EAMCET – 2013

1. If E, M, J and G respectively denote energy, mass, angular momentum and universal gravitational constant, the quantity, which has the same dimensions as the dimensions $\frac{EJ^2}{M^5G^2}$

(a) time (b) angle

(c) mass (d) length

2. The work done in moving an object from origin to a point whose position vector is $r = 3\hat{i} + 2\hat{j} - 5\hat{k}$ by a force $F = 2\hat{i} - \hat{j} - \hat{k}$ is

(a) 1 unit (b) 9 units

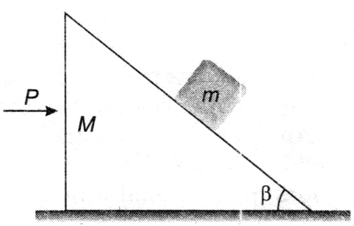
(c) 13 units (d) 60 units

3. A particle is projected from the ground with an initial speed of υ at an angle of projection θ. The average velocity of the particle between its time of projection and time it reaches highest point of trajectory is

(a) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ (b) $\frac{v}{2}\sqrt{1+2\sin^2\theta}$

(c) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ (d) $v\cos\theta$

4. Two wooden blocks of masses M and *m* are placed on a smooth horizontal surface as shown in figure. If a force P is applied to the system as shown in figure such that the mass *m* remains stationary with respect to block of mass M, then the magnitude of the force P is



(a) (M + m) g tan β

(b) g tan β

(c) mg cos β (d) (M + m) g cosec β

5. A ball at rest is dropped from a height of 12. It losses 25% of its kinetic energy on striking the ground and bounces back to a height 'h'.

Then value of 'h' is

(a) 3 m (b) 6 m (c) 9 m (d) 12 m

6. Two bodies of mass 4 kg and 5 kg are moving along east and north directions with velocities 5 m/s and 3 m/s respectively. Magnitude of the velocity of centre of mass of the system is

(a) $\frac{25}{9}$ m/s (b) $\frac{9}{25}$ m/s

(c) $\frac{41}{9}$ m/s (d) $\frac{16}{9}$ m/s

7. A mass of 2.9 kg is suspended from a string of length 50 cm and is at rest. Another body of mass 100 g, which is moving horizontally with a velocity of 150 m/s strikes and sticks to it. Subsequently when the string makes an angle of 60° with the vertical, the tension in the string is ($g = 10\ m/s^2$)

(a) 140 N (b) 135 N

(c) 125 N (d) 90 N

8. The upper half of an inclined plane with an angle of inclination ϕ, is smooth while the lower half is rough. A body starting from rest at the top of the inclined plane comes to rest at the bottom of the inclined plane. Then the coefficient of friction for the lower half is

(a) 2 tan ϕ (b) tan ϕ

(c) 2 sin ϕ (d) 2 cos ϕ

9. Moment of inertia of a body about an axis is 4 kg-m^2. The body is initially at rest and a torque of 8 N-m starts acting on it along the same axis. Work done by the torque in 20 s, in joules, is

(a) 40 (b) 640

(c) 2560 (d) 3200

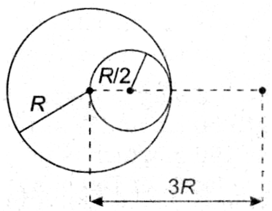
10. A uniform circular disc of radius R, lying on a frictionless horizontal plane is rotating with an angular velocity 'ω' about is its own axis. Another identical circular disc is gently placed on the top of the first disc coaxially. The loss in rotational kinetic

energy due to friction between the two discs, as they acquire common angular velocity is (I is moment of inertia of the disc)

(a) $\frac{1}{8}/\omega^2$ (b) $\frac{1}{4}/\omega^2$

(c) $\frac{1}{2}/\omega^2$ (d) $/\omega^2$

11. The gravitational force acting on a particle, due to a solid sphere of uniform density and radius R, at a distance of 3R from the centre of the sphere is Fj. A spherical hole of radius (R/2) is now made in the sphere as shown in the figure. The sphere with hole now exerts a force F_2 on the same particle. Ratio of F_1 and F_2 is



(a) $\frac{50}{41}$ (b) $\frac{41}{50}$ (c) $\frac{41}{42}$ (d) $\frac{25}{41}$

12. Two particles A and B of masses '*m*' and '2*m*' are suspended from massless springs of force constants K_1 and K_2. During their oscillation, if their maximum velocities are equal, then the ratio of amplitudes of A and B is

(a) $\sqrt{\frac{K_1}{K_2}}$ (b) $\sqrt{\frac{K_2}{2K_1}}$

(c) $\sqrt{\frac{K_2}{K_1}}$ (d) $\sqrt{\frac{2K_1}{K_2}}$

13. A tension of 20 N is applied to a copper wire of cross sectional area 0.01 cm^2, Young's modulus of copper is 1.1×10^{11} N/m^2 and Poisson's ratio is 0.32. The decrease in cross sectional area of the wire is

(a) 1.16×10^{-6} cm^2 (b) 1.16×10^{-5} m^2

(c) 1.16×10^{-4} m^2 (d) 1.16×10^{-3} m^2

14. A capillary tube of radius V is immersed in water and water rises to a height of '*h*'. Mass of water in the capillary tube is 5×10^{-3} kg. The same capillary tube is now immersed in a liquid whose surface tension is $\sqrt{2}$ times the surface tension of water. The angle of contact between the capillary tube and this liquid is 45°. The mass of liquid which rises into the capillary tube now is, (in kg)

(a) 5×10^{-3} (b) 2.5×10^{-3}

(c) $5\sqrt{2}\times10^{-3}$ (d) 3.5×10^{-3}

15. The terminal velocity of a liquid drop of radius '*r*' falling through air is *v*. If two such drops are combined to form a bigger drop, the terminal velocity with which the bigger drop falls through air is (ignore any buoyant force due to air)

(a) $\sqrt{2}$ v (b) 2 v

(c) $\sqrt[3]{4}$ v (d) $\sqrt[3]{2}$ v

16. A glass flask of volume one litre is filled completely with mercury at 0°C. The flask is now heated to 100°C. Coefficient of volume expansion of mercury is 1.82×10^{-4}/°C and coefficient of linear expansion of glass is 0.1×10^{-4}/°C. During this process, amount of mercury which overflows is

(a) 21.2cc (b) 15.2 cc

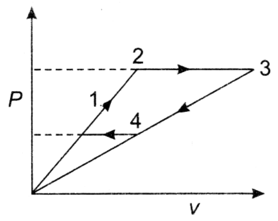
(c) 2.12 cc (d) 18.2 cc

17. On a temperature scale Y, water freezes at –160° Y and boils at –50° Y. On this Y scale, a temperature of 340 K is

(a) –160.3° Y (b) –96.3° V

(c) –86.3° Y (d) –76.3° V

18. Three moles of an ideal monoatomic gas undergoes a cyclic process as shown in the figure. The temperature of the gas in different states marked as 1, 2, 3 and 4 are 400 K, 700 K, 2500 K and 1100 K respectively. The work done by the gas during the process 1-2-3-4-1 is (universal gas constant is R)



(a) 1650 R (b) 550 R

(c) 1100 R (d) 2200 R

19. Efficiency of a heat engine whose sink is at temperature of 300 K is 40%. To increase the efficiency to 60%, keeping the sink temperature constant, the source temperature must be increased by

(a) 750 K (b) 500 K

(c) 250 K (d) 1000 K

20. Two bodies A and B of equal surface area have thermal emissivities of 0.01 and 0.81 respectively. The two bodies are radiating energy at the same rate. Maximum energy is radiated from the two bodies A and B at wavelengths λ_A and λ_B respectively. Difference in these two wavelengths is $1\mu m$. If the temperature of the body A is 5802 K, then value of λ_B is

(a) $\frac{1}{2}\mu m$ (b) $1\mu m$

(c) $2\mu m$ (d) $\frac{3}{2}\mu m$

21. An air column in a tube 32 cm long, closed at one end, is in resonance with a tuning fork. The air column in another tube, open at both ends, of length 66 cm is in resonance with another tuning fork. When these two tuning forks are sounded together, they produce 8 beats per second. Then the frequencies of the two tuning forks are, (Consider fundamental frequencies only)

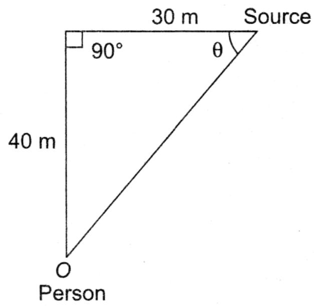
(a) 250 Hz, 258 Hz

(b) 240 Hz, 248 Hz

(c) 264 Hz, 256 Hz

(d) 280 Hz, 272 Hz

22. A source of sound of frequency 640 Hz is moving at a velocity of $\frac{100}{3}$ m/s along a road, and is at an instant 30 m away from a point A on the road (as shown in figure). A person standing at O, 40 m away from the road hears sound of apparent frequency ν'. The value of ν' is (velocity of sound = 340 m/s)



(a) 620 Hz (b) 680 Hz

(c) 720 Hz (d) 840 Hz

23. The two surfaces of a concave lens, made of glass of refractive index 1.5 have the same radii of curvature R. It is now immersed in a medium of refractive index 1.75, then the lens

(a) becomes a convergent lens of focal length 3.5ft

(b) becomes a convergent lens of focal length 3.0ft

(c) changes as a divergent lens of focal length 3.5ft

(d) changes as a divergent lens of focal length 3.0ft

24. A microscope consists of an objective of focal length 1.9 cm and eye piece of focal length 5 cm. The two lenses are kept at a distance of 10.5 cm. If the image is to be formed at the least distance of distinct vision, the distance at which the object is to be placed before the objective is (least distance of distinct vision is 25 cm)

(a) 6.2 cm (b) 2.7 cm

(c) 21.0 cm (d) 4.17 cm

25. Fresnel diffraction is produced due to light rays falling on a small obstacle. The intensity of light at a point on a screen beyond an obstacle depends on

(a) the focal length of lens used for observation

(b) the number of half-period zones that superpose at the point

(c) the square of the sum of the number of half period zones

(d) the thickness of the obstacle

26. A short bar magnet having magnetic moment 4 Am^2, placed in a vibrating magnetometer, vibrates with a time period of 8 s. Another short bar magnet having a magnetic moment 8 Am^2 vibrates with a time period of 6 s. If the moment of inertia of the second magnet is 9×10^{-2} kg-m^2, the moment of inertia of the first magnet is (assume that both magnets are kept in the same uniform magnetic induction field.)

(a) 9×10^{-2} kg-m^2

(b) 8×10^{-2} kg-m^2

(c) 5.33×10^{-2} kg-m^2

(d) 12.2×10^{-2} kg-m^2

27. Two short bar magnets have their magnetic moments 1.2 Am^2 and 1.0 Am^2. They are placed on a horizontal table parallel to each other at a distance of 20 cm between their centres, such that their north poles pointing towards geographic south. They have common magnetic equatorial line. Horizontal component of earth's field is 3.6×10^{-5} T. Then, the resultant horizontal magnetic induction at mid point of the line joining their centers is $\left(\frac{\mu_0}{4\pi} = 10^{-7}\,\text{N/m}\right)$

(a) 3.6×10^{-5} T (b) 1.84×10^{-4} T

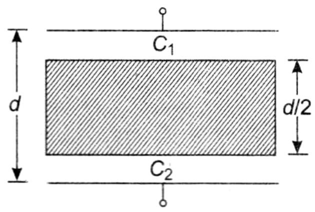
(c) 2.56×10^{-4} T (d) 5.8×10^{-5} T

28. A deflection magnetometer is adjusted and a magnet of magnetic moment M is placed on it in the usual manner and the observed deflection is 9. The period of oscillation of the needle before settling of the deflection is T. When the magnet is removed, the period of oscillation of the needle is T_0 before settling to 0° – 0°. If the earth's induced magnetic field is B_H, the relation between T and T_0 is

(a) $T^2 = T_0^2 \cos\theta$ (b) $T^2 = \frac{T_0^2}{\cos\theta}$

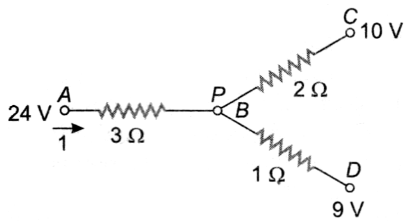
(c) $T = T_0 \cos\theta$ (d) $T = \frac{T_0}{\cos\theta}$

29. Two metal plates each of area 'A' form a parallel plate capacitor with air in between the plates. The distance between the plates is 'd'. A metal plate of thickness $\frac{d}{2}$ and of same area A is inserted between the plates to form two capacitors of capacitances C_1 and C_2 as shown in the figure. If the effective capacitance of the two capacitors is C' and the capacitance of the capacitor initially is C, then $\frac{C'}{C}$ is



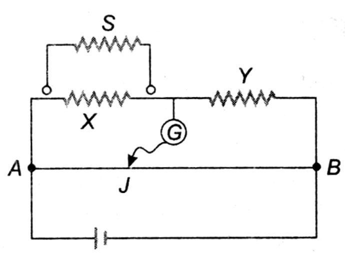
(a) 4 (b) 2 (c) 6 (d) 1

30. In the circuit shown in the figure, the current 'I' is



(a) 6 A (b) 2 A (c) 4 A (d) 7 A

31. In the meter bridge experiment, the length AB of the wire is 1 m. The resistors X and Y have values 5Ω and 2Ω respectively. When a shunt resistance S is connected to X, the balancing point is found to be 0.625 m from A. Then, the resistance of the shunt is



(a) 5Ω (b) 10Ω

(c) 7.5Ω (d) 12.5Ω

32. The ends of an element of zinc wire are kept at a small temperature difference ΔT and a small current (I) is passed through the wire. Then, the heat developed per unit time

(a) is proportional to ΔT and I

(b) is proportional to I^3 and ΔT

(c) is proportional to Thomson coefficient of the metal

(d) is proportional to ΔT only

33. A series LCR circuit is connected across a source of alternating emf of changing frequency and resonates at frequency f_0. Keeping capacitance constant, if the inductance (L) is increased by $\sqrt{3}$ times and resistance is increased (R) by 1.4 times, the resonant frequency now is

(a) $3^{1/4}f_0$ (b) $\sqrt{3}f_0$

(c) $\left(\sqrt{3}-1\right)^{1/4} f_0$ (d) $\left(\frac{3}{1}\right)^{1/4} f_0$

34. The sensitivity of a galvanometer that measures current is decreased by $\frac{1}{40}$ times by using shunt resistance of $10\,\Omega$. Then, the value of the resistance of the galvanometer is

(a) 400 Ω (b) 410 Ω

(c) 30 Ω (d) 390 Ω

35. Initially a photon of wavelength λ_1 falls on photocathode and emits an electron of maximum energy E_1. If the wavelength of the incident photon is changed to λ_2, the maximum energy of the electron emitted becomes E_2. Then value of *he* (*h* = Planck's constant, c = velocity of light) is

(a) $hc = \frac{(E_1 + E_2)\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$

(b) $hc = \frac{E_1 - E_2}{\lambda_2 - \lambda_1}.(\lambda_1\lambda_2)$

(c) $hc = \frac{(E_1 - E_2)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$

(d) $hc = \frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2 E_2}.E_1$

36. The work function of a metal is 2 eV. If a radiation of wavelength $3000\,\mathring{A}$ is incident on it, the maximum kinetic energy of the emitted photoelectrons is (Planck's constant $h = 6.6 \times 10^{-34}$ Js; velocity of light $c = 3 \times 10^8$ m/s; 1eV = 1.6×10^{-19} J)

(a) 4.4×10^{-19} J (b) 5.6×10^{-19} J

(c) 3.4×10^{-19} J (d) 2.5×10^{-19} J

37. The radius of $_{72}Te^{125}$ nucleus is 6 fermi. The radius of $_{13}Al^{27}$ nucleus in meters is

(a) 3.6×10^{-12} m (b) 3.6×10^{-15} m

(c) 7.2×10^{-8} m (d) 7.2×10^{-15} m

38. A U^{235} nuclear reactor generates energy at a rate of 3.70×10^7 J/s. Each fission liberates 185 MeV useful energy. If the reactor has to operate for 144×10^4 s, then, the mass of the fuel needed is (Assume Avogadro's number = 6×10^{23} mol^{-1}, 1 eV = 1.6×10^{23} J)

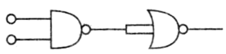
(a) 70.5 kg (b) 0.705 kg

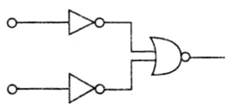
(c) 13.1 kg (d) 1.31 kg

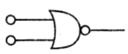
39. The base current in a transistor circuit changes from $45\,\mu A$ to $140\,\mu A$. Accordingly, the collector current changes from 0.2 mA to 0.400 mA. The gain in current is

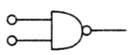
(a) 9.5 (b) 1 (c) 40 (d) 20

40. Of the following, NAND gate is

(a) 

(b) 

(c) 

(d) 

Engineering Entrance Exam Solved Paper 2013

Chemistry

1. The number of radial nodes of 3s and 2 p orbitals respectively are

   (a) 0, 2 (b) 2, 0

   (p) 1, 2 (d) 2, 1

2. The basis of quantum mechanical model of an atom is

   (a) angular momentum of electron

   (b) quantum numbers

   (c) dual nature of electron

   (d) blck body radiation

3. Thi number of elements present in the fourth period is

   (a) 32 (b) 8 (c) 18 (d) 2

4. Identify the correct set.

	Molecule	Hybridisation of central atom	Shape
(a)	PCI_5	dsp^3	square pyramidal
(b)	$[Ni(CN)_4]^{2-}$	sp^3	tetrahedral
(c)	SF_6	sp^3d^2	octahedral
(d)	IF_3	dsp^3	pyramidal

5. Which one of the following statements is correct?

   (a) Hybrid orbitals do not form a bonds

   (b) Lateral overlap of p-orbitals or p- and d-orbitals produces π-bonds

   (c) The strength of bonds follows the order

   $$\sigma_{p-p} < \sigma_{s-s} < \pi_{p-p}$$

   (d) s-orbitals do not form σ bonds

6. Which one of the following is an example of disproportionation reaction?

   (a) $3Cl_2(g) + 6OH^-(aq) \rightarrow ClO_3^-(aq) + 5Cl^-(aq) + 3H_2O(l)$

   (b) $Ag^{2+}(aq) + Ag(s) \rightarrow 2Ag^+(aq)$

   (c) $Zn(s) + CuSO_4(aq) \rightarrow Cu(s) + ZnSO_4(aq)$

   (d) $2KCIO_3(s) \rightarrow 2KCI(s) + 3O_2(g)$

7. At T(K), the ratio of kinetic energies of 4 g of $H_2(g)$ and 8 g of $O_2(g)$ is

   (a) 1 : 4 (b) 4 : 1 (c) 2 : 1 (d) 8 : 1

8. Which one of the following is an isotonic pair of solutions?

   (a) 0.15 M NaCI and 0.1 M Na_2SO_4

   (b) 0.2 M Urea and 0.1 M Sugar

   (c) 0.1 M $BaCI_2$ and 0.2 M Urea

   (d) 0.4 M $MgSO_4$ and 0.1 M NH_4CI

9. The vapour pressure in mm of Hg, of an aqueous solution obtained by adding 18 g of glucose ($C_6H_{12}O_6$) to 180 g of water at 100°C is

   (a) 7.60 (b) 76.0

   (c) 759 (d) 752.4

10. During the electrolysis of copper sulphate aqueous solution using copper electrode, the reaction takine place at the cathode is

    (a) $Cu \rightarrow Cu^{2+}(aq) + 2e^-$

    (b) $Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$

    (c) $H^+(aq) + e^- \rightarrow \frac{1}{2} H_2(g)$

    (d) $SO_4^{2-}(aq) \rightarrow SO_3(g) + \frac{1}{2} O_2(g) + 2e^-$

11. The extent of charge of lead accumulator is determined by

    (a) amount of $PbSO_4$ in the battery

    (b) amount of PbO_2 in the battery

    (c) specific gravity of H_2SO_4 of the battery

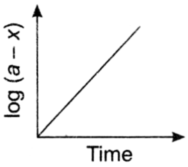
    (d) amount of Pb in the battery

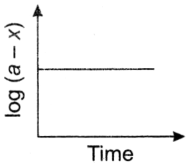
12. The number of octahedral and tetrahedral holes respectively present in a hexagonal close packed (*hcp*) crystal of 'X' atoms are

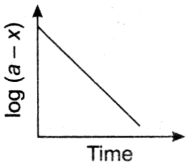
    (a) X, 2X (b) X, X

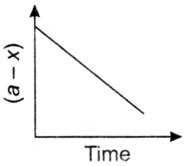
    (c) 2X, X (d) 2X, 2X

13. Which one of the following plots is correct for a first order reaction?

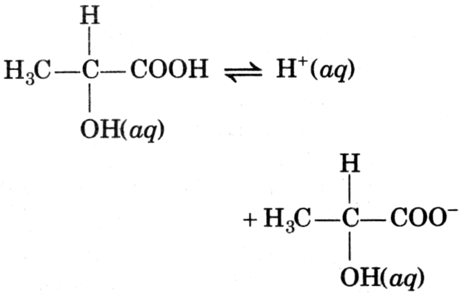
(a) 

(b) 

(c) 

(d) 

14. The degree of ionization of 0.10 M lactic acid is 4.0%



The value of K_c is

(a) $1.66 \times 1CT^5$ (b) 1.66×10^{-4}

(c) $1.66 \times 1CT^3$ (d) 1.66×10^{-2}

15. The pH of a buffer solution made by mixing 25 mL of 0.02 MNH_4OH and 25 mL of 0.2 M NH_4C1 at 25° is (pK_b of $NH_4OH = 4.8$)

(a) 5.8 (b) 8.2 (c) 4.8 (d) 3.8

16. For which one of the following reactions, the entropy change is positive?

(a) $H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_zO(/)$

(b) $Na^+ (g) + CI^-(g) \rightarrow NaCI(s)$

(c) $NaCI(/) \rightarrow NaCI(s)$

(d) $H_2O(/) \rightarrow H_2O(g)$

17. Match the following.

	List I		List II
(A)	Solid dispersed in liquid	(I)	Emulsion
(B)	Liquid dispersed in liquid	(II)	Foam
(C)	Gas dispersed in liquid	(III)	Gel
(D)	Liquid dispersed in solid	(IV)	Sol
		(V)	Aerosol

The correct match is

	(A)	(B)	(C)	(D)
(a)	(IV)	(I)	(N)	(III)
(b)	(III)	(I)	(V)	(II)
(c)	(III)	(I)	(II)	(IV)
(d)	(IV)	(I)	(V)	(III)

18. Observe the following statements

1. Heavy water is harmful for the growth of animals.

2. Heavy water reacts with $A1_4C_3$ and forms deuterated acetylene.

3. $BaCl_2 .2D_2O$ is an example of interstitial deuterate.

The correct statements are

(a) 1 and 3 (b) 1 and 2

(c) 1,2 and 3 (d) 2 and 3

19. Solution "X" contains Na_2CO_3 and $NaHCO_3$. 20 mL of X when titrated using methyl orange indicator consumed 60 mL of 0.1 M HC1 solution. In another experiment, 20 mL of X solution when titrated using phenolphthalein consumed 20 mL of 0.1 M HC1 solution. The concentrations (in mol L^{-1}) of Na_2CO_3 and $NaHCO_3$ in X are respectively

(a) 0.01.0.02 (b) 0.1, 0.1

(c) 0.01,0.01 (d) 0.1, 0.01

20. Diborane reacts with HC1 in the presence of $A1C1_3$ and liberates

(a) H_2 (b) Cl_2

(c) BCI_3 (d) Cl_2 and BCI_3

21. How many corners of SiO_4 units are shared in the formation of three dimensional silicates?

(a) 3 (b) 2 (C) 4 (d) 1

22. Which one of the following is not correct?

(a) Pyrophosphoric acid is a tetrabasic acid

(b) Pyrophosphoric acid contains P—O—P linkage

(c) Pyrophosphoric acid contains two P—H bonds

(d) Orthophosphoric acid can be prepared by dissolving P_4O_{10} in water

23. $Na_2S_2O_3$ reacts with moist Cl_2 to form Na_2SO_4, HC1 and X. Which one of the following is X?

(a) H_2S (b) SO_2

(c) SO_3 (d) S

24. The role of copper diaphragm in Whytlaw-Gray's method is

(a) preventing the corrosion of electrolytic cell

(b) preventing the mixing of H_2 and F_2

(c) as anode

(d) as cathode

25. Liquid *X* is used in bubble chamber to detect neutral mesons and gamma photons. Then, X is

(a) He (b) Ne

(c) Kr (d) Xe

26. A compound absorbs light in the wavelength region 490-500 nm. Its complementary colour is

(a) red (b) blue

(c) orange (d) blue-green

27. Which of the following is not added during the extraction of silver by cyanide process?

(a) NaCN (b) Air

(c) Zn (d) $Na_2S_2O_3$

28. Cataract and skin cancer are caused by

(a) depletion of nitric oxide

(b) depletion of ozone layer

(c) increase in methane

(d) depletion of nitrous oxide

29. Which one of the following gives Prussian blue colour?

(a) $Fe_2[Fe(CN)_6]$ (b) $Na_4[Fe(CN)_6]$

(c) $Fe_3[Fe(CN)_6]_3$ (d) $Fe_4[Fe(CN)_6]_3$

30. $C_2H_6 \xrightarrow{450°C} C_2H_4 + H_2$

Above reaction is called as

(a) combustion (b) rearrangement

(c) pyrolysis (d) cleavage

31. Assertion (A) —NH_2 group of aniline is *ortho, para* directing in electrophilic substitutions.

Reason (R) —NH_2 group stabilises the arenium ion formed by the *ortho, para* attack of the electrophile.

The correct answer is

(a) Both (A) and (R) are correct, (R) is the correct explanation of (A)

(b) Both (A) and (R) are correct, (R) is not the correct explanation of (A)

(c) (A) is correct, but (R) is not correct

(d) (A) is not correct, but (R) is correct

32. In which of the following properties, the two enantiomers of lactic acid differ from each other?

(a) Sign of specific rotation

(b) Density

(c) Melting point

(d) Refractive index

33. Heating chloroform with aqueous sodium hydroxide solution forms

(a) sodium acetate (b) sodium oxalate

(c) sodium formate (d) chloral

34. The products formed in the reaction of phenol with Br_2 dissolved in CS_2 at 0°C are

(a) o-bromo, m-bromo and p-bromophenols

(b) o-bromo and p-bromophenols

(c) 2,4,6-tribromo and 2,3,6-tribromophenols

(d) 2,4-dibromo and 2,6-dibromophenols

35. The structure of PCC is

(a) $C_6H_5\overset{\oplus}{N}HCrO_2Cl^{\ominus}$

(b) $C_6H_5\overset{\oplus}{N}HCrO_3Cl^{\ominus}$

(c) $C_5H_5\overset{\oplus}{N}HCrO_2Cl^{\ominus}$

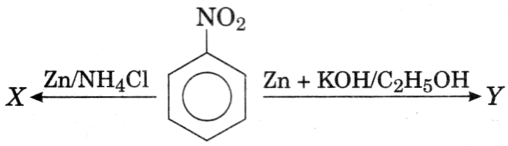
(d) $C_5H_5\overset{\oplus}{N}HCrO_3Cl^{\ominus}$

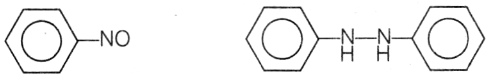
36. The pK_a values of four carboxylic acids are given below. Identify the weakest carboxylic acid.

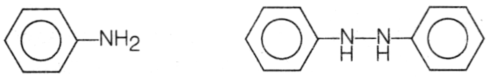
(a) 4.89 (b) 1.28

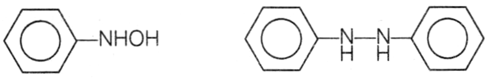
(c) 4.76 (d) 2.56

37. Identify X and Fin the following reactions



(a) 

(b) 

(c) 

(d) 

38. Example of a biodegradable polymer pair is

(a) nylon-6,6 and terylene

(b) PHBV and dextron

(c) bakelite and PVC

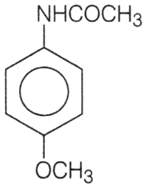
(d) PET and polyethylene

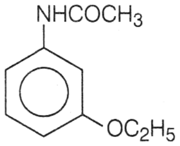
39. The number of hydrogen bonds between guanine and cytosine; and between adenine and thymine in DNA is

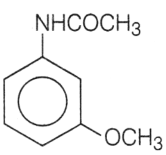
(a) 1, 2 (b) 3, 2

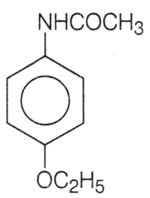
(c) 3, 1 (d) 2, 1

40. Identify phenacetin from the following.

(a) 

(b) 

(c) 

(d) 

Mathematics

1. If $f(x) = (p - x^n)^{1/n}$, $p > 0$and n is a positive integer, then f[f(x)] is equal to

(a) x (b) x^n

(c) $p^{1/n}$ (d) $p - x^n$

2. The value of

(a) $(-\infty, -1) \cup (7, \infty)$

(b) (–1, 5)

(c) (1, 7) (d) (–1, 7)

3. If I is the identity matrix of order 2 and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then for $n \geq 1$, mathematical induction gives

(a) $A^n = nA - (n-1)I$

(b) $A^n = nA + (n-1)I$

(c) $A^n = 2^n A - (n+1)I$

(d) $A^n = 2^{n-1} A - (n-1)I$

4. If ${}^nC_{r-1} = 330$, ${}^nC_r = 462$, and ${}^nC_{r+1} = 462$, then r is equal to

(a) 3 (b) 4

(c) 5 (d) 6

5. 10 men and 6 women are to be seated in a row so that no two women sit together. The number of ways they can be seated, is

(a) 11!10! (b) $\frac{11!}{6!5!}$

(c) $\frac{10!9!}{5!}$ (d) $\frac{11!10!}{5!}$

6. If t_n denotes the number of triangles formed with n points in a plane, no three of which are collinear and if $t_{n+1} - t_n = 36$, then n is equal to

(a) 7 (b) 8

(c) 9 (d) 10

7. The term independent of $x\left(x>0,\ x\neq 1\right)$ in the expansion of $\left[\frac{(x+1)}{\left(x^{2/3}-x^{1/3}+1\right)}-\frac{(x-1)}{\left(x-\sqrt{x}\right)}\right]^{10}$ is

(a) 105 (b) 210

(c) 315 (d) 420

8. If x is small, so that x^2 and higher powers can be neglected, then the approximate value for $\frac{(1-2x)^{-1}(1-3x)^{-2}}{(1-4x)^{-3}}$ is

(a) $1 - 2x$ (b) $1 - 3x$

(c) $1 - 4x$ (d) $1 - 5x$

9. If $\frac{1}{x^4+x^2+1}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$, then C + D is equal to

(a) – 1 (b) 1

(c) 2 (d) 0

10. $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\frac{1}{8.9}+\ldots$ is equal to

(a) $\log\left(\frac{2}{e}\right)$ (b) $\log\left(\frac{e}{2}\right)$

(c) log(2 e) (d) e – 1

11. If the harmonic mean between the roots of $\left(5+\sqrt{2}\right)x^2-bx+\left(8+2\sqrt{5}\right)=0$ is 4, then the value of b is

(a) 2 (b) 3

(c) $4-\sqrt{5}$ (d) $4+\sqrt{5}$

12. The set of solutions satisfying both $x^2 + 5x + 6 \geq 0$ and $x^2 + 3x - 4 < 0$ is

(a) (– 4, 1)

(b) (– 4, – 3] $\cup$ [– 2, 1)

(c) (– 4, – 3) $\cup$ (– 2, 1)

(d) [– 4, – 3] $\cup$ [– 2, 1]

13. If the roots of $x^3 - 42x^2 + 336x - 512 = 0$, are in increasing geometric progression, then its common ratio is

(a) 2 : 1 (b) 3 : 1

(c) 4 : 1 (d) 6 : 1

14. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then $\alpha^9+\beta^9$ is equal to

(a) -2^8 (b) 2^9

(c) -2^{10} (d) 2^{10}

15. If $A=\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix}$ satisfies the equation $x^2 + 4x - p = 0$, then p is equal to

(a) 64 (b) 42

(c) 36 (d) 24

16. $\begin{vmatrix}x+2 & x+3 & x+5\\ x+4 & x+6 & x+9\\ x+8 & x+11 & x+15\end{vmatrix}$ is equal to

(a) $3x^2 + 4x + 5$ (b) $x^3 + 8x + 2$

(c) 0 (d) –2

17. The system of equations 3x + 2y + z = 6, 3x + 4y + 3z = 14 and 6x + 10y +8z = a, has infinite number of solutions, if a is equal to

(a) 8 (b) 12

(c) 24 (d) 36

18. The number of real values off such that the system of homogeneous equations

tx + (t + l)y + (t – 1)z = 0

(t + l)x + ty + (t + 2)z = 0

(t – 1)x + (t + 2)y + tz = 0

has non-trivial solutions is equal to

(a) 3 (b) 2 (c) 1 (d) 4

19. $\left(\frac{1+i}{1-i}\right)^4+\left(\frac{1-i}{1+i}\right)^4$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4

20. If a complex number z satisfies $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) the real axis (b) the imaginary axis

(c) y = x (d) a circle

21. If $\frac{(1+i)x-i}{2+i}+\frac{(1+2i)y+i}{2-i}=1$, then (x, y) is equal to

(a) $\left(\frac{7}{3},\frac{-7}{15}\right)$ (b) $\left(\frac{7}{3},\frac{7}{15}\right)$

(c) $\left(\frac{7}{5},\frac{-7}{15}\right)$ (d) $\left(\frac{7}{5},\frac{7}{15}\right)$

22. The period of $f(x) = \cos\left(\frac{x}{3}\right)+\sin\left(\frac{x}{2}\right)$ is

(a) 2π (b) 4π (c) 8π (d) 12π

23. If $\sin\theta+\cos\theta = p$ and $\sin^3\theta+\cos^3\theta = q$ then $p(p^2 - 3)$ is equal to

(a) q (b) 2q

(c) – q (d) – 2q

24. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then a value of $\cos\left(\theta-\frac{\pi}{4}\right)$ among the following is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. The set of solutions of the system of equations

$$x + y = \frac{2\pi}{3}$$

and $$\cos x + \cos y = \frac{3}{2},$$

where x, y are real, is

(a) $\left\{(x, y) : \cos\left(\frac{x-y}{2}\right)=\frac{1}{2}\right\}$

(b) $\left\{(x, y) : \sin\left(\frac{x-y}{2}\right)=\frac{1}{2}\right\}$

(c) $\left\{(x, y) : \cos(x-y)=\frac{1}{2}\right\}$

(d) Epmty set

26. If $\cos^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\cos^{-1}x$, then x is equal to

(a) $\frac{3}{65}$ (b) $\frac{-36}{65}$

(c) $\frac{-33}{65}$ (d) –1

27. $\tanh^{-1}\left(\frac{1}{2}\right)+\coth^{-1}(2)$ is equal to

(a) $\frac{1}{2}\log 3$ (b) $\frac{1}{2}\log 6$

(c) $\frac{1}{2}\log 12$ (d) log 3

28. In any ΔABC, $r_1r_2 + r_2r_3 + r_3r_1$ is equal to

(a) $\frac{\Delta^2}{r^2}$ (b) $\frac{\Delta}{r}$

(c) $\frac{2\Delta}{r}$ (d) Δ^2

29. If in a ΔABC, $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$, then $\angle C$ is equal to

(a) 30° (b) 45°

(c) 60° (d) 90°

30. A person observes the top of a tower from a point A on the ground. The elevation of the tower from this point is 60°. He moves 60 m in the direction perpendicular to the line joining A and base of the tower. The angle of elevation of the tower from

this point is 45°. Then, the height of the tower (in metres) is

(a) $60\sqrt{\frac{3}{2}}$ (b) $60\sqrt{2}$

(c) $60\sqrt{3}$ (d) $60\sqrt{\frac{2}{3}}$

31. The points whose position vectors are 2i + 3j+ 4k,3i+ 4j + 2kand4i + 2j + 3kare the vertices of

(a) an isosceles triangle

(b) right angled triangle

(c) equilateral triangle

(d) right angled isosceles triangle

32. P, Q, R and S are four points with the position vectors 3i – 4j + 5k, – 4i + 5j + k and – 3i + 4j + 3k, respectively. Then, the line PQ meets the line RS at the point

(a) 3i + 4j + 3k (b) –3i + 4j + 3k

(c) –i + 4j + k (d) i + j + k

33. If $a \neq 0$, $b \neq 0$, $c \neq 0$, a × b = 0 and b × c =0, then a × c is equal to

(a) b (b) a

(c) 0 (d) i + j + k

34. The shortest distance between the lines

r = 3i + 5j + 7k + λ(i + 2j+k) and

r = – i – j – k + μ(7i – 6 j + k) is

(a) $\frac{16}{5\sqrt{5}}$ (b) $\frac{26}{5\sqrt{5}}$

(c) $\frac{36}{5\sqrt{5}}$ (d) $\frac{46}{5\sqrt{5}}$

35. A unit vector coplanar with i + j + 3k and i + 3 j + k and perpendicular to i + j + k is

(a) $\frac{1}{\sqrt{2}}$ (j + k) (b) $\frac{1}{\sqrt{3}}$(i – j + k)

(c) $\frac{1}{\sqrt{2}}$ (j – k) (d) $\frac{1}{\sqrt{3}}$(i + j – k)

36. If a and b are two non-zero perpendicular vectors, then a vector y satisfying equations a× y = c (where, c is scalar) and a x y = b is

(a) $|a|^2$[ca – (a × b)]

(b) $|a|^2$.[ca + (a × b)]

(c) $\frac{1}{|a|^2}$[ca – (a × b)]

(d) $\frac{1}{|a|^2}$[ca + (a × b)]

37. Two numbers are chosen at random from {1,2,3,4,5,6,7,8} at a time. The probability that smaller of the two numbers is less than 4 is

(a) $\frac{7}{14}$ (b) $\frac{8}{14}$

(c) $\frac{9}{14}$ (d) $\frac{10}{14}$

38. Two fair dice are rolled. The probability of the sum of digits on their faces to be greater than or equal to 10 is

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{6}$

39. A bag contains 2n + 1 coins. It is known that n of these coins have a head on both sides, whereas the remaining n + 1 coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is $\frac{31}{42}$, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

40. The randm variable takes the values 1, 2, 3, ..., m. If P(X = n) = $\frac{1}{m}$ to each n, then the variance of X is

(a) $\frac{(m+1)(2m+1)}{6}$

(b) $\frac{m^2-1}{12}$

(c) $\frac{m+1}{2}$ (d) $\frac{m^2+1}{12}$

41. If X is a poisson variate P(X= 1) = 2P(X= 2), then P(X = 3) is equal to

(a) $\frac{e^{-1}}{6}$ (b) $\frac{e^{-2}}{2}$

(c) $\frac{e^{-1}}{2}$ (d) $\frac{e^{-1}}{3}$

42. The origin is translated to (1,2). The point (7,5) in the old system undergoes the following transformations successively.

I. Moves to the new point under the given translation of origin.

II. Translated through 2 units along the negative direction of the new X-axis.

III. Rotated through an angle $\frac{\pi}{4}$ about the origin of new system in the clockwise direction. The final position of the point (7,5) is

(a) $\left(\frac{9}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $\left(\frac{7}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (d) $\left(\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

43. If p and q are the perpendicular distances from the origin to the straight lines $x\sec\theta - y\,\text{cosec}\,\theta = a$ and $x\cos\theta + y\sin\theta = a\cos 2\theta$, then

(a) $4p^2 + q^2 = a^2$ (c) $p^2 + 2q^2 = a^2$

(b) $p^2 + q^2 - a$ (d) $4p^2 + q^2 = 2a^2$

44. If 2x + 3y = 5 is the perpendicular bisector of the line segment joining the points $A\left(1, \frac{1}{3}\right)$ and B, then B is equal to

(a) $\left(\frac{21}{13}, \frac{49}{39}\right)$ (b) $\left(\frac{17}{13}, \frac{31}{39}\right)$

(c) $\left(\frac{7}{13}, \frac{49}{39}\right)$ (d) $\left(\frac{21}{13}, \frac{31}{39}\right)$

45. If the points (1,2) and (3, 4) lie on the same side of the straight line 3x – 5y + a = 0, then a lies in the set

(a) [7, 11] (b) R – [7,11]

(c) $[7, \infty)$ (d) $(-\infty, 11]$

46. The equation of the pair of lines passing through the origin whose sum and product of slopes are respectively the arithmetic mean and geometric mean of 4 and 9 is

(a) $12x^2 - 13xy + 2y^2 = 0$

(b) $12x^2 + 13xy + 2y^2 = 0$

(c) $12x^2 - 15xy + 2y^2 = 0$

(d) $12x^2 + 15xy - 2y^2 = 0$

47. The equation $x^2 - 5xy + py^2 + 3x - 8y + 2 = 0$ represents a pair of straight lines. If θ is the angle between them, then $\sin\theta$ is equal to

(a) $\frac{1}{\sqrt{50}}$ (b) $\frac{1}{7}$

(c) $\frac{1}{5}$ (d) $\frac{1}{\sqrt{10}}$

48. If the equations $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, then the square of the distance of their point of intersection from the origin is

(a) $\frac{c(a+b)-af^2-bg^2}{ab-h^2}$

(b) $\frac{c(a+b)+f^2+g^2}{ab-h^2}$

(c) $\frac{c(a+b)-f^2-g^2}{ab-h^2}$

(d) $\dfrac{c(a+b)-f^2-g^2}{(ab-h^2)^2}$

49. The circle $4x^2 + 4y^2 - 12x - 12y + 9 = 0$

(a) touches both the axes

(b) touches the x-axis only

(c) touches the y-axis only

(d) does not touch the axes

50. For the circle C with the equation $x + y^2 - 16x - 12y + 64 = 0$ match the List I with the List II given below.

	List I		List II
i)	The equation of the polar of (-5,1) with respect to C	A)	y = 6
ii)	The equation of the tangent at (8, 0) to C	B)	y = 0
iii)	The equation of the normal at (2, 6) to C	C)	x + y = 7
iv)	The equation of the diameter of C through (8,12)	D)	13x + 5y = 98
		E)	x = 8

The correct match is

	(i)	(ii)	(iii)	(iv)
(a)	(D)	(B)	(A)	(E)
(b)	(D)	(A)	(B)	(E)
(c)	(C)	(D)	(A)	(B)
(d)	(C)	(E)	(B)	(A)

51. If the length of the tangent from (h, k) to the circle $x^2 + y^2 = 16$ is twice the length of the tangent from the same point to the circle $x^2 + y^2 + 2x + 2y = 0$, then

(a) $h^2 + k^2 + 4h + 4k + 16 = 0$

(b) $h^2 + k^2 + 3h + 3k = 0$

(c) $3h^2 + 3k^2 + 8h + 8k + 16 = 0$

(d) $3h^2 + 3k^2 + 4h + 4k + 16 = 0$

52. (a, 0) and (b, 0) are centres of two circles belonging to a coaxial system of which y-axis is the radical axis. If radius of one of the circles is 'r', then the radius of the other circle is

(a) $(r^2 + b^2 + a^2)^{1/2}$ (b) $(r^2 + b^2 - a^2)^{1/2}$

(c) $(r^2 + b^2 - a^2)^{1/3}$ (d) $(r^2 + b^2 + a^2)^{1/3}$

53. If the circle $x^2 + y^2 + 4x - 6y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$, then c is equal to

(a) 16 (b) 24

(c) –42 (d) –62

54. A circle of radius 4, drawn on a chord of the parabola $y^2 = 8x$ as diameter, touches the axis of the parabola. Then, the slope of the chord is

(a) $\dfrac{1}{2}$ (b) $\dfrac{3}{4}$

(c) 1 (d) 2

55. The mid-point of a chord of the ellipse $x^2 + 4y^2 - 2x + 20y = 0$ is (2, –4). The equation of the chord is

(a) $x - 6y = 26$ (b) $x + 6y = 26$

(c) $6x - y = 26$ (d) $6x + y = 26$

56. If the focii of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$ and the hyperbola $\dfrac{x^2}{4}-\dfrac{y^2}{b^2}=1$ coincide, then b^2 is equal to

(a) 4 (b) 5 (c) 8 (d) 9

57. If x = 9 is a chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the tangent at one of the points of contact is

(a) $x+\sqrt{3}y+2=0$

(b) $3x+2\sqrt{2}y-3=0$

(c) $3x-\sqrt{2}y+6=0$

(d) $x-\sqrt{3}y+2=0$

58. The perpendicular distance from the point $(1, \pi)$ to the line joining (1, 0°) and $\left(1,\dfrac{\pi}{2}\right)$, (in polar coordinates) is

(a) 2 (b) $\sqrt{3}$ (c) 1 (d) $\sqrt{2}$

59. If D(2, 1, 0), E(2, 0, 0) and F(0,1,0) are mid-points of the sides BC, CA and AB of ΔABC, respectively. Then, the centroid of ΔABC is

(a) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (b) $\left(\frac{4}{3}, \frac{2}{3}, 0\right)$

(c) $\left(-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (d) $\left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

60. The direction ratios of the two lines AB and AC are 1, –1, –1 and 2, –1, 1. The direction ratios of the normal to the plane ABC are

(a) 2, 3, –1 (b) 2, 2, 1

(c) 3, 2, –1 (d) –1, 2, 3

61. A plane passing through(l, 2, 3) and whose normal makes equal angles with the coordinate axes is

(a) $x + y + z + 4 = 0$

(b) $x - y + z + 4 = 0$

(c) $x + y + z - 4 = 0$

(d) $x + y + z = 0$

62. A variable plane passes through a fixed point (1,2,3). Then, the foot of the perpendicular from the origin to the plane lies on

(a) a circle (b) a sphere

(c) an ellipse (d) a parabola

63. Let f be a non-zero real valued continuous function satisfying $f(x + y) = f(x).f(y)$ for all $x, y \in R$. If f(2) = 9, then f(6) is equal to

(a) 3^2 (b) 3^6 (c) 3^4 (d) 3^3

64. $\lim\limits_{x\to 0} \frac{\tan^3 x - \sin^3 x}{x^5}$ is equal to

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

65. If $f(x) = \frac{1}{1+\frac{1}{x}}$ and $g(x) = \frac{1}{1+\frac{1}{f(x)}}$ then $g'(2)$ is equal to

(a) $\frac{1}{5}$ (b) $\frac{1}{25}$ (c) 5 (d) $\frac{1}{16}$

66. If $\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} = 2$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x^2+y^2}{x+y}$ (b) $\frac{x^2-y^2}{x+y}$

(c) 1 (d) 2

67. If $\frac{d}{dx}[(x + l)(x^2 + l)(x^4 + l)(x^8 + 1)]$ $= (15x^p - 16x^q + l)(x - 1)^{-2}$, then (p, q) is equal to

(a) (12, 11) (b) (15, 14)

(C) (16, 14) (d) (16, 15)

68. If $\cos^{-1}\left(\frac{y}{b}\right) = 21og\left(\frac{x}{2}\right)$, where $x > 0$, then

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to

(b) 4y (b) – 4y

(c) 0 (d) – 8y

69. The relation between pressure p and volume V is given by $_pV^{1/4}$ = constant. If the percentage decrease in volume is $\frac{1}{2}$, then the percentage increase in pressure is

(a) $-\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

70. If the curves $x^2 + py^2 = 1$ and $qx^2 + y^2 = l$ are orthogonal to each other, then

(a) $p - q = 2$ (b) $\frac{1}{p} - \frac{1}{q} = 2$

(c) $\frac{1}{p} + \frac{1}{q} = -2$ (d) $\frac{1}{p} + \frac{1}{q} = 2$

71. The focal length of a mirror is given by $\frac{2}{f}=\frac{1}{v}-\frac{1}{u}$. In finding the values of u and v, the errors are equal to 'p'. Then, the relative error in f is

(a) $\frac{p}{2}\left(\frac{1}{u}+\frac{1}{v}\right)$ (b) $p\left(\frac{1}{u}+\frac{1}{v}\right)$

(c) $\frac{p}{2}\left(\frac{1}{u}-\frac{1}{v}\right)$ (d) $p\left(\frac{1}{u}-\frac{1}{v}\right)$

72. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then $(x + y + z)(u_x + u_y + u_z)$ is equal to

(a) 0 (b) $x - y + z$

(c) 2 (d) 3

73. $\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$ is equal to

(a) $e^x \cot x + C$ (b) $2e^x \sec^2 x + C$

(c) $e^x \cos 2x + C$ (d) $e^x \tan x + C$

74. If

$$\int\frac{x-\sin x}{1+\cos x}dx = x\tan\left(\frac{x}{2}\right)+p\log\left|\sec\left(\frac{x}{2}\right)\right|+C$$

then p is equal to

(a) – 4 (b) 4 (c) 2 (d) – 2

75. If $\int\frac{dx}{x(\log x-2)(\log x-3)}=I+C$, then I is equal to

(a) $\frac{1}{x}\log\left|\frac{\log x-3}{\log x-2}\right|$

(b) $\log\left|\frac{\log x-3}{\log x-2}\right|$

(c) $\log\left|\frac{\log x-2}{\log x-3}\right|$

(d) $\log|(\log x-3)(\log x-2)|$

76. If $\int_0^b\frac{dx}{1+x^2}=\int_b^\infty\frac{dx}{1+x^2}$, then b is equal to

(a) $\tan^{-1}\left(\frac{1}{3}\right)$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{2}$ (d) 1

77. The area (in sq units) bounded by the curves $x = -2y^2$ and $x = 1 - 3y^2$ is

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{4}{3}$ (d) $\frac{4}{3}$

78. The approximate value of $\int_1^3\frac{dx}{2+3x}$ using Simpson's rule and dividing the interval [1, 3] into two equal parts is

(a) $\frac{1}{3}\log\left(\frac{11}{5}\right)$ (b) $\frac{107}{110}$

(c) $\frac{29}{110}$ (d) $\frac{119}{440}$

79. An integrating factor of the equation $(1 + y + x^2y)\,dx + (x + x^3)dy - 0$ is

(a) e^x (b) x^2 (c) $\frac{1}{x}$ (d) x

80. The solution of the differential equation $\frac{dy}{dx}-2y\tan 2x = e^x \sec 2x$ is

(a) $y \sin 2x = e^x + C$

(b) $y \cos 2x = e^x + C$

(c) $y = e^x \cos 2x + C$

(d) $y \cos 2x + e^x = C$

Answers

Physics

1. (b)	2. (b)	3. (c)	4. (a)	5. (c)
6. (a)	7. (b)	8. (a)	9. (d)	10. (b)
11. (a)	12. (b)	13. (a)	14. (a)	15. (c)
16. (b)	17. (c)	18. (a)	19. (c)	20. (d)

21. (c)	22. (b)	23. (a)	24. (b)	25. (b)
26. (b)	27. (c)	28. (a)	29. (b)	30. (a)
31. (b)	32. (a)	33. (d)	34. (d)	35. (b)
36. (c)	37. (b)	38. (b)	39. (c)	40. (d)

Chemistry

1. (b)	2. (c)	3. (c)	4. (c)	5. (b)
6. (a)	7. (d)	8. (a)	9. (d)	10. (b)
11. (c)	12. (a)	13. (c)	14. (b)	15. (b)
16. (d)	17. (a)	18. (a)	19. (b)	20. (a)
21. (c)	22. (c)	23. (d)	24. (b)	25. (d)
26. (a)	27. (d)	28. (b)	29. (d)	30. (c)
31. (a)	32. (a)	33. (c)	34. (b)	35. (d)
36. (a)	37. (c)	38. (b)	39. (b)	40. (d)

Mathematics

1. (a)	2. (a)	3. (a)	4. (c)	5. (d)
6. (c)	7. (b)	8. (c)	9. (d)	10. (b)
11. (d)	12. (b)	13. (c)	14. (c)	15. (b)
16. (d)	17. (d)	18. (*)	19. (c)	20. (b)
21. (a)	22. (d)	23. (d)	24. (a)	25. (d)
26. (c)	27. (d)	28. (a)	29. (c)	30. (a)
31. (c)	32. (b)	33. (c)	34. (d)	35. (c)
36. (c)	37. (c)	38. (d)	39. (a)	40. (b)
41. (a)	42. (c)	43. (a)	44. (a)	45. (a)
46. (a)	47. (a)	48. (c)	49. (a)	50. (b)
51. (c)	52. (b)	53. (d)	54. (c)	55. (a)
56. (b)	57. (b)	58. (d)	59. (b)	60. (a)
61. (c)	62. (b)	63. (b)	64. (b)	65. (b)
66. (c)	67. (d)	68. (b)	69. (d)	70. (d)
71. (b)	72. (d)	73. (d)	74. (a)	75. (b)
76. (d)	77. (c)	78. (c)	79. (c)	80. (b)

(*) None of the correct option.

Hints and Solutions

Physics

1. Given quantity is $\frac{EJ^2}{M^5G^2}$ ...(i)

where dimensions of the various given quantities are

Dimensions of E = $[ML^2T^{-2}]$

Dimensions of J = $[ML^2T^{-1}]$

Dimension of M = [M]

Dimension of G = $[M^{-1}L^3T^{-2}]$

Now, on putting these dimensions in Eq. (i), we have

$$= \frac{[ML^2T^{-2}][ML^2T^{-1}]^2}{[M^5][M^{-1}L^3T^{-2}]^2}$$

$$= \frac{[M^3L^6T^{-2}]}{[M^3L^6T^{-2}]} = \text{dimensionless}$$

Since, angle is a dimensionless quantity

2. We know, work done W = F.d

Given, force, $F = 2\hat{i} - \hat{j} - \hat{k}$,

and position vector, $d = 3\hat{i} + 2\hat{j} - 5\hat{k}$

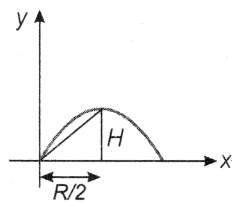
Using vector identify $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

Hence, W = F.d = $(2\hat{i} - \hat{j} - \hat{k}).(3\hat{i} + 2\hat{j} - 5\hat{k})$

= 6 – 2 + 5 = 9units

3. We know, average velocity = $\frac{\text{displacement}}{\text{time}}$

$$v_{av} = \frac{\sqrt{\mu^2 + R^2/4}}{T/2} \quad ... (i)$$



where, H = maximum height $= \dfrac{v^2 \sin^2 \theta}{2g}$ ... (ii)

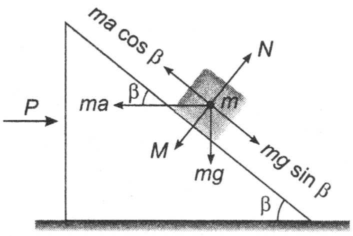
$$\text{Range } R = \frac{v^2 \sin 2\theta}{g} \quad \text{... (iii)}$$

Time of flight $T = \dfrac{2v \sin \theta}{g}$

Putting the values of Eqs. (ii), (iii) and (iv) in Eq. (i) we have

$$v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2 \theta}$$

4. The free body diagram of the given situation is



(given)

Force = Mass × Acceleration

$\therefore \quad P = (M + m)a$

$ma \cos \beta = mg \sin \beta$

$\Rightarrow \quad a = g\dfrac{\sin \beta}{\cos \beta}$

$= g \tan \beta$

$\therefore \quad P = (M + m)\, g \tan \beta$

5. Energy of balls at rest, $K_1 = mgh_1$ and $K_2 = mgh_2$

percentage loss in KE $= \dfrac{K_1 - K_2}{K_1} \times 100$

$$\frac{25}{100} = \left(\frac{12 - h^2}{12}\right)$$

$\Rightarrow \quad \dfrac{25 \times 12}{100} = 12 - h_2$

$\Rightarrow \quad h_2 = 12 - 3 = 9\text{m}$

6. Velocity of centre of mass is

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Given, $m_1 = 4$ kg, $m_2 = 5$kg, $v_1 = 5\hat{j}$ m/s

$v_2 = 3\hat{j}$ m/s

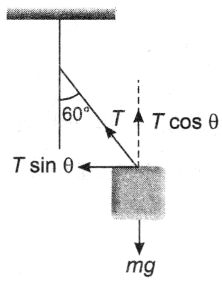
$\therefore \quad v_{CM} = \dfrac{4 \times 5\hat{j} + 5 \times 3\hat{i}}{5 + 4}$

$$= \frac{20\hat{i}}{9} + \frac{15}{9}\hat{j}$$

Hence, magnitude $|v_{CM}| = \sqrt{\left(\dfrac{20}{9}\right)^2 + \left(\dfrac{15}{9}\right)^2}$

$$= \frac{25}{9} \text{ m/s}$$

7. From law of conservation of momentum, we known,



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$u_1 = 0$, $u_2 = 150$ m/s.$m_1 = 2.9$kg and $m_2 = 0.1$ kg

So, $\quad 2.9 \times 150 = (2.9 + 0.1)\, v$

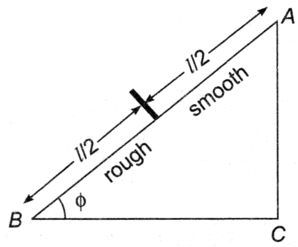
$\Rightarrow \quad \dfrac{2.9 \times 150}{3} = v$

$\Rightarrow \quad v = 145$ m/s

Also, $\quad T \sin \theta = \dfrac{mv^2}{r}$

Putting the values and solving, we get T = 135N.

8. For upper half



From equation,

$$v^2 = u^2 + 2as$$

we have, u = 0 (from rest), s = $l/2$

$$v^2 = 0 + 2\,(g \sin\phi).\frac{l}{2}$$

For lower half,

v = 0 and a = $g\left(\sin\phi - \mu\cos\phi\right)$,

$\Rightarrow$ $$0 = u^2 + 2g\left(\sin\phi - \mu\cos\phi\right).\frac{l}{2}$$

$\Rightarrow -gl\sin\phi = gl\left(\sin\phi - \mu\cos\phi\right)$

$\Rightarrow \mu\cos\phi = 2\sin\phi \quad \Rightarrow \mu = 2\tan\phi$

9. Given, $I = 4\text{kg} - \text{m}^2$, $\tau = 8\text{N} - \text{m}$ and t = 20s

$$\tau = I\alpha$$

$\Rightarrow$ $$\alpha = \frac{\tau}{I} = \frac{8}{4} = 2$$

$$\theta = \frac{1}{2}\alpha t^2$$

$\Rightarrow$ $$\theta = \frac{1}{2} \times 2 \times 20 \times 20 = 400$$

$$\omega = \tau\theta = 8 \times 400 = 3200 \text{ J}$$

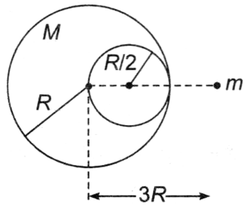
10. We know the KE of a rotational circular disc

$$KE = \frac{1}{2}I\omega^2 \text{ and } I = \frac{1}{2}MR^2$$

Hence, the resultant loss rotational KE will be the addition of both energy loss is $= \frac{1}{4}I\omega^2$

11. Gravitational force due to solid sphere is

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2}$$



where, M and m are mass of solid sphere and particle respectively. Gravitational force on particle due to sphere with cavity

$$F_2 = \frac{GMm}{9R^2} - \frac{G\left(\frac{M}{8}\right)m}{(5R/2)^2}$$

$$= \frac{GMm}{R^2}\left[\frac{1}{9} - \frac{4}{8 \times 25}\right]$$

$$= \frac{GMm}{R^2}\left[\frac{41}{50 \times 9}\right]$$

$\therefore$ $$\frac{F_1}{F_2} = \frac{50}{41}$$

12. We knows, maximum velocity

$$V_{max} = A\omega = A\sqrt{\frac{K}{m}}$$

Given, K_1, m_1 = m, K_2, m_2 = 2 m

$$(V_{max})A = (V_{max})B$$

$$A_A\sqrt{\frac{K_1}{M}} = A_B\sqrt{\frac{K_2}{2m}}$$

$\Rightarrow$ $$\frac{A_A}{A_B} = \sqrt{\frac{K_2}{2K_1}}$$

13. Given, σ = 0.32, F= 20 N

$A = 0.01 \text{ cm}^2 = 0.01 \times 10^{-3}\text{m}$

and $Y = 1 - 1\times10^{11} \text{ N/m}^2$

We know that

$$\frac{\Delta l}{l} = \frac{F}{AY} = \frac{20}{0.01\times10^{-3}\times1.1\times10^{11}} = 18.1\times10^{-7}$$

and we also known

$$\sigma = \frac{\Delta r / r}{\Delta l / l}$$

$$-\frac{\Delta r}{r} = 0.32 \times 18.1 \times 10^{-7} = 5.79 \times 10^{-7}$$

Hence, decrease in cross reactional area of wire is

$$\Delta A = 2\frac{\Delta r}{r}\times A = 2\times5.79\times10^{-7}\times 0.01 \times10^{-3}$$

$$= 0.158 \times 10^{-10}\ m^2$$

$$= 1.26 \times 10^{-6}\ cm^2$$

14. We knows height of water rise in a capillary tube

$$h = \frac{2T\cos\theta}{rdg}$$

$$h_1 = \frac{2T_1\cos\theta_1}{rdg},\ h_2 = \frac{2T_2\cos\theta_2}{rdg}$$

Given, $h_1 = h$, $T_1 = T$, $\theta_1 = 0$

$$\therefore \quad h = \frac{2T}{rdg} \quad ...(i)$$

Given, $T_2 = \sqrt{2}T$, $\theta = 45°$, $\cos 45° = \frac{1}{\sqrt{2}}$

$$\therefore \quad h_2 = \frac{2\sqrt{2}T\times\frac{1}{\sqrt{2}}}{rdg} \quad ...(i)$$

From Eqs. (i) and (ii), we observe

$$h_2 = h.$$

Hence, same mass of liquid rises into the capillary as before 5×10^{-3} kg.

15. Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

When, the two drops of same radius r coalesce then radius of new drop is R

$$\therefore \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 + \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R = 2^{1/3}.r$$

Critical velocity $\propto r^2$

$$\therefore \quad \frac{v}{v_1} = \frac{r^2}{2^{2/3}.r^2}$$

$$\Rightarrow \quad v_1 = \sqrt[3]{4}.v$$

16. Due to volume expansion of both mercury and flask, the change in volume of mercury relative to flask is given by

$$\Delta V = V_0\left[\gamma_L - \gamma_g\right]\Delta\theta$$

$$= V\left[\gamma_m - 3\alpha_g\right]\Delta\theta$$

Given, $\gamma_m = 182 \times10^{-6}/°C$, $\alpha_g = 10 \times10^{-6}/°C$

$\Delta\theta = 100°C$, $V = 1L$

$$\therefore \quad \Delta V = 1[(182 \times10^{-6} - 3\times10\times10^{-6})]\times100$$

$$\Delta V = 15.2CC$$

17. In given condition

$$\frac{Y+160}{-50+160} = \frac{340-273}{373-273}$$

$$\frac{Y+160}{110} = \frac{67}{100}$$

$$Y+160 = \frac{67\times110}{100}$$

$$Y = 73.7 - 160$$

$$y = -86.3°y$$

18. We knows

$$dQ = du + dw$$

and we also known du = 0 for cyclio process so that

$$dQ = dw$$

Here, in given condition the work done during is a basic process

$W_{2-3} = P_2(V_3 - V_2)$

$W_{4-1} = p_1(v_1 - v_4)$

Total work done = $p_2(v_3 - v_2) + p_1(v_1 - v_4)$

From gas equation $pV = nRT = \frac{3\times T}{2}$

Hence, total work done

$= \frac{3R}{2}$ (400 + 2500 – 700 – 1100)

$= \frac{3}{2}$ R (2900 –1800)

$= \frac{3}{2}$ R(1100) $= \frac{3300R}{2}$

= 1650R

19. $\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{3}{5}$

$\Rightarrow \quad T_1 = \frac{5}{3}T_2$

$\Rightarrow \quad T_1 = \frac{5}{3} \times 300 = 500$ K

New efficiency $\eta' = 60\%$

$\frac{T_2}{T_1'} = 1 - \eta' = 1 - \frac{60}{100} = \frac{2}{5}$

$\Rightarrow \quad T_1' = \frac{5}{2} \times 300 = 700$ K

Increase in temperature = 750 – 500 = 250 K

20. We knows from Stefan's law,

$E = eA\sigma T^4$

Here, $E_1 = e_1 A\sigma T_1^4$

$E_2 = e_2 A\sigma T_2^4$

so, $E_1 = E_2$

$\therefore \quad e_1 T_1^4 = e_2 T_2^4$

$\Rightarrow \quad T_2 = \left(\frac{e_1}{e_2}T_1^4\right)^{1/4} = \left(\frac{1}{81}\times(5802)^4\right)^{1/4}$

$\Rightarrow \quad T_B = 1934K$

From Wein's law, $\lambda_{AT_A} = \lambda_B T_B$

$\Rightarrow \quad \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A}$

$\Rightarrow \quad \frac{\lambda_B - \lambda_A}{\lambda_B} = \frac{T_B - T_B}{T_A}$

$\Rightarrow \quad \frac{1}{\lambda_B} = \frac{5802 - 1934}{5802} = \frac{3968}{5802}$

$\Rightarrow \quad \lambda_B = \frac{3}{2}\mu m$

21. We knows frequency of a closed end an column

$n_1 = \frac{v}{4l_1}$

We knows frequency of a open end an column

$n_2 = \frac{v}{2l_2}$

Given, $l_1 = 32$ cm, $l_2 = 66$ cm

and $\quad n_1 - n_2 = 8$ heat/s

So, $\quad n_1 = \frac{v}{4\times 32} = \frac{v}{128}$

and $\quad n_2 = \frac{v}{2\times 66} = \frac{v}{132}$

In given condition,

$\frac{v}{128} - \frac{v}{132} = 8$

v = 8448 × 4

v = 33792

Hence, $\quad n_1 = \frac{33792}{128}$

$$n_1 = 264 \text{ Hz}$$

and $$n_2 = \frac{33792}{132}$$

$$n_2 = 256 \text{ Hz}$$

22. We know that,

$$n' = n\left[\frac{v}{v - v_s \cos\theta}\right]$$

Hence, $$n' = 640\left[\frac{340}{340 - \frac{100}{3} \times \frac{3}{5}}\right]$$

$$n' = 640\left[\frac{340}{340 - \frac{100}{5}}\right]$$

$$n' = 640 \times \frac{340}{320} = 2 \times 340$$

$$= 680 \text{ Hz}$$

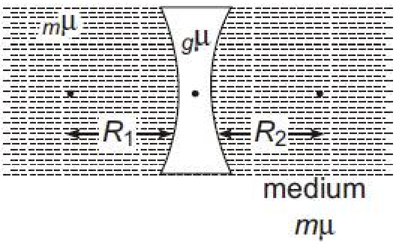
23. From lens maker's formula

$$\frac{1}{f} = \left({}_g^m\mu - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now, $${}_g^m\mu = \frac{{}_g\mu}{{}_m\mu} = \frac{1.5}{1.75}$$

For concave lens as shown in the figure, in this case

$$R_1 = -R \text{ and } R_2 = +R$$



$$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = +\frac{0.25 \times 2}{1.75R}$$

$$\Rightarrow \quad f = +3.5R$$

The positive sign shows that the lens behaves as a convergent lens.

24. For eye piece

$$V_e = -25 \text{ cm}, f_e = 5 \text{ cm}$$

$$\Rightarrow \quad \frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$$\Rightarrow \quad u_e = -\frac{25}{6} \text{cm}$$

$$v_0 = L - |u_e| = 10.5 - \frac{25}{6} = \frac{38}{6} \text{cm}$$

For objective

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\frac{1}{38/6} - \frac{1}{u_0} = \frac{1}{1.9}$$

$$\Rightarrow \quad \frac{1}{u_0} = \frac{6}{38} - \frac{1}{1.9}$$

$$\Rightarrow \quad u_0 = 2.7 \text{ cm}$$

25. In Fresnel diffraction, no lenses are required for rendering light rays parallel and also the diffraction pattern may be dark or bright depending upon the number of half-period zones that superpose at the point. Hence, the intensity of light at a point a screen beyond or obstacle depends on the number of half-periods zones that superpose at the point.

26. We know that the time period of a vibrating bar magnet

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Given, $T_1 = 8s$, $I_1 = I$, $M = 4 \text{ Am}^2$

$$8 = 2\pi\sqrt{\frac{I}{4 \times B_H}} \quad \text{... (i)}$$

Given, $T_2 = 6s$, $I_2 = 9 \times 10^{-2} \text{ kg–m}^2$, $M = 8 \text{ Am}^2$

$$6 = 2\pi\sqrt{\frac{9\times10^{-2}}{8\times B_H}} \qquad \text{... (ii)}$$

Dividing Eqs. (i) and (ii)

$$\frac{8}{6} = \sqrt{\frac{2I}{9\times10^{-2}}}$$

Squaring both sides and solving, we have

$$I = 8 \times 10^{-2}\ \text{kg} - \text{m}^2$$

27. We knows, $B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

Hence the resultant horizontal magnetic induction point of the line joining their conters is

$B = B_1 + B_2 + B_H$

$$= \frac{10^{-7}\times1.2}{\left(10\times10^{-2}\right)^3} + \frac{10^{-7}\times1}{\left(10\times10^{-2}\right)^3} + 3.6\times10^{-5}$$

$= 1.2 \times 10^{-4} + 1\times 10^{-4} + 0.36 \times 10^{-4}$

$= 2.56 \times 10^{-4}$ T

28. In deflection magnetometer, field due to magnet

F and horizontal component BH of earth's field are perpendicular to each other.

$\therefore$ Net field is $\sqrt{F^2 + B_H^2}$

So the time period

$$T = 2\pi\sqrt{\frac{I}{M\sqrt{F^2 + B_H}}} \qquad \text{... (i)}$$

When magnet is removed

$$T_0 = 2\pi\sqrt{\frac{I}{MB_H}} \qquad \text{... (ii)}$$

Also, $\frac{F}{B_H} = \tan\theta$

Dividing Eqs. (i) by (ii), we get

$$\frac{T}{T_0} = \sqrt{\frac{B_H}{\sqrt{F^2 + B_H^2}}}$$

$$= \sqrt{\frac{B_H}{\sqrt{B_H^2 + \tan^2\theta + B_H^2}}} = \sqrt{\frac{B_H}{B_H\sqrt{\sec^2\theta}}}$$

$$= \sqrt{\cos\theta}$$

$$\Rightarrow \quad T^2 = T_0^2\cos\theta$$

29. We knows capacitance $C = \frac{\varepsilon_0 A}{d}$

When plate is inserted

$$C' = \frac{\varepsilon_0 A}{d - \frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$$

$$\frac{C'}{C} = \frac{2}{1}$$

30. Applying junction law

We have

$$I = I_1 + I_2$$

$$\frac{24 - V}{3} = \frac{10 - V}{2} + \frac{9 - V}{1}$$

$$\Rightarrow \quad \frac{24 - V}{3} = \frac{28 - 3V}{2}$$

$\Rightarrow \quad 2(24 - V) = 3(28 - 3V)$

$\Rightarrow \quad 48 - 2V = 84 - 9V$

$\Rightarrow \quad 7V = 36$

$\Rightarrow \quad V = 5.14$ V

From Ohm's law

$$\Delta V = IR$$

$\Delta V = 24 - 5.14 = 18.86,\ R = 3\Omega$

$$\therefore \quad I = \frac{18.86}{3} \approx 6\ \text{A}$$

31. Here in given condition, we have

$$\frac{\frac{bx}{b+x}}{2} = \frac{0.625}{0.375}$$

$$\frac{bx}{(b+x)2} = \frac{25}{15}$$

$$\frac{5b}{(b+5)2} = \frac{5}{3}$$

$$\frac{b}{2b+10} = \frac{1}{3}$$

$\Rightarrow$ $3b - 2b = 10$

$$b = 10\Omega$$

32. The heat developed per unit time in given condition is proportional to ΔT and I.

33. We knows in LCR circuits

$$f = \frac{1}{2\pi\sqrt{LC}}$$

and $f \propto \frac{1}{\sqrt{L}}$

Here, in given condition

$$\frac{f_1}{f_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{L}{\sqrt{3}L}}$$

$$\Rightarrow \quad f_1 = \left(\frac{1}{3}\right)^{1/4} f_2$$

34. $$i_g = \frac{S}{S+G} i$$

$$\frac{1}{40} = \frac{10}{10+x}$$

$\Rightarrow$ $10 + x = 400$

$\Rightarrow$ $x = 39\Omega$

35. From equation of photoelectric effect, we have

$$E_1 = \frac{hc}{\lambda_1} - W \quad \text{... (i)}$$

$$E_2 = \frac{hc}{\lambda_2} - W \quad \text{... (ii)}$$

where, W is work function.

$$E_1 + W = \frac{hc}{\lambda_1} \quad \text{... (i)}$$

$$E_2 + W = \frac{hc}{\lambda_2} \quad \text{... (ii)}$$

From Eq. (iv)

$$W = \frac{hc}{\lambda_2} - E_2$$

$\therefore$ Putting this value in Eq. (iii), we have

$$E_1 + \frac{hc}{\lambda_1} - E_2 - \frac{hc}{\lambda_1}$$

$$\Rightarrow \quad E_1 - E_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad E_1 - E_2 = hc\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$$

$$\Rightarrow \quad hc = \frac{(E_1 - E_2)\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)}$$

36. Maximum $KE = \frac{hc}{\lambda} - \phi_0$

Given, λ = 3000 Å = 3000×10^{-10} m,

$h = 6.6 \times 10^{-34}$ J – s, $c = 3 \times 10^8$ m/s, ϕ = 2 eV

Maximum KE

$$= \frac{6.6\times10^{-34}\times3\times10^8}{3000\times10^{-10}} \times \frac{1}{1.6\times10^{-12}} - 2$$

= 4.13 – 2 = 2.13 eV

In Joules : $2.13 \times 1.6 \times 10^{-19} = 3.41 \times 10^{-19}$ J

37. The relation between radius (R) and atomic number (A) is

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

Given, R_1 = 6 fermi, A_1 = 125, A_2 = 27

$$\frac{6}{R_2} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3}$$

$$\Rightarrow \quad R_2 = \frac{6\times 3}{5} = \frac{18}{5} = 3.6 \text{ fermi}$$

$= 3.6 \times 10^{-15}$ m

38. In 1 s, energy generated is 3.7×10^7 J

In 144×10^4 s, energy generated is

=3.7 × 107 × 144 × 104 J

Also energy released in one fission is

= 185meV

= 185 × 106 × 1.6 × 10^{-19} J

$$\text{Number of fission} = \frac{3.7\times 10^7 \times 144\times 10^4}{185\times 10^6 \times 1.6\times 10^{-19}}$$

$= 1.8 \times 10^{24}$ of U^{235} atoms.

Mass contained in 1.8×10^{24} atoms of U^{235} is

$$= \frac{235\times 1.8\times 10^{24}}{6.023\times 10^{23}} = 702.3\text{g } 0.70 \text{ kg}$$

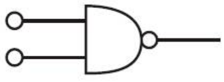
39. Current gain $\beta = \frac{\Delta i_c}{\Delta i_b}$

$$\Delta i_c = (4 - 0.2)\text{mA} = 3.8 \times 10^{-3} \text{ A}$$

$$\Delta i_b = (140 - 45)\mu\text{A} = 95 \times 10^{-6} \text{ A}$$

$$\therefore \quad \beta = \frac{3.8\times 10^{-3}}{95\times 10^{-5}} = 40$$

40. The logic symbol of NAND gate is



Chemistry

1. Number of spherical/radial nodes in any orbital

$= n - I - 1$

For s = orbitals, $I = 0$.

$\therefore$ Number of radial nodes in 3s-orbital

$= 3 - 0 - 1 = 2$

For p – orbitals, l = 1

$\therefore$ Number of radial nodes in 2p-orbitals

$= 2 - 1 - 1 = 0$

2. The quantum or wave mechanical model of atom is based upon the dual nature of electron, i.e., the electron is not only a particle but has a wave character. The wave character of electron has partical significance since its wavelength is easily observed in electromagnetic spectrum.

3. For 4th period, n = 4.

Orbitals being filled = 4s, 3d, 4p

Number of elements in the period = 2, 10, 6 = 18

4.

Molecule	**Hybridisation**	**Shape**
PCl_5	sp^3d	Trigonal
	dsp^2	bipyramidal
		Square planar
$[Ni(CN)_4]^{2-}$		
IF_3	dsp^3	Trigonal
		bipyramidal
		(bent T shaped)

5. (a) Hybridised orbitals show only head on overlapping and thus form only σ bonds.

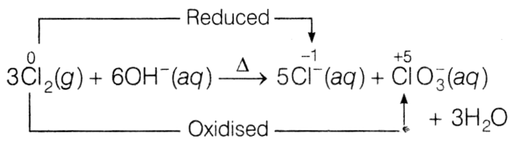
They never form π bonds.

$$\underbrace{\pi_{p-p}}_{\text{lateral overlapping}} < \underbrace{\sigma_{s-s} < \sigma_{s-p} < \sigma_{p-p}}_{\text{head on overlapping of same shell}}$$

(c) Head on overlapping is stronger than lateral or sideways overlapping. Therefore, the strength of bonds follows the order

(d) s -orbitals are spherically symmetrical and thus show only head on overlapping and form only σ bonds.

6. A reaction in which the same species is simultaneously oxidised as well as reduced is called a disproportionation reaction.



7. $KE = \frac{3}{2}RT$ for 1 mole of the gas.

$\because$ 4 g of H2 gas has 2 moles of H_2

$\therefore$ It has 2 times KE as compared to 1 mole of gas

But 8 g of O_2 gas has 1/4 moles of O_2 ;

$\therefore$ It has one fourth part of the KE as compared to 1 mole of gas.

Hence, ratio of KE of H_2 and O_2

$$KE_{H_2} : KE_{O_2} = 2 : \frac{1}{4} = 8 : 1$$

8. Two solutions are isotonic if they have same molar concentrations of the particles.

(a) 0.15 M NaCl and 0.1 M Na_2SO_4

NaCl is an electrolyte which dissociates to give 2 ions, thus concentration of ions in the solution 0.30 M.

Similarly for Na_2SO_4 (3 ions), the concentration of ions in the solution = 0.30 M. Hence, both are isotonic.

9. According to Raoult's law

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n_2}{n_1 + n_2}$$

where,

p° = vapour pressure of pure water at 100°C

= 760 mmHg.

p_s = vapour pressure of solution at 100°C

n_2 = moles of solute = $\frac{w_2}{M_2} = \frac{18}{180} = 0.1\text{mol}$

n_1 = moles of solvent = $\frac{w_1}{M_1} = \frac{180}{18} = 10\text{mol}$

By putting these values in the formula

$$\frac{p^\circ - p_s}{p^\circ} = \frac{0.1}{10 + 0.1}$$

or $10.1(p^\circ - ps) = 0.1p^\circ$

or $10p^\circ = 10.1ps$

or $p_s = \frac{10 \times 760}{10.1} = 752.4\text{mmHg}.$

10. During the electrolysis of an aqueous solution of copper sulphate using copper electrodes, both Cu^{2+} and H^+ ions move towards cathode, but the discharge potential of Cu^{2+} ions is lower than that of H^+ ions, therefore Cu^{2+} ions are discharged in preference to H^+ ions and copper is deposited on the cathode.

Cu^{2+} (aq) + $2e^-$ $\longrightarrow$ Cu(s) (at cathode)

11. In a fully charged lead accumulator or lead storage battery, sulphuric acid has a specific gravity (i.e., density that varies from 1.260 to 1.285. But during discharge (i.e., when the battery is in use). H_2SO_4 is used up.

$$Pb(s) + PbO_2(s) + 4H^+ (aq) + 2SO_4^{2-} \longrightarrow 2PbSO_4(l) + 2H_2O$$

As a result, the specific gravity of H_2SO_4 falls when it falls below 1.230 the battery needs recharging.

12. In a close packed structure (hcp or ccp)

(i) Number of octahedral voids = Number of particles present in the close packing

(ii) Number of tetrahedral voids = 2 × Number of octahedral voids.

13. Integrated rate equation for first order reaction

$$k = \frac{2.303}{t}\log\frac{a}{(a - x)}$$

or $k = \frac{k}{2.303}\log\frac{a}{(a - x)}$

$= \log a - \log (a - x)$

or $\log(a - k) = -\frac{k}{2.303}t + \log a$

Thus, if log (a – x) values are plotted against time 't' the graph obtained should be a straight line.

14. % dissociation = 4%

degree of dissociation $(\alpha) = \frac{4}{100} = 0.04$

For lactic acid

$CH_3CH(OH)COOH \rightleftharpoons CH_3CH(OH)COO^- + H^+$

Initial concentration

C mol L^{-1} 0 0

At. equilibrium $C(1-\alpha)$ $C\alpha$ $C\alpha$

$$\therefore \quad K_C = \frac{C\alpha.C\alpha}{C(1-\alpha)} = \frac{C\alpha^2}{(1-\alpha)}$$

$$= \frac{0.1 \times 0.04 \times 0.04}{(1-0.04)}$$

$$= \frac{1.6 \times 10^{-4}}{0.96} = 1.66 \times 10^{-4}$$

15. As a mixture of NH_4OH and NH_4Cl acts as a basic buffer, so its pH must be basic, (i.e., greater than 7), hence the answer must be 2nd. It can also be find by calculations :

$$pOH\ p = K_b + \log \frac{[salt]}{[base]}$$

$$= 4.8 + \log \frac{0.2M}{0.02M}$$

$$= 4.8 \times \log 10$$

$$pOH = 4.8 + 1 = 5.8$$

$$\therefore \quad pH = 14 - pOH = 14 - 5.8 = 8.2$$

16. Evaporation of water in an open vessel is a process which takes place by itself, by absorption of heat from the surroundings, because the gaseous water molecules are more random than the liquid water molecules.

$H_2O(I) \rightarrow H_2O(g); \ \Delta H = +40.8\ kJmol^{-1}$

In other words, the process is spontaneous because it is accompanied by increase of entropy, which is further a measure of randomness or disorder of the system.

17. The examples of colloidal systems are sols (solids in liquids), gels (liquids in solids), emulsions (liquids in liquids) and foams (gases in liquids) whereas aerosols are the colloidal system in which dispersed phase is liquid and dispersion medium is gas.

18. (a) Heavy water is injurous to human beings, plants and animals since it slows down the rates of reactions occuring in them.

(b) Heavy water reacts with aluminium carbide forming deuteromethane.

$$\underset{\text{aluminium carbide}}{Al_4C_3} + 12D_2O \longrightarrow 4Al(OD)_3 + \underset{\text{deuteromethane}}{CD_4}$$

(c) In interstitial hydrates or deuterates, water molecules are present in interstitial sites or voids in the crystal lattice. e.g., $BaCl_2 . 2H_2O$ and similarly $BaCl_2 . 2D_2O$ are interstitial compounds.

19. For titration of a basic solution of Na_2CO_3 and $NaHCO_3$ against HCl, if phenolphthalein is used as indicator, the end point is indicated only for half neutralization of Na_2CO_3, *i.e.,* (upto $NaHCO_3$).

$Na_2CO_3 + HCl \longrightarrow NaHCO_3 + NaCl$

The remaining solution then contains the unreacted $NaHCO_3$ from this reaction plus the unreacted $NaHCO_3$ originally in the solution. At the phenolphthalein end point, there is no reaction between HCl and $NaHCO_3$.

From the equations

Mol of HCl consumed = mol of Na_2CO_3

20 mL of 0.1 M = 20 mL of 0.1 M

$\therefore$ The concentration of Na_2CO_3 in solution

$X = 0.1M.$

Note that for a quantity of Na2CO3, exactly half volume of the HCl is used at the phenolphthalein end point and the second half volume of the HCl is required for complete neutralization of Na_2CO_3 at methyl orange end point.

$NaHCO_3 + HCl \longrightarrow NaCl + CO_2 + H_2O$

$\therefore$ Volume of HCl required to neutralize

Na_2CO_3 in original sample = 2×20 mL

= 40 mL

If methyl orange is used, the end point is indicated when all the alkali is neutralized.

$NaHCO_3 + HCl \longrightarrow NaCl + CO_2 + H_2O$

As 40 mL of 0.1 M HCl is consumed in complete neutralization of Na_2CO_3 at methyl orange end point, so the volume of HCl used to neutralized

$NaHCO_3$ from the original sample would be

Remaining HCl = 60 – 40 = 20 mL of 0.1 M

As per equation = 1 mol of $NaHCO_3$ = 1 mol of HCl

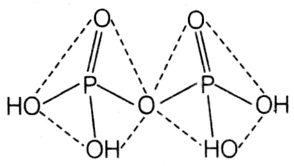
$\therefore$ 0.1 mol of $NaHCO_3$ = 0.1 mol of HCl,

20. Diborane reacts with HCl in the presence of $AlCl_3$ catalyst and liberates H_2 gas.

$$\underset{\text{diborane}}{B_2H_6} + HCl \xrightarrow{AlCl_3} B_2H_5Cl + H_2 \uparrow$$

21. If all the four corners, *i.e.,* all the four oxygen atoms of each tetrahedra (SiO_4) are shared with other tetrahedra, three-dimensional network structure is obtained. *i.e.,* different forms of silica such as quartz, tridymite and crystobalite.

22. In pyrophosphoric acid ($H_4P_2O_7$), the phosphorus is bonded in tetrahedral manner with four sp^3 bonds. It has +5 oxidation state and it has four P — OH bonds, two P == O bonds and one P — O — P linkage.



pyrophosphoric acid ($H_4P_2O_7$)

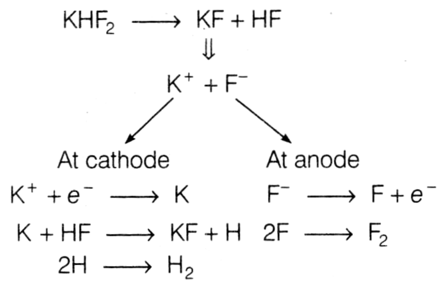
$$P_4O_{10} + 6H_2O \longrightarrow \underset{\text{orthphosphoric acid}}{4H_3PO_4}$$

23. Sodium thiosulphate is oxidised by moist Cl_2 or chlorine water and precipitates sulphur,

$$\underset{\text{sodium thiosulphate}}{Na_2S_2O_3} + Cl_2 + H_2O \longrightarrow Na_2SO_4 + 2HCl + \underset{\text{sulphur}}{S}$$

24. In Whytlaw-Gray's method for preparation of fluorine, the copper diaphragm is used to prevent the mixing of H_2 and F_2 liberated at cathode and anode respectively.

Reactions in the electrolytic cell



25. Liquid xenon is used in bubble chamber for the detection of g-photon and neutral mesons.

26. When a compound absorbs a certain wavelength (i.e., 490 nm - 500 nm) from the visible light which corresponds to the blue-green light in the visible spectrum, the colour or light transmitted by it does not contain the colour of absorbed radiation and thus shows the complementary colour, i.e., red.

27. In the extraction of silver from cyanide process (also Mac-Arthur Forrest process), the siliver compound dissolve in NaCN solution forming a soluble complex salt, in presence of air, then silver is precipitated from this complex salt by the addition of Zn, because Zn being more reactive displaces the Ag.

$$Ag_2S + 4NaCN \rightleftharpoons 2Na[Ag(CN)_2] + Na_2S \downarrow$$

$$Na_2S + 2O_2 \longrightarrow Na_2SO_4$$

$$2Na[Ag(CN)_2] + Zn \longrightarrow Na_2[Zn(CN)_4] + 2Ag \downarrow$$

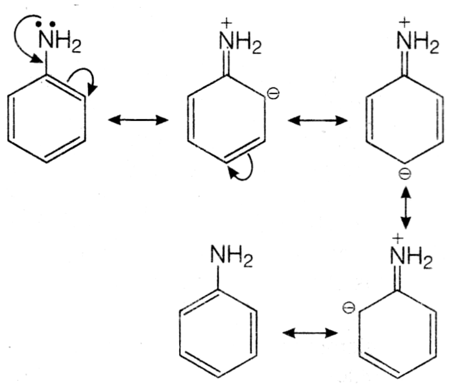
28. The most serious effect of the delpetion of ozone layer is that the UV rays coming from the sun can pass through the stratosphere and reach the surface of the earth. IT has been found that with increase in the exposure to UV rays, the chance for occurance of skin cancer, increases, also exposure of eye to UV rays damages the cornea and lens of the eye and may cause cataract and even blindness.

29. Ferric salts (such as FeCl3) form Prussian blue (blue ppt or colouration) with potassium ferrocyanide.

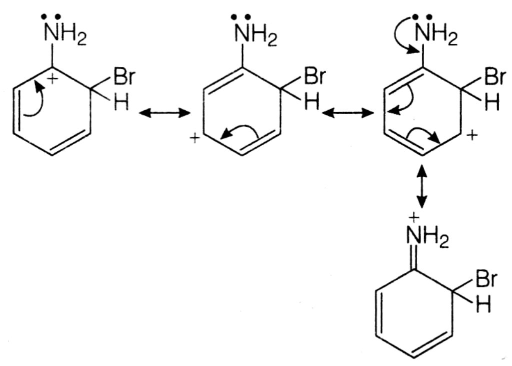
$$4FeCl_3 + 3K_4[Fe(CN)_6] \rightarrow \underset{\substack{\text{Prussian blue}\\\text{(ferric ferrocyanide)}}}{Fe_4\left[Fe(CN)_6\right]_3} + 12KCl$$

30. Decomposition of a compound by application of heat is called pyrolysis and pyrolysis of higher alkanes into a mixture of lower alkanes, alkenes, etc. is also called cracking.

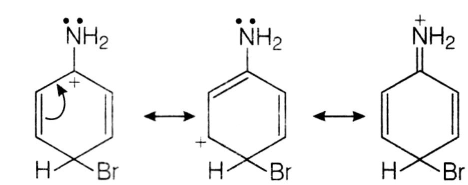
31. The —NH_2 group of aniline is a very strong electron donor (+ M effect), hence it activates the benzene ring thoroughly and the electrophilic aromatic substitutions on the benzene ring are very easy to take place at ortho and para-positions.



***o*-bromination**

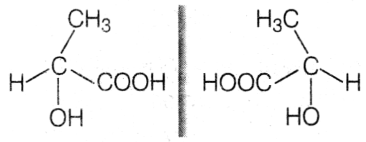


***p*-bromination**



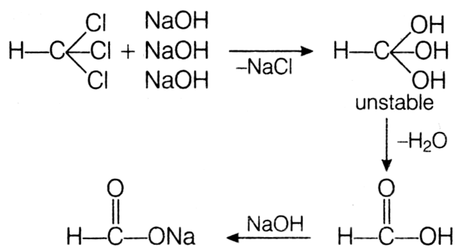
In addition to the usual resonating structures, that stabilizes the intermediate carbonium ion, the resonating strucutres formed by the interaction of lone pair electrons of nitrogen with the positively charged carbon of the ring also increase the stability of the carbonium ion formed during the atttack of Br^+ ion (electrophile) on o- and p-positions.

32. The compound which is non-superimposable on its mirror image, is called optically active or chiral and its two non-superimposable mirror images are called enantiomers which have all physical and chemical properties same and also rotate the plane polarised light upto same extent but in opposite direction.



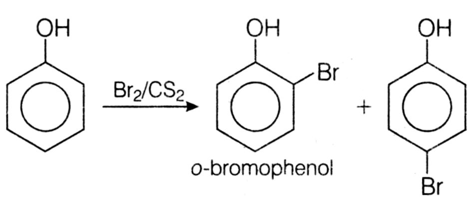
enantiomers of lactic acid

33. On heating chloroform with concentrated aqueous or alcoholic NaOH, we get sodium formate,

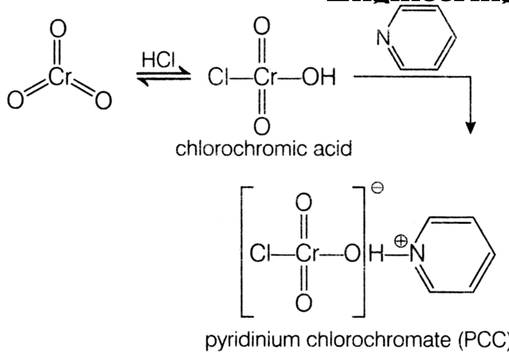


34. Phenol when treated with Br_2 in the presence of non-polar solvent CS_2, it gives only o- and p-bromophenols instead the trisubstituted products. Reason for the above observation is supression of phenoxide ion in non-polar solvent.

Thus, we get only mono substituted products.



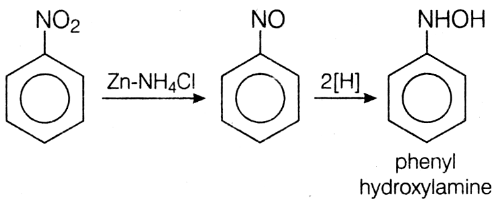
35. Pyridinium chlorochromate (PCC) can be prepared by the dissolution of chromium trioxide in aqueous HCl. Addition of pyridine gives pridinium chlorochromate as orange cryotol.



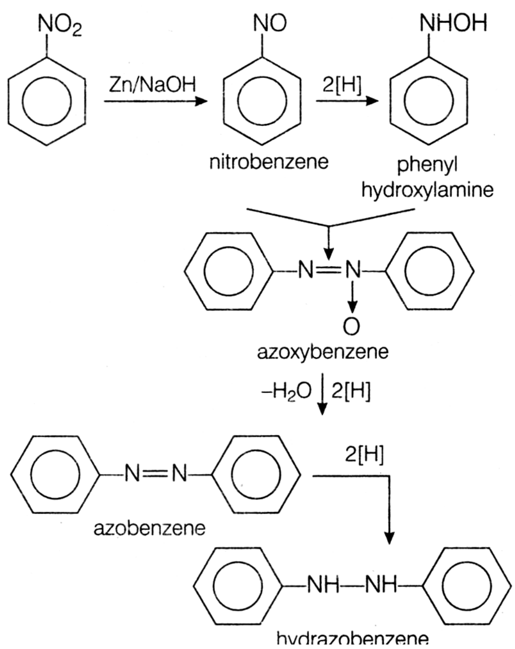
PCC offers the advantage of the selective oxidation of alcohols to aldehydes, whereas many other reagents are less selective.

36. The more the value of pK_a, compound will be less acidic indicating the less value of K_a and smaller the value of pKa, the compound will be more acidic, i.e., there will be more the value of K_a.

37. (i) On reduction in neutral media, using Zn dust and NH_4Cl solution nitrobenzene gives phenyl hydroxylamine.

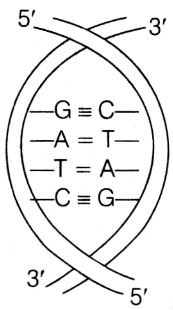


(ii) While in alkaline medium, using Zn-NaOH, mononuclear intermediate products (nitrobenzene and phenylhydroxyl amine) interact to each other to give dinuclear product. Final product using Zn-NaOH is hydrazobenzene, which in a formed viz the formation of azoxybenzene and azobenzene.



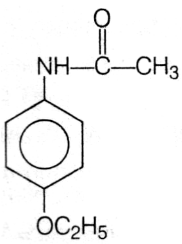
38. The polymers which disintegrate by themselves during a certain period of time by enzymatic hydrolysis and to some extent by oxidation, are known as biodegradable polymers. Example $\rightarrow$ Poly-b-hydroxy-butyrate-co-β-hydroxyvalerate (PHBV), which is used in orthopaedic devices and in controlled drug release, and poly (glycollic acid) poly (lactic acid) or commonly known as dextron, which is used for stitching of wounds after operation.

39. The only possible pairing in DNA are between G (guanine) and C (cytosine) through three H-bonds i.e., $(C \equiv G)$ and between A (adenine) and T (thymine) through two H-bonds (i.e., A = T) as shown in figure.



The double α-helix structure of DNA.

40. Phenacetin is a derivative of p-aminophenol and used as analgesic (pain killer). The main limitation of this drug is that it may act on red blood cells and thus may be harmful even in moderate dose.



phenacetin

Engineering Entrance Exam Solved Paper 2013

Mathematics

1. Given, $f(x) = (p - x^n)^{1/n}, p > 0$

   Now, $f[f(x)] = f[(p - x^n)^{1/n}]$

   $= \{p - (p - x^n)^{1/n \times n}\}^{1/n}$

   $= (x^n)^{1/n} = x$

2. $x \in R \,|\log [(1.6)^{1-x2} - (0.625)^{6(1+x)}] \in R \}$

   Now, $(1.6)^{1-x2} > (0.625)^{6(1+x)}$

   $\Rightarrow \quad (1.6)^{1-x2} > (0.625)^{6(1+x)}$

   $$= \left(\frac{8}{5}\right)^{1-x^2} > \left(\frac{8}{5}\right)^{-6(1+x)}$$

   $\therefore \quad 1 - x^2 > -6(1 + x)$

   $\Rightarrow \quad x^2 - 6x - 7 < 0$

   $\Rightarrow \quad (x - 7)(x + 1) < 0$

   $\Rightarrow \quad x \in (-\infty, -1) \cup (7, \infty)$

   Hence,

   $x \in R \,|\log [(1.6)^{1-x2} - (0.625)^{6(1+x)}] \in R \}$

   $= (-\infty, -1) \cup (7, \infty)$

3. Given $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

   Now, $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

   $$= \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

   Similarly,

   $$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

   $$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

   We have, $nA - (n - 1) I$

   $$= \begin{bmatrix} n & n \\ 0 & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$$

   $$= \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = A^n$$

   $A^n = nA - (n - 1) I$ is true

4. Given,

   ${}^nC_{r-1} = 330,\ {}^nC_r = 462$

   and ${}^nC_{r+1} = 462$

   Now, $\dfrac{{}^nC_{r+1}}{{}^nC_r} = 1$

   $$\Rightarrow \quad \frac{\dfrac{n!}{(r+1)!(n-r-1)!}}{\dfrac{n!}{r!(n-r)!}} = 1$$

   $$\Rightarrow \quad \frac{r!(n-r)(n-r-1)!}{(r+1)r!(n-r-1)!} = 1$$

   $\Rightarrow \quad \dfrac{n-r}{r+1} = 1 \quad \Rightarrow n - 2r = 1 \quad$ ... (i)

   Again, $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{462}{330} = \dfrac{77}{55}$

   $$\Rightarrow \quad \frac{\dfrac{n!}{r!(n-r)!}}{\dfrac{n!}{(r-1)!(n-r+1)!}} = \frac{77}{55}$$

   $$\Rightarrow \quad \frac{(r-1)!(n-r+1)(n-r)!}{r(r-1)!(n-r)!} = \frac{77}{55}$$

   $\Rightarrow \quad 55n - 132r + 55 = 0 \quad$ ...(ii)

   From Eqs. (i) and (ii), we get

   $22r = 110$

   $\therefore \quad r = 5$

5. M M M M M M M M M M

W W W W W W W W W W W

First, we arrange 10 men in a row at alternate position.

So, number of ways formula = 10 !

Now, 6 women can arrange in 11 positions

So, number of ways for women = $^{11}P_6$

Required number of ways =10 ! × $^{11}P_6$

$$= \frac{10!11!}{5!}$$

6. t_n = The number of triangles formed with n points in a plane, no three of which are collinear.

i.e., $t_n = {}^nC_3$

$\Rightarrow$ $t_{n+1} = {}^{n+1}C_3$

Now, $t_{n+1} - = {}^{n+1}C_3$

$\Rightarrow$ $^{n+1}C_3 - {}^nC_3 = 36$

$$\Rightarrow \frac{(n+1)!}{(n-2)!3!} = \frac{n!}{(n-3)!3!} = 36$$

$$\Rightarrow \frac{(n+1)(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 36$$

$\Rightarrow$ n(n – 1)(n + 1 – n + 2) = 36 ×6

$\Rightarrow$ 3n(n – 1) = 36 × 6

$\Rightarrow$ $n^2 - n - 72 = 0$

$\Rightarrow$ $n^2 - 9n + 8n - 72 = 0$

$\Rightarrow$ (x – 1) n(n – 9)+ 8(n – 9) = 0

(n – 9)(n + 8) = 0

$\therefore$ n = 9 $(\because n \neq -3)$

7. $$\left[\frac{(x+1)}{(x^{2/3}-x^{1/3}+1)} - \frac{(x-1)}{(x-\sqrt{x})}\right]^{10}$$

$$= \left[\frac{(x^{1/3})+1^3}{(x^{2/3}-x^{1/3}+1)} - \frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$= \left[\frac{(x^{1/3}+1)+(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)} - \frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$= \left[(x^{1/3}+1) - \frac{(\sqrt{x}+1)}{\sqrt{x}}\right]^{10} = (x^{1/3} - x^{-1/2})^{10}$$

The general term is

For the term independent of x, put

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$\Rightarrow$ 20 – 2r – 3r = 0

$\Rightarrow$ 20 = 5r $\Rightarrow$ r = 4

$$\therefore\ T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

8. Given expression is

$$E = \frac{(1-2x)^{-1}(1-3x)^{-2}}{(1-4x)^{-3}}$$

$$= \frac{(1+2x+2x^2+...)(1+6x+...)}{(1+12x+...)}$$

$$= \frac{(1+2x+6x+...)}{(1+12x)}$$

$= (1+ 8x)(1 + 12x)^{-1}$

(neglect higher powers)

= (1 + 8x)(1 – 12x...)

= (1 + 8x – 12x...)

= (1 – 4x)

($\because$ neglecting the higher term)

9. Given,

$$\frac{1}{x^4+x^2+1} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{x^2-x+1}$$

$\Rightarrow$ $1 = (Ax + 8)(x^2 - x + 1) + (Cx + D)(x^2 + x + 1)$

$\Rightarrow$ $1 = (Ax^3 + Ax^{2+} Ax + 6x^2 - 6x + B)$

$+ (Cx^3 + Cx^2 + Cx + Dx^2 + Dx + D)$

$\Rightarrow$ $1 = (A + C)x^3 + (-A + 6 + C + D)x^2$

$+ (A - B + C + D)x + (6 + D)$

On comparing, the coefficient of like powers on both sides, we get

$$A + C = 0 \quad \text{... (i)}$$

$$-A + 8 + C + D = 0 \quad \text{... (ii)}$$

$$A - 6 + C + D = 0 \quad \text{... (iii)}$$

and $$6 + D = 1 \quad \text{... (iv)}$$

On adding Eqs. (ii) and (iii), we get

$$2(C + D) = 0$$

$\Rightarrow$ $$C + D = 0$$

10. $$\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\frac{1}{8.9}+\ldots\infty$$

$$=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\left(\frac{1}{8}-\frac{1}{9}\right)+\ldots\infty$$

$= 1 - \log_e 2$

$= \log_e e - \log_e 2$

$$=\log\left(\frac{e}{2}\right)$$

11. Given equation is

$$\left(5+\sqrt{2}\right)x^2 - bx + \left(8+2\sqrt{5}\right) = 0$$

Let a and p be the roots of this equation.

$\therefore$ $$\alpha+\beta=\frac{b}{5+\sqrt{2}}$$

and $$\alpha\beta=\frac{8+2\sqrt{3}}{5+\sqrt{2}}$$

Given that harmonic mean between the roots of the given equation is 4.

$\therefore$ $$\frac{2\alpha\beta}{\alpha+\beta}=4$$

$\Rightarrow$ $$\frac{8+2\sqrt{5}}{5+\sqrt{2}}\times\frac{5+\sqrt{2}}{b}=2$$

$\therefore$ $$b=\frac{8+2\sqrt{5}}{2}=4+\sqrt{5}$$

12. Given, $x^2 + 5x + 6 \geq 0$ and $x^2 + 3x - 4 < 0$

$\Rightarrow$ $x^2 + 2x + 3x + 6 \geq 0$

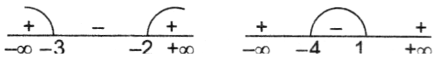
and $x^2 + 4x - x - 4 < 0$

$\Rightarrow$ $x(x + 2) + 3(x + 2) \geq 0$

and $x(x + 4) - 1(x + 4) < 0$

$\Rightarrow$ $(x + 2)(x + 3) \geq 0$

and $(x + 4)(x - 1) < 0$



$\Rightarrow$ $X \in (-\infty, -3][-2, \infty)$ and $X \in (-4, 1)$

Common condition is

$$X \in (-4, -3] \cup [-2, 1)$$

13. Given, cubic equation is

$$x^3 - 42x^2 + 336x - 512 = 0$$

$\Rightarrow$ $x^2(x - 2) - 40x(x - 2) + 256(x - 2) = 0$

$\Rightarrow$ $(x - 2)(x^2 - 40x + 256) = 0$

$\Rightarrow$ $(x - 2)\{x^2 - 32x - 8x + 256\} = 0$

$\Rightarrow$ $(x - 2)\{x(x - 32) - 8(x - 32)\} = 0$

$\Rightarrow$ $(x - 2)(x - 32)(x - 8) = 0$

$\Rightarrow$ $(x - 2)(x - 8)(x - 32) = 0$

$\Rightarrow$ $x = 2, 8, 32$

Which represents a geometric progression in increasing order.

Common ratio $= \dfrac{8}{2} = 4{:}1\ 2$

14. Given quadratic equation is

$$x^2 - 2x + 4 = 0$$

whose roots are α and β.

$\therefore$ $\alpha+\beta = 2$ and $\alpha\beta = 4$ ... (i)

Now,

$$\alpha^9+\beta^9=\left(\alpha^3\right)^3+\left(\beta^3\right)^3$$

$$=\left(\alpha^3+\beta^3\right)=\left(\alpha^6+\beta^6-\alpha^3\beta^3\right)$$

$$=\left(\alpha+\beta\right)\left(\alpha^2-\alpha\beta^6+\beta^2\right)$$

$$\left[\left(\alpha^2\right)^3+\left(\beta^2\right)^3-\alpha^3+\beta^3\right]$$

$$=\left(\alpha+\beta\right)\left[\left(\alpha+\beta\right)^2+3\alpha\beta\right]$$

$$\left[\left(\alpha^2+\beta^2\right)\left(\alpha^4+\beta^4-\alpha^2\beta^2\right)-\alpha^3\beta^3\right]$$

$$=\left(\alpha+\beta\right)\left[\left(\alpha+\beta\right)^2-3\alpha\beta\right]\left[\left\{\left(\alpha+\beta\right)^2-2\alpha\beta\right\}\right.$$

$$\left.\left\{\left(\alpha^2+\beta^2\right)^2-3\alpha^2\beta^2\right\}-\alpha^3\beta^3\right]$$

$$=\left(\alpha+\beta\right)\left[\left(\alpha+\beta\right)^2-3\alpha\beta\right]\left[\left\{\left(\alpha+\beta\right)^2-2\alpha\beta\right\}\right.$$

$$\left[\left\{\left(\alpha+\beta\right)^2-\left(2\alpha\beta\right)\right]^2-3\alpha^2\beta^2\right\}-\alpha^3\beta^3\right]$$

$= 2[4 - 12][\{4 - 8\}\{(4 - 8)^2 - 48\} - 64]$

[from Eq. (i)]

$= 2(-8)\{(-4)(-32)(-64)\}$

$= 2(-8)(128 - 64)$

$= 2(-8)(64) = -2^{10}$

15. Given, $A=\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix}$

$$A^2=\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix}\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}64+10 & -40+20\\ -16+8 & 10+16\end{bmatrix}$$

$$=\begin{bmatrix}74 & -20\\ -8 & 26\end{bmatrix}$$

$$4A=\begin{bmatrix}-32 & 20\\ 8 & 16\end{bmatrix}$$

$$-pl=\begin{bmatrix}-p & 0\\ 0 & -p\end{bmatrix}$$

Since, the matrix A satisfies the equation

$x^2 + 4x - p = 0$, then

$A^2 + 4A - pl = 0$

$$\Rightarrow \begin{bmatrix}74 & -20\\ -8 & 26\end{bmatrix}+\begin{bmatrix}-32 & 20\\ 8 & 16\end{bmatrix}+\begin{bmatrix}-p & 0\\ 0 & -p\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}74-32-p & -20+20+0\\ -8+8+0 & 26+16-p\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}42-p & 0\\ 0 & 42-p\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

On comparing, we get

$42 - p = 0 \Rightarrow p = 42$

16. Let $\Delta=\begin{vmatrix}x+2 & x+3 & x+5\\ x+4 & x+6 & x+9\\ x+8 & x+11 & x+15\end{vmatrix}$

Apply operations $R_2 \to R_2 \to R_1, R_3 \to R_3$ we get

$$\Delta=\begin{vmatrix}x+2 & x+3 & x+5\\ 2 & 3 & 4\\ 6 & 8 & 10\end{vmatrix}$$

Again, apply operation $C_2 \to C_2 - C_1$,

$C_3 \to C_3 - C_1$, we get x + 2

$$\Delta=\begin{vmatrix}x+2 & 1 & 3\\ 2 & 1 & 2\\ 6 & 2 & 4\end{vmatrix}$$

Expand along R_1, we get

$\Delta = (x + 2)(4 - 4) - 1(8 - 12) + 3(4 - 6)$

$= 0 + 4 + 3(-2) = 4 - 6 = -2$

17. Given system of equation is

$$3x + 2y + z = 6$$

$$3x + 4y + 3z = 14$$

$6x + 10y + 8z = a$

Here, $A = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 4 & 3 \\ 6 & 10 & 8 \end{vmatrix}, B = \begin{vmatrix} 6 \\ 14 \\ a \end{vmatrix}$

$C_{11} = (32 - 30) = 2, C_{12} = -(24 - 18) = -6,$
$C_{13} = (30 - 24) = 6$
$C_{21} = -(16 - 10) = -6, C_{22} = (24 - 6) = 18,$
$C_{23} = -(30 - 12) = -18$
$C_{31} = (6 - 4) = 2, C_{32} = -(9 - 3) = -6,$
$C_{33} = (12 - 6) = 6$

$$\text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}' = \begin{bmatrix} 2 & -6 & 6 \\ -6 & 18 & -18 \\ 2 & -6 & 6 \end{bmatrix}'$$

$$= \begin{bmatrix} 2 & -6 & 2 \\ -6 & 18 & -6 \\ 6 & -18 & 6 \end{bmatrix}$$

So, $(\text{adj}\,A)B = \begin{bmatrix} 2 & -6 & 2 \\ -6 & 18 & -6 \\ 6 & -18 & 6 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ a \end{bmatrix}$

$$= \begin{bmatrix} 12 - 84 + 2a \\ -36 + 252 - 6a \\ 36 - 252 + 6a \end{bmatrix}\begin{bmatrix} -72 + 2a \\ 216 - 6a \\ -216 + 6a \end{bmatrix}$$

and $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 4 & 3 \\ 6 & 10 & 8 \end{vmatrix}$

$= 3(32 - 30) - 2(24 - 18) + 1(30 - 24)$
$= 3(2) - 2(6) + 6 = 6 - 12 + 6 = 0$

We know that, if $|A| = 0$ and $(\text{adj}\,A).B = 0$, then the system of equations is consistent and has an infinite number of solutions.

$(\text{adj}A).6 = 0$

$$\Rightarrow \begin{bmatrix} -72 + 2a \\ 216 + 6a \\ -216 + 6a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On comparing, we get

$2a - 72 = 0$

$\therefore \quad a = 36$

18. Given,

$fx + (r + 1)y + (f - 1)z = 0$
$(f + 1)x + ty + (f + 2)z = 0$
$(f - 1)x + (t + 2)y + fz = 0$

Here,

Coefficient matrix, $A = \begin{bmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{bmatrix}$

If $|A| = 0$, then system of equations has non-trivial solution and it has infinite solutions.

$$|A| = \begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix} = 0$$

Apply operation $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ we get

$$= \begin{vmatrix} t & t+1 & t-1 \\ 1 & -1 & 3 \\ -1 & 1 & 1 \end{vmatrix} = 0$$

Apply $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$, we get

$$\begin{vmatrix} t & 1 & -1 \\ 1 & -2 & 2 \\ -1 & 2 & 2 \end{vmatrix} = 0$$

Expanding along R_1, we get

$t(-4 - 4) - 1(2 + 2) - 1(2 - 2) = 0$

$\Rightarrow \quad -8t - 4 = 0$

$\therefore \quad t = \frac{-1}{2}$

So, no option is correct.

19. $\left(\frac{1+i}{1-i}\right)^4 + \left(\frac{1-i}{1+i}\right)^4$

$$= \left\{\frac{(1+i)(1+i)}{(1-i)(1+i)}\right\}^4 + \left\{\frac{(1-i)(1-i)}{(1+i)(1-i)}\right\}^4$$

$$= \left\{\frac{(1+i)^2}{1-i^2}\right\}^4 + \left\{\frac{(1-i)^2}{1-i^2}\right\}^4$$

$$= \left\{\frac{1+i^2+2i}{1+1}\right\}^4 + \left\{\frac{1+i^2-2i}{1+1}\right\}^4$$

$$= \left\{\frac{1-1+2i}{2}\right\}^4 + \left\{\frac{1-1-2i}{2}\right\}^4$$

$= (i)^4 + (-i)^4$

$= i^4 + i^4 = 2i^4 = 2(1) = 2$

20. Given, $|z^2 - 1| = |z|^2 + 1$

Let $z = x + iy$

$\Rightarrow |(x+iy)^2 - 1| = |x+iy|^2 + 1$

$\Rightarrow |x^2 - y^2 + 2ixy - 1| = (x^2 + y^2) + 1$

$\Rightarrow |(x^2 - y^2 - 1) + 2ixy| = (x^2 + y^2 + 1)$

$\Rightarrow \sqrt{(x^2 - y^2 - 1)^2 + 4x^2y^2} = x^2 + y^2 + 1$

$\Rightarrow (x^2 - y^2)^2 + 1 - 2(x^2 - y^2) + 4x^2y^2$

$= (x^2 + y^2 + 1)^2$

$= x^4 + y^4 + 2x^2y^2 + 1 + 2x^2 + 2y^2$

$\Rightarrow -2x^2y^2 - 2x^2 + 4x^2y^2 = 2x^2y^2 + 2x^2$

$\Rightarrow -2x^2 = 2x^2$

$\Rightarrow 4x^2 = 0 \Rightarrow x = 0$

$\therefore z = x + iy = 0 + iy$

$\Rightarrow z = iy \Rightarrow (x, y) = (0, y)$

Hence, z lies on the imaginary axis.

21. $\frac{(1+i)x - i}{2+i} + \frac{(1+2i)y + i}{2-i} = 1$

$$\Rightarrow \frac{[(1+i)x - i](2-i)}{(4-i^2)} + \frac{[(1+2i)y + i](2+i)}{(4-i^2)} = 1$$

$$\Rightarrow \frac{2(1+i)x - 2i - i(1+i)x + i^2}{4+1} + \frac{2(1+2i)y + 2i + i(1+2i)y + i^2}{(4+1)} = 1$$

$$\Rightarrow \frac{(2+2i-i-i^2)x - 2i + i^2}{5} + \frac{(4i+2+i+2i^2)y + 2i + i^2}{5} = 1$$

$$\Rightarrow \frac{(2+i+1)x - 2i - i}{5} + \frac{(5i+2-2)y + 2i - i^2}{5} = 1$$

$\Rightarrow (3+i)x - 2i - 1 + (5i)y + 2i - 1 = 5$

$\Rightarrow (3+i)x + 5iy = 7$

$\Rightarrow 3x + ix + 5iy - 7 = 0$

$\Rightarrow (3x - 7) + (x + 5y)i = 0 + i0$

On comparing, we get

$3x - 7 = 0$

$\Rightarrow x = \frac{7}{3}$ and $x + 5y = 0$

$\Rightarrow y = \frac{-7}{15}$

Hence, $(x, y) = \left(\frac{7}{3}, \frac{-7}{15}\right)$

22. Given, $f(x) = \cos\left(\frac{x}{3}\right) + \sin\left(\frac{x}{2}\right)$

Period of cos x and sin x are 2π.

$\therefore$ Period of f(x) = Period of $\left[\cos\frac{x}{3} + \sin\frac{x}{2}\right]$

$= \text{Period of } \cos\frac{x}{3} + \text{Period of } \sin\frac{x}{2}$

$$= \frac{2\pi}{\left(\frac{1}{3}\right)} + \frac{2\pi}{\left(\frac{1}{2}\right)} = \frac{6\pi}{1} + \frac{4\pi}{1}$$

$$= \frac{\text{LCM of }(6\pi, 4\pi)}{\text{LCM of }(1,1)} = \frac{12\pi}{1} = 12\pi$$

23. Given, $\sin\theta + \cos\theta = p$ ... (i)

and $\sin^3\theta + \cos^3\theta = q$ ... (ii)

$\Rightarrow (\sin\theta + \cos\theta)$

$(\sin^2\theta - \sin\theta.\cos\theta + \cos^2\theta) = q$

$\Rightarrow p(1 - \sin\theta.\cos\theta) = q$

[From Eq. (i) and $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow 1 - \sin\theta.\cos\theta = \frac{q}{p}$

$\Rightarrow \sin\theta.\cos\theta = 1 - \frac{q}{p}$ ... (iii)

On squaring both sides of Eq. (i), we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta + \cos\theta = p^2$

$\Rightarrow 1 + 2\left(1 - \frac{q}{p}\right) = p^2$ [from Eq. (iii)]

$\Rightarrow p + 2(p - q) = p^3$

$\Rightarrow 3p - 2q = p^3$

$\Rightarrow p^3 - 3p = -2q$

$\Rightarrow p(p^2 - 3) = -2g$

24. Given, $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$\Rightarrow \tan(\pi\cos\theta) = \tan\left\{\frac{\pi}{2} - \pi\sin\theta\right\}$

$\Rightarrow \pi\cos\theta = \frac{\pi}{2} - \pi\sin\theta$

$\Rightarrow \sin\theta + \cos\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\theta.\cos\frac{\pi}{4} + \sin\theta.\sin\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

25. Given system of equation is

$x + y = \frac{2\pi}{3}$ ... (i)

and $\cos x + \cos y = \frac{3}{2}$, where x, y are real.

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\Rightarrow \cos\left(\frac{1}{2}.\frac{2\pi}{3}\right).\cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$

[from Eq. (i)]

$\Rightarrow \cos\left(\frac{\pi}{3}\right).\cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$

$\Rightarrow \frac{1}{2}\cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$

$\Rightarrow \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ ... (ii)

Now, we have

$\cos(x - y) = 2\cos^2\left(\frac{x-y}{2}\right) - 1$

$= 2 \times \frac{9}{4} - 1 = \frac{9}{2} - 1 = \frac{7}{2}$ [from Eq. (ii)]

and $\cos(x - y) = 1 - 2\sin^2\left(\frac{x-y}{2}\right)$

$$\Rightarrow \quad \sin\left(\frac{x-y}{2}\right) < 0$$

So, system of equation have empty set of solution.

26. $\cos^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \cos^{-1} x$

$$= \cos^{-1}\left[\frac{5}{13}.\frac{3}{5} - \sqrt{1-\frac{25}{169}}.\sqrt{1-\frac{9}{25}}\right] = \cos^{-1} x$$

$$\Rightarrow \cos^{-1}\left[\frac{3}{13} - \frac{12}{13}.\frac{4}{5}\right] = \cos^{-1} x$$

$$\Rightarrow \quad \cos^{-1}\left[\frac{15-48}{65}\right] = \cos^{-1} x$$

$$\therefore \quad x = \frac{-33}{65}$$

27. $\tanh^{-1}\left(\frac{1}{2}\right) + \coth^{-1}(2)$

$$\Rightarrow \quad \tanh^{-1}\left(\frac{1}{2}\right) + \tanh^{-1}\left(\frac{1}{2}\right)$$

$[\coth^{-1} x = \tanh^{-1}(1/x)]$

$$\Rightarrow \quad 2\tanh^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow 2.\frac{1}{2}\log\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)\left[\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)\right]$$

$$= \log\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right) = \log 3$$

28. In ΔABC,

$$r_1r_2 + r_2r_3 + r_3r_1$$

$$= \frac{\Delta}{s-a}.\frac{\Delta}{s-b} + \frac{\Delta}{s-b}.\frac{\Delta}{s-c} + \frac{\Delta}{s-c}.\frac{\Delta}{s-a}$$

$$= \frac{\Delta^2}{(s-a)(s-b)(s-c)}[s - c + s - a + s - b]$$

$$= \frac{\Delta^2 3x - (a+b+c)}{(s-a)(s-b)(s-c)} \quad (\because 2s = a + b + c)$$

$$= \frac{\Delta^2(3s-2s)}{(s-a)(s-b)(s-c)} = \frac{\Delta^2.s}{\left(\frac{\Delta^2}{s}\right)}$$

$$\left[\because \frac{\Delta^2}{s} = (s-a)(s-b)(s-c)\right]$$

$$= \frac{s^2.\Delta^2}{\Delta^2} = \frac{\Delta^2}{\left(\frac{\Delta}{s}\right)^2} = \frac{\Delta^2}{r^2} \qquad \left[\because r = \frac{\Delta}{s}\right]$$

29. In ΔABC,

$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

Let $\angle C = 60°$, then

$$\cos C = \frac{\pi}{3}$$

$\Rightarrow$ $a^2 + b^2 - c^2 = ab$

$\Rightarrow$ $b^2 + bc + a^2 + ac = ab + ac + be + c^2$

$\Rightarrow$ $b(b+ c) + a(a + c) = (a + c)(b + c)$

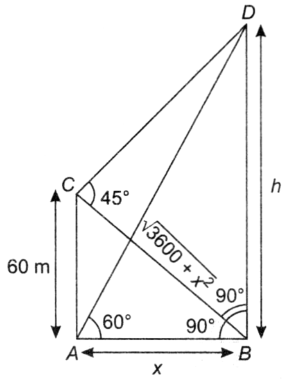
Divide by $(a + c)(b + c)$and add 2 on both sides, we get

$$1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$$

$$\Rightarrow \quad \frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

So, $\angle C$ should be 60°.

30. In ΔABC,



$$\tan 60° = \frac{h}{x} = \sqrt{3}$$

$\Rightarrow \quad h = x\sqrt{3}$ ... (i)

In ΔBAC,

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow \quad BC = \sqrt{x^2 + (60)^2}$$

$$= \sqrt{x^2 + 3600}$$

In ΔCBD, $\tan 45° = \dfrac{h}{\sqrt{3600 + x^2}}$

$$\Rightarrow \quad h = \sqrt{3600 + x^2}$$

$\Rightarrow \quad h^2 - x^2 = 3600$ ... (ii)

From Eqs. (i) and (ii), we get

$$3x^2 - x^2 = 3600$$

$$\Rightarrow \quad 2x^2 = 3600 \Rightarrow x^2 = 1800$$

From Eqs. (i) and (ii), we get

$$h^2 - 1800 = 3600$$

$$\Rightarrow \quad h^2 = 5400$$

$$\Rightarrow \quad h = 30\sqrt{6} = 30\sqrt{2}.\sqrt{3}$$

$$= 30 \times \frac{2.\sqrt{3}}{\sqrt{2}} = 60\sqrt{\frac{3}{2}}$$

31. Let a = 2i + 3j + 4k = OA

b = 3i + 4j + 2k = OB

and c = 4i + 2j + 3k = OC

AB = OB – OA = i + j – 2k

BC = OC – OB = i – 2j + k

and CA = OA – OC = –2i + j + k

Now, $AB = \sqrt{1+1+4} = \sqrt{6}$

$BC = \sqrt{1+4+1} = \sqrt{6}$

and $CA = \sqrt{4+1+1} = \sqrt{6}$

Since, the length of all three sides are equal.

So, the triangle is an equilateral triangle.

32. Let the coordinates of four points P, 0, R and S be (3, –4, 5), (0, 0, 4), (–4, 5, 1) and (–3, 4, 3) respectively.

Now, equation of line PQ is

$$\frac{x-3}{0-3} = \frac{y+4}{0+4} = \frac{z-5}{4-5}$$

$\Rightarrow \quad \dfrac{x-3}{-3} = \dfrac{y+4}{4} = \dfrac{z-5}{-1} = r_1$ (say) ...(i)

Equation of line RS is

$$\frac{x+4}{-3+4} = \frac{y-4}{4-5} = \frac{z-5}{3-1}$$

$\Rightarrow \quad \dfrac{x+4}{1} = \dfrac{y-5}{-1} = \dfrac{z-7}{2} = r_2$ (say) ...(ii)

Let $(-3r_1 + 3, 4r_1 - 4, -r_1 + 5)$ and $(r_2 - 4, -r_2 + 5, 2r_2 + 1)$ be the points on line (i) and (ii), respectively. Since, both lines intersect at a common point, then

$$-3r_1 + 3 = r_2 - 4$$

$\Rightarrow \quad 3r_1 + r_2 = 7$ ...(iii)

and $\quad -r_2 + 5 = 4r_1 - 4$

$\Rightarrow \quad 4r_1 + r_2 = 9$ ...(iv)

On subtracting Eq. (iv) from Eq. (iii), we get

$$r_1 = 2$$

On putting the value of r_1 in Eq. (iii), we get

$$3(2) + r_2 = 7 \Rightarrow r_2 = 1$$

So, required point of intersection is (–3, 4, 3) i.e., $-3i + 4j + 3k$

33. Given,

$a \neq 0,\ b \neq 0, c \neq 0$ and $a \times b = 0, b \times c = 0$

$a \times b = 0 \Rightarrow$ Vector a and b are parallel.

$\Rightarrow$ a and c are also parallel.

$\Rightarrow a \times c = 0$

34. The given lines are $r = a_1 + \lambda b_1$, $r = a_2 + \mu b_2$

where,

$a_1 = 3i + 5j + 7k$, $b_1 = i + 2j + k$

$a_2 = -i - j - k$, $b_2 = 7i - 6j + k$

$$|b_1 \times b_2| = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$\Rightarrow \quad |i(2+6) - j(1-7) + k(-6-14)|$

$\Rightarrow \quad |8i + 6j - 20k|$

$\Rightarrow \quad \sqrt{64+36+400} = \sqrt{500} = 10\sqrt{5}$

Now, $[(a_2 - a_1)b_1b_2] = (a_2 - a_1).(b_1 \times b_2)$

$= (-4i - 6j - 8k) - (8i + 6j - 20k)$

$= -32 - 36 + 160$

$= 160 - 68 = 92$

Shortest distance

$$= \frac{[(a_2 - a_1).(b_1 \times b_2)]}{|b_1 \times b_2|}$$

$$= \frac{92}{10\sqrt{5}} = \frac{46}{5\sqrt{5}}$$

35. Let the unit vector be

$r = xi + yj + zk$

and $a = i + j + 3k, b = i + 3j + k$

and $c = i + j + k$

Given, $[r, a\ b] = 0$, i.e., coplanar.

$$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad x(1-9) - y(1-3) + z(3-1) = 0$

$\Rightarrow \quad -8x + 2y + 2x = 0$

$\Rightarrow \quad -4x + y + z = 0 \quad$ ...(i)

and $r.c = 0$, i.e., perpendicular

$\Rightarrow \quad (xi + yj + zk) - (i + j + k) = 0$

$\Rightarrow \quad x + y + z = 0 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$5y + 5z = 0$

$\Rightarrow \quad y = -z \quad$ ...(iii)

$\because$ r is a unit vector.

$\therefore \ |r| = 1 = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow \quad x^2 + y^2 + z^2 = 1$

$\Rightarrow \quad x^2 + 2y^2 = 1$ [from Eq. (iii)] ...(iv)

Put $y = -z$ in Eq. (i), we get

$-4x = 0 \Rightarrow x = 0$

From Eq. (iv), we get

$$2y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{2}}$$

Required vector is

$r = xi + yj + zk$

$$= 0i \mp \frac{1}{\sqrt{2}} j \mp \frac{1}{\sqrt{2}} k$$

$$= \frac{j-k}{\sqrt{2}} \text{ or } \frac{-j+k}{\sqrt{2}}$$

36. Since, a, b and a x b are non-coplanar, hence

$y = ax + tbx + (axb)z$

For some scalars x, f and z,

Now,

$b = a \times y$

$\Rightarrow$ b = (a × a)x + (a × b)r + a ×(a × b)z

= 0x + (a × b)t + [(a.b)a – (a.a)b]z

[$\because$ a × a = 0, a.b = 0]

$\Rightarrow$ b = (a × b)f – (a.a)b.z

$\Rightarrow$ t = 0 and $z = \frac{-1}{|a|^2}$

Also, c =a.y = a.ax + a.bt + a.(a × b)z

= | a $|^2$ x + 0t + 0z

= | a $|^2$ x [$\because$ a – b = 0,(aab) = 0]

$\Rightarrow$ $x = \frac{c}{|a|^2}$

$\therefore$ y = ax + bt + (a × b)z

$$y = \frac{ac}{|a|^2} + b0 + (a \times b)\left\{\frac{-1}{|a|^2}\right\}$$

$$\Rightarrow \quad y = \frac{1}{|a|^2}\left[ac - (a \times b)\right]$$

37. **Case I** When smaller of the two numbers is 1.

Then, total number of cases

$= 1 \times {}^7C_1 = 7$

Case II When smaller of two numbers is 2.

Then, total number of cases

$= 1 \times {}^6C_1 = 6$

Case III When smaller of two numbers is 3.

Then, total number of cases

$= 1 \times {}^5C_1 = 5$

Total favourable cases = 7 + 6+ 5 = 18

Total case = ${}^8C_2 = 28$

$\therefore$ Required probability $= \frac{18}{28} = \frac{9}{14}$

38. Total samle points, n(S) = 6 × 6 = 36

Favourable events

= [(6, 4), (6 5), (6, 6), (5, 5), (5, 6), (4, 6)]

Total favourable events, n(E) = 6

Required probability

$$= \frac{n(E)}{n(S)} \frac{6}{36} = \frac{1}{6}$$

39. The probability that the toss results is a tail

$$= \frac{(n+1)}{2(2n+1)}$$

$\therefore$ $1 - \frac{(n+1)}{2(2n+1)}$ is the probability that the toss result is a head.

$$\therefore \quad 1 - \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

$$\Rightarrow \quad \frac{4n+2-n-1}{4n+2} = \frac{31}{42}$$

$\Rightarrow$ 126n + 42 = 124n + 62

$\Rightarrow$ 2n = 20

$\Rightarrow$ n =10

40. $\text{var}(X) = \sum_{i=2}^{\infty} P_i \left(X_i - \overline{X}\right)^2$

var (X) = {P(X = 1) + P(X = 2) + ... + P(X = m)}

$$= \left\{\frac{1}{m}\left(1 - \frac{m+1}{2}\right)^2 + \frac{1}{m}\left(2 - \frac{m+1}{2}\right)^2 + \ldots + \frac{1}{m}\left(m - \frac{m+1}{2}\right)^2\right\}$$

$$= \frac{1}{m}\left\{\left(1^2 + \frac{(m+1)^2}{4} - 2.1\frac{(m+1)}{2}\right) + \left(2^2 + \frac{(m+1)^2}{4} - 2.2\frac{m+1}{2}\right) + \ldots + \left(m^2 + \frac{(m+1)^2}{4} - m.2\frac{(m+1)}{2}\right)\right\}$$

$$= \frac{1}{m}\left\{\left(1^2 + 2^2 + ... + m^2\right) + \frac{(m+1)^2}{4}(1+1+...\right.$$

$$\left. +m \text{ times}) - (m+1)(1+2+3+...+m)\right\}$$

$$= \frac{1}{m}\left\{\frac{m(m+1)(2m+1)}{6} + \frac{m(m+1)^2}{4}\right.$$

$$\left. -\frac{(m+1)(m+1)m}{2}\right\}$$

$$= \frac{1}{12}(m+1)[2(2m+1) + 3(m+1) - 6m + 1]$$

$$= \frac{1}{12}(m+1)(4m + 2 + 3m + 3 - 6m - 6)$$

$$= \frac{1}{12}(m+1)(m-1) = \frac{1}{12}(m^2 - 1)$$

41. In poission distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad ...(i)$$

$$P(X = 1) = \frac{\lambda e^{-\lambda}}{1!} = \lambda e^{-\lambda}$$

$$P(X = 2) = \frac{\lambda^2 e^{-\lambda}}{2!} = \frac{\lambda^2 e^{-\lambda}}{2}$$

$$P(X = 2) = P(X = 1)$$

$$\Rightarrow \quad \lambda e^{-\lambda} = 2 \times \frac{\lambda^2 e^{-\lambda}}{2}$$

$$\Rightarrow \quad \lambda(\lambda - 1) = 0$$

$$\lambda \neq 0$$

$$\therefore \quad \lambda = 1$$

Hence, $P(X = 3) = \frac{(1)^3 e^{-1}}{3!} = \frac{e^{-1}}{6} \quad (\because \lambda = 1)$

42. Under the translation of origin to (1,2) the point (7, 5) undergoes to $(7 - 1, 5 - 2) \equiv (6, 3)$

Under the translation through 2 units along the negative direction of the newx-axis, the point (6, 3) undergoes to $(6 - 2, 3) \equiv (4, 3)$

Under the rotation throw an angle $\frac{\pi}{4}$ about the origin of new system in the clockwise direction, the final position of point (7, 5)

$$= \left(4\cos\frac{\pi}{4} + 3\sin\frac{\pi}{4}, -4\sin\frac{\pi}{4} + 3\cos\frac{\pi}{4}\right)$$

$$= \left(\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}, -\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}\right) = \left(\frac{7}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

43. Given equations of straight lines are

$$x\sec\theta - y\operatorname{cosec}\theta = a \quad ...(i)$$

$$x\cos\theta + y\sin\theta = a\cos 2\theta \quad ...(ii)$$

Also,

p = Perpendicular distance from the origin to the line (i)

$$P = \frac{|0 - 0 - a|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} = \frac{a\sin\theta.\cos\theta}{\sqrt{1}}$$

$$= a\sin\theta.\cos\theta = \frac{a}{2}\sin 2\theta$$

$$\Rightarrow \quad 2p = a\sin 2\theta$$

and q = perpendicular distance from the origin to the line (ii)

$$q = \frac{|0 - 0 - a\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = \frac{a\cos 2\theta}{\sqrt{1}} = a\cos 2\theta$$

Now,

$$4p^2 + q^2 = a^2\sin^2 2\theta + a^2\cos^2 2\theta$$

$$= a^2(\sin^2 2\theta + \cos^2 2\theta)$$

$$= a^2(1) = a^2$$

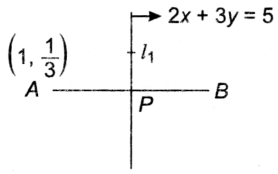
44. Let $l_1 \equiv 2x + 3y = 5$

Since, line $AB \perp l_1$

$\therefore$ Slope of l_1 is

$$m_1 \text{ say} = \frac{-2}{3}$$

$\therefore$ Slope of $AB = \frac{-1}{(-2/3)} = \frac{3}{2}$



Equation of line AS is

$$\left(y - \frac{1}{3}\right) = \frac{3}{2}(x-1)$$

$$\Rightarrow \quad 3x - 2y = \frac{7}{3} \qquad \text{... (i)}$$

Equation of line l_1, is

$$2x + 3y = 5 \qquad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$13x = 17$$

$$\Rightarrow \quad x = \frac{17}{13}$$

From Eq. (i), we get

$$3y = 5 - \frac{34}{13} \Rightarrow y = \frac{65-34}{13 \times 3} = \frac{31}{39}$$

So, mid-point $P \to \left(\frac{17}{13}, \frac{31}{39}\right)$

Coordinate of point B

$$= \left(\frac{17}{13} \times 2 - 1, \frac{31}{39} \times 2 - \frac{1}{3}\right)$$

$$= \left(\frac{34-13}{13}, \frac{62-13}{39}\right) = \left(\frac{21}{13}, \frac{49}{39}\right)$$

45. Since, the points (1, 2) and (3, 4) lie on the same side of the line $3x - 5y + a = 0$

$\therefore$ $3(1) - 5(2) + a \geq 0$ or ≤ 0

$\Rightarrow$ $a - 7 \geq 0$ or ≤ 0

$\Rightarrow$ $a \geq 7$ or $a < 7$

and $3(3) - 5(4) + a \geq 0$ or ≤ 0

$\Rightarrow$ $a - 11 \geq 0$ or ≤ 0

$\Rightarrow$ $a \geq 11$ or $a \leq 11$

So, common condition is [7, 11].

46. Let m, and m_2 be the slopes of lines.

Then, $m_1 + m_2$ = arithmetic mean $= \frac{13}{2}$ ... (i)

and $m_1 m_2$ = geometric mean $= \sqrt{36} = 6$ ... (ii)

Now, equation of the pair of lines passing through the origin is

$$(y - m_1 x)(y - m_2 x) = 0$$

$$\Rightarrow y^2 - (m_1 + m_2)\, xy + m_1 m_2 x^2 = 0$$

Using Eq. (i) and (ii), we get

$$y^2 - \frac{13}{2} xy + 6x^2 = 0$$

$$\Rightarrow \quad 12x^2 - 13xy + 2y^2 = 0$$

47. Comparing the given equation

$$x^2 - 5xy + py^2 + 3x - 8y + 2 = 0 \text{ ...(i)}$$

with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ... (ii)

we get

$a = 1$, $h = \frac{-5}{2}$, $b = p$, $g = \frac{3}{2}$, $f = -4$ and $c = 2$

Eq. (i) represents a pair of straight lines, if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

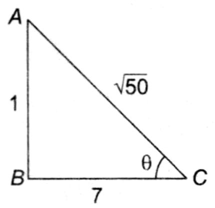
$$\Rightarrow 1 \times p \times 2 + 2 \times (-4) \times \frac{3}{2} \times \left(\frac{-5}{2}\right) - 1 \times (-4)^2$$

$$-p \times \left(\frac{3}{2}\right)^2 - 2 \times \left(\frac{-5}{2}\right)^2 = 0$$

$$\Rightarrow \quad p = 6 \quad \therefore \quad p = 6$$

$\therefore$ Required angle,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{\left(\frac{-5}{2}\right)^2 - 1\times 6}}{1+6}$$



$\Rightarrow \tan\theta = \frac{1}{7}$ $\quad\Rightarrow \theta = \tan^{-1} = \frac{1}{7}$

$\therefore \quad \sin\theta = \frac{1}{\sqrt{50}}$

48. We know that the point of intersection of the pair of straight line is

$$\left(\sqrt{\frac{f^2 - bc}{h^2 - ab}}, \sqrt{\frac{b^2 - ac}{h^2 - ab}}\right)$$

Required distance

$$= \left[\sqrt{\left(\sqrt{\frac{f^2 - bc}{h^2 - ab}} - 0\right)^2 + \left(\sqrt{\frac{g^2 - ac}{h^2 - ab}} - 0\right)^2}\right]^2$$

$$= \frac{f^2 - bc}{h^2 - ab} + \frac{g^2 - ac}{h^2 - ab}$$

$$= \frac{f^2 + g^2 - c(a+b)}{h^2 - ab}$$

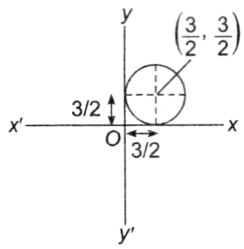
$$= \frac{c(a+b) - f^2 - g^2}{ab - h^2}$$

49. Given circle is

$$4x^2 + 4y^2 - 12x - 12y + 9 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 3x - 3y + \frac{9}{4} = 0$$

$$\Rightarrow \quad (x^2 - 3x) + (y^2 - 3y) = -\frac{9}{4}$$



Hence, centre = $\left(\frac{3}{2}, \frac{3}{2}\right)$ and radius = $\frac{3}{2}$

So, the given circle touches both the axes.

50. Given equation of circle is

$$C = x^2 + y^2 - 16x - 12y + 64 = 0 \quad \text{... (i)}$$

(i) Equation of polar at (–5, 1) w.r.t. to C is

$x(-5) + y(1) - 8(x - 5) - 6(y + 1) + 64 = 0$

$\Rightarrow \quad -5x + y - 8x + 40 - 6y - 6 + 64 = 0$

$\Rightarrow \quad -13x - 5y + 98 = 0$

$\Rightarrow \quad 13x + 5y = 98$

(ii) On differentiating Eq. (i) w.r.t. tox, we get

$$2x + 2y\frac{dy}{dx} - 16 - 12\frac{dy}{dx} + 0 = 0$$

$$\Rightarrow \quad (2y - 12)\frac{dy}{dx} = (16 - 2x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{8 - x}{y - 6}\right)$$

At (8, 0) $\quad \left(\frac{dy}{dx}\right)_{(8,0)} = \frac{8-8}{0-6} = 0$

$\therefore$ Equation of tangent at (8 0) is $(y - 0) = 0(x - 8)$

$\Rightarrow \quad y = 0$

(iii) Slope of normal is

$$\frac{dy}{dx} = \frac{8 - y}{8 - x}$$

$$\left(\frac{dy}{dx}\right)_{(2,6)} = \frac{8-6}{8-2} = 0$$

Equation of normal is

$(y-6) = 0(x-2)$

$\Rightarrow \quad y = 6$

(iv) Equation of the diameter of circle through (8,12) is

$x = 8$

51. Given equations of circle is

$$x^2 + y^2 = 16$$

and $\quad x^2 + y^2 + 2x + 2y = 0$

According to the questions,

Length of the tangent from (h, k)to the circle

$x^2 + y^2 = 16$

= 2 × Length of the tangent from (h, k) to the circle $x^2 + y^2 + 2x + 2y = 0$

$\Rightarrow \quad \sqrt{h^2 + k^2 - 16} = 2 \times \sqrt{h^2 + k^2 + 2h + 2k}$

On squaring both sides, we get

$h^2 + k^2 - 16 = 4 \times (h^2 + k^2 + 2h + 2k)$

$\Rightarrow \quad 3h^2 + 3k^2 + 8h + 8k + 16 = 0$

52. Let the equation of circle whose centre (a, 0) and radius (r) is

$(x-a)^2 + (y-0)^2 = r^2$

$\Rightarrow \quad S_1 \equiv x^2 + a^d - 2ax + y^z - r^d = 0$

and the equation of circle whose centre (b, 0)and radius R is

$(x-b)^2 + (y-0)^2 = R^2$

$\Rightarrow \quad S_2 \equiv x^2 + b^2 - 2bx + y^2 - R^2 = 0$

$\therefore$ Equation of radical axis is

$S_1 - S_2 = 0$

$\Rightarrow \quad a^2 - b^2 + 2bx - 2ax + R^2 - r^2 = 0$

$\Rightarrow \quad R^2 = r^2 - a^2 + b^2 - 2bx + 2ax \quad \text{... (i)}$

Since, radical axis is y-axis.

Therefore, putting x = 0 in Eq. (i), we get

$R^2 = r^2 - a^2 + to^2 - 0 + 0$

$\Rightarrow \quad R = (r^2 + b^2 - a^2)^{1/2}$

53. The common chord of the given circle is

$S_1 - S_2 = 0$

$\Rightarrow \quad (x^2 + y^2 + 4x - 6y + c)$

$\quad -(x^2 + y^2 - 6x + 4y - 12) = 0$

$\Rightarrow \quad 10x - 10y + c + 12 = 0 \quad \text{...(i)}$

Since, circle $x^2 + y^2 + 4x + 6y + c = 0$ bisects the circumference of the circle.

$x^2 + y^2 - 6x + 4y - 12 = 0$

Therefore, Eq. (i) passes through the centre of second circle i.e., (3, –2)

$\therefore \quad 10(3) - 10(-2) + c + 12 = 0$

$\Rightarrow \quad 30 + 20 + c + 12 = 0$

$\Rightarrow \quad C = -62$

54. Given, equation of parabola is

$y^2 = 8x \quad \text{...(i)}$

$\therefore \quad a = 2$

Let (h, 4) be the coordinate of mid-point of chord.

Then, equation of chord is

$y - 4 = m(x - h) \quad \text{...(ii)}$

If line (ii) passes through the point $P\left(2t_1^2, 4t_1\right)$ and $Q\left(2t_2^2, 4t_2\right)$ on parabola Eq. (i), then

$y(t_1 + t_2) - 2x - 4t_1t_2 = 0 \quad \text{...(iii)}$

having slope

$$m = \frac{2}{t_1 + t_2} \quad \text{...(iv)}$$

Since, (h, 4) is the mid-point of PQ. Therefore,

$2 \times 4 = 4(t_1 + f_2)$

$\Rightarrow \quad t_1 + t_2 = 2$

Hence, slope of chord PQ is

$$m = \frac{2}{2} = 1 \quad \text{[using Eq. (iv)]}$$

55. Equation of chord at (2, – 4) is

$T = S$

$\Rightarrow \quad 2x + 4y(-4) - (x + 2) + 10(y - 4)$

$\quad = (2)^2 + 4(-4)^2 + 2(2) + 20(-4)$

$\Rightarrow \quad 2x - 16y - x - 2 + 10y - 40$

$= 4 + 64 - 4 - 80$

$\Rightarrow \quad x - 6y = 42 - 16 \Rightarrow 26$

56. Given, equation of ellipse is

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

and equation of hyperbola is

$$\frac{x^2}{4}-\frac{y^2}{b^2}=1$$

eccentricity of ellipse

$b^2 = a^2(1 - e^2)$

$\Rightarrow \quad 16 = 25(1 - e^2)$

$\Rightarrow \quad e^2 = 1-\frac{16}{25}=\frac{9}{25}$

$\Rightarrow \quad e=\pm\frac{3}{5}$

Focii of the ellipse =(± ae, 0) = (± 3, 0) which coincide with focii of the hyperbola.

Let e_1, be the eccentricity of the hyperbola.

$\therefore \quad \pm ae, = \pm 3$

$\Rightarrow \quad e_1=\frac{3}{2}$

Now, $\quad b^2=a^2\left(e_1^2-1\right)$

$\Rightarrow \quad b^2=4\left(\frac{9}{4}-1\right)=4\times\frac{5}{4}$

$\Rightarrow \quad b^2 = 5$

57. Given that, x = 9 is a chord of contact of hyperbola.

$x^2 - y^2 = 9 \qquad ...(i)$

put x = 9, $\quad 81 - y^2 = 9$

$\Rightarrow \quad y^2 = 72$

$\Rightarrow \quad y=6\sqrt{2}$ or $-6\sqrt{2}$

$\therefore$ Points.are $\left(9,\ 6\sqrt{2}\right)$ and $\left(9,-6\sqrt{2}\right)$

Now, differentiating Eq. (i) w.r.t. x, we get

$$2x-2y\frac{dy}{dx}=0$$

$\Rightarrow \quad \frac{dy}{dx}=\frac{x}{y}$

at $\left(9,6\sqrt{2}\right)\left(\frac{dy}{dx}\right)_{\left(9,6\sqrt{2}\right)}=\frac{9}{6\sqrt{2}}=\frac{3}{2\sqrt{2}}$

and $\left(9-6\sqrt{2}\right)\left(\frac{dy}{dx}\right)_{\left(9,-6\sqrt{2}\right)}=-\frac{3}{2\sqrt{2}}$

$\therefore$ Equation of tangent at $\left(9,6\sqrt{2}\right)$ is

$$\left(y-6\sqrt{2}\right)=\frac{3}{2\sqrt{2}}(x-9)$$

$\Rightarrow \quad 2\sqrt{2}-24=3x-27$

$\Rightarrow \quad 3x-2\sqrt{2}y-3=0$

and equation of tangent at $\left(9,-6\sqrt{2}\right)$ is

$$\left(y+6\sqrt{2}\right)=\frac{-3}{2\sqrt{2}}(x-9)$$

$\Rightarrow \quad 2\sqrt{2}y+24=-3x+27$

$\Rightarrow \quad 3x+2\sqrt{2}y-3=0$

58. Given points $(1, \pi)$, $(1, 0°)$ and $\left(1,\frac{\pi}{2}\right)$ are in polar form.

Now, change in cartesian form,

$(1,\pi)\to(1.\cos\pi,1.\sin\pi)\to(-1,0)$

$(1,0°)\to(1.\cos 0°,1.\sin 0°)\to(1,0)$

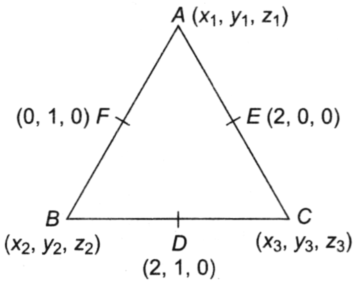
and $\left(1,\frac{\pi}{2}\right)\to\left(1.\cos\frac{\pi}{2},1.\sin\frac{\pi}{2}\right)\to(0,1)$

Now. equation of the line passing through (1, 0) and (0,1) is

So, the perpendicular distance from the point (–1,0) to the line (i) is

$$=\frac{|-1+0-1|}{\sqrt{1+1}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

59. Let $A\equiv(x_1,y_1,z_1)$, $B\equiv(x_2,y_2,z_2)$ and $C\equiv(x_3,y_3,z_3)$



Since, F is the mid-point of A8.

$$\therefore \quad \left.\begin{aligned} x_1+x_2&=0\\ y_1+y_2&=0\\ z_1+z_2&=0\end{aligned}\right\} \quad \text{... (i)}$$

Since, D is the mid-point of SC.

$$\therefore \quad \left.\begin{aligned} x_2+x_3&=4\\ y_2+y_3&=2\\ z_2+z_3&=0\end{aligned}\right\} \quad \text{... (ii)}$$

and £ is the mid-point of AC

$$\therefore \quad \left.\begin{aligned} x_3+x_1&=4\\ y_3+y_1&=0\\ z_3+z_1&=0\end{aligned}\right\} \quad \text{... (iii)}$$

$\therefore \quad x_3 = 4,\ y_3 = 0,\ z_3 = 0$

$x_1 = 0,\ y_1 = 0,\ z_1 = 0$

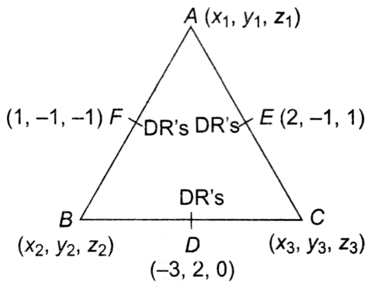
and $\quad x_2 = 0,\ y_2 = 2,\ z_2 = 0$

$\therefore$ Centroid of ΔABC

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

$$=\left(\frac{4}{3},\frac{2}{3},0\right)$$

60. Direction ratios of AB and /AC are < 1, –1, –1 > and <2, –1, 1 >, respectively.



$$\therefore \quad \left.\begin{aligned} x_2+x_1&=0\\ y_2+y_1&=-1\\ z_2+z_1&=1\end{aligned}\right\} \quad \text{... (i)}$$

$$\text{and} \quad \left.\begin{aligned} x_1+x_3&=2\\ y_1+y_3&=-1\\ z_1+z_3&=1\end{aligned}\right\} \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$\left.\begin{aligned} x_2-x_3&=3 \Rightarrow x_3-x_2=-3\\ y_2-y_3&=-2 \Rightarrow y_3-y_2=2\\ z_2+z_3&=0 \Rightarrow z_3-z_2=0\end{aligned}\right\} \quad \text{... (iii)}$$

Let < a, b, c > be the direction ratio of the normal to the plane ABC.

Then, $\quad a – b – c = 0$

and $\quad 2a – b + c = 0$

By cross-multiplication method,

$$\frac{a}{-1-1}=\frac{b}{-2-1}=\frac{c}{-1+2}$$

$$\frac{a}{-2}=\frac{b}{-3}=\frac{c}{1}$$

$$\Rightarrow \quad \frac{a}{2}=\frac{b}{3}=\frac{c}{-1}$$

So, the direction ratio of the normal to the plane ABC is < 2, 3, –1 >.

61. A plane passing through the point (–1, 2, 3), then its equation is

$a(x + 1) + b(y - 2) + c(z - 3) = 0$ ...(i)

where < a, b, c > are direction ratios of normal to the plane ABC.

So, the normal makes equal angles with coordinate axes

i.e., $(a, b, c) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Now, from Eq. (i),

$$\frac{1}{\sqrt{3}}(x+1)+\frac{1}{\sqrt{3}}(y-2)+\frac{1}{\sqrt{3}}(z-1)=0$$

$\Rightarrow \quad x + y + z - 4 = 0$

62. The foot of the propendicular from the origin to the plane lies on a sphere.

63. $\because\ f(x + y) = f(x)f(y),\ \forall x, y \in R$ ...(i)

Put x = y = 1, we get

$f(2) = f(1).f(1) = 9 \quad [\because f(2) = 9]$

$\Rightarrow \quad f(1)^2 = 9 \Rightarrow f(1) = 3$

Now, put x = 2 and y = 1 in Eq. (i), we get

$f(3) = f(2) . f(1) = 3^2. 3 = 3^3$

Now, put x = 3 and y = 1 in Eq. (i), we get

$f(4) = f(3) . f(1) = 3^3 . 3 = 3^4$

Again, put x = 4 and y = 2 in Eq. (i), we get

$f(6) = f(4).f(2) = 3^4 . 3^2 = 3^6$

Alternative Method

We have,

$f(x + y) = f(x) f(y),\ \forall x,\ y \in R$

and $f(2) = 9$ ...(i)

Now. $f(1 + 1) = f(1).(1)$

$\Rightarrow \quad f(2) = \{f(1)\}^2$

$\Rightarrow \quad \{f(1)\} = \sqrt{\{f(2)\}}$ ...(ii)

Now, $f(6) = f(1+1+1+1 + 1+1)$

$= f(1). f(1) . f(1) . f(1) . f(1). f(1)$

$= \{f(1)\}^6$ [using Eq. (ii)]

$= \left[\sqrt{\{f(2)\}}\right]^6 = \left[f(2)\right]^3$

$= (9)^3$ [using Eq. (i)]

$= (3)^6$

64. $\lim_{x\to 0}\frac{\tan^3 x - \sin^3 x}{x^5}$

$$=\lim_{x\to 0}\frac{\left[\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+\ldots\right)^3-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}x^5+\ldots\right)^3\right]}{x^5}$$

$$=\lim_{x\to 0}\frac{\left[\left(1+\frac{x^2}{2}+\frac{2}{15}x^4+\ldots\right)^3-\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}+\ldots\right)^3\right]}{x^2}$$

$$\left\{\left(1+\frac{x^2}{3}+\frac{2}{15}x^4+\ldots\right)-\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}+\ldots\right)\right\}$$

$$=\lim_{x\to 0}\frac{\left[\left(1+\frac{x^2}{3}+\ldots\right)^2+\left(1-\frac{x^2}{3!}+\ldots\right)^2+\left(1+\frac{x^2}{3}+\ldots\right)\left(1-\frac{x^2}{3!}+\ldots\right)\right]}{x^2}$$

[using $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$]

$$=\lim_{x\to 0}\frac{\left(\frac{x^2}{2}+\frac{x^4}{8}\ldots\right)\left\{\left(1+\frac{x^2}{3}+\ldots\right)^2+\left(1-\frac{x^2}{3!}+\ldots\right)^2+\left(1+\frac{x^2}{3}+\ldots\right)\left(1-\frac{x^2}{3}+\ldots\right)\right\}}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{1}{2}+\frac{x^2}{8}\right)\left\{\begin{matrix}\left(1+\frac{x^2}{3}+...\right)^2+\left(1-\frac{x^2}{3!}+...\right)^2 \\ +\left(1+\frac{x^2}{3}+...\right)\left(1-\frac{x^2}{3!}+...\right)\end{matrix}\right\}$$

$$= \left(\frac{1}{2}+0\right)\{(1+0+...)^2+(1-0+...)^2$$

$$+ (1+0)(1-0)\}$$

$$= \frac{1}{2}(1+1+1) = \frac{3}{2}$$

65. $f(x) = \frac{1}{1+\frac{1}{x}} + \frac{x}{1+x}$

and $g(x) = \frac{1}{1+\frac{1}{f(x)}} + \frac{1}{1+\frac{1+x}{x}} = \frac{x}{2x+1}$

$\therefore$ $g'(x) = \frac{(2x+1).1 - x(2)}{(2x+1)^2} = \frac{1}{(2x+1)^2}$

Now, $g'(2) = \frac{1}{(4+1)^2} = \frac{1}{25}$

66. We have $\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} = 2$

$\Rightarrow$ $y + x = 2\sqrt{x}.\sqrt{y}$

On squaring both sides, we get

$x^2 + y^2 + 2xy = 4xy$

$\Rightarrow$ $x^2 + y^3 - 2xy = 0$

$\Rightarrow$ $(x - y)^2 = 0 \Rightarrow y = x$

$\therefore$ $\frac{dy}{dx} = 1$

67. $\frac{d}{dx}[(x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1)]$

$$= \frac{(15x^p - 16x^q + 1)}{(x-1)^2} \quad ... (i)$$

$$\text{LHS} = \frac{d}{dx}\left[\frac{(x^2-1)(x^2+1)(x^4+1)(x^8+1)}{(x-1)}\right]$$

$$= \frac{d}{dx}\left[\frac{(x^4-1)(x^4+1)(x^8+1)}{(x-1)}\right]$$

$$= \frac{d}{dx}\left[\frac{(x^8-1)(x^8+1)}{(x-1)}\right]$$

$$= \frac{d}{dx}\left[\frac{(x^{16}-1)}{(x-1)}\right]$$

$$= \frac{(x-1)(16x^{15}) - (x^{16}-1)}{(x-1)^2}$$

$$= \frac{16x^{16} - 16x^{15} - x^{16} + 1}{(x-1)^2}$$

$$= \frac{15x^{16} - 16x^{15} + 1}{(x-1)^2}$$

On comparing LHS = RHS,

we get p = 16 and q = 15 (p,g) = (16,15)

On differentiating w.r.t. x we get

68. $\cos^{-1}\left(\frac{y}{b}\right) = 2\log\left(\frac{x}{2}\right)$, $x > 0$

On differentiating w.r.t., x we get

$$-\frac{1}{\sqrt{1-\frac{y^2}{b^2}}}.\frac{1}{b}\frac{dy}{dx} = 2.\frac{1}{\left(\frac{x}{2}\right)}.\frac{1}{2}$$

$$\Rightarrow \quad -\frac{1}{\sqrt{b^2-y^2}}.\frac{dy}{dx}=\frac{2}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx}=-2\sqrt{b^2-y^2} \quad \text{... (ii)}$$

Again, differentiating w.r.t. x we get,

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-2.\frac{1}{2}\left(b^2-y^2\right)^{-1/2}(-2y)\frac{dy}{dx}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{2y}{\sqrt{b^2-y^2}}.\frac{dy}{dx}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{2y}{-\frac{x}{2}.\frac{dy}{dx}}.\frac{dy}{dx}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-4y$$

69. Given, $pV^{1/4}=a$, where a is constant.

$$\Rightarrow \quad p=\frac{a}{V^{1/4}}$$

$\therefore$ Decreased volume

$$=V^{1/4}-\frac{V^{1/4}}{200}=\frac{199}{200}V^{1/4}$$

$$\text{Increased pressure } =\frac{a}{\frac{199}{200}V^{1/4}}=\frac{200a}{199V^{1/4}}$$

$\therefore$ Percentage increase in pressure

$$=\frac{\frac{200a}{199V^{1/4}}-\frac{a}{V^{1/4}}}{\frac{a}{V^{1/4}}}\times 100$$

$$=\left(\frac{200}{199}-1\right)\times 100$$

$$=\frac{100}{199}\approx\frac{1}{2} \text{ (aproximate)}$$

70. Given curves are

$$x^2+py^2=1 \quad \text{... (i)}$$

and $$qx^2+y^2=1 \quad \text{... (ii)}$$

On differentiating Eq. (i), w.r.t., x we get

$$2x+2y\,p\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{dy}{dx}=m_1=-\frac{x}{py}$$

On putting x and y in Eq. (ii) we get

$$2qx+2y\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{dy}{dx}=m_2=-\frac{-qx}{y}$$

Since, both the curves are orthogonal to each other,

Then, $\quad m_1m_2=-1$

$$\Rightarrow \quad \frac{-x}{py}.\frac{-qx}{y}=-1$$

$$\Rightarrow \quad qx^2=-py^2 \quad \text{... (iii)}$$

$$\Rightarrow \quad q(1-py)^2=-py^2 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad q-pqy^2=-py^2$$

$$\Rightarrow \quad q=(pq-p)=y^2$$

$$\therefore \quad y^2=\frac{q}{pq-p}$$

and $$x^2=\frac{-p}{pq-p}$$

On putting x^2 and y^2 in Eq. (ii) we get

$$-\frac{pq}{pq-q}+\frac{q}{pq-q}=1$$

$$\Rightarrow \quad -pq+q=pq-p \Rightarrow p+q=2pq$$

$$\Rightarrow \quad \frac{1}{p}+\frac{1}{q}=2$$

71. Given, equation is $\frac{2}{f} = \frac{1}{v} - \frac{1}{u}$ ... (i)

Differentiating the given equation, we have

$$-\frac{2}{f^2}df = -\frac{1}{v^2}dv + \left(-\frac{1}{u^2}\right)du$$

$$= -p\left(\frac{1}{v} - \frac{1}{u}\right)\left(\frac{1}{v} + \frac{1}{u}\right) \left(\because \frac{dv}{v} = \frac{du}{u} = p\right)$$

$$= \frac{-2p}{f}\left(\frac{1}{v} + \frac{1}{u}\right) \quad \text{[using Eq. (i)]}$$

$$\therefore \frac{df}{f} = p\left(\frac{1}{v} + \frac{1}{u}\right)$$

72. Given, $u = \log(x^3 + y^3 + z^3 - 3xyz)$

$$u_x = \frac{du}{dx} = \frac{3x^2 - 3yz}{(x^3 + y^3 + z^3 - 3xyz)}$$

$$u_y = \frac{du}{dy} = \frac{3y^2 - 3xz}{x^3 + y^3 + z^3 - 3xyz}$$

and $$u_z = \frac{du}{dz} = \frac{3z^2 - 3xy}{x^3 + y^3 + z^3 - 3xyz}$$

$u_x + u_y + u_z$

$$= \frac{3(x^2 + y^2 + z^2 - xy - yz - zx)}{(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)}$$

$\Rightarrow (x + y + z)(u_x + u_y + u_z) = 3$

73. Let $$I = \int e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x}\right)dx$$

$$= \int e^x \left\{\frac{2}{1 + \cos 2x} + \frac{\sin 2x}{1 + \cos 2x}\right\}dx$$

$$= \int e^x \left\{\frac{2}{2\cos^2 x} + \frac{2\sin x \cos x}{2\cos^2 x}\right\}dx$$

$$= \int e^x (\sec^2 x + \tan x)dx$$

$$= \int e^x \sec^2 x\, dx + \int e^x \tan x\, dx$$

$$= e^x \tan x - \int e^x \tan x\, dx + \int e^x \tan x\, dx$$

$$= e^x \tan x + C$$

74. Let $$I = \int \frac{x - \sin x}{1 + \cos x}dx$$

$$= \int \frac{x}{1 + \cos x}dx - \int \frac{\sin x}{1 + \cos x}dx$$

$$= \frac{1}{2}\int \frac{x}{\cos x^2\left(\frac{x}{2}\right)}dx - \int \frac{2\sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}dx$$

$$= \frac{1}{2}\int x\sec^2\left(\frac{x}{2}\right)dx - \int \tan\left(\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left\{x.2\tan\left(\frac{x}{2}\right) - \int 2\tan\left(\frac{x}{2}\right)dx\right\}$$

$$- \int \tan\left(\frac{x}{2}\right)dx$$

$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2}dx - \int \tan\left(\frac{x}{2}\right) + C$$

$$= x\tan\frac{x}{2} - 4\log\left|\sec\frac{x}{2}\right| + C$$

but given,

$$\int \frac{x - \sin x}{1 + \cos x}dx = x\tan\frac{x}{2} + p\log\left|\sec\frac{x}{2}\right| + C$$

On comparing, we get

$$p = -4$$

75. Let $$I_1 = \int \frac{dx}{x(\log x - 2)(\log x - 3)}$$

Let $t = \log x \Rightarrow dt = \frac{dx}{x}$

$$\therefore \quad I_1 = \int \frac{dt}{(t-2)(t-3)}$$

$$= \int \left[\frac{1}{(t-3)} - \frac{1}{(t-2)}\right] dt$$

$$= \log|(f-3)| - \log|(f-2)| + C$$

$$= \log\left|\frac{\log x - 3}{\log x - 2}\right| + C$$

$$\Rightarrow \quad I_1 = \int \frac{dx}{x(\log x - 2)(\log x - 3)}$$

$$= \log\left|\frac{\log x - 3}{\log x - 2}\right| + C$$

But given, $\int \frac{dx}{x(\log x - 2)(\log x - 3)} = I + C$

comparing, we get

$$I = \log\left|\frac{\log x - 3}{\log x - 2}\right|$$

76. We have,

$$\int_0^b \frac{dx}{1+x^2} = \int_b^\infty \frac{dx}{1+x^2}$$

$$\Rightarrow \quad \left[\tan^{-1} x\right]_0^b = \left[\tan^{-1} x\right]_b^\infty$$

$$\Rightarrow \tan^{-1}(b) - \tan^{-1}(0) = \tan^{-1}(\infty) - \tan^{-1}(b)$$

$$\Rightarrow \quad 2\tan^{-1}(b) - 0 = \frac{\pi}{2}$$

$$\Rightarrow \quad \tan^{-1}(b) = \frac{\pi}{2} \Rightarrow b = \tan\left(\frac{\pi}{4}\right)$$

$$\therefore \quad b = 1$$

77. Given curves,

$x = -2y^2$ ... (i)

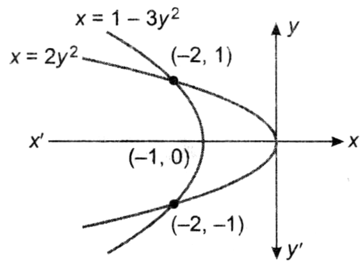
and $x = 1 - 3y^2$ ... (ii)

On solving both curves, we get

$$-2y^2 = 1 - 3y^2$$

$$\Rightarrow \quad y = \pm 1 \text{ and } x = -2$$

So, the intersection points are (–2,1) and (–2, –1).



$\therefore$ Required area

$$= 2\int_0^1 \{(1-3y^2) - (-2y^2)\} dy$$

$$= 2\int_0^1 (1-y^2) dy$$

$$= 2\left(y - \frac{y^3}{3}\right)_0^1 = 2\left(1 - \frac{1}{3}\right)$$

$$= 2 \times \frac{2}{3} = \frac{4}{3}$$

78. Here, $h = \frac{3-1}{2} = 1$

x	1.0	2.0	3.0
y	0.2	0.125	0.09

$$\therefore \int_1^3 \frac{dx}{2+3x} = \frac{1}{3}(0.2 + 4x0.125 + 0.09)$$

$$= \frac{1}{3}(0.2 + 0.500 + 0.09)$$

$$= \frac{1}{3} \times 0.79 = 0.263 = \frac{22}{110}$$

79. Given, differential equation is,

$$dx(1 + y + x^2y) + (x + x^3)dy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left[\frac{1 + y + x^2y}{x + x^3}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left\{\frac{1}{x(1+x^2)} + \frac{(1+x^2)y}{x(1+x^2)}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = -\frac{1}{x(1+x^2)}$$

Here, $p = -\frac{1}{2}, Q = -\frac{1}{x(1+x^2)}$

Integrating factor $\equiv e^{\int p\,dx}$

$$= e^{\int\left(-\frac{1}{x}\right)dx}$$

$$= e^{-\log x}$$

$$= e^{\log\left(-\frac{1}{x}\right)}$$

$$= \frac{1}{x}$$

80. Given, differential equation is

$$\frac{dy}{dx} - 2y \tan 2x = e^x \sec 2x$$

Here, P = 2 tan 2x, Q = e^x sec 2

$$\therefore \quad IF = e^{\int -2\tan 2x dx}$$

$$= e^{\frac{-2\log\sec 2x}{2}}$$

= e–log sec 2x

$$= \frac{1}{\sec 2x}$$

$\therefore$ Required solution is

$$\frac{y}{\sec 2x} = \int e^x . \frac{1}{\sec 2x} .\sec 2x\, dx + C$$

$$\Rightarrow \quad y \cos 2x = \int e^x .1 dx + C$$

$$\Rightarrow \quad y \cos 2x = e^x + C$$

where, C is the constant of integration.