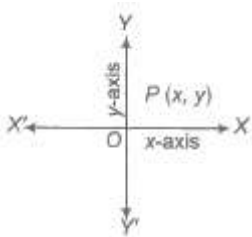# Class 11 Maths Chapter 10. Straight Lines

Coordinate Geometry

The branch of Mathematics in which geometrical problem are solved through algebra by using the coordinate system, is known as coordinate geometry.

Rectangular Axis

Let XOX' and YOY' be two fixed straight lines, which meet at right angles at O. Then,

(i) X'OX is called axis of X or the X-axis or abscissa.

(ii) Y'OY is called axis of Yor the Y-axis or ordinate.

(iii) The ordered pair of real numbers (x, y) is called cartesian coordinate .

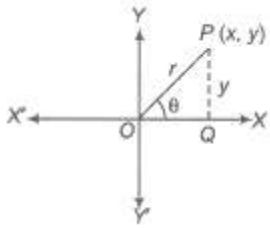**Quadrants**

The X and Y-axes divide the coordinate plane into four parts, each part is called a quadrant which is given below.
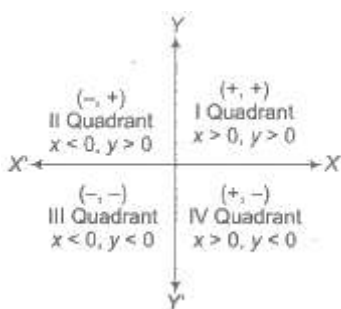
Polar Coordinates

In ΔOPQ,

$\cos \theta = x / r$ and $\sin \theta = y / r \Rightarrow x = r \cos \theta$ and $y = r \sin \theta$

where, $r = \sqrt{x^2 + y^2}$

The polar coordinate is represented by the symbol $P(r, \theta)$.

Distance Formula

(i) Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, is

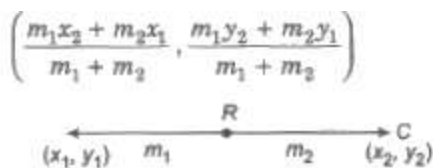$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(ii) If points are (r_1, θ_1) arid (r_2, θ_2), then distance between them is

$\sqrt{r^2_1 + r^2_2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}$.

(iii) Distance of a point (x_1, y_1) from the origin is $\sqrt{x^2_1 + y^2_1}$.

Section Formula

(i) The coordinate of the point which divides the joint of (x_1, y_1) and (x_2, y_2) in the ratio $m_1 : m_2$ internally, is

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

and externally is $\left(\dfrac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \dfrac{m_1 y_2 - m_2 y_1}{m_1 - m_2} \right)$

(ii) X-axis divides the line segment joining (x_1, y_1) and (x_2, y_2) in the ratio $-y_1 : y_2$.

Similarly, Y-axis divides the same line segment in the ratio $-x_1 : x_2$.

(iii) Mid-point of the joint of (x_1, y_1) and (x_2, y_2) is $(x_1 + x_2 / 2, y_1 + y_2 / 2)$

(iv) Centroid of ΔABC with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3), is

$(x_1 + x_2 + x_3 / 3 , y_1 + y_2 + y_3 / 3)$.

(v) Circumcentre of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, is

$$\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C} , \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C} \right).$$

(vi) Incentre of Δ ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ and whose sides are a, band c, is

$(ax_1 + bx_2 + cx_3 / a + b + c , ay_1 + by_2 + cy_3 / a + b + c)$.

(vii) Excentre of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ and whose sides are a, band c, is given by
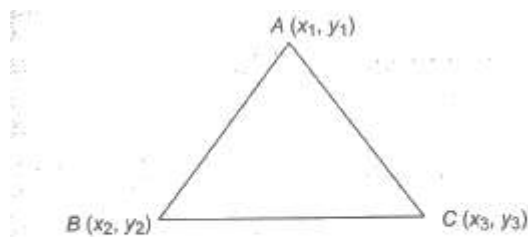
$$I_1 = \left(\frac{- ax_1 + bx_2 + cx_3}{- a + b + c} , \frac{-ay_1 + by_2 + cy_3}{- a + b + c} \right),$$

$$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a - b + c} , \frac{ay_1 - by_2 + cy_3}{a - b + c} \right)$$

$$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a + b - c} , \frac{ay_1 + by_2 - cy_3}{a + b - c} \right)$$

Area of Triangle/Quadrilateral

(i) Area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{vmatrix}$$

These points A, Band C will be collinear, if $\Delta = O$.

(ii) Area of the quadrilateral formed by joining the vertices

$(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) is $\dfrac{1}{2} \begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$
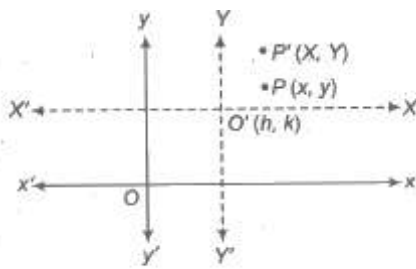
(iii) Area of trapezium formed by joining the vertices

$(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) is

$$\frac{1}{2}[(y_2 + y_1)(x_1 - x_2) + (y_1 + y_3)(x_3 - x_1) + (y_2 + y_3)(x_4 - x_2)]$$

Shifting/Rotation of Origin/Axes

Shifting of Origin

Let the origin is shifted to a point O'(h, k). If P(x, y) are coordinates of a point referred to old axes and P' (X, Y) are the coordinates of the same points referred to new axes, then
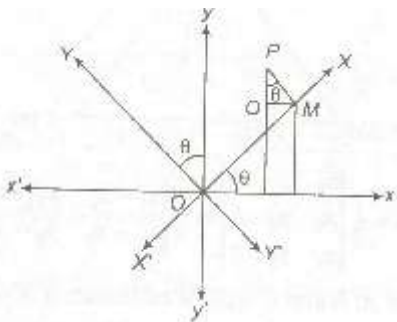

$$x = X + h, y = Y + k$$

Rotation of Axes

Let (x, y) be the coordinates of any point P referred to the old axes and (X, Y) be its coordinates referred to the new axes (after rotating the old axes by angle θ). Then,

$$X = x \cos \theta + y \sin \theta \text{ and } Y = y \cos \theta + x \sin \theta$$

Shifting of Origin and Rotation of Axes

If origin is shifted to point (h, k) and system is also rotated by an angle θ in anti-clockwise, then coordinate of new point P' (x', y') is obtained by replacing

$$x' = h + x \cos \theta + y \sin \theta$$

and $y' = k - x \sin \theta + y \cos \theta$

Locus

The curve described by a point which moves under given condition(s) is called its locus.

Equation of Locus

The equation of the locus of a point which is satisfied by the coordinates of every point.

Algorithm to Find the Locus of a Point

Step I Assume the coordinates of the point say (h,k) whose locus is to be found.

Step II Write the given condition in mathematical form involving h, k.

Step III Eliminate the variable(s), if any.

Step IV Replace h by x and k by y in the result obtained in step III. The equation so obtained is the locus of the point, which moves under some stated condition(s).

Straight Line

Any curve is said to be a straight line, if two points are taken on the curve such that every point on the line segment joining any two points on it lies on the curve.

General equation of a line is ax + by + c = o.

Slope (Gradient) of a Line)

The trigonometric tangent of the angle that a line makes with the positive direction of the X-axis in anti-clockwise sense is called the slope or gradient of the line.

So, slope of a line, $m = \tan \theta$

where, θ is the angle made by the line with positive direction of X-axis.

Important Results on Slope of Line

(i) Slope of a line parallel to X-axis, $m = 0$.
(ii) Slope of a line parallel to Y-axis, $m = \infty$.
(iii) Slope of a line equally inclined with axes is 1 or -1 as it makes an angle of 45° or 135°, with X-axis.
(iV) Slope of a line passing through (x, y,) and (x_2, y_2) is given by

$m = \tan \theta = y_2 - y_1 / x_2 - x_1$.

Angle between Two Lines

The angle e between two lines having slopes m_1 and m_2 is

$$\tan\theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|.$$

(i) Two lines are parallel, iff $m_1 = m_2$.
(ii) Two lines are perpendicular to each other, iff $m_1 m_2 = -1$.

Equation of a Straight Line

General equation of a straight line is $Ax + By + C = 0$.

(i) The equation of a line parallel to X-axis at a distance b from it, is given by

$y = b$

(ii) The equation of a line parallel to Y-axis at a distance a from it, is given by

$x = a$

(iii) Equation of X-axis is

$y = 0$

(iv) Equation of Y-axis is

$x = 0$

Different Form of the Equation of a Straight Line

(i) **Slope Intercept Form** The equation of a line with slope m and making an intercept c on Y-axis, is

$y = mx + c$

If the line passes through the origin, then its equation will be

$y = mx$

(ii) **One Point Slope Form** The equation of a line which passes through the point (x_1, y_1) and has the slope of m is given by

$(y - y_1) = m(x - x_1)$

(iii) **Two Points Form** The equation of a line' passing through the points (x_1, y_1) and (x_2, y_2) is given by

$(y - y_1) = (y_2 - y_1 / x_2 - x_1)(x - x_1)$

This equation can also be determined by the determinant method, that is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(iv) **The Intercept Form** The equation of a line which cuts off intercept a and b respectively on the X and Y-axes is given by
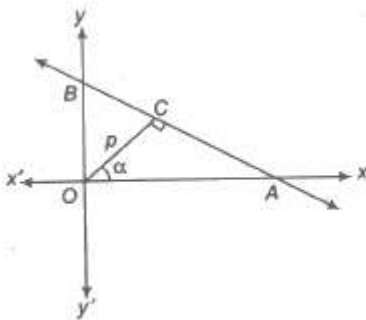
x / a + y / b = 1

The general equation Ax + By + C = 0 can be converted into the intercept form, as

x / – (C A) + y / – (C B) = 1

(v) **The Normal Form** The equation of a straight line upon which the length of the perpendicular from the origin is p and angle made by this perpendicular to the X-axis is α, is given by
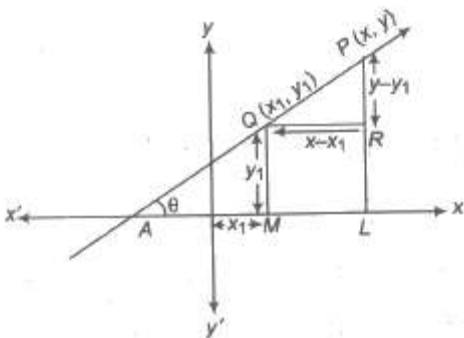
x cos α + Y sin α = p

(vi) **The Distance** (Parametric) **Form** The equation of a straight line passing through (x_1, y_1) and making an angle θ with the positive direction of x-axis, is

x – x₁ / cos θ = y – y₁ / sin θ = r

where, r is the distance between two points P(x, y) and Q(x₁, y₁).

Thus, the coordinates of any point on the line at a distance r from the given point (x_1, y_1) are $(x_1 + r \cos \theta, y_1 + r \sin \theta)$. If P is on the right side of (x_1, y_1) then r is positive and if P is on the left side of (x_1, y_1) then r is negative.

Position of Point(s) Relative to a Given Line

Let the equation of the given line be $ax + by + C = 0$ and let the Coordinates of the two given points be $P(x_1, y_1)$ and $Q(x_2, y_2)$.

(i) The two points are on the same side of the straight line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same sign.

(ii) The two points are on the opposite side of the straight line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have opposite sign.

(iii) A point (x_1, y_1) will lie on the side of the origin relative to a line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and c have the same sign.

(iv) A point (x_1, y_1) will lie on the opposite side of the origin relative to a line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and c have the opposite sign.

(v) Condition of concurrency for three given lines $ax_1 + by_1 + c_1 = 0$, $ax_2 + by_2 + c_2$ and $ax_3 + by_3 + c_3 = 0$ is $a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - a_1c_2) + c_3(a_1b_2 - a_2b_1) = 0$

or $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

(vi) **Point of Intersection of Two Lines** Let equation of lines be $ax_1 + by_1 + c_1 = 0$ and $ax_2 + by_2 + c_2 = 0$, then their point of intersection is

$(b_1c_2 - b_2c_1 / a_1b_2 - a_2b_1, c_1a_2 - c_2a_1 / a_1b_2 - a_2b_1)$.

Line Parallel and Perpendicular to a Given Line

(i) The equation of a line parallel to a given line $ax + by + c = 0$ is $ax + by + \lambda = 0$, where λ is a constant.

(ii) The equation of a line perpendicular to a given line $ax + by + c =$ is $bx - ay + \lambda = 0$, where λ is a constant.

Image of a Point with Respect to a Line

Let the image of a point (x_1, y_1) with respect to $ax + by + c = 0$ be (x_2, y_2), then

$x_2 - x_1 / a = y_2 - y_1 / b = -2 (ax_1 + by_1 + c) / a^2 + b^2$

(i) The image of the point $P(x_1, y_1)$ with respect to X-axis is $Q(x_1 - y_1)$.

(ii) The image of the point $P(x_1, y_1)$ with respect to Y-axis is $Q(-x_1, y_1)$.

(iii) The image of the point $P(x_1, y_1)$ with respect to mirror $Y = x$ is $Q(y_1, x_1)$.

(iv) The image. of the point $P(x_1, y_1)$ with respect to the line mirror $y == x \tan \theta$ is

$x = x_1 \cos 2\theta + y_1 \sin 2\theta$
$Y = x_1 \sin 2\theta - y_1 \cos 2\theta$

(v) The image of the point $P(x_1, y_1)$ with respect to the origin is the point $(-x_1, y_1)$.

(vi) The length of perpendicular from a point (x_1, y_1) to a line $ax + by + c = 0$ is

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Equation of the Bisectors

The equation of the bisectors of the angle between the lines

$a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

are given by

$a_1x + b_1y + c_1 / \sqrt{a^2}_1 + b^2_1 = \&plusmn\ a_2x + b_2y + c_2 / \sqrt{a^2}_2 + b^2_2$

(i) If $a_1\ a_2 + b_1\ b_2 > 0$, then we take positive sign for obtuse and negative sign for acute.

(ii) If $a_1\ a_2 + b_1\ b_2 < 0$, then we take negative sign for obtuse and positive sign for acute .

Pair of Lines

General equation of a pair of straight lines is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Homogeneous Equation of Second Degree

A rational, integral, algebraic equation in two variables x and y is said to be a homogeneous equation of the second degree, if the sum of the indices of x and y in each term is equal to 2.

The general form of homogeneous equation of the second degree x and y is $ax^2 + 2hxy + by^2 = 0$, which passes through the origin.

Important Properties

If $ax^2 + 2hxy + by^2 = 0$ be an equation of pair of straight lines.

(i) Slope of first line, $m_1 = -h + \sqrt{h^2 - ab} / b$

and slope of second line, $m_2 = -h - \sqrt{h^2 - ab} / b$

$m_1 + m_2 = -2h / b = -$ Coefficient of xy / Coefficient of y^2

and $m_1 m_2 = a / b =$ Coefficient of x^2 / Coefficient of y^2

Here, m_1 and m_2 are

(a) real and distinct, if $h^2 > ab$.

(b) coincident, if $h^2 = ab$.

(c) imaginary, if $h^2 < ab$.

(ii) Angle between the pair of lines is given by

$\tan \theta = {}^2\sqrt{h^2 - ab} / a + b$

(a) If lines are coincident, then $h^2 = ab$
(b) If lines are perpendicular, then $a + b = $ o.

(iii) The joint equation of bisector of the angles between the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ is

$x^2 - b^2 / a - b = xy / h \Rightarrow hx^2 - (a - b)xy - hy^2 = 0$.

(iv) The necessary and sufficient condition $ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$ to represent a pair of straight lines, if $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

or $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

(v) The equation of the bisectors of the angles between the lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are given by

$(x - x^1)^2 - (y - y^1)^2 / a - b = (x - x^1)(y - y^1) / h$,

where, (x^1, y^1) is the point of intersection of the lines represented by the given equation.

(vi) The general equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$ will represent two parallel lines, if $g^2 - ac > 0$ and $a/h = h/b = g/f$ and the distance between them is $2\sqrt{g^2 - ac}/a(a + b)$ or $2\sqrt{f^2 - bc}/b(a + b)$.

(vii) If the equation of a pair of straight lines is $ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$, then the point of intersection is given by

$(hf - bg/ab - h^2, gh - af/ab - h^2)$.

(viii) The equation of the pair of lines through the origin and perpendicular to the pair of lines given by $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.

(ix) Equation of the straight lines having the origin to the points of intersection of a second degree curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and a straight line $Lx + my + n = 0$ is

$ax^2 + 2hxy + by^2 + 2gx(Lx + my/-n) + 2fy(Lx + my/-n) + c\ (Lx + my/-n)^2 = 0$.

Important Points to be Remembered

1. A triangle is an isosceles, if any two of its median are equal.

2. In an equilateral triangle, orthocentre, centroid, circumcentre, incentre coincide.

3. The circumcentre of a right angled triangle is the mid-point of the hypotenuse.

4. Orthocentre, centroid, circumcentre of a triangle are collinear, Centroid divides the line joining orthocentre and circumcentre in the ratio 2: 1.

5. If D, E and F are the mid-point of the sides BC, CA and AB of MBC, then the centroid of Δ ABC = centroid of Δ DEF.

6. Orthocentre of the right angled Δ ABC, right angled at A is A

7. Circumcentre of the right angled Δ ABC, right angled at A is B + C / 2.

8. The distance of a point (x^1, y^1) from the $ax + by + c = 0$ is

$d = |ax_1 + by_1 + c / \sqrt{a^2 + b^2}|$

9. Distance between two parallel lines $a_1x + b_1y + c_1 = 0$ and $a_1x + b_1y + c_2 = 0$ is given by

$d = |c_2 - c_1 / \sqrt{a^2 + b^2}|$.

10. The area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ is .

$$\Delta = \frac{1}{2} \left| \sum \frac{(c_1 - c_2)^2}{m_1 - m_2} \right|$$

.

11. Area of the triangle formed by the line $ax + by + c = 0$ with the coordinate axes is $\Delta = c^2 / 2|ab|$.

12. The foot of the perpendicular(h,k) from (x_1, y_1) to the line $ax + by + c = 0$ is given by $h - x_1 / a = k - y_1 / b = - (ax_1 + by_1 + c) / a^2 + b^2$.

13. Area of rhombus formed by $ax \pm by \pm c = 0$ is $|2c^2 / ab|$.

14. Area of the parallelogram formed by the lines

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_1x + b_1y + d_1 = 0$ and $a_2x + b_2y + d_2 = 0$ is

$|(d_1 - c_1) (d_2 - c_2 / a_1b_2 - a_2b_1|$.

15. (a) Foot of the perpendicular from (a, b) on $x - y = 0$ is

$(a + b / 2, a + b / 2)$.

(b) Foot of the perpendicular from (a,b) on $x + y = 0$ is

$(a - b / 2, a - b / 2)$.

16. The image of the line $a_1x + b_1y + c_1 = 0$ about the line $ax + by + c = 0$ is .

$2(aa_1 + bb_1) (ax + by + c) = (a^2 + b^2) (a_1x + b_1y + c_1)$.

17. Given two vertices (x_1, y_1) and (x_2, y_2) of an equilateral MBC, then its third vertex is given by.

$[x_1 + x_2 \pm \sqrt{3} (y_1 - y_2) / 2, y_1 + y_2 \mp \sqrt{3} (x_1 - x_2) / 2]$

18. The equation of the straight line which passes through a given point (x_1, y_1) and makes an angle α with the given straight line $y = mx + c$ are

$$(y - y_1) = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$
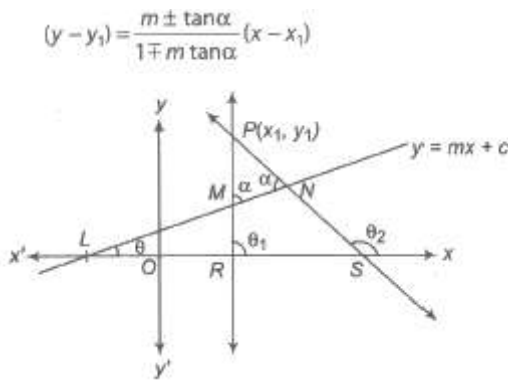
19. The equation of the family of lines passing through the intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is

$(a_1x + b_1y + c_1) + l(a_2x + b_2y + c_2) = 0$

where, λ is any real number.

20. Line $ax + by + c = 0$ divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $\lambda : 1$, then $\lambda = - (a_1x + b_1y + c / a_2x + b_2y + c)$.

If λ is positive it divides internally and if λ. is negative, then it divides externally.

21. Area of a polygon of n-sides with vertices $A_1(x_1, y_1)$, $A_2 (x_2, y_2)$,... ,$A_n(x_n, y_n)$

$$= \frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \dots + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right]$$

22. Equation of the pair of lines through (α, β) and perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$ is $b (x - \alpha\}^2 - 2h (x - \alpha)(y - \beta) + a (y - \beta)^2 = 0$.