Mock Test 10

Time : 3 hrs. **Max. Marks : 300**

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. There are three parts in the question paper consisting of Physics (Q.no.1 to 30), Chemistry (Q.no.31 to 60) and Mathematics (Q. no.61 to 90). Each part is divided into two sections, Section A consists of 20 multiple choice questions & Section B consists of 10 Numerical value answer Questions.
   In Section B, candidates have to attempt **only 5 questions out of 10**.
5. There will be only one correct choice in the given four choices in Section A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for unattempted question. For Section B 4 marks will be awarded for correct answer and zero for marked for each review / unattempted/incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations / written work should be done in the rough sheet provided.

PHYSICS

Section - A

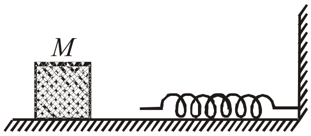
1. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and 3R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal :

(1) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (2) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

(3) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (4) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

2. The block of mass *M* moving on the frictionless horizontal surface collides with the spring of spring constant *k* and compresses it by length

L. The maximum momentum of the block after collision is



(1) $\dfrac{kL^2}{2M}$ (2) $\sqrt{Mk}\ L$

(3) $\dfrac{ML^2}{k}$ (4) Zero

3. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\dfrac{A_1}{A_2}\right)$ is:

(1) $\dfrac{M+m}{M}$ (2) $\left(\dfrac{M}{M+m}\right)^{\frac{1}{2}}$

(3) $\left(\dfrac{M+m}{M}\right)^{\frac{1}{2}}$ (4) $\dfrac{M}{M+m}$

4. Three particles of equal mass m are situated at the vertices of an equilateral triangle of side L. What should be the velocity of each particle so that they move on a circular path without changing L?

(1) $\sqrt{(GM/2L)}$ (2) $\sqrt{(GM/L)}$

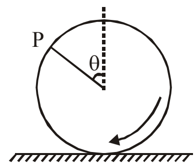
(3) $\sqrt{(2GM/L)}$ (4) $\sqrt{(GM/3L)}$

5. A charge Q is distributed over two concentric hollow spheres of radii r and R (>r) such that the surface densities are equal and placed on the same axial points. Then the potential at the common centre is

(1) $\dfrac{Q(R^2+r^2)}{4\pi\varepsilon_0(R+r)}$ (2) $\dfrac{Q}{R+r}$

(3) Zero (4) $\dfrac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$

6. A wheel is rolling straight on ground without slipping. If the axis of the wheel has speed v, the instantenous velocity of a point P on the rim, defined by angle θ, relative to the ground will be



(1) $v\cos\left(\frac{1}{2}\theta\right)$ (2) $2v\cos\left(\frac{1}{2}\theta\right)$

(3) $v(1+\sin\theta)$ (4) $v(1+\cos\theta)$

7. In the Bohr model an electron moves in a circular orbit around the proton. Considering the orbiting electron to be a circular current loop, the magnetic moment of the hydrogen atom, when the electron is in n^{th} excited state, is :

(1) $\left(\dfrac{e}{2m}\dfrac{n^2h}{2\pi}\right)$ (2) $\left(\dfrac{e}{m}\right)\dfrac{nh}{2\pi}$

(3) $\left(\dfrac{e}{2m}\right)\dfrac{nh}{2\pi}$ (4) $\left(\dfrac{e}{m}\right)\dfrac{n^2h}{2\pi}$

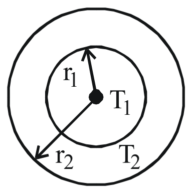
8. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path.

The relation between H, u and n is:

(1) $2gH = n^2u^2$ (2) $gH = (n-2)^2 u^2 d$

(3) $2gH = nu^2 (n-2)$ (4) $gH = (n-2)u^2$

9. The figure shows a system of two concentric spheres of radii r_1 and r_2 are kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



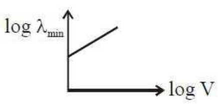
(1) $In\left(\frac{r_2}{r_1}\right)$

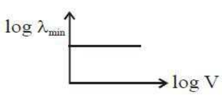
(2) $\frac{(r_2 - r_1)}{(r_1 r_2)}$

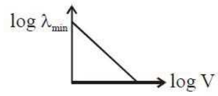
(3) $(r_2 - r_1)$

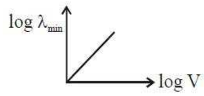
(4) $\frac{r_1 r_2}{(r_2 - r_1)}$

10. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in :

(1) 

(2) 

(3) 

(4) 

11. A cell of e.m.f. E is connected across a reistance R. The potential difference between the terminals of the cell is found to be V. The internal resistance of the cell must be

(1) [2(E–V)V]/R (2) [2(E–V)/V]R

(3) [(E–V)/V]R (4) (E–V)R

12. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius ‘a’. The time in which it will be emptied is:

(1) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$ (2) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(3) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$ (4) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

13. Two straight long conductors AOB and COD are perpendicular to each other and carry currents I_1 and I_2 respectively. The magnitude of the magnetic induction at a point P at a distance ‘a’ from the point O where the two conductors intersect, in a direction perpendicular to the plane ABCD is

(1) $\frac{\mu_0}{2\pi a}(I_1 + I_2)$ (2) $\frac{\mu_0}{2\pi a}(I_1 - I_2)$

(3) $\frac{\mu_0}{2\pi a}\left[I_1^2 + I_2^2\right]^{1/2}$ (4) $\frac{\mu_0}{2\pi a}\left(\frac{I_1 I_2}{I_1 + I_2}\right)$

14. In a series resonant circuit, having L, C and R as its elements, the resonant current is i. The power dissipated in the circuit at resonance is

(1) $\dfrac{i^2R}{\left(\omega L - \dfrac{1}{\omega C}\right)}$ (2) zero

(3) $i^2\omega L$ (4) i^2R

15. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by

(1) $3t\sqrt{\alpha^2+\beta^2}$ (2) $3t^2\sqrt{\alpha^2+\beta^2}$

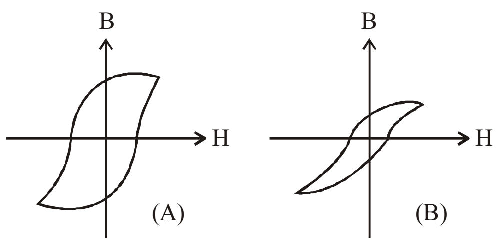
(3) $t^2\sqrt{\alpha^2+\beta^2}$ (4) $\sqrt{\alpha^2+\beta^2}$

16. The mutual inductance of a pair of coils, each of N turns, is M henry. If a current of I ampere in one of the coils is brought to zero in t second, the emf induced per turn in the other coil, in volt, will be

(1) $\dfrac{MI}{t}$ (2) $\dfrac{NMI}{t}$

(3) $\dfrac{MN}{It}$ (4) $\dfrac{MI}{Nt}$

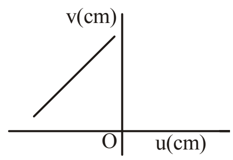
17. Hysteresis loops for two magnetic materials A and B are given below :



These materials are used to make magnets for elecric generators, transformer core and electromagnet core. Then it is proper to use :

(1) A for transformers and B for electric generators.
(2) B for electromagnets and transformers.
(3) A for electric generators and trasformers.
(4) A for electromagnets and B for electric generators.

18. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like

(1)


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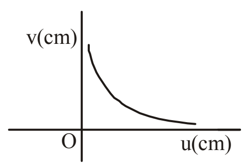

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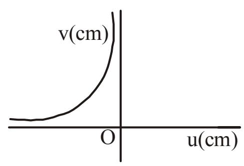

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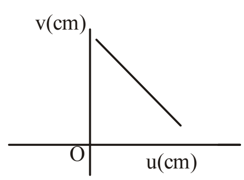
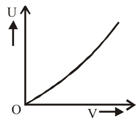
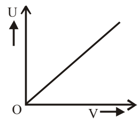
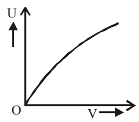
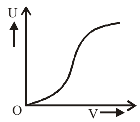


19. A battery is connected across series combination of a capacitor and a resistor, at t = 0. If at an instant t potential difference across the capacitor be ‘V’ and energy stored in it be U then which of the following graph is correct?

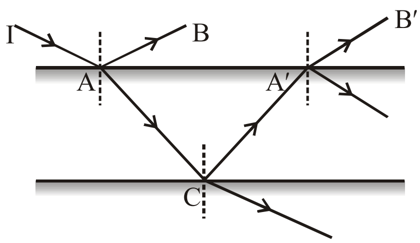
(1) 

(2) 

(3) 

(4) 

20. A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in fig. it undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays AB and A′B′ undergo interference. The ratio I_{max} / I_{min} is



(1) 4 : 1 (2) 8 : 1
(3) 7 : 1 (4) 49 : 1

Section - B

21. The time taken by a body in sliding down a rough inclined plane of angle of inclination 45° is n times the time taken by the same body in slipping down a similar frictionless plane. The coefficient of dynamic friction between the body and the plane is $\mu = 1 - \left(\frac{1}{n^x}\right)$. Find the value of x.

22. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field (in V/m) associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly ______

23. A Carnot’s engine works as a refrigerator between 250 K and 300 K. If it receives 750 calories of heat from the reservoir at the lower temperature, the amount of heat rejected at the higher temperature is ______ cal.

24. The electric field in a region of space is given by, $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$ where E_o = 100 N/C. The flux of the field through a circular surface of radius 0.02 m parallel to the Y-Z plane is nearly ____ Nm^2/C

25. A gas molecule of mass M at the surface of the Earth has kinetic energy equivalent to 0°C. If it were to go up straight without colliding with any other molecules, it rises to $h = \frac{819k_B}{xMg}$. Find the value of x. Assume that the height attained is much less than radius of the earth. (k_B is Boltzmann constant).

26. In a npn transistor 10^{10} electrons enter the emitter in 10^{-6} s. 4% of the electrons are lost in the base. The current transfer ratio will be

27. 56 tuning forks are so arranged in series that each fork gives 4 beats per sec with the previous one. The frequency of the last fork is 3 times that of the first. The frequency of the first fork is ______ Hz.

28. For sky wave propagation of a 10 MHz signal, what should be the minimum electron density (in m^{-3}) in ionosphere?

29. In an experiment the angles are required to be measured using an instrument, 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half- a degree (= 0.5°), then the least count of the instrument is ______ minute.

30. The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed, as counts per second at t = 6 seconds will be

CHEMISTRY

Section - A

31. If the shortest wavelength in Lyman series of hydrogen atom is A, then the longest wavelength in Paschen series of He^+ is :

(1) $\frac{5A}{9}$ (2) $\frac{9A}{5}$

(3) $\frac{36A}{5}$ (4) $\frac{36A}{7}$

32. The value of $\frac{t_{0.875}}{t_{0.50}}$ for n^{th} order reaction is

(1) $2^{(2n-2)}$ (2) $2^{(2n-2)-1}$

(3) $\frac{8^{n-1}-1}{2^{n-1}-1}$ (4) None of these

33. Urea and hydrazine react to form ammonia gas along with compound X which reacts with aldehydes and ketones to form specific crystalline derivatives. X is

(1) phenylhydrazine (2) semicarbazide

(3) biuret (4) acetylurea

34. The IUPAC name of $\underset{\displaystyle CN}{\underset{|}{CH_2}} - \underset{\displaystyle CN}{\underset{|}{CH}} - \underset{\displaystyle CN}{\underset{|}{CH_2}}$ is

(1) 1, 2, 3-propanetrinitrile

(2) 1, 2, 3-tricyanopropane

(3) 3-cyano-1, 5-dinitrilepentane

(4) 1, 2, 3-propanetricarbonitrile

35. The catenation tendency of C, Si and Ge is in the order Ge < Si < C. The bond energies (in $kJmol^{-1}$) of C – C, Si – Si and Ge –Ge bonds are respectively;

(1) 348, 297, 260 (2) 297, 348, 260

(3) 348, 260, 297 (4) 260, 297, 348

36. The repeating units of acrilan are

(1) $H_2C=\overset{\overset{\large H}{|}}{C}-COOCH_3$

(2) $H_2C=\overset{\overset{\large H}{|}}{C}-COOC_2H_5$

(3) $H_2C=\overset{\overset{\large H}{|}}{C}-CN$

(4) $H_2C=\overset{\overset{\large CH_3}{|}}{C}-COOCH_3$

37. A metal X on heating in nitrogen gas gives Y. Y on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a blue colour. Y is

(1) $Mg(NO_3)_2$ (2) Mg_3N_2
(3) NH_3 (4) MgO

38. The turbidity of a polymer solution measures

(1) the light scattered by solution
(2) the light absorbed by a solution
(3) the light transmitted by a solution
(4) None of these

39. Which of the following has a shape different from others?

(1) $[Zn(NH_3)_4]^{2+}$ (2) $Ni(CO)_4$
(3) $[Cd(CN)_4]^{2+}$ (4) $[Cu(NH_3)_4]^{2+}$

40. Di-n-propyl ether and diallyl ether can be distinguished by

(1) Acetic acid
(2) Sodium Metal
(3) Cold dilute $KMnO_4$ solution
(4) PCl_5

41. In which of the following cases, the stability of two oxidation states is correctly represented

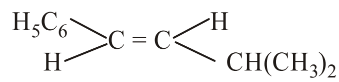
(1) $Ti^{3+} > Ti^{4+}$ (2) $Mn^{2+} > Mn^{3+}$
(3) $Fe^{2+} > Fe^{3+}$ (4) $Cu^{+} > Cu^{2+}$

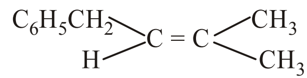
42. Which one of the following oxides of chlorine is obtained by passing dry chlorine over silver chlorate at 90°C ?

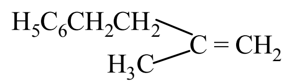
(1) Cl_2O (2) ClO_3
(3) ClO_2 (4) ClO_4

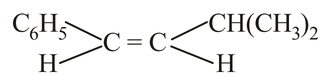
43. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc.}H_2SO_4} ?$$

(1) 

(2) 

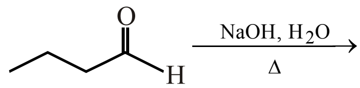
(3) 

(4) 

44. Zirconium phosphate $[Zr_3(PO_4)_4]$ dissociates into three zirconium cations of charge + 4 and four phosphate anions of charge – 3. If molar solubility of zirconium phosphate is denoted by S and its solubility product by K_{sp} then which of the following relationship between S and K_{sp} is correct?

(1) $S = [K_{sp}/(6912)^{1/7}]$
(2) $S = [K_{sp}/144]^{1/7}$
(3) $S = [K_{sp}/6912]^{1/7}$
(4) $S = [K_{sp}/6912[^{7}$

45. The final product of the following sequence of reactions



$$B \xrightarrow[H_2O]{LiCu(CH_3)_2} C \xrightarrow[H_2O]{LiAlH_4} D$$

(1) 3-ethyl-2-methyl-1-hexanol

(2) 2, 3-dimethyl-1-pentanol

(3) 2-ethyl-3-methyl-1-hexanol

(4) 3, 3-dimethyl-1-pentanol

46. A solution containing As^{3+}, Cd^{2+}, Ni^{2+} and Zn^{2+} is made alkaline with dilute NH_4OH and treated with H_2S. The precipitate obtained will consist of

(1) As_2S_3 and CdS

(2) CdS, NiS and ZnS

(3) NiS and ZnS

(4) Sulphide of all ions

47. The intermediate formed during the addition of HCl to propene in the presence of peroxide is

(1) $CH_3\dot{C}HCH_2Cl$ (2) $CH_3\overset{+}{C}HCH_3$

(3) $CH_3CH_2\dot{C}H_2$ (4) $CH_3CH_2\overset{+}{C}H_2$

48. An organic compound 'A' having molecular formula C_2H_3N on reduction gave another compound 'B'. Upon treatment with nitrous acid, 'B' gave ethyl alcohol. On warming with chloroform and alcoholic KOH, B formed an offensive smelling compound 'C'. The compound 'C' is

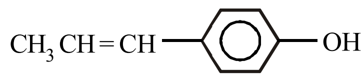
(1) $CH_3CH_2NH_2$

(2) $CH_3CH_2N \overset{\rightarrow}{=} C$

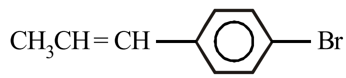
(3) $CH_3C \equiv N$

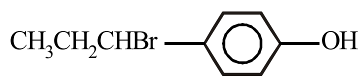
(4) CH_2CH_2OH

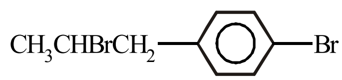
49. The reaction of

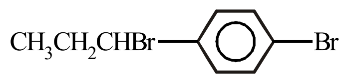


with HBr gives

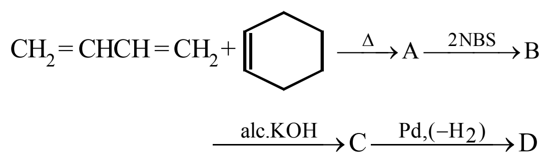
(1) 

(2) 

(3) 

(4) 

50. Final product (D) formed in the following reaction sequence is



(1) Naphthalene (2) Tetralin

(3) Benzene (4) Anthracene

Section - B

51. Determine enthalpy of formation for $H_2O_2(l)$, using listed enthalpies of reaction :

$$N_2H_4(l)+2H_2O_2(l)\longrightarrow N_2(g)+4H_2O(l);\quad \Delta_rH_1^\circ=-818\ kJ/mol$$

$$N_2H_4(l)+O_2(g)\longrightarrow N_2(g)+2H_2O(l);\quad \Delta_rH_2^\circ=-622\ kJ/mol$$

$$H_2(g)+1/2O_2(g)\longrightarrow H_2O(l);\quad \Delta_rH_3^\circ=-285\ kJ/mol$$

52. A metallic element exists as cubic lattice. Each edge of the unit cell is 2.88 Å. The density of the metal is 7.20 g cm^{-3}. How many unit cell will be present in 100 g of the metal in the multiple of 10^{23}?

53. The solution containing 4.0 % PVC in one litre of dioxane was found to have osmotic pressure of 6.0×10^{-4} atm at 300 K. Find the molecular mass of polymer in terms of $x \times 10^5$.

54. For a chemical reaction $X \rightarrow Y$, the rate of reaction increases by a factor of 1.837 when the concentration of X is increased by 1.5 times. Find the order of the reaction with respect to X

55. M_2SO_4 (M^+ is a monovalent metal ion) has a K_{sp} of 3.2×10^{-5} at 298 K. The maximum concentration of SO_4^{-2} ion that could be attained in a saturated solution of this solid at 298 K is $x \times 10^{-2}$ M. Find the value of x.

56. The colour of $KMnO_4$ solution is decolourised by Fe^{2+} solution, one mole of Fe^{2+} reacts with x moles of $KMnO_4$. Find x.

57. For the coagulation of 500 mL of arsenious sulphide sol, 2 mL of 1 M NaCl is required. What is the flocculation value of NaCl?

58. The standard reduction potential for Cu^{2+}/Cu is + 0.34. Calculate the reduction potential at pH = 14 for the above couple.
($K_{sp}\ Cu(OH)_2 = 1 \times 10^{-19}$)

59. A current of 2.0A when passed for 5 hours through a molten metal salt deposits 22.2g of metal of atomic weight 177. Find the oxidation state of the metal in the metal salt

60. A gaseous mixture of three gases A, B and C has a pressure of 10 atm. The total number of moles of all the gases is 10. If the partial pressures of A and B are 3.0 and 1.0 atm respectively and if 'C' has mol. wt of 2.0 what is the weight of 'C' in g present in the mixture

MATHEMATICS

Section - A

61. ${}^{n+4}C_r - {}^nC_r - 3.{}^nC_{r-1} - 3.{}^nC_{r-2} - {}^nC_{r-3}$ is equal to
(1) ${}^{n+1}C_{r-1}$
(2) ${}^{n+2}C_{r-1}$
(3) ${}^{n+3}C_{r-1}$
(4) ${}^{n+4}C_{r-1}$

62. Let $\alpha/(\alpha-1)$ and $\beta/(\beta-1)$ be the roots of $x^2+ax+b=0$. Then $1/\alpha$ and $1/\beta$ are the roots of
(1) $bx^2+ax+1=0$
(2) $bx^2-ax+1=0$
(3) $bx^2+(a+2b)x+a+b+1=0$
(4) $bx^2-(a+2b)x+a+b+1=0$

63. For any real θ, the maximum value of $\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is

(1) 1 (2) $1 + \sin^2 1$
(3) $1 + \cos^2 1$ (4) 1/2

64. If tan(A/2), tan(B/2), tan(C/2) are in A.P. then sec A, sec B, sec C are in

(1) A.P. (2) G.P.
(3) H.P. (4) None

65. The equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is given by

(1) $xa^{1/3} + yb^{1/3} + a^{2/3}b^{2/3} = 0$
(2) $xb^{1/3} + ya^{1/3} + a^{2/3}b^{2/3} = 0$
(3) $xa^{1/3} + yb^{1/3} - a^{2/3}b^{2/3} = 0$
(4) None of these

66. If tangents be drawn to the cricle $x^2 + y^2 = 12$ at its points of intersection with the circle $x^2 + y^2 - 5x + 3y - 2 = 0$, then the tangents intersect at the point

(1) $\left(-6, \frac{18}{5}\right)$ (2) $\left(6, \frac{18}{5}\right)$
(3) $\left(-6, -\frac{18}{5}\right)$ (4) $\left(6, -\frac{18}{5}\right)$

67. Let $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is :

(1) reflexive but neither symmetric nor transitive.
(2) symmetric and transitive.
(3) reflexive and symmetric,
(4) reflexive and transitive.

68. The value of

$$\lim_{x\to\pi/6}(4 - 3\sin x - 2\cos^2 x)^{\frac{1}{2\sin x - 1}} \text{ is}$$

(1) 1 (2) e
(3) $\sqrt{e}$ (4) $e^{-1/2}$

69. The function $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3$, $(0 \le x \le \pi)$ is –

(1) Increasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(2) Increasing in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$
(3) Decreasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(4) All of the above

70. Value of $\int_0^{\pi} \sin^n x \cos^{2m+1} x\, dx$ (where m, n $\in$ N) is

(1) $\frac{(2m+1)!}{n!}$ (2) $\frac{(2m+1)!}{n^2}$
(3) $\int_0^{\pi} \cos^{2m+1} x\, dx$ (4) None of these

71. If $f'(x) = f(x)$ and $f(0) = 2$, then

$$\int \frac{f(x)}{3 + 4f(x)} dx =$$

(1) $\log(3 + 8e^x) + C$
(2) $\frac{1}{4}\log(3 + 8e^x) + C$
(3) $\frac{1}{2}\log(3 + 8e^x) + C$
(4) None of these

72. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$, and above x-axis is :

(1) $2\sqrt{2}$

(2) $2\sqrt{2} - 2$

(3) $2\sqrt{2} + 2$

(4) None of these

73. Let coordinates of a point 'p' with respect to the system of non-coplanar vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is (3, 2, 1). Then coordinates of 'p' with respect to the system of vectors $\vec{a} + \vec{b} + \vec{c}$, $\vec{a} - \vec{b} + \vec{c}$ and $\vec{a} + \vec{b} - \vec{c}$ is

(1) $\left(\frac{3}{2}, \frac{1}{2}, 1\right)$ (2) $\left(\frac{3}{2}, 1, \frac{1}{2}\right)$

(3) $\left(\frac{1}{2}, \frac{3}{2}, 1\right)$ (4) None

74. Equation of the line through the point (2,3,l) and parallel to the line of intersection of the planes $x - 2y - z + 5 = 0$ and $x + y + 3z = 6$ is

(1) $\frac{x-2}{-5} = \frac{y-3}{-4} = \frac{z-1}{3}$

(2) $\frac{x-2}{5} = \frac{y-3}{-4} = \frac{z-1}{3}$

(3) $\frac{x-2}{5} = \frac{y-3}{4} = \frac{z-1}{3}$

(4) $\frac{x-2}{4} = \frac{y-3}{3} = \frac{z-1}{2}$

75. If $x + 2 = 6$, then $x = 4$. So, which statement is its converse?

(1) If $x \neq 4$, then $x + 2 \neq 6$

(2) If $x = 4$, then $x + 2 \neq 6$

(3) If $x = 4$, then $x + 2 = 6$

(4) If, $x \neq 4$ then $x + 2 = 6$

76. If y = f(x) be a monotonically increasing or decreasing function of x and M is the median of variable x, then the median of y is

(1) f(M) (2) M/2

(3) f^{-1}(M) (4) None of these

77. A fair die is tossed eight times. The probability that a third six is observed on the eighth throw is

(1) ${}^7C_2 \frac{5^5}{6^8}$ (2) ${}^7C_3 \frac{5^3}{6^8}$

(3) ${}^7C_6 \frac{5^6}{6^8}$ (4) None of these

78. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then

$[\lambda(\vec{a} + \vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\quad \vec{b} + \vec{c}\quad \vec{b}]$ for

(1) exactly one value of λ

(2) no value of λ

(3) exactly three values of λ

(4) exactly two values of λ

79. $S = \tan^{-1}\left(\frac{1}{n^2 + n + 1}\right) + \tan^{-1}\left(\frac{1}{n^2 + 3n + 3}\right) + \ldots$

$+ \tan^{-1}\left(\frac{1}{1 + (n+19)(n+20)}\right)$, then $\tan S$ is equal to :

(1) $\frac{20}{401 + 20n}$ (2) $\frac{n}{n^2 + 20n + 1}$

(3) $\frac{20}{n^2 + 20n + 1}$ (4) $\frac{n}{401 + 20n}$

80. The middle term in the expansion of $\left(1-\frac{1}{x}\right)^n (1-x)^n$ in powers of x is

(1) $-{}^{2n}C_{n-1}$ (2) $-{}^{2n}C_n$

(3) ${}^{2n}C_{n-1}$ (4) ${}^{2n}C_n$

Section - B

81. The value of $S = \frac{5}{1^2 \cdot 4^2} + \frac{11}{4^2 \cdot 7^2} + \frac{17}{7^2 \cdot 10^2} + \ldots \infty$ is ______ .

82. The line $2x + y = 3$ cuts the ellipse $4x^2 + y^2 = 5$ at P and Q. If θ be the angle between the normals at these points, then value of $\tan\theta$ is ______ .

83. Matrix M_r is defined as $M_r = \begin{pmatrix} r & r-1 \\ r-1 & r \end{pmatrix}$, $r \in \mathbf{N}$. If value of $\det(M_1) + \det(M_2) + \det(M_3) + \ldots + \det(M_{2007}) = (k)^2$, then value of k is ____ .

84. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and

$$(10)B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}.$$

If B is the inverse of matrix A, then value of α is ______ .

85. The derivative of $\tan^{-1}[(3x^2 - 1)/(3x - x^3)]$ with respect to $\sin^{-1}[(x^2 - 1)/(x^2 + 1)]$ is ______ .

86. If $2a + 3b + 6c = 0$ and at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (m, n), then value of m + n is ______ .

87. If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$ is a purely imaginary number, then $\left|\frac{2Z_1 + 3Z_2}{2Z_1 - 3Z_2}\right|$ is ______ .

88. If the straight lines

$x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$

and $x = \frac{t}{2}, y = 1 + t, z = 2 - t,$ with parameters s and t respectively, are co-planar, then value of λ is ______ .

89. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. The most probability of occurrence of A is ______ .

90. If X and Y are independent binomial variates $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$, then P (X + Y = 3) is ______ .

Mock Test-10

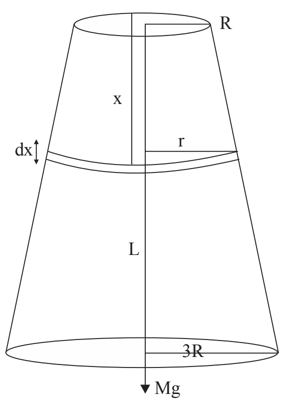
ANSWER KEY											
1	(3)	**16**	(2)	**31**	(4)	**46**	(4)	**61**	(3)	**76**	(1)
2	(2)	**17**	(2)	**32**	(3)	**47**	(2)	**62**	(4)	**77**	(1)
3	(3)	**18**	(3)	**33**	(2)	**48**	(2)	**63**	(2)	**78**	(2)
4	(2)	**19**	(1)	**34**	(4)	**49**	(2)	**64**	(3)	**79**	(3)
5	(4)	**20**	(4)	**35**	(1)	**50**	(1)	**65**	(1)	**80**	(4)
6	(2)	**21**	(2)	**36**	(3)	**51**	(187)	**66**	(4)	**81**	(0.33)
7	(3)	**22**	(2.68)	**37**	(2)	**52**	(5.82)	**67**	(4)	**82**	(0.60)
8	(3)	**23**	(900)	**38**	(1)	**53**	(1.6)	**68**	(4)	**83**	(2007)
9	(4)	**24**	(0.125)	**39**	(4)	**54**	(1.5)	**69**	(1)	**84**	(5.00)
10	(3)	**25**	(2)	**40**	(3)	**55**	(2)	**70**	(4)	**85**	(1.50)
11	(3)	**26**	(0.96)	**41**	(2)	**56**	(0.2)	**71**	(2)	**86**	(1.00)
12	(2)	**27**	(110)	**42**	(3)	**57**	(4)	**72**	(2)	**87**	(1.00)
13	(3)	**28**	(1.2×10^{12})	**43**	(1)	**58**	(0.22)	**73**	(1)	**88**	(2.00)
14	(4)	**29**	(1)	**44**	(3)	**59**	(3)	**74**	(1)	**89**	(0.50)
15	(2)	**30**	(200)	**45**	(3)	**60**	(12)	**75**	(3)	**90**	(0.05)

Solutions

PHYSICS

1. (3) Consider a small element dx of radius r,

$$r = \frac{2R}{L}x + R$$



At equilibrium change in length of the wire

$$\int_0^L dL = \int \frac{Mg\,dx}{\pi\left[\frac{2R}{L}x + R\right]^2 Y}$$

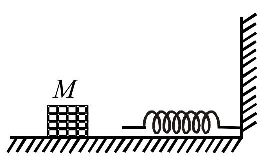
Taking limit from 0 to L

$$\Delta L = \frac{Mg}{\pi Y}\left[-\frac{1}{\left[\frac{2Rx}{L}+R\right]_0^L} \times \frac{L}{2R}\right] = \frac{MgL}{3\pi R^2 Y}$$

The equilibrium extended length of wire = L + ΔL

$$= L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{1}{3}\frac{Mg}{\pi Y R^2}\right)$$

2. (2)



$$\frac{1}{2}Mv^2 = \frac{1}{2}kL^2 \Rightarrow v = \sqrt{\frac{k}{M}}.L$$

$$\text{Momentum} = M \times v = M \times \sqrt{\frac{k}{M}}.L = \sqrt{kM}\,.L$$

3. (3) The net force becomes zero at the mean point.

Therefore, linear momentum must be conserved.

$\therefore Mv_1 = (M+m)v_2$

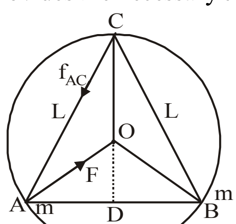
$$MA_1\sqrt{\frac{k}{M}} = (M+m)A_2\sqrt{\frac{k}{m+M}}$$

$$\therefore \left(V = A\sqrt{\frac{k}{M}}\right)$$

$$\because \; A_1\sqrt{M} = A_2\sqrt{M+m}$$

$$\therefore \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

4. (2) The resultant gravitational force on each particle provides the necessary centripetal force.



$\therefore \quad mv^2/r = \sqrt{(F^2 + F^2 + 2F^2 \cos 60°)} = \sqrt{3}\, F$

But $r = (\sqrt{3}\, L/2) \times (2/3) = L/\sqrt{3}$

$\therefore \quad mv^2\sqrt{3}/L = \sqrt{3}GM^2/L^2 \Rightarrow v = \sqrt{(GM/L)}$

5. (4) Let the charges on spheres of radii r and R be q_1 and q_2 respectively. Then, $Q = q_1 + q_2$. The potential at the centre will be :

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

As surface densities are equal, hence

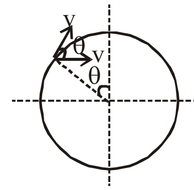
$$\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \text{ or } q_2 = q_1\left(\frac{R^2}{r^2}\right)$$

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r} + q_1\frac{R}{r^2}\right) = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1}{r^2}(r+R)$$

Now, $q_1 + q_2 = Q$ or $q_1 + q_1 \dfrac{R^2}{r^2} = Q$

$$\Rightarrow \frac{q_1}{r^2} = \frac{Q}{R^2 + r^2}; V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{R+r}{R^2+r^2}\right)$$

6. (2) $v_R = \sqrt{v^2 + v^2 + 2v^2\cos\theta}$



$$= \sqrt{2v^2(1+\cos\theta)} = 2v\cos\frac{\theta}{2}$$

7. (3) Magnetic moment of the hydrogen atom, when the electron is in n^{th} excited state, i.e., $n' = (n+1)$

As magnetic moment $M_n = I_n A = i_n(\pi r_n^2)$

$$i_n = eV_n = \frac{mz^2e^5}{4\varepsilon_0^2 n^3 h^3}$$

$$r_n = \frac{n^2h^2}{4\pi^2 kzme^2}\left(k = \frac{1}{4\pi \in_0}\right)$$

Solving we get magnetic moment of the hydrogen atom for n^{th} excited state

$$M_{n'} = \left(\frac{e}{2m}\right)\frac{nh}{2\pi}$$

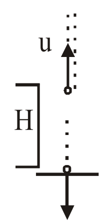
8. (3) Speed on reaching ground

$$v = \sqrt{u^2 + 2gh}$$

Now, $v = u + at$

$\Rightarrow \quad \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \dfrac{u}{g}$,



$$\Rightarrow \quad t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g} \text{ (from question)}$$

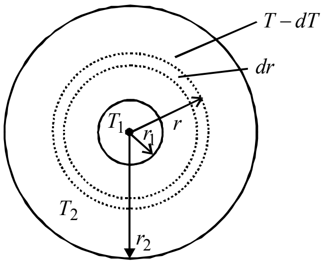
$\Rightarrow \quad 2gH = n(n-2)u^2$

9. (4) Consider a shell of thickness (dr) and of radii (r) and the temperature of inner and outer surfaces of this shell be T, $(T - dT)$

$\dfrac{dQ}{dt}$ = rate of flow of heat through it

$$= \frac{KA[(T-dT)-T]}{dr} = \frac{-KAdT}{dr}$$

$$= -4\pi Kr^2 \frac{dT}{dr} \quad (\because\ A = 4\pi r^2)$$



To measure the radial rate of heat flow, integration technique is used, since the area of the surface through which heat will flow is not constant.

$$\text{Then, } \left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2}\frac{1}{r^2}dr = -4\pi K\int_{T_1}^{T_2}dT$$

$$\frac{dQ}{dt}\left[\frac{1}{r_1}-\frac{1}{r_2}\right] = -4\pi K[T_2 - T_1]$$

$$\text{or} \quad \frac{dQ}{dt} = \frac{-4\pi K r_1 r_2 (T_2 - T_1)}{(r_2 - r_1)}$$

$$\therefore \quad \frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

10. (3) In X-ray tube, $\lambda_{\min} = \frac{hc}{eV}$

$$\text{In } \lambda_{\min} = In\left(\frac{hc}{e}\right) - InV$$

Clearly, log $\lambda_{\min}$ versus log V graph slope is negative hence option (3) correctly depicts.

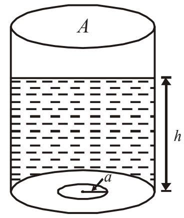
11. (3) Let r be the internal resistance of the cell. Terminal potential difference (V) < EMF of a cell (E).

$$V = E - Ir \text{ or } r = \frac{E-V}{I} = \left(\frac{E-V}{V}\right)R$$

12. (2) Let the rate of falling water level be $-\frac{dh}{dt}$

Initially at $t = 0;\ h = h$

$t = t;\ h = 0$



$$\text{Then, } A\left(-\frac{dh}{dt}\right) = \pi a^2 . v$$

$$dt = -\frac{A}{\pi a^2\sqrt{2gh}}dh$$

[$\because$ velocity of efflux of liquid $v = \sqrt{2gh}$]

Integrating both sides

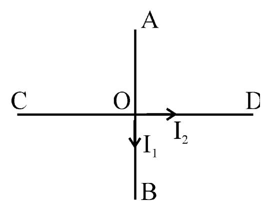
$$\int_0^t dt = -\frac{A}{\sqrt{2g}\pi a^2}\int_h^0 h^{-1/2}dh$$

$$[t]_0^t = -\frac{A}{\sqrt{2g}\pi a^2}.\left[\frac{h^{1/2}}{1/2}\right]_h^0$$

$$t = \frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$$

13. (3) $\vec{B}$ due to AOB and COD are $\perp$ to each other. Hence, net $\vec{B}^2 = B_1^2 + B_2^2$

$$B = \frac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2} \quad (\perp \text{ to plane ABCD})$$



14. (4) At resonance $L\omega = \frac{1}{C\omega}$, $\omega = \frac{1}{\sqrt{LC}}$

Current through circuit $i = \frac{E}{R}$

Power dissipated at Resonance = i^2R

15. (2) $x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$= 3t^2\sqrt{\alpha^2+\beta^2}$

16. (2) $\beta = \frac{\lambda D}{2d} = \frac{6.5\times10^{-7}}{10^{-3}}\times 1 = 0.65\times10^{-3}$

$= 0.65$ mm

The distance between fifth bright fringe from third dark fringe $= 2.5\,\beta = 2.5\times .65$

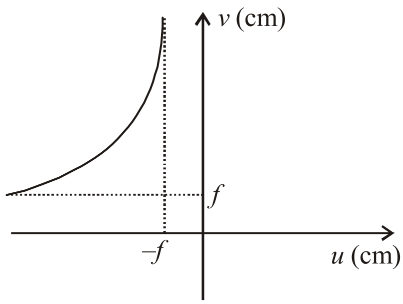
$= 1.63$ mm.

17. (2) Graph [A] is for material used for making permanent magnets (high coercivity)

Graph [B] is for making electromagnets and transformers.

18. (3) This graph suggest that when

$u = -f, v = +\infty$

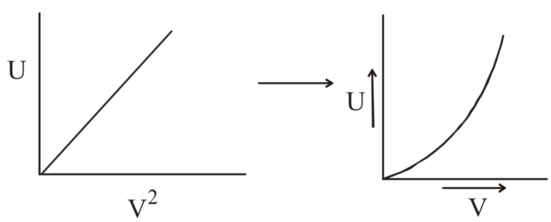


When the object is moved further away from the lens, v decreases but remains positive. When u is at $-\infty$, $v = f$.

This is how image formation takes place for different positions of the object in case of a convex lens.

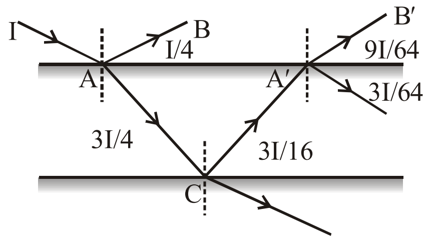
19. (1) $U = \frac{1}{2}CV^2$

From this, we get $U \propto V^2$



20. (4) From figure $I_1 = \frac{I}{4}$ and

$I_2 = \frac{9I}{64} \Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$



By using

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right)^2 = \left(\frac{\sqrt{\frac{9}{16}}+1}{\sqrt{\frac{9}{16}}-1}\right)^2 = \frac{49}{1}$$

21. (2) Acceleration when there is no friction is a = $g\sin\theta$ and acceleration when friction is there is $a' = (g\sin\theta - \mu g\cos\theta)$

We know that $s = ut + \frac{1}{2}at^2$, under condition of $u = 0$, gives, $t = \sqrt{(2s/a)}$.

Therefore, $t'/t = \sqrt{(a/a')}$.

For our case we get $n = \sqrt{[g\sin\theta/(g\sin\theta - \mu g\cos\theta)]}$

$\Rightarrow n^2 = g\sin\theta/(g\sin\theta - \mu g\cos\theta) = 1/(1-\mu\cot\theta)$

But $\theta = 45°$

$\Rightarrow \cot\theta = 1 \therefore n^2 = 1/(1-\mu)$ or $\mu = 1-(1/n^2)$

22. (2.68) Wavelength of monochromatic green light $= 5.5\times10^{-5}$ cm

Intensity $I = \frac{\text{Power}}{\text{Area}}$

$= \frac{100\times(3/100)}{4\pi(5)^2} = \frac{3}{100\pi}\,\text{Wm}^{-2}$

Now, half of this intensity (I) belongs to electric field and half of that to magnetic field, therefore,

$\frac{I}{2} = \frac{1}{4}\varepsilon_0 E_0^2 C$ or $E_0 = \sqrt{\frac{2I}{\varepsilon_0 C}}$

$$= \sqrt{\frac{2\times\left(\frac{3}{100}\pi\right)}{\left(\frac{1}{4\pi\times9\times10^9}\right)\times\left(3\times10^8\right)}}$$

$$= \sqrt{\frac{6}{25}\times30} = \sqrt{7.2}$$

$\therefore \quad E_0 = 2.68\ V/m$

23. (900) We know that

$$\eta = 1 - \frac{T_2}{T_1} = \frac{300-250}{300} = \frac{50}{300} = \frac{1}{6}$$

or $\eta = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{1}{6} = \frac{Q_1 - 750}{Q_1}$

$Q_1 = 6Q_1 - 4500$

$\Rightarrow -5Q_1 = -4500 \Rightarrow Q_1 = 900\ Cal$

24. (0.125) $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$

Given, $E_0 = 100 N/c$

So, $\vec{E} = 100\hat{i} + 200\hat{j}$

Radius of circular surface = 0.02 m

Area = $\pi r^2 = \frac{22}{7}\times0.02\times0.02$

$= 1.25\times10^{-3}\hat{i}\ m^2$ [Loop is parallel to Y-Z plane]

Now, flux (ϕ) = EA cosθ

$= \left(100\hat{i} + 200\hat{j}\right).1.25\times10^{-3}\hat{i}\cos\theta°$ [$\theta = 0°$]

$= 125 \times 10^{-3}\ Nm^2/c$

$= 0.125\ Nm^2/c$

25. (2) Kinetic energy of each molecule,

$$K.E. = \frac{3}{2}K_BT$$

In the given problem,

Temperature, T = 0°C = 273 K

Height attained by the gas molecule, h = ?

$$K.E. = \frac{3}{2}K_B\left(273\right) = \frac{819K_B}{2}$$

K.E. = P.E.

$$\Rightarrow \frac{819K_B}{2} = Mgh$$

or $h = \frac{819K_B}{2Mg}$

26. (0.96) No. of electrons reaching the collector,

$$n_C = \frac{96}{100}\times10^{10} = 0.96\times10^{10}$$

Emitter current, $I_E = \frac{n_E\times e}{t}$

Collector current, $I_C = \frac{n_C\times e}{t}$

$\therefore$ Current transfer ratio,

$$\alpha = \frac{I_C}{I_E} = \frac{n_C}{n_E} = \frac{0.96\times10^{10}}{10^{10}} = 0.96$$

27. (110) $n_{last} = n_{first} + \left(n-1\right)\times d$

$\Rightarrow 3n = n + \left(56-1\right)\times4$

or $2n = 55\times4 \Rightarrow n = 110\,Hz$

28. (1.2×10^{12}) The critical frequency of a sky wave for reflection from a layer of atmosphere is given by $f_c = 9(N_{max})^{1/2}$

$\Rightarrow 10\times10^6 = 9(N_{max})^{1/2}$

$$\Rightarrow N_{max} = \left(\frac{10\times10^6}{9}\right)^2 \simeq 1.2\times10^{12}\,m^{-3}$$

29. (1) 30 Divisions of vernier scale coincide with 29 divisions of main scales

Therefore 1 V.S.D = $\frac{29}{30}$ MSD

Least count = 1 MSD – 1VSD

$= 1\ MSD - \frac{29}{30}\ MSD$

$= \frac{1}{30}\ MSD$

$= \frac{1}{30}\times0.5° = 1$ minute.

30. (200) After 8 sec., the counting rate falls to

$$\frac{100}{1600} = \frac{1}{16} = \left(\frac{1}{2}\right)^4 \text{ th Part ;}$$

Time period = $\frac{8}{4}$ sec. = 2 sec.

Therefore, after 6 seconds, the counting rate should be $\left(\frac{1}{2}\right)^3$ th part of 1600, i.e. $\frac{1}{8} \times 1600 = 200$.

CHEMISTRY

31. (4) For Lyman series (shortest wavelength)

$n_1 = 1, n_2 = \infty$

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{A} = 1^2 R\left(\frac{1}{1} - \frac{1}{\infty}\right) \Rightarrow \frac{1}{A} = R$$

Longest wavelength = 1st line

$n_1 = 3, n_2 = 4$

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \Rightarrow \frac{1}{\lambda} = \frac{R7}{36}$$

$$R = \frac{1}{A}$$

$$\frac{1}{\lambda} = \frac{\frac{1}{A} \times 7}{36} \Rightarrow \frac{1}{\lambda} = \frac{7}{36A} \Rightarrow \lambda = \frac{36A}{7}$$

32. (3) $kt = \frac{1}{n-1}\left[\frac{1}{A_t^{n-1}} - \frac{1}{A_0^{n-1}}\right]$

$$kt_{0.5} = \frac{1}{n-1}\left[\frac{1}{\left(\frac{A_0}{2}\right)^{n-1}} - \frac{1}{A_0^{n-1}}\right] = \frac{1}{n-1}\left[\frac{2^{n-1}-1}{A_0^{n-1}}\right]$$

$$kt_{0.875} = \frac{1}{n-1}\left[\frac{1}{\left(\frac{A_0}{8}\right)^{n-1}} - \frac{1}{A_0^{n-1}}\right] = \frac{1}{n-1}\left[\frac{8^{n-1}-1}{A_0^{n-1}}\right]$$

$$\frac{t_{0.875}}{t_{0.5}} = \frac{8^{n-1}-1}{2^{n-1}-1}$$

33. (2) $NH_2CONH_2 + NH_2–NH_2 \longrightarrow NH_2NHCONH_2 + NH_3$

(Semicarbazide)

34. (4)

$$\begin{array}{ccccc} \overset{1}{H_2C} & - & \overset{2}{CH} & - & \overset{3}{CH_2} \\ | & & | & & | \\ CN & & CN & & CN \end{array}$$

1,2,3-propanetricarbonitrile

35. (1) The linking of identical atoms with each other to form long chains is called catenation. However, this property decreases from carbon to lead. Decrease of this property is associated with M-M bond energy which decreases from carbon to lead.

36. (3) Polyacrylonitrile (PAN), acrilan or orlon

$$\underset{\text{Acrylonitrile}}{n\,CH_2{=}\underset{|}{\overset{CN}{\overset{|}{CH}}}} \longrightarrow \underset{\text{Orlon}}{\left(CH_2 - \overset{CN}{\overset{|}{CH}}\right)_n}$$

It is hard used in preparing cloths, carpets etc.

37. (2) $\underset{X}{3Mg} + N_2 \xrightarrow{\Delta} \underset{Y}{Mg_3N_2};$

$$Mg_3N_2 + 6H_2O \longrightarrow 3Mg(OH)_2 + \underset{\text{Colourless}}{2NH_3} \uparrow$$

$$CuSO_4 + 4NH_3 \longrightarrow \underset{\text{Blue complex}}{[Cu(NH_3)_4]SO_4}$$

38. (1) Molecules of a polymer, being large in size, scatter light.

39. (4) $[Zn(NH_3)_4]^{2+}$, $Ni(CO)_4$ and $[Cd(CN)_4]^{2-}$ all form tetrahedral structures where the central atom uses sp^3 hybridisation.

Whereas $[Cu(NH_3)_4]^{2+}$ uses sp^2d hybridisation and forms square planar structure. Although Cu should use both tetrahedral (sp^3) and square planar (sp^2d) hybridisations but the later one is confirmed through X-ray analysis.

40. (3) Diallyl ether has double bond which shows addition reaction with $KMnO_4$ (change its colour) while di-n-propyl ether does not react with $KMnO_4$.

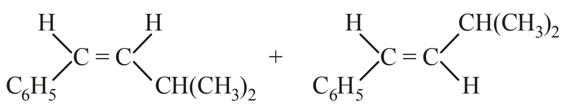
41. (2) Mn^{2+} ($3d^5$) is more stable then Mn^{3+} ($3d^4$). due to half filled orbitals.

42. (3) Pure ClO_2 is obtained by passing dry Cl_2 over $AgClO_3$ at 90°C.

$$2AgClO_3 + Cl_2(\text{dry}) \xrightarrow{90°C} 2AgCl + 2ClO_2 + O_2$$

43. (1) Whenever dehydration can produce two different alkenes, major product is formed according to **Saytzeff rule** *i.e.* more substituted alkene (alkene having lesser number of hydrogen atoms on the two doubly bonded carbon atoms) is the major product.

$$C_6H_5-CH_2-\underset{\substack{|\\ OH}}{CH}-\underset{\substack{|\\ CH_3}}{CH}-CH_3 \xrightarrow{\text{Conc. } H_2SO_4}$$



cis (minor) *trans* (major)

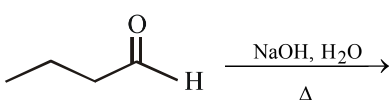
44. (3) $\left[Zr_3(PO_4)_4\right] \rightleftharpoons \underset{3S}{3Zr^{4+}} + \underset{4S}{4PO_4^{3-}}$

$K_{sp} = (3S)^3(4S)^4$

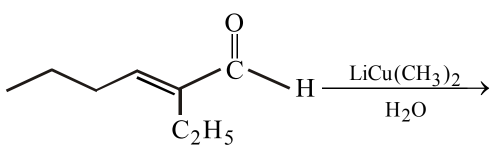
$= 27S^3 \times 256\,S^4$

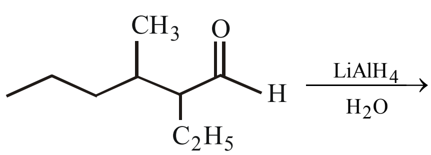
$= 6912\,S^7.$

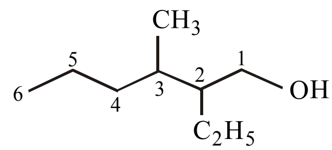
$\therefore \quad S = \left(\dfrac{K_{sp}}{6912}\right)^{1/7}$

45. (3) 

Butanal







2-Ethyl-3 methyl-1-hexanol

46. (4) As^{3+} and Cd^{2+} are the radicals of group II, whereas Ni^{2+} and Zn^{2+} are the radicals of group IV. The solubility product of group IV radicals is higher as compared to group II. NH_4OH increases the ionisation of H_2S by removing H^+ of H_2S as unionisable water.

$$H_2S \rightleftharpoons 2H^+ + S^{2-};$$

$$H^+ + OH^- \longrightarrow H_2O$$

Thus, excess of sulphide ions are present which leads to the precipitation of all the four ions.

Note : HCl decreases ionisation of H_2S whereas NH_4OH increases the ionisation of H_2S.

47. (2) The reaction proceed through Markownikoff's rule by ionic mechanism. The initial carbonium ion $(CH_3CH_2\overset{+}{C}H_2)$ rearrange into more stable $(CH_3\overset{+}{C}HCH_3)$ carbonium ion.

48. (2)

$$\underset{(C_2H_3N)}{A} \xrightarrow{\text{reduction}} B \xrightarrow{HNO_2} C_2H_5OH$$

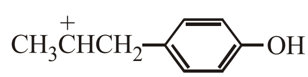
$$B \xrightarrow{\text{Carbylamine reaction}} \text{Offensive smell (C)}$$

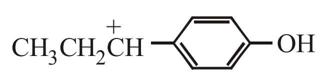
Given reactions indicate that B has 1° NH_2 group, and thus A, C_2H_3N, should be $CH_3C\equiv N$. Hence C should be CH_3CH_2NC

$$\underset{A}{CH_3C\equiv N} \xrightarrow{\text{reduction}} \underset{B}{CH_3CH_2NH_2}$$

$$\xrightarrow[KOH]{CHCl_3} CH_3CH_2N \overset{\rightarrow}{=} \underset{C}{C}$$

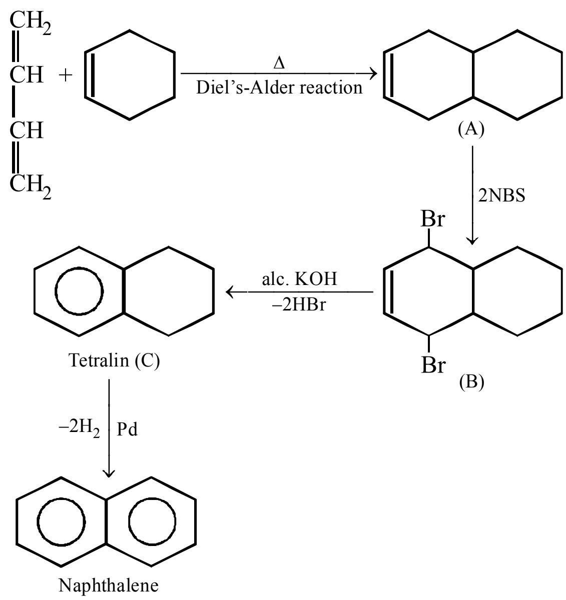
49. (2) Of the two possible carbocations, benzylic type (II) is more stable than the I.

 ......(I)

 ......(II)

hence bromination occurs on 3rd C atom

50. (1)



51. (187) For $H_2(g) + O_2(g) \longrightarrow H_2O_2(l)$

$$\Delta_f H^\circ(H_2O_2, l) = \Delta_r H_3^\circ + \frac{\Delta_r H_2^\circ}{2} - \frac{\Delta_r H_1^\circ}{2}$$

$$= (-285) + \left(\frac{-622}{2}\right) - \left(\frac{-818}{2}\right)$$

$= -187$ kJ/mol

52. (5.82) The volume of the unit cell $= (2.88\ \text{Å})^3 = 23.9 \times 10^{-24}\ cm^3$.

The volume of 100 g of the metal

$$= \frac{m}{\rho} = \frac{100}{7.20} = 13.9\ cm^3$$

Number of unit cells in this volume

$$= \frac{13.9\ cm^3}{23.9 \times 10^{-24}\ cm^3} = 5.82 \times 10^{23}$$

53. (1.6) $W_2 = 4g$, $V = 1000\ mL = 1\ L$,

$\pi = 6.0 \times 10^{-4}$ atm

$T = 300K$, $R = 0.0821$ Latm. $K^{-1}\ mol^{-1}$

$\pi = W_2 \times RT/(M_2 V)$

$\therefore M_2 = 4 \times 0.0821 \times 300/(6 \times 10^{-14} \times 1)$

$= 16.42 \times 10^4 = 1.6 \times 10^5$

54. (1.5) $(1.5)^n = 1.837$, $n = \dfrac{\log 1.837}{\log 1.5} = 1.5$

55. (2) The maximum concentration of SO_4^{2-} will be same as the solubility of M_2SO_4;

If solubility is x mol L^{-1}

$M_2SO_4 \rightleftharpoons 2M^+ + SO_4^{2-}$

$K_{sp} = 4x^3$, therefore $x = (K_{sp}/4)^{1/3}$

$[SO_4^{-2}] = x = (K_{sp}/4)^{1/3} = [(3.2 \times 10^{-5})/4]^{1/3}$

$= 2 \times 10^{-2}$ M

56. (0.2) $MnO_4^- + 5Fe^{2+} + 8H^+ \longrightarrow 5Fe^{3+} + Mn^{2+} + 4H_2O$

In the above reaction 1 mole of MnO_4^- reacts with 5 mole of Fe^{2+}.

or 1 mole of Fe^{2+} reacts with 0.2 mole of MnO_4^-.

57. (4) 2 mL of 1 M NaCl contains

$NaCl = \dfrac{2}{1000} = 2$ m mole Thus 500 mL of As_2S_3 sol require NaCl for complete coagulation = 2 m mole

Hence 1 L, i.e., 1000 mL of the sol require NaCl for complete coagulation = 4 m mole

Therefore, flocculation value of NaCl = 4.

58. (0.22) When pH = 14

$[H^+] = 10^{-14}$

$$[OH^-] = \frac{Kw}{[H^+]} = \frac{10^{-14}\ m}{10^{-14}\ m} = 1\ m$$

$$K_{sp} = [Cu^{2+}][OH^-]^2 = 10^{-19}$$

$$\therefore [Cu^{2+}] = \frac{10^{-19}}{[OH^-]^2} = 10^{-19}$$

The half cell reaction

$$Cu^{2+} + 2e^- \longrightarrow Cu$$

$$E = E^\circ - \frac{0.059}{2} \log \frac{1}{[Cu^{2+}]}$$

$$= 0.34 - \frac{0.059}{2} \log \frac{1}{10^{-19}} = -0.22\ V$$

59. (3) F_{metal} = (Wt of metal $\times$ 96500)/No of coulombs

$$= \frac{(22.2 \times 96500)}{(2 \times 5 \times 60 \times 60)} \approx 59.5$$

Oxidation no. of metal $= \dfrac{177}{59.5} = +3$

60. (12) Total pressure of mixture of gases A,B & C = 10 atm.

Partial pressure of

A = (No. of moles of A × 10)/ 10

or (No. of moles of A × 10)/ 10 = 3

Hence, no. of moles of A = 3

∴ Partial pressure of

B = (No. of moles of B × 10)/ 10

(since partial pressure of B = 1atm)

∴ No.of moles of B = 1

Now the no. of moles of C = 10 – (3+1) = 6;

1 mole of C weighs = 2 g.

∴ 6 mole of C will weigh = 2 × 6 = 12

MATHEMATICS

61. (3)

$^{n+4}C_r - (^nC_r + 3.^nC_{r-1} + 3.^nC_{r-2} + ^nC_{r-3})$

$= ^{n+4}C_r - (^nC_r + ^nC_{r-1}) + (^nC_{r-1} + ^nC_{r-2})$

$\quad + (^nC_{r-1} + ^nC_{r-2}) + (^nC_{r-2} + ^nC_{r-3})$

$= ^{n+4}C_r - ^{n+1}C_r + ^{n+1}C_{r-1} + ^{n+1}C_{r-1} + ^{n+1}C_{r-2}$

$= ^{n+4}C_r - ^{n+2}C_r + ^{n+2}C_{r-1}$

$= ^{n+4}C_r - ^{n+3}C_r = ^{n+3}C_{r-1}$

62. (4) Let $\alpha' = \alpha/(\alpha - 1)$, $\beta' = \beta/(\beta - 1)$

$\Rightarrow \alpha = \alpha'/(\alpha' - 1)$, $\beta = \beta'/(\beta' - 1)$

$\Rightarrow 1/\alpha = (\alpha' - 1)/\alpha'$, $1/\beta = (\beta' - 1)/\beta'$

Then equation whose roots are $1/\alpha$, $1/\beta$, is

$x^2 - (1/\alpha + 1/\beta)x + 1/\alpha\beta = 0$

$\Rightarrow \quad x^2 - [(\alpha' - 1)/\alpha' + (\beta' - 1)/\beta']x$

$\quad + [(\alpha' - 1)(\beta' - 1)] / \alpha'\beta' = 0$

$\Rightarrow \alpha'\beta'x^2 - [2\alpha'\beta' - (\alpha' + \beta')]x + \alpha'\beta' - (\alpha' + \beta') + 1 = 0$

$\Rightarrow bx^2 - (a + 2b)x + a + b + 1 = 0$

63. (2) $f(\theta) = \cos^2(\cos\theta) + \sin^2(\sin\theta)$

Now, put $\theta = \pi/2$ & 0

$f(0) = \cos^2 1 + 0 = \cos^2 1$

$f(\pi/2) = 1 + \sin^2 1$

Since, $\sin^2 1 > \cos^2 1$

$$[\because \sin\theta > \cos\theta \ \forall \ \frac{\pi}{4} < \theta < \frac{\pi}{2} \ \& \ 1 > \frac{\pi}{4}]$$

Now, seeing the options $1 + \sin^2 1$ is greater than all other options.

64. (3) tanA/2, tanB/2, tanC/2 are in A.P.

$\Rightarrow \tan A/2 - \tan B/2 = \tan B/2 - \tan C/2$

$$\Rightarrow \frac{\sin A/2}{\cos A/2} - \frac{\sin B/2}{\cos B/2} = \frac{\sin B/2}{\cos B/2} - \frac{\sin C/2}{\cos C/2}$$

$\Rightarrow \cos(C/2)\sin(A/2 - B/2)$

$= \cos A/2 \sin(B/2 - C/2)$

$\Rightarrow \cos B - \cos A = \cos C - \cos B$

$\Rightarrow 2\cos B = \cos A + \cos C$

$\Rightarrow$ secA, secB, secC are in H.P.

65. (1) Any tangent to the parabola $y^2 = 4ax$ is

$$y = mx + \frac{a}{m} \quad \text{...(1)}$$

meets $x^2 = 4by$ where $x^2 = 4b\left(mx + \frac{a}{m}\right)$

or $mx^2 - 4bm^2x - 4ab = 0$ ...(2)

If the line (1) be a tangent to the second parabola, then roots of (2) must be equal. The condition for this is

$$16b^2m^4 + 16abm = 0 \quad \text{or} \quad m^3 = -\frac{a}{b}$$

$$\therefore \quad m = -\left(\frac{a}{b}\right)^{1/3} = -\frac{a^{1/3}}{b^{1/3}}$$

Substituting this value of m in (1), we get

$$y = -\frac{a^{1/3}}{b^{1/3}}x + a\left(-\frac{b^{1/3}}{a^{1/3}}\right)$$

i.e. $xa^{1/3} + yb^{1/3} + a^{2/3}b^{2/3} = 0$

which is the required equation of the common tangent.

66. (4) Let the point of contact be (h,k); equation of chord of contact is T = 0

$\Rightarrow xh + yk - 12 = 0$ ....(1)

Equation of common chord $C_1 - C_2 = 0$

$\Rightarrow 5x - 3y - 10 = 0$ ....(2)

(1) and (2) represent the same line

$\Rightarrow h/5 = k/(-3) = -12/(-10)$

$\Rightarrow h = 6, k = -18/5$

67. (4) $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$

Now, $x^2 - 4xy + 3y^2 = 0$

$\Rightarrow (x-y)(x-3y) = 0$

$\therefore x = y$ or $x = 3y$

$\therefore R = \{(1, 1), (3, 1), (2, 2), (6, 2), (3, 3), (9, 3), ...\}$

Since (1, 1), (2, 2), (3, 3),...... are present in the relation, therefore R is reflexive.

Since (3, 1) is an element of R but (1, 3) is not the element of R, therefore R is not symmetric

Here $(3, 1) \in R$ and $(1, 1) \in R \Rightarrow (3, 1) \in R$

$(6, 2) \in R$ and $(2, 2) \in R \Rightarrow (6, 2) \in R$

For all such $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

Hence R is transitive.

68. (4) $\lim\limits_{x\to\pi/6}\left(4 - 3\sin x - 2\cos^2 x\right)^{1/(2\sin x - 1)}$

$$\lim_{x\to\pi/6}\left[1 + (2\sin x - 1)(\sin x - 1)\right]^{\frac{\sin x - 1}{(2\sin x - 1)(\sin x - 1)}} = e^{-1/2}$$

69. (1) $f'(x) = -12\cos^3 x \sin x - 30\cos^2 x \sin x - 12\cos x \sin x = -6\sin x \cos x(\cos x + 2)(2\cos x + 1)$

$f'(x) = 0$, for $x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$

Clearly, $f'(x) > 0$ for $\frac{\pi}{2} < x < \frac{2\pi}{3}$

And $f'(x) < 0$; for $0 < x < \frac{\pi}{2}$ or $\frac{2\pi}{3} < x < \pi$

70. (4) $I = \int_0^{\pi} \sin^n x \cos^{2m+1} x\, dx$

$= \int_0^{\pi} \sin^n(\pi - x).\cos^{2m+1}(\pi - x)\, dx$

$= -\int_0^{\pi} \sin^n x \cos^{2m+1} x\, dx = -I \Rightarrow I = 0$

Similarly, one can show that

$\int_0^{\pi} \cos^{2m+1} x\, dx = 0$

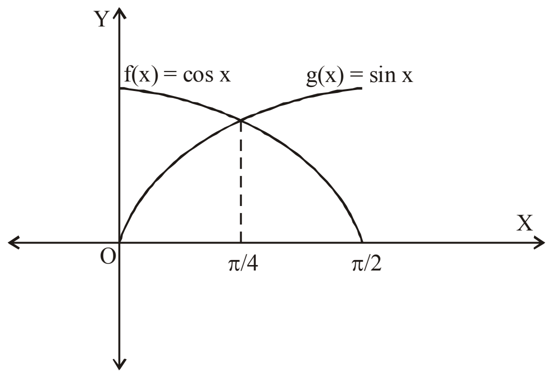
71. (2) Since, $f'(x) = f(x)$, therefore, $f(x) = \alpha e^x$. Since, $f(0) = 2$, therefore, $f(x) = 2e^x$.

$\therefore I = 2\int \frac{e^x}{3 + 8e^x} dx$

Put $e^x = t$, $\therefore dt = e^x dx$

$\therefore I = 2\int \frac{dt}{3 + 8t} \Rightarrow I = \frac{1}{4}\log(3 + 8e^x) + C$

72. (2) $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= 2\left[\sin x + \cos x\right]_0^{\pi/4}$

$= 2\left[\frac{2}{\sqrt{2}} - 1\right] = (2\sqrt{2} - 2)$ sq. units

73. (1) Any vector (x, y, z) can be represented as a linear combination of three non coplanar vectors.

$\overrightarrow{OP} = x\vec{a} + y\vec{b} + z\vec{c}$

But $\vec{P}$ is $(3,2,1) \Rightarrow \overrightarrow{OP} = 3\vec{a} + 2\vec{b} + \vec{c}$

Now if coordinate axis changes then the vector is represented by

$x(\vec{a} + \vec{b} + \vec{c}) + y(\vec{a} - \vec{b} + \vec{c}) + 2(\vec{a} + \vec{b} - \vec{c})$

$= 3\vec{a} + 2\vec{b} + \vec{c}$

Comparing coeff of $\vec{a}, \vec{b}$ & $\vec{c}$, we get

$x + y + z = 3,\ x - y + z = 2,\ x + y - z = 1$

On solving, we get

$x = \frac{3}{2}, y = \frac{1}{2}, z = 1$

Hence the new co-ordinates of point P are $\left(\frac{3}{2},\frac{1}{2},1\right)$

74. (1) Let DR of the line of intersection of the planes be a, b, c

$\Rightarrow \ a-2b-c=0$ ....(1)

and $a+b+3c=0$ ....(2)

on solving, we get $\frac{a}{-5}=\frac{b}{-4}=\frac{c}{3}=\text{k}$

$\Rightarrow \ a=-5k, b=-4k, c=3k$

Thus required eq^n of line is

$$\frac{x-2}{-5}=\frac{y-3}{-4}=\frac{z-1}{3}$$

75. (3) $x+2=6 \Rightarrow x=4$, Its converse is $x=4 \Rightarrow x+2=6$ i.e. if $x=4$ then $x+2=6$

76. (1) Since, $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ if $x_1, x_2, ---, x_n$ are in ascending order

$\Rightarrow f(x_1), f(x_2), --- f(x_n)$ are also in ascending order.

If M is the median along x, then f (M) is the median along y

77. (1) The required event occurs if two sixes are observed in the first seven throws and a six is observed on the eighth throw. If p is the probability that a six shows on the die, the number of throws n is 7, and X is the number of times a six is observed, then $X \sim B(7,p)$. Therefore the required probability equals $P(X = 2)$ times the probability of getting a six on the eighth throw, i.e., it equals

$$({}^7C_2p^2q^5)(p)=({}^7C_2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)$$

$$=\frac{{}^7C_2(5^5)}{6^8}$$

78. (2) Let us consider

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k} \quad \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

then as per question

$$\left[\lambda\left(\vec{a}+\vec{b}\right)\ \lambda^2\vec{b}\ \lambda\vec{c}\right]=\left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$$

$$\Rightarrow \begin{vmatrix} \lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3) \\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda^2 b_3 \\ \lambda c_1 & \lambda c_2 & \lambda c_3 \end{vmatrix}$$

$$=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\Rightarrow \lambda^4\begin{vmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\Rightarrow \lambda^4\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\Rightarrow \lambda^4=-1$$

Hence λ has no real values.

79. (3)

$$\tan^{-1}\frac{1}{1+n(n+1)}+\tan^{-1}\frac{1}{1+(n+1)(n+2)}+......+$$

$$\tan^{-1}\frac{1}{1+(n+19)\ (n+20)}$$

$$=\tan^{-1}\frac{n+19}{n+21}-\tan^{-1}\frac{n-1}{n+1}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{n^2+n+1}\right)+\tan^{-1}\left(\frac{1}{n^2+3n+3}\right)+......+$$

$$\tan^{-1}\frac{1}{1+(n+19)\ (n+20)}$$

$$= \tan^{-1}\left(\frac{\frac{n+19}{n+21}-\frac{n-1}{n+1}}{1+\frac{n+19}{n+21}\times\frac{n-1}{n+1}}\right)$$

$$= \tan^{-1}\frac{20}{n^2+20n+1} = S$$

$$\therefore \tan S = \frac{20}{n^2+20n+1}$$

80. (4) Given expansion can be written as

$\left(\frac{x-1}{x}\right)^n.(1-x)^n = (-1)^n x^{-n} (1-x)^{2n}$

Total number of terms will be $2n+1$ which is odd ($\because 2n$ is always even)

$\therefore$ Middle term $= \frac{2n+1+1}{2} = (n+1)$ th

Now, $T_{r+1} = {}^nC_r (1)^r x^{n-r}$

So, $\frac{{}^{2n}C_n \cdot x^{2n-n}}{x^n.(-1)^n} = {}^{2n}C_n .(-1)^n$

Middle term is an odd term. So, $n+1$ will be odd.

So, n will be even.

$\therefore$ Required answer is ${}^{2n}C_n$.

81. (0.33) $S = \frac{5}{1^2\cdot4^2} + \frac{11}{4^2\cdot7^2} + \frac{17}{7^2\cdot10^2} + \ldots\infty.$

$3S = \frac{3\cdot5}{1^2\cdot4^2} + \frac{3\cdot11}{4^2\cdot7^2} + \frac{3\cdot17}{7^2\cdot10^2} + \ldots\infty$

$\Rightarrow 3S = \frac{(4-1)(4+1)}{1^2\cdot4^2} + \frac{(7-4)(7+4)}{4^2\cdot7^2} + \frac{(10-7)(10+7)}{7^2\cdot10^2} + \ldots\infty$

$\Rightarrow S = \frac{1}{3} = 0.33$

82. (0.60) Solving $2x + y = 3$ i.e. $y = 3 - 2x$ and $4x^2 + y^2 = 5$, we get

$4x^2 + (3-2x)^2 = 5 \Rightarrow 8x^2 - 12x + 4 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0 \Rightarrow (2x-1)(x-1) = 0$

$\therefore x = \frac{1}{2}, 1$

When $x = 1$, $y = 3 - 2 = 1$ and when $x = \frac{1}{2}$,

$y = 3 - 1 = 2$

$\therefore$ Points of intersection are $P\left(\frac{1}{2}, 2\right)$ and $Q(1, 1)$.

Equation of tangent at $P\left(\frac{1}{2}, 2\right)$ is

$4x\left(\frac{1}{2}\right) + y(2) = 5$ i.e. $2x + 2y = 5$

Its slope $= -1$ $\therefore$ Slope of the normal $= 1$

Again, equation of tangent at $Q(1, 1)$ is

$4x(1) + y(1) = 5$ i.e. $4x + y = 5$

Its slope $= -4$ $\therefore$ Slope of the normal $= \frac{1}{4}$

$$\therefore \tan\theta = \frac{1-\frac{1}{4}}{1+(1)\left(\frac{1}{4}\right)} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5} = 0.6$$

83. (2007) $\det(M_r) = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix} = 2r-1$

$$\sum_{r=1}^{2007} \det(M_r) = 2\sum_{r=1}^{2007} r - 2007$$

$$= 2\times\frac{2007\times2008}{2} - 2007 = (2007)^2$$

$= k^2 \Rightarrow k = 2007$

84. (5.00) $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

Cofactors of various entries are

$4, -5, 1; 2, 0, -2; 2, 5, 3$

$|A| = 1\times4 + (-1)\times-5 + 1\times1 = 10$

Cofactor Matrix $C = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$

$$\therefore \text{Adj } A = C^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{Adj}A}{|A|} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Comparing , we get $\alpha = 5$

85. (1.50) $y = \tan^{-1}\left(\frac{3x^2-1}{3x-x^3}\right)$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{3x-x^3}{3x^2-1}\right) = \frac{\pi}{2} + 3\tan^{-1} x$$

$$\frac{dy}{dx} = \frac{3}{1+x^2}$$

$$u = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$$

$$= -\frac{\pi}{2} + 2\tan^{-1} x$$

$$\frac{du}{dx} = \frac{2}{1+x^2}$$

$$\frac{dy}{dx} = \frac{dy/du}{du/dx} = \frac{3}{2} = 1.50$$

86. (1.00) Let $f(x) = ax^2 + bx + c$

$$g(x) = \int f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$$

$$\therefore\ g(1) = \frac{a}{3} + \frac{b}{2} + c = 2a + 3b + c = 0$$

$g(0) = 0 = g(1)$

$\therefore$ g(x) is continuous apply Rolle's theorem

$\Rightarrow g(x) = 0$ for some value $x \in (0,1)$

$\Rightarrow g'(x) = f(x) = ax^2 + bx + c = 0$

has a root in $(0, 1) = (m, n)$

$\Rightarrow\ m + n = 1.$

87. (1.00) Let $z_1 = 1 + i$ and $z_2 = 1 - i$

$$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$$

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{2 + 3\left(\frac{z_2}{z_1}\right)}{2 - 3\left(\frac{z_2}{z_1}\right)} = \frac{2-3i}{2+3i}$$

$$\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right| = \left|\frac{2-3i}{2+3i}\right| = \left|\frac{2-3i}{2+3i}\right|$$

$$\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1$$

88. (2.00) The given lines are

$$x - 1 = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s \qquad ......(1)$$

and $2x = y - 1 = \frac{z-2}{-1} = t \qquad ......(2)$

The lines are coplanar, if

$$\begin{vmatrix} 0-(-1) & -1-3 & -2-(-1) \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

$$c_2 \to c_2 + c_3;\ \begin{vmatrix} 1 & -5 & -1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 5(-1 - \frac{\lambda}{2}) = 0 \Rightarrow \lambda = -2$$

$\Rightarrow\ |\lambda| = 2$

89. (0.50) Since A and B are two independent events

$\therefore\ P(A \cap B) = P(A)\,P(B)$

$\Rightarrow\ P(A)\,P(B) = \frac{1}{6}$ (given) ...(i) and P (neither of A nor B) = $P\,\overline{(A \cup B)} = \frac{1}{3}$

$\Rightarrow P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{3} = \frac{2}{3}$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{2}{3} = P(A) + P(B) - \frac{1}{6}$

$\Rightarrow P(A) + P(B) = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$

$\Rightarrow P(A) + \frac{1}{6P(A)} = \frac{5}{6}$ [from (i)]

$\Rightarrow 6\,[P(A)]^2 - 5P(A) + 1 = 0$

$\Rightarrow (2P(A) - 1)(3P(A) - 1) = 0$

$\Rightarrow P(A) = \frac{1}{2}, \frac{1}{3}$

$\therefore$ Most $P(A) = \frac{1}{2}$

90. (0.05) $B\left(5, \frac{1}{2}\right) \Rightarrow n = 5, p = \frac{1}{2}, q = \frac{1}{2}$

$B\left(7, \frac{1}{2}\right) \Rightarrow n = 7, p = \frac{1}{2}, q = \frac{1}{2}$

Since X and Y are independent events

$\Rightarrow X + Y = 3$

$\Rightarrow X = 0, Y = 3; X = 1, Y = 2; X = 2, Y = 1; X = 0, Y = 3$

$\therefore P(X + Y = 3)$

$$= {}^5C_0\left(\frac{1}{2}\right)^5 \cdot {}^7C_3\left(\frac{1}{2}\right)^7 + {}^5C_1\left(\frac{1}{2}\right)^5 {}^7C_2\left(\frac{1}{2}\right)^7$$

$$+ {}^5C_2\left(\frac{1}{2}\right)^5 {}^7C_1\left(\frac{1}{2}\right)^7 + {}^5C_3\left(\frac{1}{2}\right)^5 {}^7C_0\left(\frac{1}{2}\right)^7$$

$$= \frac{55}{1024} = 0.05$$