)$

Hence it is proportional to R.

9. **(b)** Only option (b) is false since acceleration vector is always radial (i.e . towards the center) for uniform circular motion.

10. **(d)** Speed, $u = 60 \times \frac{5}{18}\text{m/s} = \frac{50}{3}\text{m/s}$

$d = 20\text{m}, u' = 120 \times \frac{5}{18} = \frac{100}{3}\text{m/s}$

Let declaration be a then $(0)^2 - u^2 = -2ad$

or $u^2 = 2ad \quad \ldots (1)$

and $(0)^2 - u'^2 = -2ad'$ or $u'^2 = 2ad' \quad \ldots(2)$

(2) divided by (1) gives, $4 = \frac{d'}{d} \Rightarrow d' = 4 \times 20 = 80\text{m}$

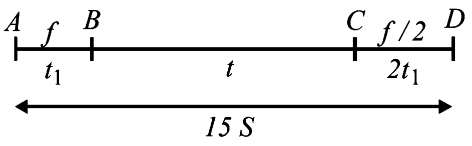
11. **(c)** Yes, the person can catch the ball when horizontal velocity is equal to the horizontal component of ball's velocity, the motion of ball will be only in vertical direction with respect to person for that,

$\frac{v_o}{2} = v_o \cos\theta$ or $\theta = 60°$

12. **(d)** Distance from A to $B = S = \frac{1}{2}ft_1^2 \Rightarrow ft_1^2 = 2S$

Distance from B to $C = (ft_1)t$

Distance from C to $D = \frac{u^2}{2a} = \frac{(ft_1)^2}{2(f/2)} = ft_1^2 = 2S$



$\Rightarrow S + f t_1 t + 2S = 15S \Rightarrow f t_1 t = 12S$

But $\frac{1}{2}f t_1^2 = S$

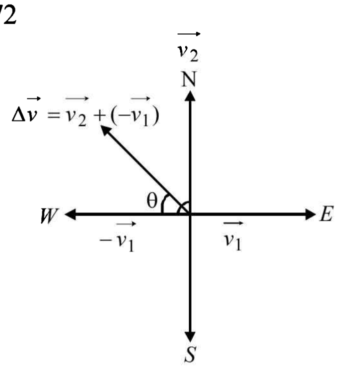
On dividing the above two equations, we get $t_1 = \frac{t}{6}$

$\Rightarrow S = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$

13. **(c)** Average acceleration

$= \frac{\text{change in velocity}}{\text{time interval}}$

$= \frac{\Delta \vec{v}}{t}$



$\vec{v_1} = 5\hat{i}, \vec{v_2} = 5\hat{j}$

$\therefore \vec{a} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{\hat{j} - \hat{i}}{2}$

$\therefore a = \frac{\sqrt{1^2 + (-1)^2}}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}\text{ms}^{-2}$

$\tan\theta = \frac{v_2}{v_1} = \frac{5}{5} = 1 \quad \therefore \theta = 45°$

Therefore the directon is North-west.

14. **(d)** $t = ax^2 + bx$; Diff. with respect to time (t)

$\frac{d}{dt}(t) = a\frac{d}{dt}(x^2) + b\frac{dx}{dt} = a.2x\frac{dx}{dt} + b.\frac{dx}{dt}$

$1 = 2axv + bv = v(2ax + b) \Rightarrow 2ax + b = \frac{1}{v}$.

Again differentiating, $2a\frac{dx}{dt} + 0 = -\frac{1}{v^2}\frac{dv}{dt}$

$\Rightarrow \frac{dv}{dt} = f = -2av^3 \quad \left(\because \frac{dv}{dt} = f = acc\right)$

15. **(a)** $v = \alpha\sqrt{x}$, $\frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha\, dt$

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt \Rightarrow \left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t \Rightarrow 2\sqrt{x} = \alpha t \Rightarrow x = \frac{\alpha^2}{4}t^2$$

16. **(d)** Let u be the velocity with which the particle is thrown and m be the mass of the particle. Then

$$K = \frac{1}{2}mu^2. \qquad ...(1)$$

At the highest point the velocity is u cos 60° (only the horizontal component remains, the vertical component being zero at the top-most point). Therefore kinetic energy at the highest point.

$$K' = \frac{1}{2}m(u\cos 60°)^2 = \frac{1}{2}mu^2\cos^2 60° = \frac{K}{4} \quad \text{[From 1]}$$

17. **(c)** We know that, $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$

Integrating, $\int_0^x dx = \int_0^t v\,dt$

or $x = \int_0^t (v_0 + gt + ft^2)\,dt = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$

or, $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$, $x = v_0 + \frac{g}{2} + \frac{f}{3}$.

18. **(b)** **For the body starting from rest**

$x_1 = 0 + \frac{1}{2}at^2 \Rightarrow x_1 = \frac{1}{2}at^2$

For the body moving with constant speed

$x_2 = vt$

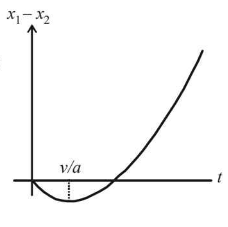
$\therefore\ x_1 - x_2 = \frac{1}{2}at^2 - vt \Rightarrow \frac{d(x_1 - x_2)}{dt} = at - v$

at $t = 0$, $\qquad x_1 - x_2 = 0$

For $t < \frac{v}{a}$; the slope is negative

For $t = \frac{v}{a}$; the slope is zero

For $t > \frac{v}{a}$; the slope is positive



These characteristics are represented by graph (b).

19. **(b)** For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \frac{1}{2}at^2$

We get $y - h = -\frac{1}{2}gt^2 \Rightarrow y = h - \frac{1}{2}gt^2$

The graph between y and t is a parabola with $y = h$ at $t = 0$. As time increases y decreases.

For upward motion.

The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocity just after collision. As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2}gt^2$

All these characteristics are represented by graph (b).

20. **(a)** Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, $t = 10$ s

$$\vec{v} = \vec{u} + \vec{a}t = 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j}) \times 10 = 7\hat{i} + 7\hat{j}$$

$\therefore |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

21. **(d)** $\vec{v} = k(y\hat{i} + x\hat{j}) = v_x\hat{i} + v_y\hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

$\therefore \frac{dx}{dt} = ky$ and $\therefore \frac{dy}{dt} = kx$

$\therefore \frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx \Rightarrow y^2 = x^2 + \text{constant}$

22. **(d)** $s = t^3 + 5 \Rightarrow$ velocity, $v = \frac{ds}{dt} = 3t^2$

Tangential acceleration $a_t = \frac{dv}{dt} = 6t$

Radial acceleration $a_c = \frac{v^2}{R} = \frac{9t^4}{R}$

At $t = 2s$, $a_t = 6 \times 2 = 12\ \text{m/s}^2$

$a_c = \frac{9 \times 16}{20} = 7.2\ \text{m/s}^2$

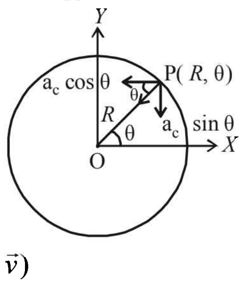
$\therefore$ Resultant acceleration

$= \sqrt{a_t^2 + a_c^2} = \sqrt{(12)^2 + (7.2)^2} = \sqrt{144 + 51.84}$

$= \sqrt{195.84} = 14\ \text{m/s}^2$

23. **(c)** Clearly $\vec{a} = a_c\cos\theta(-\hat{i}) + a_c\sin\theta(-\hat{j})$

$= \frac{-v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$



24. **(c)** $\vec{L} = m(\vec{r} \times \vec{v})$

$$\vec{L} = m\left[v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}\right] \times \left[v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\hat{j}\right]$$

$$= mv_0\cos\theta t\left[-\frac{1}{2}gt\right]\hat{k} = -\frac{1}{2}mgv_0 t^2\cos\theta\,\hat{k}$$

25. (a) $\frac{dv}{dt} = -2.5\sqrt{v} \Rightarrow \frac{dv}{\sqrt{v}} = -2.5\, dt$

Integrating, $\int_{6.25}^{0} v^{-1/2} dv = -2.5\int_0^t dt$

$\Rightarrow \left[\frac{v^{+1/2}}{(1/2)}\right]_{6.25}^{0} = -2.5[t]_0^t$

$\Rightarrow -2(6.25)^{1/2} = -2.5t \Rightarrow t = 2$ sec

26. (a) Total area around fountain

$A = \pi R_{max}^2 = \pi \frac{v^4}{g^2}$

$[\because R_{max} = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin 90°}{g} = \frac{v^2}{g}]$

27. (d) $R = \frac{u^2 \sin^2\theta}{g}$, $H = \frac{u^2 \sin^2\theta}{2g}$; H_{max} at $2\theta = 90$

$H_{max} = \frac{u^2}{2g}$; $= 10 \Rightarrow u^2 = 10g \times 2$

$R = \frac{u^2 \sin 2\theta}{(g)} \Rightarrow R_{max} = \frac{u^2}{g}$

$R_{max} = \frac{10 \times g \times 2}{g} = 20$ meter

28. (c) $a = r w^2 = r \times \left(\frac{2\pi}{T}\right)^2$

$\therefore \frac{a_1}{a_2} = \frac{r_1}{r_2}$ [$\because$ T is same]

29. (b) Given that $F(t) = F_0 e^{-bt} \Rightarrow m\frac{dv}{dt} = F_0 e^{-bt}$

$\frac{dv}{dt} = \frac{F_0}{m} e^{-bt} \Rightarrow \int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt} dt$

$v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_0}{mb}\left[-\left(e^{-bt} - e^{-0}\right)\right]$

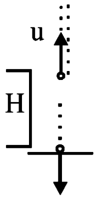
$\Rightarrow v = \frac{F_0}{mb}\left[1 - e^{-bt}\right]$

30. (b) $\vec{u} = \hat{i} + 2\hat{j} = u_x\hat{i} + u_y\hat{j} \Rightarrow u\cos\theta = 1,\ u\sin\theta = 2$

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u_x^2}$

$\therefore y = 2x - \frac{1}{2}gx^2 = 2x - 5x^2$

31. (c) Speed on reaching ground $v = \sqrt{u^2 + 2gh}$



Now, $v = u + at$

$\Rightarrow \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \frac{u}{g}$,

$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$ (from question)

$\Rightarrow 2gH = n(n-2)u^2$

32. (b) $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

for $y_1 = -240$m, $t = 8$s

$\therefore$ $y_2 - y_1 = 30t$ for $t \le 8$s.

for $t > 8$s,

$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$