Quadratic Equations

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1. Check whether the following are Quadratic Equations.
(i) $(x+1)^2=2(x-3)$
(ii) $x^2-2x=(-2)(3-x)$
(iii) $(x-2)(x+1)=(x-1)(x+3)$
(iv) $(x-3)(2x+1)=x(x+5)$
(v) $(2x-1)(x-3)=(x+5)(x-1)$
(vi) $x^2+3x+1=(x-2)^2$
(vii) $(x+2)^3=2x(x^2-1)$
(viii) $x^3-4x^2-x+1=(x-2)^3$

Ans. (i) $(x+1)^2=2(x-3)$ $\{(a+b)^2=a^2+2ab+b^2\}$
$\Rightarrow x^2+1+2x=2x-6$
$\Rightarrow x^2+7=0$
Here, degree of equation is 2.
Therefore, it is a Quadratic Equation.

(ii) $x^2-2x=(-2)(3-x)$
$\Rightarrow x^2-2x=-6+2x$
$\Rightarrow x^2-2x-2x+6=0$
$\Rightarrow x^2-4x+6=0$
Here, degree of equation is 2.
Therefore, it is a Quadratic Equation.

(iii) $(x-2)(x+1)=(x-1)(x+3)$
$\Rightarrow x^2+x-2x-2=x^2+3x-x-3=0$
$\Rightarrow x^2+x-2x-2-x^2-3x+x+3=0$
$\Rightarrow x-2x-2-3x+x+3=0$
$\Rightarrow -3x+1=0$
Here, degree of equation is 1.
Therefore, it is not a Quadratic Equation.

(iv) $(x-3)(2x+1)=x(x+5)$
$\Rightarrow 2x^2+x-6x-3=x^2+5x$
$\Rightarrow 2x^2+x-6x-3-x^2-5x=0$
$\Rightarrow x^2-10x-3=0$
Here, degree of equation is 2.
Therefore, it is a quadratic equation.

(v) $(2x-1)(x-3)=(x+5)(x-1)$
$\Rightarrow 2x^2-6x-x+3=x^2-x+5x-5$
$\Rightarrow 2x^2-7x+3-x^2+x-5x+5=0$
$\Rightarrow x^2-11x+8=0$
Here, degree of Equation is 2.
Therefore, it is a Quadratic Equation.

(vi) $x^2+3x+1=(x-2)^2$ $\{(a-b)^2=a^2-2ab+b^2\}$
$\Rightarrow x^2+3x+1=x^2+4-4x$
$\Rightarrow x^2+3x+1-x^2+4x-4=0$
$\Rightarrow 7x-3=0$
Here, degree of equation is 1.
Therefore, it is not a Quadratic Equation.

(vii) $(x+2)^3=2x(x^2-1)$ $\{(a+b)^3=a^3+b^3+3ab(a+b)\}$
$\Rightarrow x^3+2^3+3(x)(2)(x+2)=2x(x^2-1)$
$\Rightarrow x^3+8+6x(x+2)=2x^3-2x$
$\Rightarrow 2x^3-2x-x^3-8-6x^2-12x=0$
$\Rightarrow x^3-6x^2-14x-8=0$
Here, degree of Equation is 3.
Therefore, it is not a quadratic Equation.

(viii) $x^3-4x^2-x+1=(x-2)^3$ $\{(a-b)^3=a^3-b^3-3ab(a-b)\}$
$\Rightarrow x^3-4x^2-x+1=x^3-2^3-3(x)(2)(x-2)$
$\Rightarrow -4x^2-x+1=-8-6x^2+12x$
$\Rightarrow 2x^2-13x+9=0$
Here, degree of Equation is 2.
Therefore, it is a Quadratic Equation.

2. Represent the following situations in the form of Quadratic Equations:
(i) The area of rectangular plot is 528 m2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
(ii) The product of two consecutive numbers is 306. We need to find the integers.
(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) after 3 years will be 360. We would like to find Rohan's present age.
(iv) A train travels a distance of 480 km at uniform speed. If, the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find speed of the train.

Ans. (i) We are given that area of a rectangular plot is 528 m^2.
Let breadth of rectangular plot be x metres
Length is one more than twice its breadth.
Therefore, length of rectangular plot is $(2x+1)$ metres
Area of rectangle = length × breadth

$\Rightarrow 528=x(2x+1)$
$\Rightarrow 528=2x^2+x$
$\Rightarrow 2x^2+x-528=0$
This is a Quadratic Equation.

(ii) Let two consecutive numbers be x and (x+1).
It is given that $x(x+1)=306$
$\Rightarrow x^2+x=306$
$\Rightarrow x^2+x-306=0$
This is a Quadratic Equation.

(iii) Let present age of Rohan = x years
Let present age of Rohan's mother = (x +26) years
Age of Rohan after 3 years = (x+3) years
Age of Rohan's mother after 3 years = x+26+3 = (x+29) years
According to given condition:
$(x+3)(x+29)=360$
$\Rightarrow x^2+29x+3x+87=360$
$\Rightarrow x^2+32x-273=0$
This is a Quadratic Equation.

(iv) Let speed of train be x km/h
Time taken by train to cover 480 km = $480x$ hours
If, speed had been 8km/h less then time taken would be $(480x-8)$ hours
According to given condition, if speed had been 8km/h less then time taken is 3 hours less.
Therefore, $480x-8=480x+3$
$\Rightarrow 480(1x-8-1x)=3$
$\Rightarrow 480(x-x+8)(x)(x-8)=3$
$\Rightarrow 480\times 8=3(x)(x-8)$
$\Rightarrow 3840=3x^2-24x$
$\Rightarrow 3x^2-24x-3840=0$
Dividing equation by 3, we get
$\Rightarrow x^2-8x-1280=0$
This is a Quadratic Equation.

3. Find the roots of the following equations:

(i) $\frac{x-1}{x}=3, x\neq 0$

(ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x\neq -4,7$

Ans. (i) $x-\frac{1}{x}=3 \text{ where } x \neq 0$

$\Rightarrow \frac{x^2-1}{x}=3$

$\Rightarrow x^2 - 1=3x$

$\Rightarrow x^2-3x- 1=0$

Comparing equation $x^2-3x- 1=0$ with general form $ax^2+bx+c=0$,

We get $a=1, b=-3$ and $c=-1$

Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$x=\frac{3\pm\sqrt{(3)^2-4(1)(-1)}}{2\times1}$

$\Rightarrow x=\frac{3\pm\sqrt{13}}{2}$

$\Rightarrow x=\frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30} \text{ where } x \neq -4,7$

$\Rightarrow \frac{(x-7)-(x+4)}{(x-4)(x-7)}=\frac{11}{30}$

$\Rightarrow \frac{-11}{(x-4)(x-7)}=\frac{11}{30}$

$\Rightarrow -30=x^2-7x+4x-28$

$\Rightarrow x^2-3x+2=0$

Comparing equation $x^2-3x+2=0$ with general form $ax^2+bx+c=0$,

We get $a=1, b=-3$ and $c=2$

Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$x=\frac{3\pm\sqrt{(3)^2-4(1)(2)}}{2\times1}$

$\Rightarrow x=\frac{3\pm\sqrt{1}}{2}$

$\Rightarrow x=\frac{3+\sqrt{1}}{2}, \frac{3-\sqrt{1}}{2}$

$\Rightarrow x=2,1$

4. The sum of reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 13. Find his present age.

Ans. Let present age of Rehman= x years
Age of Rehman 3 years ago = $(x-3)$ years.
Age of Rehman after 5 years = $(x+5)$ years
According to the given condition:

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3}$$

⇒

⇒ $3(2x+2) = (x-3)(x+5)$
⇒ $6x+6=x^2-3x+5x-15$
⇒ $x^2-4x-15-6=0$
⇒ $x^2-4x-21=0$
Comparing quadratic equation $x^2-4x-21=0$ with general form $ax^2+bx+c=0$,
We get $a=1, b=-4$ and $c=-21$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Using quadratic formula

$$x=\frac{4\pm\sqrt{(4)^2-4(1)(-21)}}{2\times1}$$

$$x=\frac{4\pm\sqrt{16+84}}{2}$$

⇒

$$x=\frac{4\pm\sqrt{100}}{2}=\frac{4\pm10}{2}$$

⇒

$$x=\frac{4+10}{2},\frac{4-10}{2}$$

⇒

⇒ $x=7,-3$
We discard $x=-3$.Since age cannot be in negative.
Therefore, present age of Rehman is 7 years.

5. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.

(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2-4\sqrt{3}x+4=0$
(iii) $2x^2 - 6x + 3 = 0$

Ans. (i) $2x^2 - 3x + 5 = 0$
Comparing this equation with general equation $ax^2+bx+c=0$,

We get $a=2, b=-3$ and $c=5$
Discriminant = $b^2-4ac=(-3)^2 - 4(2)(5)=9 - 40=-31$
Discriminant is less than 0 which means equation has no real roots.

(ii) $3x^2-4\sqrt{3}x+4=0$
Comparing this equation with general equation $ax^2+bx+c=0$,
We get $a=3, b=-4\sqrt{3}$ and c=4
Discriminant = $b^2-4ac=\left(-4\sqrt{3}\right)^2 -4(3)(4)=48 - 48=0$
Discriminant is equal to zero which means equations has equal real roots.
Applying quadratic $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find roots,
$x=\frac{4\sqrt{3}\pm\sqrt{0}}{6}=\frac{2\sqrt{3}}{3}$
Because, equation has two equal roots, it means $x=\frac{2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$
Comparing equation with general equation $ax^2+bx+c=0$,
We get $a=2, b=-6$, and $c=3$
Discriminant = $b^2-4ac=(-6)^2 - 4(2)(3)=36 - 24=12$
Value of discriminant is greater than zero.
Therefore, equation has distinct and real roots.
Applying quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find roots,
$x=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm2\sqrt{3}}{4}$
$\Rightarrow x=\frac{3\pm\sqrt{3}}{2}$
$\Rightarrow x=\frac{3+\sqrt{3}}{2},\frac{3-\sqrt{3}}{2}$

6. Solve for $x: 5^{x+1}+5^{1-x}=26$

Ans.

$5^{x+1}+5^{1-x}=26$

$5^x.5^1+5^1.5^{-x}=26$

$\Rightarrow 5^x.5+\frac{5^1}{5^x}=26$

Put $5^x=y$

$\frac{5y}{1}+\frac{5}{y}=26$

$\Rightarrow 5y^2-26y+5=0$

$\Rightarrow 5y^2-25y-y+5=0$

$\Rightarrow 5y(y-5)-1(y-5)=0$

$\Rightarrow (y-5)(5y-1)=0$

$\Rightarrow y=5 \quad \text{or} \quad y=\frac{1}{5}$

But

$5^x=5^1$ and $5^x=\frac{1}{5}$

$\Rightarrow x=1$ and $5^x=5^{-1} \Rightarrow x=-1$

7. $\frac{1}{p+q+x}=\frac{1}{p}+\frac{1}{q}+\frac{1}{x}$ **solve for** x **by factorization method.**

Ans. $\frac{1}{p+q+x}=\frac{1}{p}+\frac{1}{q}+\frac{1}{x}$

$\Rightarrow \frac{1}{p+q+x}-\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

$\Rightarrow \frac{x-p-q-x}{x^2+px+qx}=\frac{p+q}{pq}$

$\Rightarrow \frac{-(p+q)}{x^2+px+qx}=\frac{p+q}{pq}$

$\Rightarrow \frac{-1}{x^2+px+qx}=\frac{1}{pq}$

$\Rightarrow x^2+px+qx=-pq$

$\Rightarrow x^2+px+qx+pq=0$

$\Rightarrow x(x+p)+q(x+p)=0$

$\Rightarrow (x+p)(x+q)=0$

$\Rightarrow x=-p \; or \; x=-q$

8. $5x^2-6x-2=0,$ **solve for** x **by the method of completing the square.**

Ans. $5x^2-6x-2=0$

$$\Rightarrow x^2-\frac{6}{5}x-\frac{2}{5}=0$$

$$\Rightarrow x^2-\frac{6}{5}x+\left(\frac{3}{5}\right)^2-\frac{2}{5}=0$$

$$\Rightarrow \left(x-\frac{3}{5}\right)^2=\frac{9}{25}+\frac{2}{5}$$

$$\Rightarrow \left(x-\frac{3}{5}\right)^2=\frac{9+10}{25}$$

$$\Rightarrow \left(x-\frac{3}{5}\right)^2=\frac{19}{25}$$

$$\Rightarrow x-\frac{3}{5}=\pm\frac{\sqrt{19}}{5}$$

$$\Rightarrow x=\frac{3}{5}\pm\frac{\sqrt{19}}{5}$$

$$\Rightarrow x=\frac{3+\sqrt{19}}{5} \text{ or } x=\frac{3-\sqrt{19}}{5}$$

9. Solve for x**:** $a^2b^2x^2+b^2x-a^2x-1=0$

Ans. $a^2b^2x^2+b^2+x-a^2x-1=0$

$$\Rightarrow b^2x\left(a^2x+1\right)-1\left(a^2x+1\right)=0$$

$$\Rightarrow \left(a^2x+1\right)\left(b^2x-1\right)=0$$

$$\Rightarrow x=\frac{-1}{a^2} \text{ or } x=\frac{1}{b^2}$$

10. Using quadratic formula, solve for x**:** $9x^2-9\left(a+b\right)x+\left(2a^2+5ab+2b^2\right)=0$

Ans. $D=b^2-4ac$

$$=\left(-9\left(a+b\right)\right)^2-4\times9\times\left(2a^2+5ab+2ab^2\right)$$

$$= 81(a+b)^2 - 36(2a^2 + 5ab + 2b^2)$$

$$= 9\left[9(a^2 + b^2 + 2ab - 8a^2 - 20ab - 8b^2)\right]$$

$$= 9\left[a^2 + b^2 - 2ab\right]$$

$$= 9(a-b)^2$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{9(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$$

$$\Rightarrow x = 3\frac{\left[3(a+b) \pm (a-b)\right]}{2 \times 9}$$

$$\Rightarrow x = \frac{(3a+3b) \pm (a-b)}{6}$$

$$\Rightarrow x = \frac{3a+3b+a-b}{6} \text{ or } x = \frac{3a+3b+a-b}{6}$$

$$\Rightarrow x = \frac{4a+2b}{6} \text{ or } x = \frac{4a+2b}{6}$$

$$\Rightarrow x = \frac{2a+b}{3} \text{ or } x = \frac{2a+b}{3}$$

11. In a cricket match, Kapil took one wicket less than twice the number of wickets taken by Ravi. If the product of the numbers of wickets taken by these two is 15, find the number of wickets taken by each.

Ans. Let no. of wicket taken by Ravi = x

No. of wicket taken by Kapil = $2x-1$

According to question,

$$(2x-1)x = 15$$

$$\Rightarrow 2x^2 - x - 15 = 0$$

$$\Rightarrow x = 3 \text{ or } x = \frac{-5}{2}$$ (Neglects)

So, no. of wickets taken by Ravi is $x = 3$

12. The sum of a number and its reciprocal is $\frac{17}{4}$. Find the number.

Ans. Let no. be x

According to question,

$$\frac{x}{1}+\frac{1}{x}=\frac{17}{4}$$

$$\Rightarrow \frac{x^2+1}{x}=\frac{17}{4}$$

$$\Rightarrow 4x^2+4=17x$$

$$\Rightarrow 4x^2-17x+4=0$$

$$\Rightarrow 4x^2-16x-x+4=0$$

$$\Rightarrow 4x(x-4)-1(x-4)=0$$

$$\Rightarrow (x-4)(4x-1)=0$$

$$\Rightarrow x=4 \text{ or } x=\frac{1}{4}$$