NUMBER SYSTEM

Prime and Composite Numbers:

- Prime numbers are numbers with only two factors, 1 and the number itself.
- Composite numbers are numbers with more than 2 factors.
- 0 and 1 are neither composite nor prime.
- There are 25 prime numbers less than 100.

Theorems on Prime numbers:

- **Fermat's Theorem:** The remainder of $x^{(p-1)}$ when divided by p is 1, where p is a prime.
- **Wilson's Theorem:** The Remainder when (p-1)! is divided by p is (p-1) where p is a prime.
- To find the number of zeroes in n! find the highest power of 5 in n!
- If all possible permutations of n distinct digits are added together the sum = (n-1)! * (sum of n digits) * (11111... n times)
- If the number can be represented as N = $a * * . . . p\ b\ q\ c\ r$ then the number of factors is (p+1) * (q+1) * (r+1)
- Sum of the factors =
- If the number of factors is odd then N is a perfect square.
- If there are n factors,$(a^{p+1}-1)/(a-1) \times (b^{q+1}-1)(b-1) \times (c^{r+1}-1)(c-1)$n2. If N is a perfect square then n number of pairs (including the square root) is $\frac{(n+1)}{2}$

- If the number can be expressed as $N = 2^p * a^q * b^r$ where the power of 2 is p and a, b are prime numbers, Then the number of even factors of N = p (1+q) (1+r)......

- The number of odd factors of N = (1+q) (1+r)...

- Number of positive integral solutions of the equation $x^2 - y^2 = k$ is given by $\frac{\text{Total number of factors of } k}{2}$ (If k is odd but not a perfect square)

- $\frac{(\text{Total number of factors of } k) - 1}{2}$ (If k is odd and a perfect square)

- $\frac{(\text{Total number of factors of } k/4)}{2}$ (If k is even and not a perfect square)

- $\frac{(\text{Total number of factors of } k/4)-1}{2}$ (If it is even and a perfect square)

- Sum of first n odd numbers is n^2

- Sum of first n even numbers is n(n+1)

- The product of the factors of N is given by $N^{a/2}$ where a is the number of factors.

- The last two digits of $a^2, (50-a)^2 (50+a)^2, (100-a)^2 \ldots..$ are the same.

- If the number is written as 2^{10n}, When n is odd, the last 2 digits are 24. When n is even, the last 2 digits are 76

Remainder Theorem:

- If x, y, z are the prime factors of N such that
- $N= x^p*y^q*z^q$ Then the number of numbers less than N and co-prime to N is $\phi(N)= N (1 - 1/x) (1 - 1/y) (1 - 1/z)$.
- This is Euler's totient function.

Sum of natural numbers

$\sum n = \frac{n(n+1)}{2}$, $\sum n$ is the sum of first n natural numbers

$\sum n^2 = \frac{n(n+1)(n+2)}{6}$, $\sum n^2$ is the sum of first n perfect squares.

$\sum n^3 = \frac{n^2(n+1)^2}{4}$, $\sum n^3$ is the sum of first n perfect cubes

Divisibility:

- Divisibility by 2: Last digit divisible by 2
- Divisibility by 3: Sum of digits divisible by 3
- Divisibility by 4: Last two digits divisible by 4
- Divisibility by 5: Unit digit should be 0 or 5
- Divisibility by 6: Number should be divisible by 3 and 2
- Divisibility by 7: Remove the last digit and double that number, subtract the doubled number from the number whose last digit is removed. If the number is identified as 2-digit multiple of 7 then it is divisible by 7, else repeat the process further and check.
- Divisibility by 8: The last three digits should be divisible by 8
- Divisibility by 9: Sum of digits divisible by 9
- Divisibility by 10: Unit digit should be 0

- Divisibility by 11: If the difference of the sum of alternative digits of a number is divisible by 11, then that number is divisible by 11 completely.

- Divisibility by 12: The number should be divisible by both 3 and 4

Divisibility properties:

- For composite divisors, check if the number is divisible by the factors individually. Hence to check if a number is divisible by 6 it must be divisible by 2 and 3.

- The equation $a^n - b^n$ is always divisible by a-b. If n is even it is divisible by a+b. If n is odd it is not divisible by a+b.

- The equation $a^n + b^n$ is always divisible by a-b. If n is odd it is divisible by a+b. If n is even it is not divisible by a-b.

- The equation $a^n - b^n$ is divisible by a+b if n is odd. If n is even it is not divisible by a+b.

HCF & LCM

- HCF * LCM of two numbers = Product of two numbers

- The greatest number dividing a, b and c leaving remainders of x_1, x_2 and x_3 is the HCF of (a - x1), (b - x2), (c - x3)

- The greatest number dividing a, b and c (a<b<c) leaving the same remainder each time is the HCF of (c-b), (c-a), (b-a).

- If a number, N is divisible by X and Y and HCF(X, Y) = 1. Then, N is divisible by X × Y

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