CBSE Class 9 Mathemaics

Important Questions

Chapter 8

Quadrilaterals

1 Marks Quetions

1. A quadrilateral ABCD is a parallelogram if

(a) AB = CD

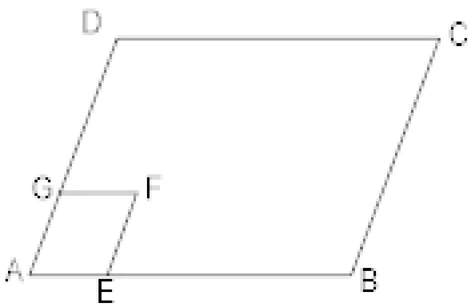
(b) AB || BC

(c) $\angle A = 60^0, \angle C = 60^0, \angle B = 120^0$

(d) AB = AD

Ans. (c) $\angle A = 60^0, \angle C = 60^0, \angle B = 120^0$

2. In figure, ABCD and AEFG are both parallelogram if $\angle C = 80^0, then \angle DGF\ is$



(a) 100^0

(b) 60^0

(c) 80^0

(d) 120^0

Ans. (c) 80^0

3. In a square ABCD, the diagonals AC and BD bisects at O. Then ΔAOB **is**

(a) acute angled

(b) obtuse angled

(c) equilateral

(d) right angled

Ans. (d) right angled

4. ABCD is a rhombus. If $\angle ACB = 30^0$, *then* $\angle ADB$ *is*

(a) 30^0

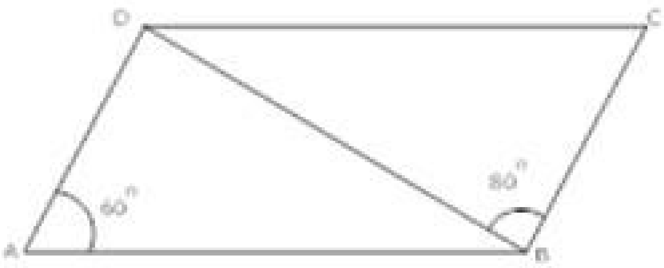
(b) 120^0

(c) 60^0

(d) 45^0

Ans. (c) 60^0

5. In fig ABCD is a parallelogram. If $\angle DAB = 60°$ **and** $\angle DBC = 80°$ **then** $\angle CDB$ **is**



$(\text{A})\,80°$

$(\text{B})\,60°$

$(\text{C})\,20°$

$(\text{D})\,40°$

Ans. (D) 40°

6. If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be.

(a) Square

(b) Parallelogram

(c) Rhombus

(d) Rectangle

Ans. (b) Parallelogram

7. The diagonal AC and BD of quadrilateral ABCD are equal and are perpendicular bisector of each other then quadrilateral ABCD is a

(a) Kite

(b) Square

(c) Trapezium

(d) Rectangle

Ans. (b) Square

8. The quadrilateral formed by joining the mid points of the sides of a quadrilateral ABCD taken in order, is a rectangle if

(a) ABCD is a parallelogram

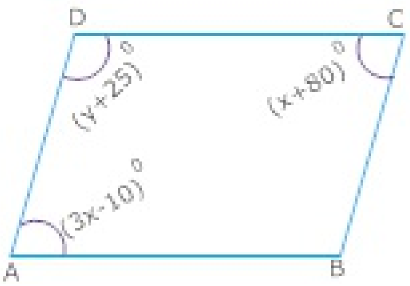
(b) ABCD is a rut angle

(c) Diagonals AC and BD are perpendicular

(d) AC=BD

Ans. (a) ABCD is a parallelogram

9. In the fig ABCD is a Parallelogram. The values of x and y are



(a) 30, 35

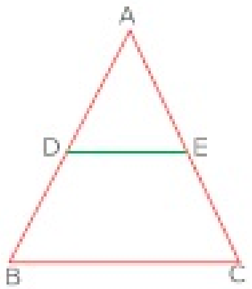
(b) 45, 30

(c) 45, 45

(d) 55, 35

Ans. (b) 45, 30

10. In fig if DE=8 cm and D is the mid-Point of AB, then the true statement is



(a) AB=AC

(b) DE||BC

(c) E is not mid-Point of AC

(d) DE $\neq$ BC

Ans. (c) E is not mid-Point of AC

11. The sides of a quadrilateral extended in order to form exterior angler. The sum of

these exterior angle is

(a) $180°$

(b) $270°$

(c) $90°$

(d) $360°$

Ans. (d) $360°$

12. ABCD is rhombus with $\angle ABC = 40°$. **The measure of** $\angle ACD$ **is**

(a) $90°$

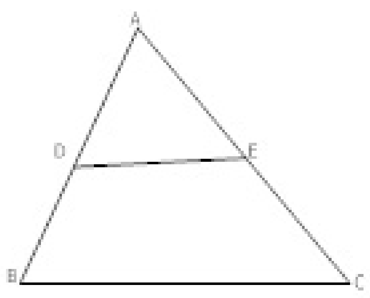
(b) $20°$

(c) $40°$

(d) $70°$

Ans. b) $20°$

13. In fig D is mid-point of AB and DE $\|$ **BC then AE is equal to**



(a) AD

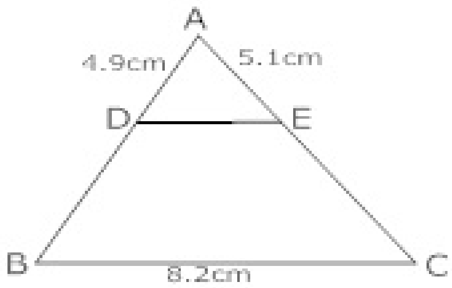
(b) EC

(c) DB

(d) BC

Ans. (b) EC

14. In fig D and E are mid-points of AB and AC respectively. The length of DE is



(a) 8.2 cm

(b) 5.1 cm

(c) 4.9 cm

(d) 4.1 cm

Ans. (d) 4.1 cm

15. A diagonal of a parallelogram divides it into

(a) two congruent triangles

(b) two similes triangles

(c) two equilateral triangles

(d) none of these

Ans. (a) two congruent triangles

16. A quadrilateral is a _________, if its opposite sides are equal:

(a) Kite

(b) trapezium

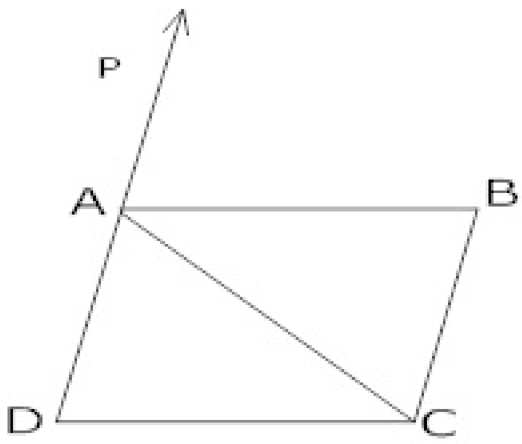
(c) cyclic quadrilateral

(d) parallelogram

Ans. (d) parallelogram

17. In the adjoining Fig. AB = AC. CD||BA and AD is the bisector of $\angle PAC$ prove that

(a) $\angle DAC = \angle BCA$ and



Ans. In ΔABC $\quad AB = AC$

$\Rightarrow \angle BCA = \angle BAC$ [Opposite angle of equal sides are equal]

$\angle CAD = \angle BCA + \angle ABC$ [Exterior angle]

$\Rightarrow \angle PAC = \angle BCA$

Now $\angle PAC = \angle BCA$

$\Rightarrow AP \parallel BC$

Also CD||BA Given)

$\therefore ABCD$ is a parallelogram

(ii) ABCD is a parallelogram

18. Which of the following is not a parallelogram?

(a) Rhombus

(b) Square

(c) Trapezium

(d) Rectangle

Ans. (c) Trapezium

19. The sum of all the four angles of a quadrilateral is

(a) 180^0

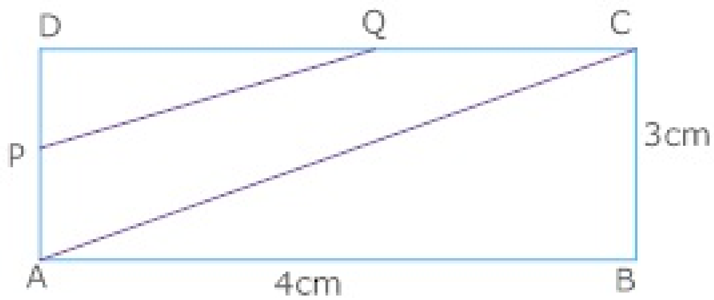
(b) 360^0

(c) 270^0

(d) 90^0

Ans. (b) 360^0

20. In Fig ABCD is a rectangle P and Q are mid-points of AD and DC respectively. Then length of PQ is



(a)5 cm

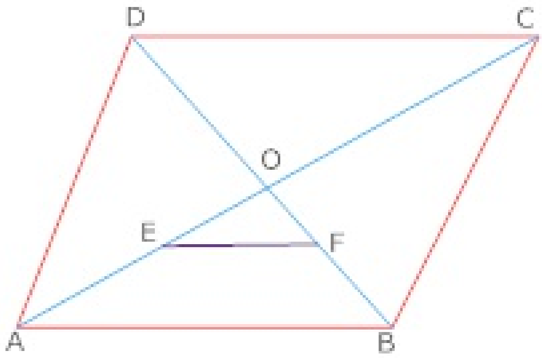
(b) 4 cm

(c) 2.5 cm

(d) 2 cm

Ans. (c) 2.5 cm

21. In Fig ABCD is a rhombus. Diagonals AC and BD intersect at O. E and F are mid points of AO and BO respectively. If AC = 16 cm and BD = 12 cm then EF is



(a)10 cm

(b) 5 cm

(c) 8 cm

(d) 6 cm

Ans. (b) 5 cm

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Quadrilaterals

2 Marks Questions

1. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all angles of the quadrilateral

Ans. Let in quadrilateral ABCD, $\angle A = 3x$, $\angle B = 5x$, $\angle C = 9x$ and $\angle D = 13x$.

Since, sum of all the angles of a quadrilateral = 360°

$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ \Rightarrow 3x + 5x + 9x + 13x = 360^\circ$

$\Rightarrow 30x = 360^\circ \Rightarrow x = 12^\circ$

Now $\angle A = 3x = 3 \times 12 = 36^\circ$

$\angle B = 5x = 5 \times 12 = 60^\circ$

$\angle C = 9x = 9 \times 12 = 108^\circ$

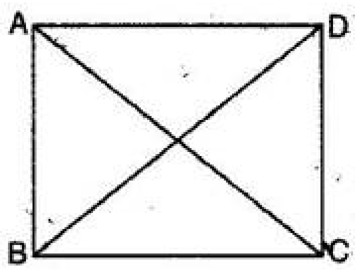
And $\angle D = 13x = 13 \times 12 = 156^\circ$

Hence angles of given quadrilateral are $36^\circ, 60^\circ, 108^\circ$ and 156°.

2. If the diagonals of a parallelogram are equal, show that it is a rectangle.

Ans. Given: ABCD is a parallelogram with diagonal AC = diagonal BD

To prove: ABCD is a rectangle.



Proof: In triangles ABC and ABD,

AB = AB [Common]

AC = BD [Given]

AD = BC [opp. Sides of a $\parallel$ gm]

$\therefore$ ΔABC $\cong$ ΔBAD [By SSS congruency]

$\Rightarrow$ $\angle$DAB = $\angle$CBA [By C.P.C.T.] ....(i)

But $\angle$DAB + $\angle$CBA = 180° ....(ii)

[$\because$ AD $\parallel$ BC and AB cuts them, the sum of the interior angles of the same side of transversal is 180°]

From eq. (i) and (ii),

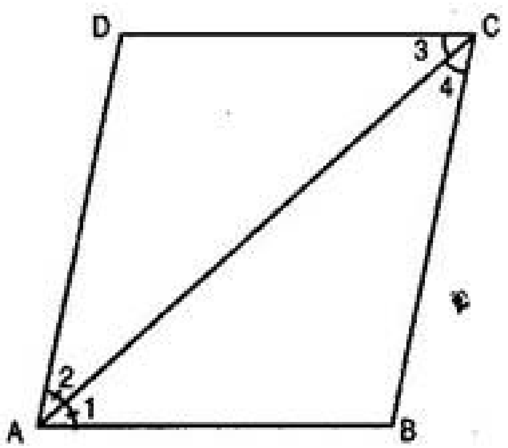
$\angle$DAB = $\angle$CBA = 90°

Hence ABCD is a rectangle.

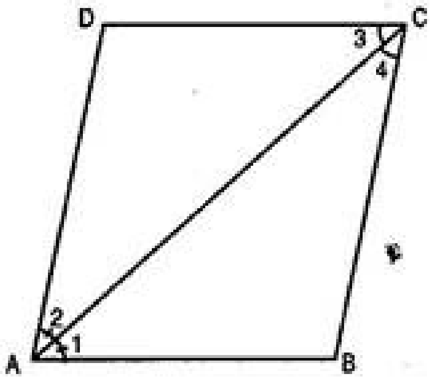
3. Diagonal AC of a parallelogram ABCD bisects $\angle$A (See figure). Show that:

(i) It bisects $\angle$C also.

(ii) ABCD is a rhombus.



Ans. Diagonal AC bisects $\angle$A of the parallelogram ABCD.



(i) Since AB $\parallel$ DC and AC intersects them.

$\therefore$ $\angle 1 = \angle 3$ [Alternate angles] ....(i)

Similarly $\angle 2 = \angle 4$ ....(ii)

But $\angle 1 = \angle 2$ [Given] ....(iii)

$\therefore$ $\angle 3 = \angle 4$ [Using eq. (i), (ii) and (iii)]

Thus AC bisects $\angle C$.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$

$\Rightarrow$ AD = CD [Sides opposite to equal angles]

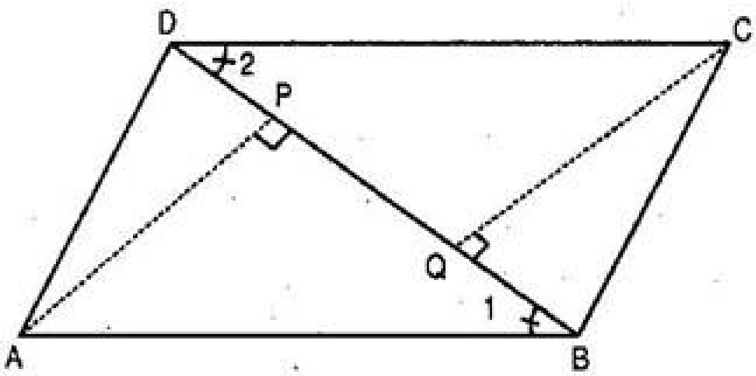
$\therefore$ AB = CD = AD = BC

Hence ABCD is a rhombus.

4. ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:

(i) $\triangle$APB $\cong$ $\triangle$CQD

(ii) AP = CQ



Ans. Given: ABCD is a parallelogram. AP $\perp$ BD and CQ $\perp$ BD

To prove: (i) $\triangle$APB $\cong$ $\triangle$CQD (ii) AP = CQ

Proof: (i) In $\triangle$APB and $\triangle$CQD,

$\angle 1 = \angle 2$ [Alternate interior angles]

AB = CD [Opposite sides of a parallelogram are equal]

$\angle APB = \angle CQD = 90^\circ$

$\therefore$ $\triangle APB \cong \triangle CQD$ [By ASA Congruency]

(ii) Since $\triangle APB \cong \triangle CQD$

AP = CQ [By C.P.C.T.]

5. ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:

(i) SR $\parallel$ AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Ans. In $\triangle$ ABC, P is the mid-point of AB and Q is the mid-point of BC.

Then PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC

(i) In $\triangle$ACD, R is the mid-point of CD and S is the mid-point of AD.

Then SR $\parallel$ AC and SR = $\frac{1}{2}$ AC

(ii) Since PQ = $\frac{1}{2}$ AC and SR = $\frac{1}{2}$ AC

Therefore, PQ = SR

(iii) Since PQ $\parallel$ AC and SR $\parallel$ AC

Therefore, PQ $\parallel$ SR [two lines parallel to given line are parallel to each other]

Now PQ = SR and PQ $\parallel$ SR

Therefore, PQRS is a parallelogram.

6. The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

Ans. Suppose angles of quadrilateral ABCD are 3x, 5x, 9x, and 13x

$\angle A + \angle B + \angle C + \angle D = 360^0$ [sum of angles of a quadrilateral is 360^0]

$30x = 360^0$

$x = 12^0$

$\therefore \angle A = 3x = 3 \times 12 = 36^0$

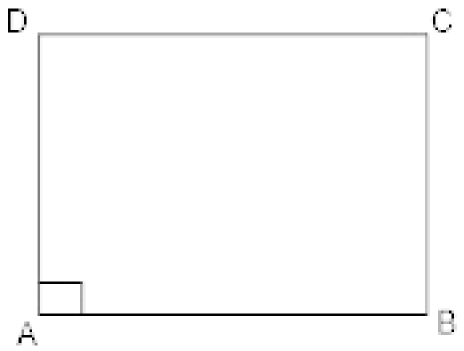
$\angle B = 5x = 5 \times 12 = 60^0$

$\angle C = 9x = 9 \times 12 = 108^0$

$\angle D = 13x = 13 \times 12 = 156^0$

7. Show that each angle of a rectangle is a right angle.

Ans. We know that rectangle is a parallelogram whose one angle is right angle.



Let ABCD be a rectangle.

$\angle A = 90^0$

To prove $\angle B = \angle C = \angle D = 90^0$

Proof: $\because AD \parallel BC$ and AB is transversal

$\therefore \angle A + \angle B = 180^0$

$90^0 + \angle B = 180^0$

$\angle B = 180^0 - 90^0 = 90^0$

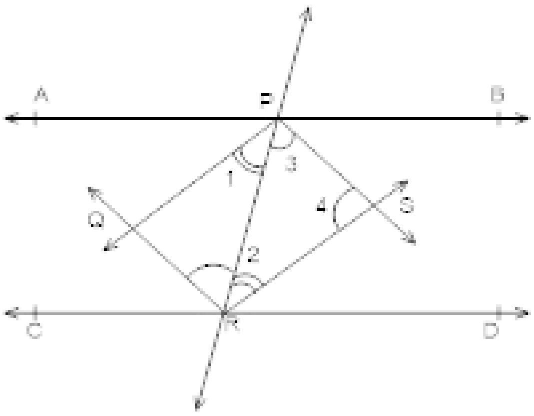
$\angle C = \angle A$

$\therefore \angle C = 90^0$

$\angle D = \angle B$

$\therefore \angle D = 90^0$

8. A transversal cuts two parallel lines prove that the bisectors of the interior angles enclose a rectangle.



Ans. $\because AB \parallel CD$ and EF cuts them at P and R.

$\therefore \angle APR = \angle PRD$ [alternate interior angles]

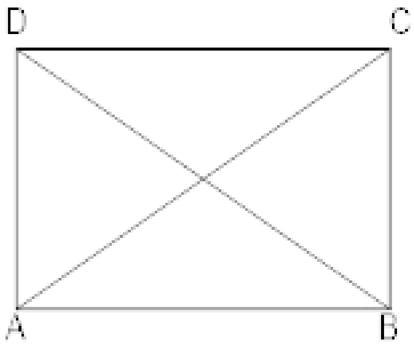
$\therefore \frac{1}{2}\angle APR = \frac{1}{2}\angle PRD$

$i.e.\ \angle 1 = \angle 2$

$\therefore PQ \parallel RS$ [alternate]

9. Prove that diagonals of a rectangle are equal in length.

Ans. ABCD is a rectangle and AC and BD are diagonals.



To prove AC = BD

Proof: $In\ \Delta\ DAB\ and\ CBA$

AD = BC [In a rectangle opposite sides are equal]

$\angle A = \angle B \qquad [90^0 each]$

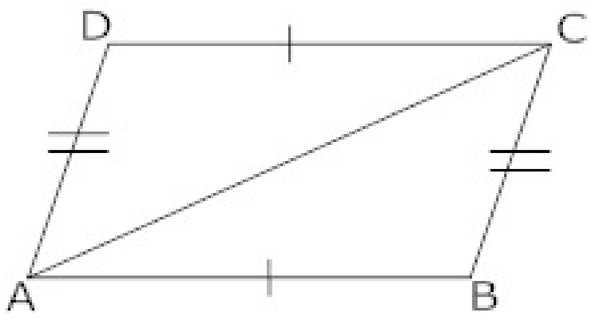
AB = AB common [common]

$\therefore \Delta DAB \cong \Delta CAB\ \ [By\ SAS]$

$\therefore AC = BD\ \ [By\ CPCT]$

10. If each pair of opposite sides of a quadrilateral is equal, then prove that it is a parallelogram.

Ans. Given A quadrilateral ABCD in which AB = DC and AD = BC



To prove: ABCD is a parallelogram

Construction: Join AC

<u>Proof:</u> In ΔABC and ΔADC

AD=BC (Given)

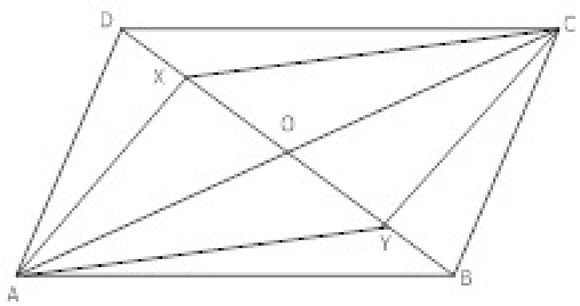
AB=DC

AC=AC [common]

$\therefore \Delta ABC \cong \Delta ADC$ [by SSS]

$\therefore \angle BAC = \angle DAC$ [By CPCT]

$\therefore ABCD$ is a parallelogram.

11. 

Ans. ABCD is a parallelogram. The diagonals of a parallelogram bisect bisect each other

$\therefore OD = OB$

But $DX = BY$ [given]

$\therefore OD - DX = OB - BY$

Or OX=OY

Now in quadrilateral AYCX, the diagonals AC and XY bisect each other

$\therefore AYCX$ is a parallelogram.

In fig ABCD is a parallelogram and x, y are the points on the diagonal BD such that Dx<By show that AYCX is a parallelogram.

12. Show that the line segments joining the mid points of opposite sides of a quadrilateral bisect each other.

Ans. Given ABCD is quadrilateral E, F, G, H are mid points of the side AB, BC, CD and DA respectively

To prove: EG and HF bisect each other.

In ΔABC, E is mid-point of AB and F is mid-point of BC

$\therefore EF \parallel AC$ And $EF = \frac{1}{2}AC ......(i)$

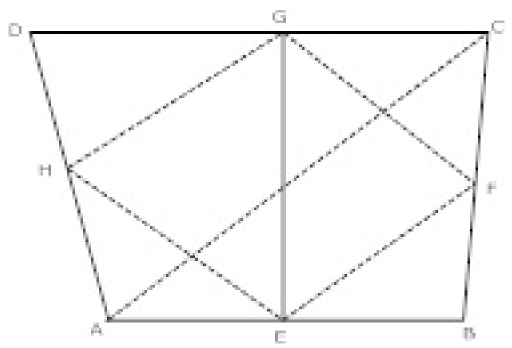
Similarly, $HG \parallel AC$ and $HG = \frac{1}{2}AC......(ii)$

From (i) and (ii), $EF \parallel HG$ and $EF = GH$

$\therefore EFGH$ is a parallelogram and EG and HF are its diagonals

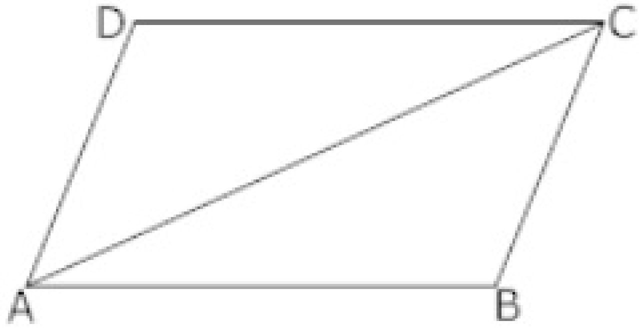
Diagonals of a parallelogram bisect each other

Thus, EG and HF bisect each other.



13. ABCD is a rhombus show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$

Ans. ABCD is a rhombus



In ΔABC and ΔADC

$AB = AD$ [Sides of a rhombus]

$BC = DC$ [Sides of a rhombus]

$AC = AC$ [Common]

$\therefore \Delta ABC \cong \Delta ADC$ [By SSS Congruency]

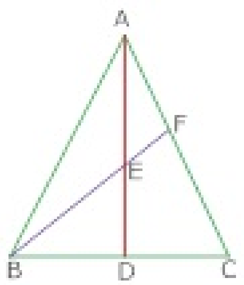
$\therefore \angle CAB = \angle CAD$ And $\angle ACB = \angle ACD$

Hence AC bisects $\angle A$ as well as $\angle C$

Similarly, by joining B to D, we can prove that $\Delta ABD \cong \Delta CBD$

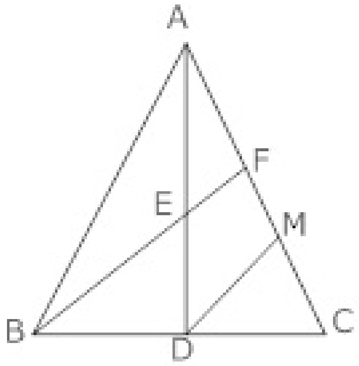
Hence BD bisects $\angle B$ as well as $\angle D$

14. In fig AD is a median of ΔABC, E is mid-Point of AD.BE produced meet AC at F. Show that $AF = \frac{1}{3}AC$



Ans. Let M is mid-Point of CF Join DM

$\therefore DM \parallel BF$.



In ΔADM, E is mid- Point of AD and

$DM \parallel EF \Rightarrow F$ is mid-point of AM

$\therefore AF = FM$

FM=MC

$\therefore AF = FM = MC$

$\therefore AC = AF + FM + MC$

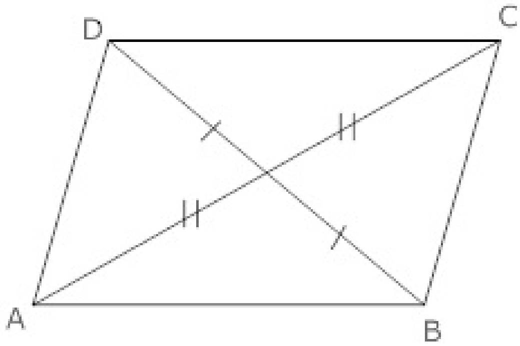
$= AF + AF + AF$

$AF = 3AF$

$\Rightarrow AF = \frac{1}{3}AC$

Hence Proved.

15. Prove that a quadrilateral is a parallelogram if the diagonals bisect each other.



Ans. ABCD is a quadrilateral in which diagonals AC and BD intersect each other at O

In ΔAOB and ΔDOC

$OA = OC$ [Given]

$OB = OD$ [Given]

And $\angle AOB = \angle COD$ [Vertically apposite angle

$\therefore \Delta AOB \cong \Delta COD$ [By SAS]

$\therefore \angle OAB = \angle OCD$ [By C.P.C.T]

But this is Pair of alternate interior angles

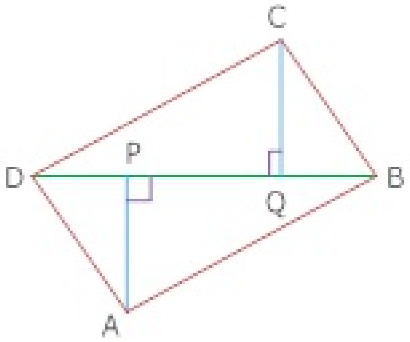
$\therefore AB \parallel CD$

$\therefore AB \parallel CD$

Similarly AD||BC

$\therefore$ Quadrilateral ABCD is a Parallelogram.

16. In fig ABCD is a Parallelogram. AP and CQ are Perpendiculars from the Vertices A and C on diagonal BD.



Show that

(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Ans. (I) in ΔAPB and ΔCQD

AB=DC [opposite sides of a Parallelogram]

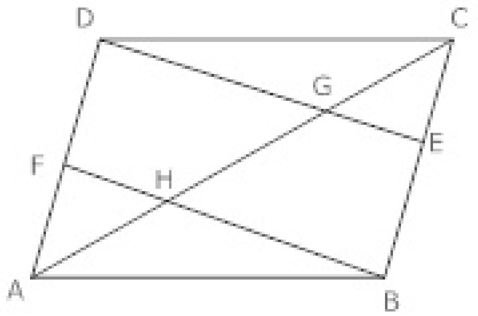
$\angle P = \angle Q$ [each 90°]

And $\angle ABP = \angle CDQ$

$\therefore \Delta APB \cong \Delta CQD$ [ASA]

(II) $\therefore AP = CQ$ (By C.P.C.T)

17. ABCD is a Parallelogram E and F are the mid-Points of BC and AD respectively. Show that the segments BF and DE trisect the diagonal AC.



Ans. FD||BE and FD=BE

$\therefore$BEDF Is a Parallelogram

EG||BH and E is the mid-Point of BC

$\therefore$G is the mid-point of HC

Or HG=GC.............(i)

Similarly AH=HG............(ii)

From (i) and (ii) we get

AH=HG=GC

Thus the segments BF and DE bisects the diagonal AC.

18. Prove that if each pair of apposite angles of a quadrilateral is equal, then it is a parallelogram.

Ans. Given: ABCD is a quadrilateral in which $\angle A = \angle C$ and $\angle B = \angle D$

To Prove: ABCD is a parallelogram

Proof: $\angle A = \angle C$ [Given]

$\angle B = \angle D$ [Given]

$\angle A + \angle B = \angle C + \angle D......(i)$

In quadrilateral. ABCD

$\angle A + \angle B + \angle C + \angle D = 360°$

$(\angle A + \angle B) + (\angle A + \angle B) = 360°$ [By....(i)]

$\angle A + \angle B = 180°$

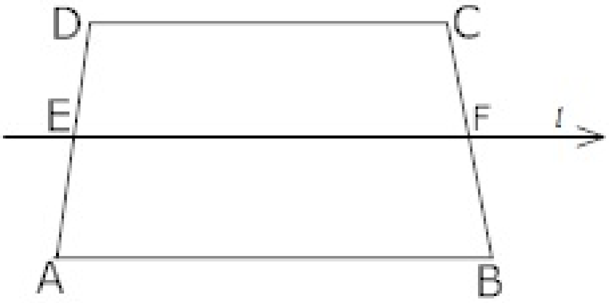
$\angle A + \angle B = \angle C + \angle D = 180°$

These are sum of interior angles on the same side of transversal

$\therefore AD \| BC$ and $AB \| DC$

$\therefore$ ABCD is a parallelogram.

19. In Fig. ABCD is a trapezium in which AB||DC E is the mid-point of AD. A line through E is parallel to AB show that l bisects the side BC



Ans. Join AC

In ΔADC

E is mid-point of AD and EO||DC

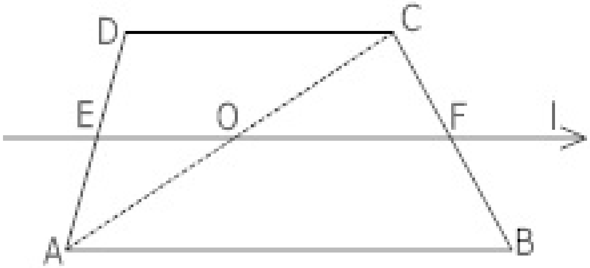
$\therefore$O is mid point of AC [A line segment joining the midpoint of one side of a Δparallel to second side and bisect the third side]

In ΔACB

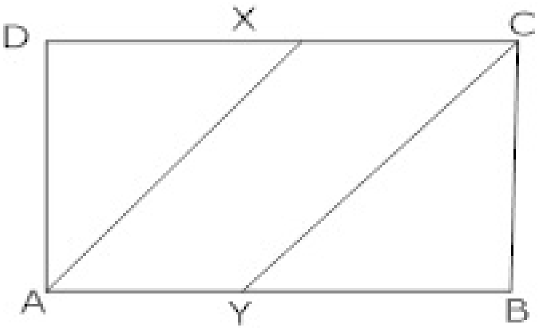
O is mid point of AC

OF||AB $\therefore$F is mid point of BC

$\therefore l$ Bisect BC



20. In Fig. ABCD is a parallelogram in which X and Y are the mid-points of the sides DC and AB respectively. Prove that AXCY is a parallelogram



Ans. In the given fig

ABCD is a parallelogram

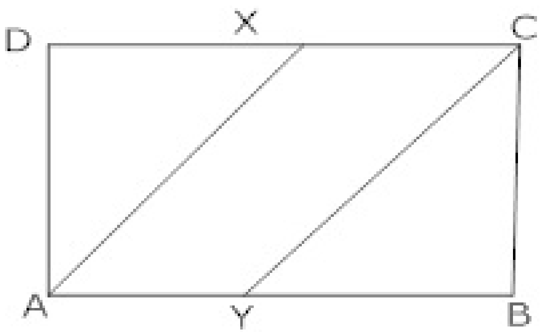
$\therefore$AB||CD and AB = CD

$\Rightarrow \frac{1}{2}AB \parallel \frac{1}{2}CD$ And $\frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow XC \parallel AY$ And $XC = AY$

[X and Y are mid-point of DC and AB respectively]

$\Rightarrow AXCY$ is a parallelogram



21. The angles of quadrilateral are in the ratio 3:5:10:12 Find all the angles of the quadrilateral.

Ans. Suppose angles of quadrilaterals are

$3x,\ 5x,\ 10x,$ and $12x$

$\therefore \angle A = 3x,\ \angle B = 5x,\ \angle C = 10x,\ \angle D = 12x$

In a quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360°$

$3x + 5x + 10x + 12x = 360°$

30x=360

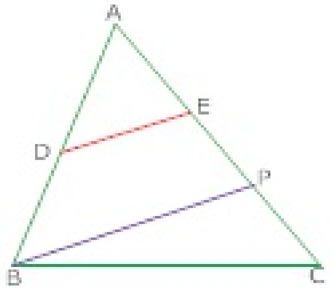
$x = \frac{360}{30} = 12$

$\angle A = 3\times 12 = 36°,\ \angle B = 5\times 12 = 60°$

$\angle C = 10\times 12 = 120°,\ \angle D = 12\times 12 = 144°$

22. In fig D is mid-points of AB. P is on AC such that $PC = \frac{1}{2}AP$ and DE||BP show that

$AE = \frac{1}{3} AC$



Ans. In ΔABP

D is mid points of AB and DE||BP

$\therefore$ E is midpoint of AP

$\therefore$ AE = EP also PC = $\frac{1}{2}$ AP

2PC = AP

2PC = 2AE

$\Rightarrow$ PC = AE

$\therefore$ AE = PE = PC

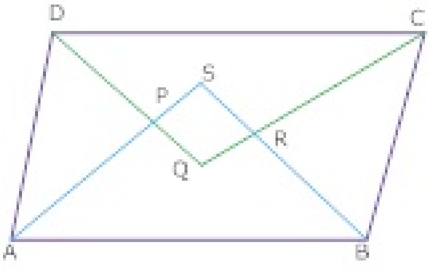
$\therefore$ AC = AE + EP + PC

AC = AE + AE + AE

$\Rightarrow$ AE = $\frac{1}{3}$ AC

Hence Proved.

23. Prove that the bisectors of the angles of a Parallelogram enclose a rectangle. It is given that adjacent sides of the parallelogram are unequal.



Ans. $\because$ ABCD is a parallelogram

$\therefore \angle A + \angle D = 180°$

or $\frac{1}{2}(\angle A+\angle D)=90^\circ$

Or $\angle APD = 90^\circ$ [Sum of angle of a $\Delta 180^\circ$]

$\therefore \angle SPQ = \angle APD = 90^\circ$

Similarly, $\angle QRS = 90^\circ$ and $\angle PQR = 90^\circ$

$\angle P + \angle Q + \angle R + \angle S = 360^\circ$

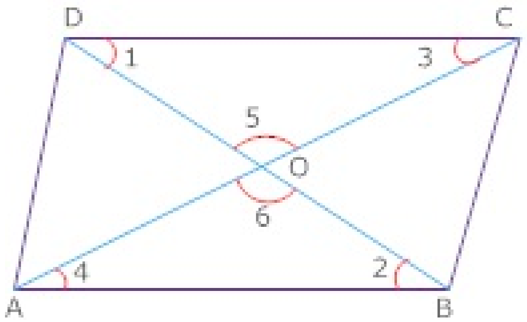
$\therefore \angle PSR = 90^\circ$. Thus each angle of quadrilateral PQRS is 90°

Hence PQRS is a rectangle.

24. Prove that a quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal

Ans. Given: ABCD is a quadrilateral in which AB||DC and BC||AD.

To Prove: ABCD is a parallelogram



Construction: Join AC and BD intersect each other at O.

Proof: $\Delta AOB \cong \Delta DOC$ [By AAA

Because $\angle 1 = \angle 2$

$\angle 3 = \angle 4$ and $\angle 5 = \angle 6$

$\therefore$AO=OC

And BO=OD

$\therefore$ ABCD is a parallelogram

$\because$Diagonals of a parallelogram bisect each other.

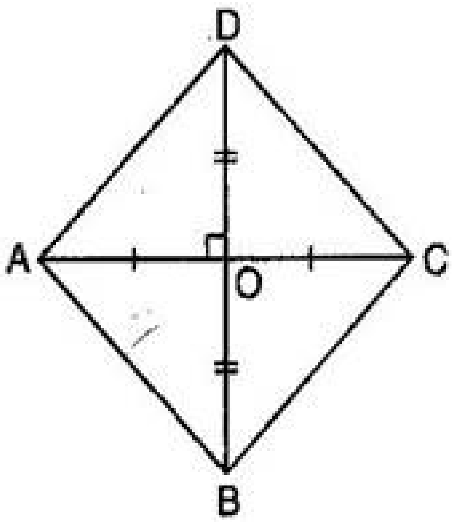
CBSE Class 9 Mathemaics
Important Questions
Chapter 8
Quadrilaterals

3 Marks Quetions

1. Show that is diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Ans. Given: Let ABCD is a quadrilateral.

Let its diagonal AC and BD bisect each other at right angle at point O.



$\therefore$ OA = OC, OB = OD

And $\angle$AOB = $\angle$BOC = $\angle$COD = $\angle$AOD = 90°

To prove: ABCD is a rhombus.

Proof: In $\triangle$AOD and $\triangle$BOC,

OA = OC[Given]

$\angle$AOD = $\angle$BOC[Given]

OB = OD[Given]

$\therefore \triangle AOD \cong \triangle COB$ [By SAS congruency]

$\Rightarrow$ AD = CB [By C.P.C.T.]..........(i)

Again, In $\triangle$ AOB and $\triangle$ COD,

OA = OC[Given]

$\angle$ AOB = $\angle$ COD[Given]

OB = OD[Given]

$\therefore \triangle AOB \cong \triangle COD$ [By SAS congruency]

$\Rightarrow$ AD = CB[By C.P.C.T.]..........(ii)

Now In $\triangle$ AOD and $\triangle$ BOC,

OA = OC[Given]

$\angle$ AOB = $\angle$ BOC[Given]

OB = OB[Common]

$\therefore \triangle AOB \cong \triangle COB$ [By SAS congruency]

$\Rightarrow$ AB = BC [By C.P.C.T.]..........(iii)

From eq. (i), (ii) and (iii),

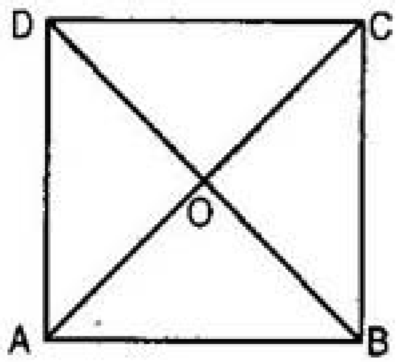
AD = BC = CD = AB

And the diagonals of quadrilateral ABCD bisect each other at right angle.

Therefore, ABCD is a rhombus.

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Ans. Given: ABCD is a square. AC and BD are its diagonals bisect each other at point O.



To prove: AC = BD and AC $\perp$ BD at point O.

Proof: In triangles ABC and BAD,

AB = AB[Common]

$\angle$ ABC = $\angle$ BAD = 90°

BC = AD [Sides of a square]

$\therefore$ Δ ABC $\cong$ Δ BAD [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.] Hence proved.

Now in triangles AOB and AOD,

AO = AO[Common]

AB = AD[Sides of a square]

OB = OD[Diagonals of a square bisect each other]

$\therefore$ Δ AOB $\cong$ Δ AOD[By SSS congruency]

$\angle$ AOB = $\angle$ AOD[By C.P.C.T.]

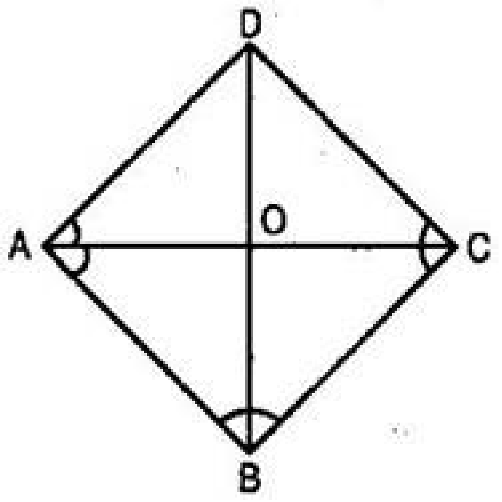
But $\angle$ AOB + $\angle$ AOD = 180° [Linear pair]

$\therefore$ $\angle$ AOB = $\angle$ AOD = 90°

$\Rightarrow$ OA $\perp$ BD or AC $\perp$ BD

Hence proved.

3. ABCD is a rhombus. Show that the diagonal AC bisects $\angle$A as well as $\angle$C and diagonal BD bisects $\angle$B as well as $\angle$D.



Ans. ABCD is a rhombus. Therefore, AB = BC = CD = AD

Let O be the point of bisection of diagonals.

$\therefore$ OA = OC and OB = OD

In $\triangle$AOB and $\triangle$AOD,

OA = OA[Common]

AB = AD[Equal sides of rhombus]

OB = OD(diagonals of rhombus bisect each other]

$\therefore$ $\triangle$AOB $\cong$ $\triangle$AOD[By SSS congruency]

$\Rightarrow$ $\angle$OAD = $\angle$OAB[By C.P.C.T.]

$\Rightarrow$ OA bisects $\angle$A..........(i)

Similarly $\triangle$BOC $\cong$ $\triangle$DOC[By SSS congruency]

$\Rightarrow$ $\angle$OCB = $\angle$OCD[By C.P.C.T.]

$\Rightarrow$ OC bisects $\angle$C..........(ii)

From eq. (i) and (ii), we can say that diagonal AC bisects $\angle$A and $\angle$C.

Now in $\triangle$AOB and $\triangle$BOC,

OB = OB[Common]

AB = BC[Equal sides of rhombus]

OA = OC(diagonals of rhombus bisect each other]

$\therefore \triangle$AOB $\cong \triangle$COB[By SSS congruency]

$\Rightarrow \angle$OBA = $\angle$OBC[By C.P.C.T.]

$\Rightarrow$ OB bisects $\angle$B..........(iii)

Similarly $\triangle$AOD $\cong \triangle$COD[By SSS congruency]

$\Rightarrow \angle$ODA = $\angle$ODC[By C.P.C.T.]

$\Rightarrow$ BD bisects $\angle$D..........(iv)

From eq. (iii) and (iv), we can say that diagonal BD bisects $\angle$B and $\angle$D

4. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (See figure). Show that:

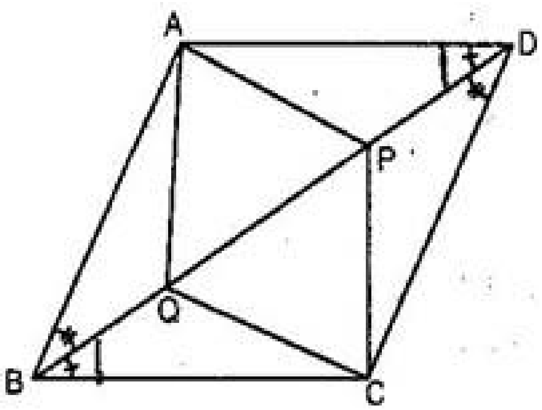
(i) $\triangle$APD $\cong \triangle$CQB

(ii) AP = CQ

(iii) $\triangle$AQB $\cong \triangle$CPD

(iv) AQ = CP

(v) APCQ is a parallelogram.



Ans. (i) In $\triangle$ APD and $\triangle$ CQB,

DP = BQ[Given]

$\angle$ ADP = $\angle$ QBC[Alternate angles (AD $\parallel$ BC and BD is transversal)]

AD = CB[Opposite sides of parallelogram]

$\therefore$ $\triangle$ APD $\cong$ $\triangle$ CQB[By SAS congruency]

(ii) Since $\triangle$ APD $\cong$ $\triangle$ CQB

$\Rightarrow$ AP = CQ[By C.P.C.T.]

(iii) In $\triangle$ AQB and $\triangle$ CPD,

BQ = DP[Given]

$\angle$ ABQ = $\angle$ PDC[Alternate angles (AB $\parallel$ CD and BD is transversal)]

AB = CD[Opposite sides of parallelogram]

$\therefore$ $\triangle$ AQB $\cong$ $\triangle$ CPD[By SAS congruency]

(iv) Since $\triangle$ AQB $\cong$ $\triangle$ CPD

$\Rightarrow$ AQ = CP[By C.P.C.T.]

(v) In quadrilateral APCQ,

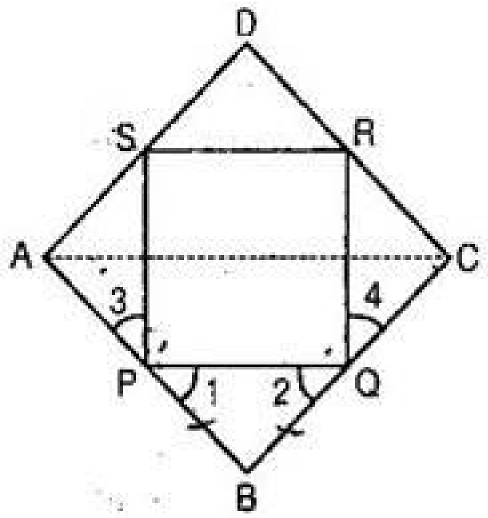
AP = CQ[proved in part (i)]

AQ = CP[proved in part (iv)]

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

5. ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.



Ans. Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In Δ ABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$ PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC ....(i)

In Δ ADC, R is the mid-point of CD and S is the mid-point of AD.

$\therefore$ SR $\parallel$ AC and SR = $\frac{1}{2}$ AC..........(ii)

From eq. (i) and (ii),PQ $\parallel$ SR and PQ = SR

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus. [Given]

$\therefore$ AB = BC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ BC $\Rightarrow$ PB = BQ

$\therefore \angle 1 = \angle 2$[Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

AP = CQ[P and Q are the mid-points of AB and BC and AB = BC]

Similarly AS = CR and PS = QR[Opposite sides of a parallelogram]

$\therefore \Delta$ APS $\cong \Delta$ CQR[By SSS congreuancy]

$\Rightarrow \angle 3 = \angle 4$[By C.P.C.T.]

Now we have $\angle 1 + \angle SPQ + \angle 3 = 180^\circ$

And $\angle 2 + \angle PQR + \angle 4 = 180^\circ$ [Linear pairs]

$\therefore \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$[Proved above]

$\therefore \angle SPQ = \angle PQR$..........(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore \angle SPQ + \angle PQR = 180^\circ$ ....(iv)[Interior angles]

Using eq. (iii) and (iv),

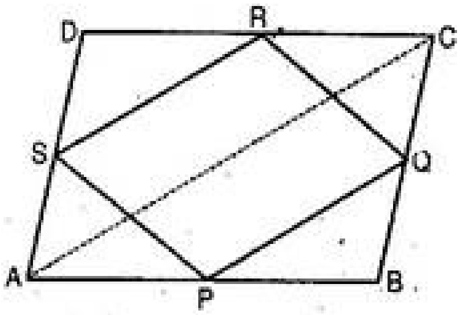
$\angle SPQ + \angle SPQ = 180^\circ \Rightarrow 2\angle SPQ = 180^\circ$

$\Rightarrow \angle SPQ = 90^\circ$

Hence PQRS is a rectangle.

6. ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans. Given: A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In $\triangle$ ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore$ PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC..........(i)

In $\triangle$ ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore$ SR $\parallel$ AC and SR = $\frac{1}{2}$ AC..........(ii)

From eq. (i) and (ii), PQ $\parallel$ SR and PQ = SR..........(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle.[Given]

$\therefore$ AD = BC

$\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC $\Rightarrow$ AS = BQ..........(iv)

In triangles APS and BPQ,

AP = BP[P is the mid-point of AB]

$\angle$PAS = $\angle$PBQ[Each 90°]

And AS = BQ[From eq. (iv)]

$\therefore \triangle$ APS $\cong \triangle$ BPQ[By SAS congruency]

$\Rightarrow$ PS = PQ[By C.P.C.T.].........(v)

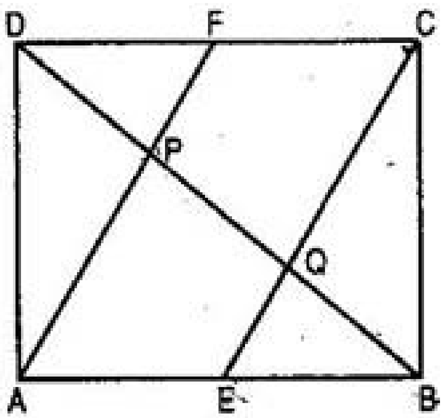
From eq. (iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

7. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Ans. Since E and F are the mid-points of AB and CD respectively.

$\therefore$AE = $\frac{1}{2}$ AB and CF = $\frac{1}{2}$ CD..........(i)

But ABCD is a parallelogram.

$\therefore$AB = CD and AB $\parallel$ DC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ CD and AB $\parallel$ DC

$\Rightarrow$ AE = FC and AE $\parallel$ FC[From eq. (i)]

$\therefore$ AECF is a parallelogram.

$\Rightarrow$ FA $\parallel$ CE $\Rightarrow$ FP $\parallel$ CQ[FP is a part of FA and CQ is a part of CE] ....(ii)

Since the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\triangle$ DCQ, F is the mid-point of CD and $\Rightarrow$ FP $\parallel$ CQ

$\therefore$ P is the mid-point of DQ.

$\Rightarrow$ DP = PQ..........(iii)

Similarly, In $\triangle$ ABP, E is the mid-point of AB and $\Rightarrow$ EQ $\parallel$ AP

$\therefore$ Q is the mid-point of BP.

$\Rightarrow$ BQ = PQ..........(iv)

From eq. (iii) and (iv),

DP = PQ = BQ.........(v)

Now BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ

$\Rightarrow$ BQ = $\frac{1}{3}$ BD..........(vi)

From eq. (v) and (vi),

DP = PQ = BQ = $\frac{1}{3}$ BD

$\Rightarrow$ Points P and Q trisects BD.

So AF and CE trisects BD.

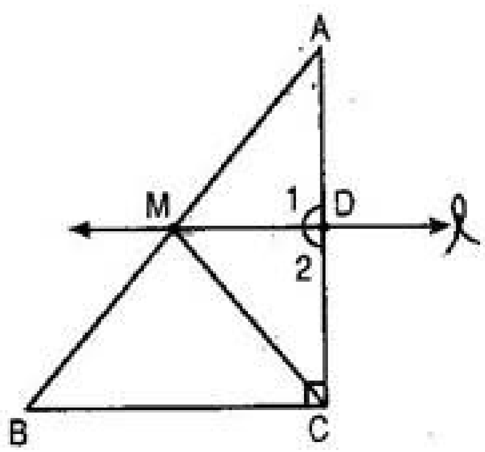
8. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

Ans. (i) In $\triangle$ABC, M is the mid-point of AB[Given]

MD $\parallel$ BC

$\therefore$ AD = DC[Converse of mid-point theorem]

Thus D is the mid-point of AC.



(ii) $l \parallel$ BC (given) consider AC as a transversal.

$\therefore \angle 1 = \angle C$[Corresponding angles]

$\Rightarrow \angle 1 = 90^\circ$ [$\angle C = 90^\circ$]

Thus MD $\perp$ AC.

(iii) In $\triangle$AMD and $\triangle$CMD,

AD = DC[proved above]

$\angle 1 = \angle 2 = 90^\circ$ [proved above]

MD = MD[common]

$\therefore \triangle AMD \cong \triangle CMD$[By SAS congruency]

$\Rightarrow$ AM = CM[By C.P.C.T.]..........(i)

Given that M is the mid-point of AB.

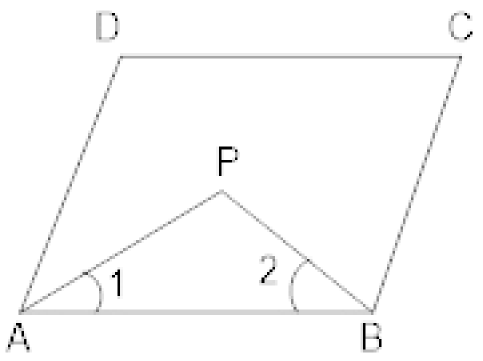
$\therefore$ AM = $\frac{1}{2}$ AB..........(ii)

From eq. (i) and (ii),

CM = AM = $\frac{1}{2}$ AB

9. In a parallelogram ABCD, bisectors of adjacent angles A and B intersect each other at P. prove that $\angle APB = 90^0$

Ans. Given ABCD is a parallelogram is and bisectors of $\angle A$ *and* $\angle B$ intersect each other at P.



To prove $\angle APB = 90^0$

Proof:

$$\angle 1 + \angle 2 = \frac{1}{2}\angle A + \frac{1}{2}\angle B$$

$$= \frac{1}{2}(\angle A + \angle B) \quad \rightarrow (i)$$

But ABCD is a parallelogram and AD || BC

$$\therefore \angle A + \angle B = 180^0$$

$\therefore \angle 1+\angle 2=\frac{1}{2}\times 180^{0}=90^{0}$

$In\ \Delta APB$

$\angle 1+\angle 2+\angle APB=180^{0}$

$90^{0}+\angle APB=180^{0}$

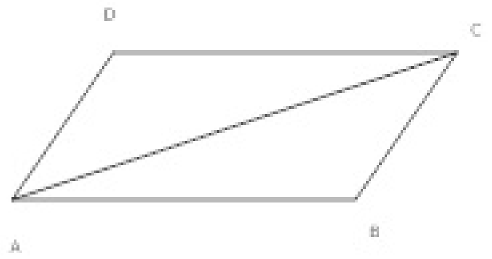
$\angle APB=90^{0}$

Hence Proved

10. In figure diagonal AC of parallelogram ABCD bisects $\angle A$ show that

(i) if bisects $\angle C$

ABCD is a rhombus



Ans.(i) AB $\|$ DC and AC is transversal

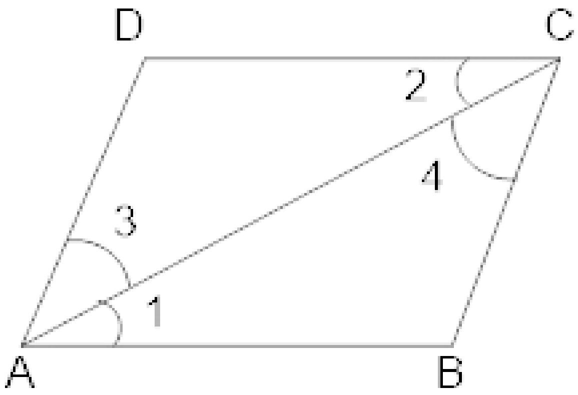
$\therefore \angle 1=\angle 2$ (Alternate angles)

And $\angle 3=\angle 4$ (Alternate angles)

But, $\angle 1=\angle 3$

$\therefore \angle 2=\angle 4$

$\therefore AC\ bi\sec sts\ \angle C$



(ii) $In\ \Delta ABC\ and\ \Delta ADC$

AC=AC [common]

$\angle 1 = \angle 3$ [given]

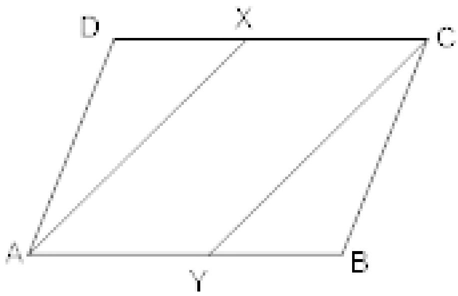
$\angle 2 = \angle 4$ [proved]

$\therefore \Delta ABC \cong \Delta ADC$

$\therefore AB = AD$ [By CPCT]

$\therefore ABCD\ is\ a$ rhombus

11. In figure ABCD is a parallelogram. AX and CY bisects angles A and C. prove that AYCX is a parallelogram.



Ans. Given in a parallelogram AX and CY bisects $\angle A\ and \angle C$ respectively and we have to show that AYCX in a parallelogram.

$In\ \Delta ADX\ and\ \Delta CBY$

$\angle D = \angle B$ ...(i) [opposite angles of parallelogram]

$\angle DAX = \frac{1}{2}\angle A$ [Given] ...(ii)

And $\angle BCY = \frac{1}{2}\angle C$ [give] .....(iii)

But $\angle A = \angle C$

$\therefore$ By (2) and (3), we get

$\angle DAX = \angle BCY \rightarrow (iv)$

Also, $AD = BC$ [opposite sides of parallelogram] ....(v)

$\therefore$ From (i), (iv) and (v), we get

$\Delta ADX \cong \Delta CBY \quad [By\ ASA]$

$\therefore DX = BY \quad [CPCT]$

But, AB =CD [opposite sides of parallelogram]

AB-BY=CD-DX

Or

Ay=CX

But $AY \parallel XC \ [\because ABCD\ is\ a \parallel gm]$

$\therefore AYCX$ is a parallelogram

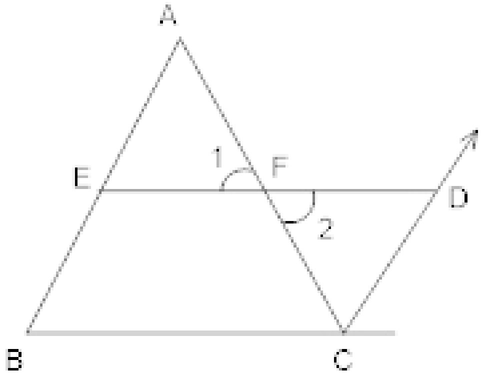
12. Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Ans. Given ΔABC in which E and F are mid points of side AB and AC respectively.

To prove: EF||BC

Construction: Produce EF to D such that EF = FD. Join CD

Proof: $In\ \Delta AEF\ and\ \Delta CDF$



AF=FC$[\because F\ is\ mid-po\text{int}\ of\ AC]$

$\angle 1 = \angle 2$ [vertically opposite angles]

EF=FD [By construction]

$\therefore \Delta AEF \cong \Delta CDF \quad [By\ SAS]$

And $\therefore AE = CD \quad [By\ CPCT]$

AE= BE[$\because E$ is the mid-point]

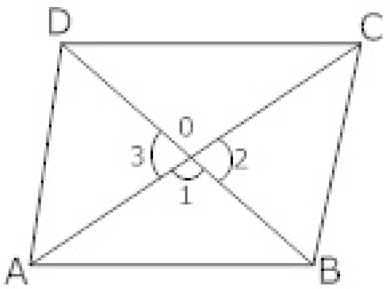
And $\therefore BE = cd$

$AB \parallel CD\ [\therefore \angle BAC = \angle ACD]$

$\therefore BCDE\ is\ a\ paralle\log ram$

$EF \parallel BC \quad Henceproved$

13. Prove that a quadrilateral is a rhombus if its diagonals bisect each other at right angles.



Ans. Given ABCD is a quadrilateral diagonals AC and BD bisect each other at O at right angles

To Prove: ABCD is a rhombus

Proof: $\because$diagonals AC and BD bisect each other at O

$\therefore OA = OC, OB = OD$ And $\angle 1 = \angle 2 = \angle 3 = 90°$

Now In ΔBOA And ΔBOC

$OA = OC$ Given

$OB = OB$ [Common]

And $\angle 1 = \angle 2 = 90°$ (Given)

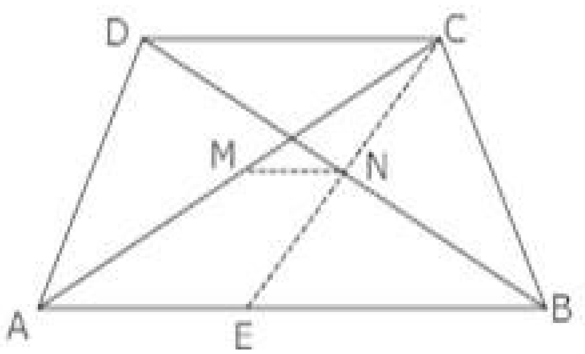
$\therefore \Delta BOA = \Delta BOC$ (SAS)

$\therefore BA = BC$ (C.P.C.T.)

Similarly, BC=CD, CD=DA and DA=AB,

Hence, ABCD is a rhombus.

14. Prove that the straight line joining the mid points of the diagonals of a trapezium is parallel to the parallel sides.



Ans. Given a trapezium ABCD in which $AB \parallel DC$ and M,N are the mid Points of the diagonals AC and BD.

We need to prove that $MN \parallel AB \parallel DC$

Join CN and let it meet AB at E

Now in ΔCDN and ΔEBN

$\angle DCN = \angle BEN$ [Alternate angles]

$\angle CDN = \angle BEN$ [Alternate angles]

And $DN = BN$ [given]

$\therefore \Delta CDN \cong \Delta EBN$ [ASA]

$\therefore CN = EN$ [By C.P.C.T]

Now in $\Delta ACE, M$ and N are the mid points of the sides AC and CE respectively.

$\therefore MN \parallel AE$ Or $MN \parallel AB$

Also $AB \parallel DC$

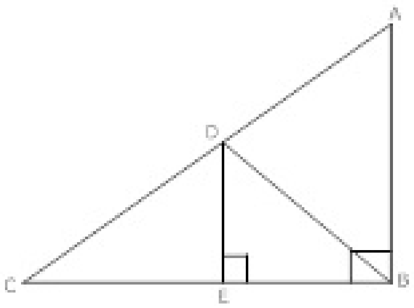
$\therefore MN \parallel AB \parallel DC$

15. In fig $\angle B$ is a right angle in $\Delta ABC.D$ is the mid-point of $AC.DE \parallel AB$ intersects BC at E. show that

(i) E is the mid-point of BC

(ii) DE $\perp$ BC

(ii) BD = AD



Ans. <u>Proof:</u> $\because DE \parallel AB$ and D is mid points of AC

In ΔDCE and ΔDBE

CE=BE

DE= DE

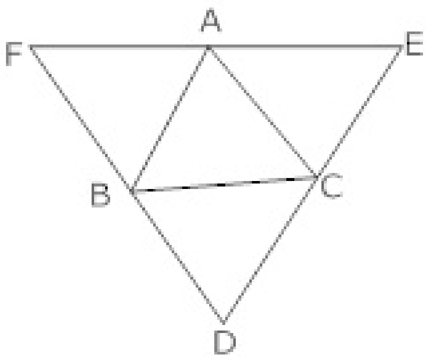
And $\angle DEC = \angle DEB = 90°$

$\therefore \Delta DCE = \Delta DBE$

$\therefore \Delta DCE \cong \Delta DBE$

$\therefore CD = BD$

16. ABC is a triangle and through vertices A, B and C lines are drawn parallel to BC, AC and AB respectively intersecting at D, E and F. prove that perimeter of ΔDEF is double the perimeter of ΔABC.



Ans. $\because BCAF$ Is a parallelogram

$\therefore BC = AF$

$\because ABCE$ Is a parallelogram

$\therefore BC = AE$

$AF + AE = 2BC$

Or $EF = 2BC$

Similarly, ED = 2AB and FD = 2AC

$\therefore$Perimeter of $\Delta ABC = AB + BC + AC$

Perimeter of $\Delta DEF = DE + EF + DF$

= 2AB+2BC+2AC

= 2[AB+BC+AC]

= 2 Perimeter of ΔABC

Hence Proved.

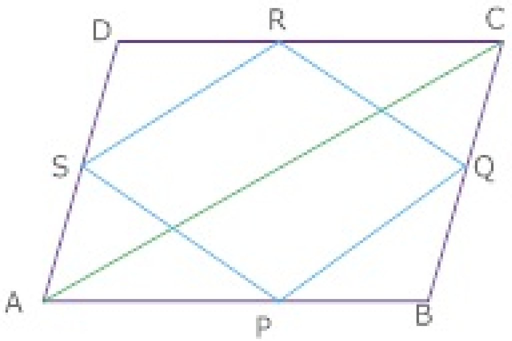
17. In fig ABCD is a quadrilateral P, Q, R and S are the mid Points of the sides AB, BC, CD and DA, AC is diagonal. Show that

(i) SR||AC

(ii) PQ=SR

(iii) PQRS is a parallelogram

(iv) PR and SQ bisect each other



Ans. In ΔABC, P and Q are the mid-points of the sides AB and BC respectively

(i) $\therefore$ PQ||AC and PQ= $\frac{1}{2}$ AC

(ii) Similarly SR||AC and SR= $\frac{1}{2}$ AC

$\therefore$ PQ||SR and PQ=SR

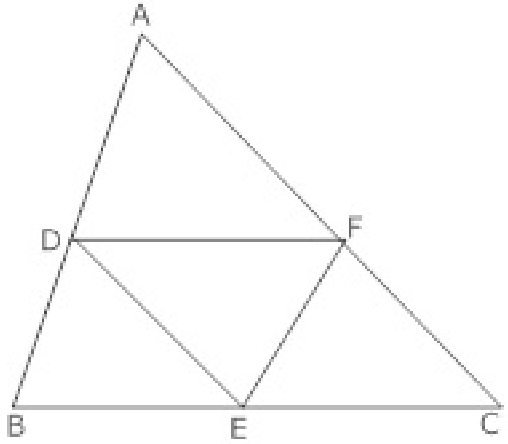
(iii) Hence PQRS is a Parallelogram.

(iv) PR and SQ bisect each other.

18. In $\Delta ABC, D, E, F$ are respectively the mid-Points of sides AB,DC and CA. show that

ΔABC is divided into four congruent triangles by Joining D,E,F.

Ans. D and E are mid-Points of sides AB and BC of ΔABC



$\therefore$DE||AC {$\because$A line segment joining the mid-Point of any two sides of a triangle parallel to third side}

Similarly, DF||BC and EF||AB

$\therefore$ADEF, BDEF and DFCE are all Parallelograms.

DE is diagonal of Parallelogram BDFE

$\therefore \Delta BDE \cong \Delta FED$

Similarly, ΔDAF$\cong\Delta$FED

And ΔEFC$\cong\Delta$FED

So all triangles are congruent

19. ABCD is a Parallelogram is which P and Q are mid-points of opposite sides AB and CD. If AQ intersect DP at S BQ intersects CP at R, show that

(i) APCQ is a Parallelogram

(ii) DPBQ is a parallelogram

(iv) PSQR is a parallelogram

Ans. **(i)** In quadrilateral APCQ

AP||QC [∵AB||CD]........(i)

AP=$\frac{1}{2}$ AB, CQ=$\frac{1}{2}$ CD (Given)

Also AB= CD

So AP=QC..........(ii)

Therefore, APCQ is a parallelogram

[It any two sides of a quadrilateral equal and parallel then quad is a parallelogram]

(ii) Similarly, quadrilateral DPBQ is a Parallelogram because DQ||PB and DQ=PB

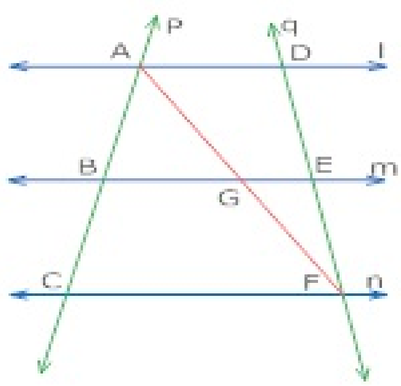
(iii) In quadrilateral PSQR,

SP||QR [SP is a part of DP and QR is a Part of QB]

Similarly, SQ||PR

So. PSQR is also parallelogram.

20. l, m, n **are three parallel lines intersected by transversals P and q such that** l, m **and** n **cut off equal intercepts AB and BC on P In fig Show that** l, m, n **cut off equal intercepts DE and EF on q also.**



Ans. In fig l, m, n are 3 parallel lines intersected by two transversal P and Q.

To Prove DE=EF

Proof: In ΔACF

B is mid-point of AC

And BG||CF

$\therefore$ G is mid-point of AF [By mid-point theorem]

Now In ΔAFD

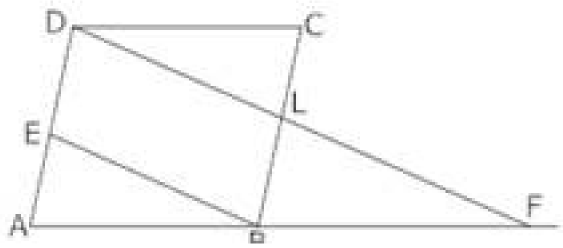
G is mid-point of AF and GE || AD

$\therefore$ E is mid-point of FD [By mid-point theorem]

$\therefore$ DE=EF

Hence Proved.

21. ABCD is a parallelogram in which E is mid-point of AD. DF||EB meeting AB produced at F and BC at L prove that DF = 2DL



Ans. In ΔAFD

$\because E$ is mid-point of AD (Given)

BE||DF (Given)

$\therefore$ By converse of mid-point theorem B is mid-point of AF

$\therefore AB = BF \ldots.(i)$

ABCD is parallelogram

$\therefore AB = CD......(ii)$

From (i) and (ii)

CD = BF

Consider ΔDLC and ΔFLB

DC = FB [Proved above]

$\angle DCL = \angle FBL$ [Alternate angles]

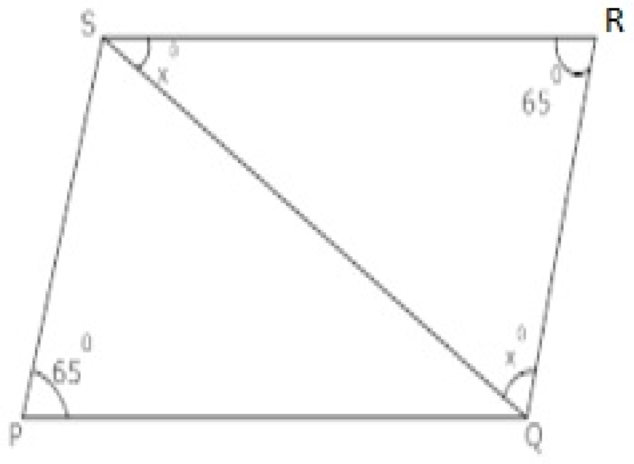
$\angle DLC = \angle FLB$ [Vertically opposite angles]

$\Delta DLC = \Delta FLB$ [ASA]

$\therefore$ DL=LF

$\therefore$ DF=2DL

22. PQRS is a rhombus if $\angle P = 65^\circ$ find $\angle RSQ$



Ans. $\angle R = \angle P = 65^\circ$ [opposite angles of a parallelogram are equal]

Let $\angle RSQ = x^\circ$

In ΔRSQ we have RS=RQ

$\angle RQS = \angle RSQ = x^\circ$ [opposite Sides of equal angles are equal]

In ΔRSQ

$\angle S+\angle Q+\angle R=180°$ [By angle sum property]

$x°+x°+65°=180°$

$2x°=180°-65°$

$2x°=115°$

$x=\frac{115}{2}=57.5°$

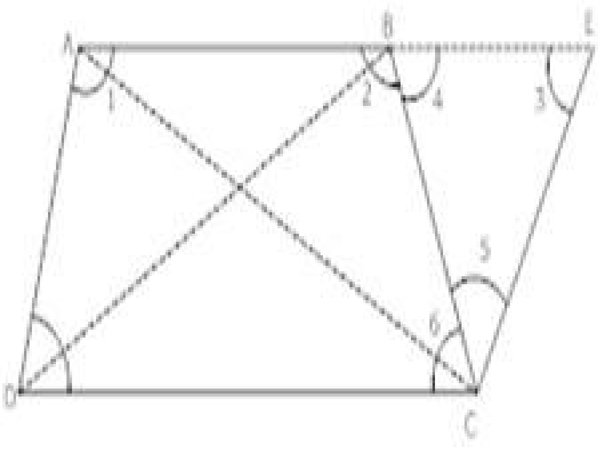
$\therefore \angle RSQ=57.5°$

23. ABCD is a trapezium in which AB||CD and AD = BC show that

(i) $\angle A=\angle B$

(ii) $\angle C=\angle D$

(iii) $\Delta ABC\cong\Delta BAD$



Ans. Produce AB and Draw a line Parallel to DA meeting at E

∵ AD||EC

$\angle 1+\angle 3=180°$ ....(i) [Sum of interior angles on the some side of transversal is $180°$]

In ΔBEC

BC=CE (given)

$\therefore \angle 3 = \angle 4$ .....(2) [in a Δ equal side to opposite angles are equal]

$\angle 2 + \angle 4 = 180°$ ......(3)

By (i) and (3)

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle 3 = \angle 4$

$\therefore \angle 1 = \angle 2$

(i) $\therefore \angle A = \angle B$

(ii) $\because AD \parallel EC$

$\angle D + \angle 6 + \angle 5 = 180°......(i)$

$AE \parallel DC$

$\angle 6 + \angle 5 + \angle 3 = 180°........(ii)$

$\angle D + \angle 6 + \angle 5 = \angle 6 + \angle 5 + \angle 3$

$\angle D = \angle 3 = \angle 4$

(iii) In ΔABC and ΔBAD

AB=AB [common]

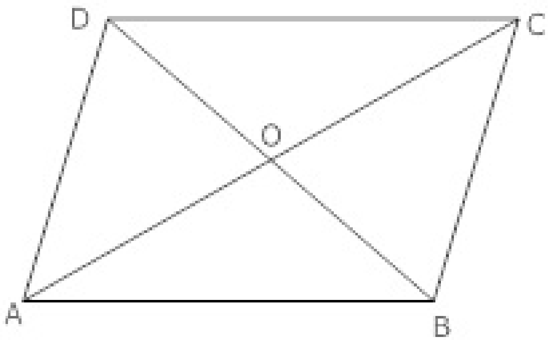
$\angle 1 = \angle 2$ [Proved above]

AD=BC [given]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS]

24. Show that diagonals of a rhombus are perpendicular to each other.

Ans. Given: A rhombus ABCD whose diagonals AC and BD intersect at a Point O



To Prove: $\angle BOC = \angle DOC = \angle AOD = \angle AOB = 90^\circ$

Proof: clearly ABCD is a Parallelogram in which

AB=BC=CD=DA

We know that diagonals of a Parallelogram bisect each other

$\therefore$ OA=OC and OB=OD

Now in ΔBOC and ΔDOC, we have

OB=OD

BC=DC

OC=OC

$\therefore \Delta$BOC $\cong \Delta$DOC [By SSS]

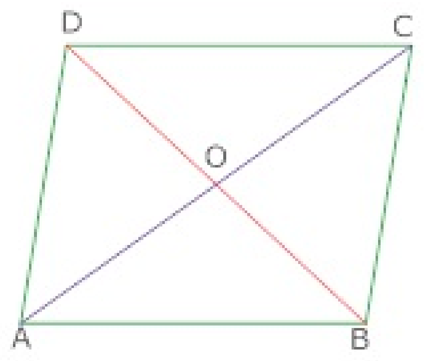
$\therefore \angle BOC = \angle DOC$ [By C.P.C.T]

But $\angle BOC + \angle DOC = 180^\circ \therefore \angle BOC = \angle DOC = 90^\circ$

Similarly, $\angle AOB = \angle AOD = 90^\circ$

Hence diagonals of a rhombus bisect each other at 90°

25. Prove that the diagonals of a rhombus bisect each other at right angles



Ans. We are given a rhombus ABCD whose diagonals AC and BD intersect each other at O.

We need to prove that OA=OC, OB=OD and $\angle AOB = 90^\circ$

In ΔAOB and ΔCOD

AB=CD [Sides of rhombus]

$\angle AOB = \angle COD$ [vertically opposite angles]

And $\angle ABO = \angle CDO$ [Alternate angles]

$\therefore \Delta$ AOB $\cong \Delta$ COD [By ASA]

$\therefore$ OA=OC

And OB=OD [By C.P.C.T]

Also in Δ AOB and Δ COB

OA=OC [Proved]

AB=CB [sides of rhombus]

And OB=OB [Common]

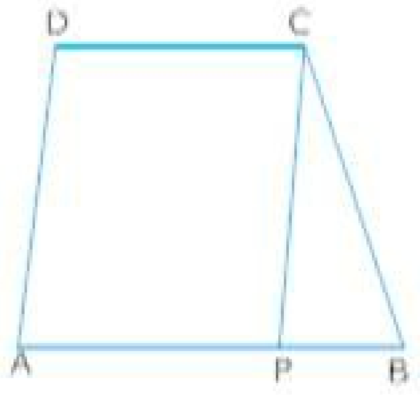
$\therefore \Delta$ AOB $\cong \Delta$ COB [By SSS]

$\therefore \angle AOB = \angle COB$ [By C.P.C.T]

But $\angle AOB + \angle COB = 180^\circ$ [linear pair]

$\therefore \angle AOB = \angle COB = 90^\circ$

26. In fig ABCD is a trapezium in which AB||DC and AD=BC. Show that $\angle A = \angle B$



Ans. To show that $\angle A = \angle B$.

Draw CP||DA meeting AB at P

$\because$ AP||DC and CP||DA

$\therefore$ APCD is a parallelogram

Again in Δ CPB

CP=CB [$\because$ BC=AD [Given]

$\angle CPB = \angle CBP ...(i)$ [Angles opposite to equal sides]

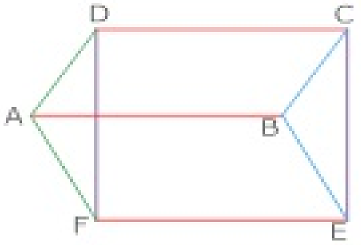
But $\angle CPA + \angle CPB = 180°$ [By linear pair]

Also $\angle A + \angle CPA = 180°$ [$\because$ APCD is a parallelogram]

$\therefore$ $\angle A + \angle CPA = \angle CPA + \angle CPB$ Or $\angle A = \angle CPB$

= $\angle$ CB

27. In fig ABCD and ABEF are Parallelogram, prove that CDFE is also a parallelogram.



Ans. ∵ABCD is a parallelogram

∴ AB=DC also AB||DC…………..(i)

Also ABEF is a parallelogram

∴AB=FE and AB||FE……….(ii)

By (i) and (ii)

AB=DC=FE

∴AB=FE

And AB||DC||FE

∴ AB||FE

∴CDEF is a parallelogram.

Hence Proved.

CBSE Class 9 Mathemaics

Important Questions

Chapter 8

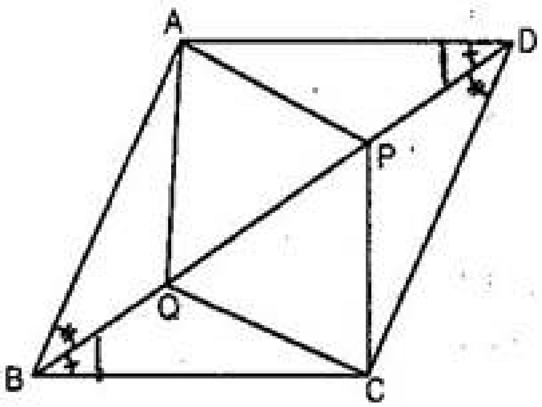
Quadrilaterals

4 Marks Quetions

1. ABCD is a rectangle in which diagonal AC bisects $\angle$A as well as $\angle$C. Show that:

(i) ABCD is a square.

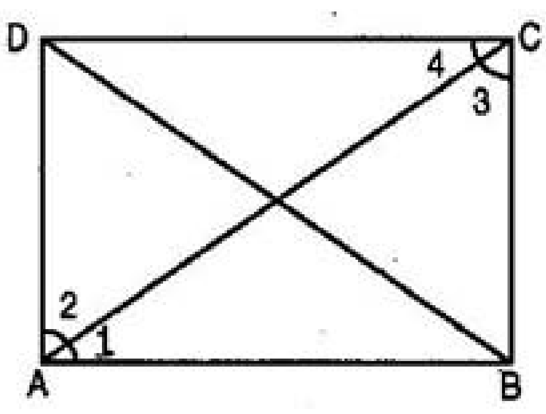
(ii) Diagonal BD bisects both $\angle$B as well as $\angle$D.



Ans. ABCD is a rectangle. Therefore AB = DC ....(i)

And BC = AD

Also $\angle$A = $\angle$B = $\angle$C = $\angle$D = 90°



(i) In $\triangle$ABC and $\triangle$ADC

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

[AC bisects $\angle$A and $\angle$C (given)]

AC = AC [Common]

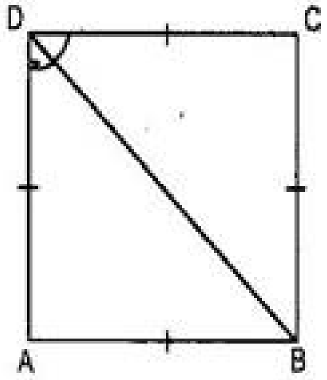
$\therefore \triangle ABC \cong \triangle ADC$ [By ASA congruency]

$\Rightarrow$ AB = AD ....(ii)

From eq. (i) and (ii), AB = BC = CD = AD

Hence ABCD is a square.

(ii) In $\triangle$ABC and $\triangle$ADC



AB = BA [Since ABCD is a square]

AD = DC [Since ABCD is a square]

BD = BD [Common]

$\therefore \triangle ABD \cong \triangle CBD$ [By SSS congruency]

$\Rightarrow \angle ABD = \angle CBD$ [By C.P.C.T.] ....(iii)

And $\angle ADB = \angle CDB$ [By C.P.C.T.] ....(iv)

From eq. (iii) and (iv), it is clear that diagonal BD bisects both $\angle$B and $\angle$D.

2. An $\triangle$ABC and $\triangle$DEF, AB = DE, AB $\parallel$ DE, BC = EF and BC $\parallel$ EF. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:

(i) Quadrilateral ABED is a parallelogram.

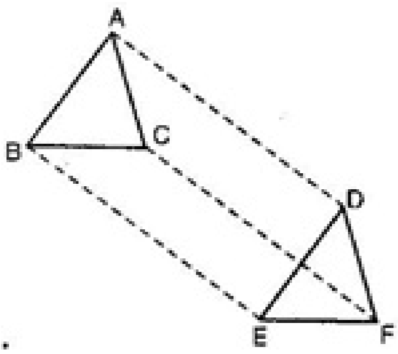
(ii) Quadrilateral BEFC is a parallelogram.

(iii) AD $\parallel$ CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram.

(v) AC = DF

(vi) $\triangle$ ABC $\cong$ $\triangle$ DEF



Ans. (i) In $\triangle$ ABC and $\triangle$ DEF

AB = DE [Given]

And AB $\parallel$ DE [Given]

$\therefore$ ABED is a parallelogram.

(ii) In $\triangle$ ABC and $\triangle$ DEF

BC = EF [Given]

And BC $\parallel$ EF [Given]

$\therefore$ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

$\therefore$ AD $\parallel$ BE and AD = BE ……….(i)

Also BEFC is a parallelogram.

$\therefore$ CF $\parallel$ BE and CF = BE ....(ii)

From (i) and (ii), we get

$\therefore$ AD $\parallel$ CF and AD = CF

(iv) As AD $\parallel$ CF and AD = CF

$\Rightarrow$ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

$\therefore$ AC = DF

(vi) In $\triangle$ ABC and $\triangle$ DEF,

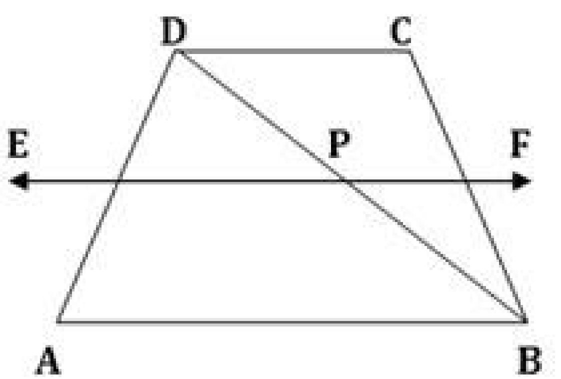
AB = DE [Given]

BC = EF [Given]

AC = DF [Proved]

$\therefore$ $\triangle$ ABC $\cong$ $\triangle$ DEF [By SSS congruency]

3. ABCD is a trapezium, in which AB $\parallel$ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Ans. Let diagonal BD intersect line EF at point P.

In $\triangle$ DAB,

E is the mid-point of AD and EP $\parallel$ AB [$\because$ EF $\parallel$ AB (given) P is the part of EF]

$\therefore$ P is the mid-point of other side, BD of $\triangle$ DAB.

[A line drawn through the mid-point of one side of a triangle, parallel to another side intersects the third side at the mid-point]

Now in $\triangle$ BCD,

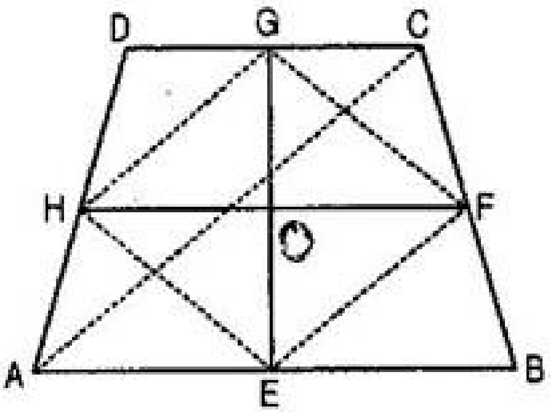
P is the mid-point of BD and PF $\parallel$ DC [$\because$ EF $\parallel$ AB (given) and AB $\parallel$ DC (given)]

$\therefore$ EF $\parallel$ DC and PF is a part of EF.

$\therefore$ F is the mid-point of other side, BC of $\triangle$ BCD. [Converse of mid-point of theorem]

4. Show that the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Ans. Given: A quadrilateral ABCD in which EG and FH are the line-segments joining the mid-points of opposite sides of a quadrilateral.



To prove: EG and FH bisect each other.

Construction: Join AC, EF, FG, GH and HE.

Proof: In $\triangle$ ABC, E and F are the mid-points of respective sides AB and BC.

$\therefore$ EF $\parallel$ AC and EF $\frac{1}{2}$ AC ....(i)

Similarly, in ΔADC,

G and H are the mid-points of respective sides CD and AD.

$\therefore$ HG $\|$ AC and HG $\frac{1}{2}$ AC ……….(ii)

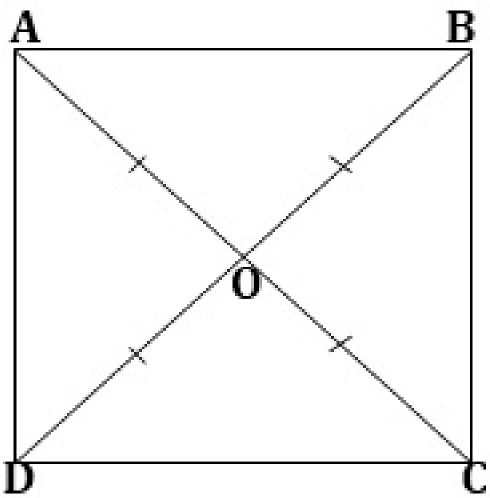
From eq. (i) and (ii),

EF $\|$ HG and EF = HG

$\therefore$ EFGH is a parallelogram.

Since the diagonals of a parallelogram bisect each other, therefore line segments (i.e. diagonals) EG and FH (of parallelogram EFGH) bisect each other.

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Ans. Let ABCD be a quadrilateral in which equal diagonals AC and BD bisect each other at right angle at point O.

We have AC = BD and OA = OC ……….(i)

And OB = OD ……….(ii)

Now OA + OC = OB + OD

$\Rightarrow$ OC + OC = OB + OB [Using (i) & (ii)]

$\Rightarrow$ 2OC = 2OB

$\Rightarrow$ OC = OB ……….(iii)

From eq. (i), (ii) and (iii), we get, OA = OB = OC = OD ……….(iv)

Now in $\triangle$AOB and $\triangle$COD,

OA = OD [proved]

$\angle$AOB = $\angle$COD [vertically opposite angles]

OB = OC [proved]

$\therefore$ $\triangle$AOB $\cong$ $\triangle$DOC [By SAS congruency]

$\Rightarrow$ AB = DC [By C.P.C.T.] ……….(v)

Similarly, $\triangle$BOC $\cong$ $\triangle$AOD [By SAS congruency]

$\Rightarrow$ BC = AD [By C.P.C.T.] ……….(vi)

From eq. (v) and (vi), it is concluded that ABCD is a parallelogram because opposite sides of a quadrilateral are equal.

Now in $\triangle$ABC and $\triangle$BAD,

AB = BA [Common]

BC = AD [proved above]

AC = BD [Given]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$BAD [By SSS congruency]

$\Rightarrow$ $\angle$ABC = $\angle$BAD [By C.P.C.T.] ……….(vii)

But $\angle$ABC + $\angle$BAD = 180° [ABCD is a parallelogram] ……….(viii)

$\therefore$ AD $\parallel$ BC and AB is a transversal.

$\Rightarrow$ $\angle$ABC + $\angle$ABC = 180° [Using eq. (vii) and (viii)]

$\Rightarrow$ 2$\angle$ABC = 180° $\Rightarrow$ $\angle$ABC = 90°

$\therefore$ $\angle$ABC = $\angle$BAD = 90° ....(ix)

Opposite angles of a parallelogram are equal.

But $\angle$ABC = $\angle$BAD =

$\therefore$ $\angle$ABC = $\angle$ADC = 90° ....(x)

$\therefore$ $\angle$BAD = $\angle$BDC = 90° ....(xi)

From eq. (x) and (xi), we get

$\angle$ABC = $\angle$ADC = $\angle$BAD = $\angle$BDC = 90° ....(xii)

Now in ΔAOB and ΔBOC,

OA = OC [Given]

$\angle$AOB = $\angle$BOC = 90° [Given]

OB = OB [Common]

$\therefore$ ΔAOB $\cong$ ΔCOB [By SAS congruency]

$\Rightarrow$ AB = BC ....(xiii)

From eq. (v), (vi) and (xiii), we get,

AB = BC = CD = AD ....(xiv)

Now, from eq. (xii) and (xiv), we have a quadrilateral whose equal diagonals bisect each other at right angle.

Also sides are equal make an angle of 90° with each other.

$\therefore$ ABCD is a square.

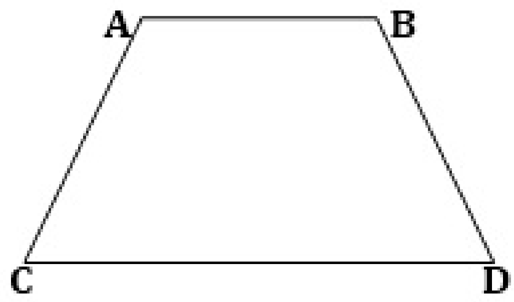
6. ABCD is a trapezium in which AB || CD and AD = BC (See figure). Show that:

(i) $\angle$A = $\angle$B

(ii) $\angle$C = $\angle$D

(iii) $\triangle$ABC $\cong$ $\triangle$BAD

(iv) Diagonal AC = Diagonal BD



Ans. Given: ABCD is a trapezium.

AB || CD and AD = BC

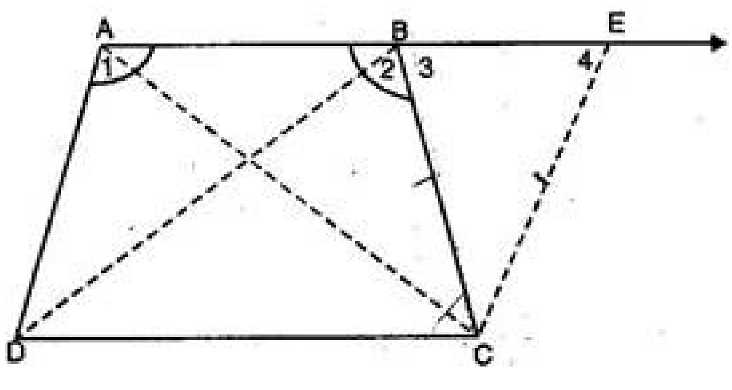
To prove: **(i)** $\angle$A = $\angle$B

(ii) $\angle$C = $\angle$D

(iii) $\triangle$ABC $\cong$ $\triangle$BAD

(iv) Diag. AC = Diag. BD

Construction: Draw CE || AD and extend AB to intersect CE at E.



Proof: (i) As AECD is a parallelogram. [By construction]

$\therefore$ AD = EC

But AD = BC [Given]

$\therefore$ BC = EC

$\Rightarrow \angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now $\angle 1 + \angle 4 = 180^\circ$ [Interior angles]

And $\angle 2 + \angle 3 = 180^\circ$ [Linear pair]

$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$ [$\because \angle 3 = \angle 4$]

$\Rightarrow \angle A = \angle B$

(ii) $\angle 3 = \angle C$ [Alternate interior angles]

And $\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [$\triangle$BCE is an isosceles triangle]

$\therefore \angle C = \angle D$

(iii) In $\triangle$ABC and $\triangle$BAD,

AB = AB [Common]

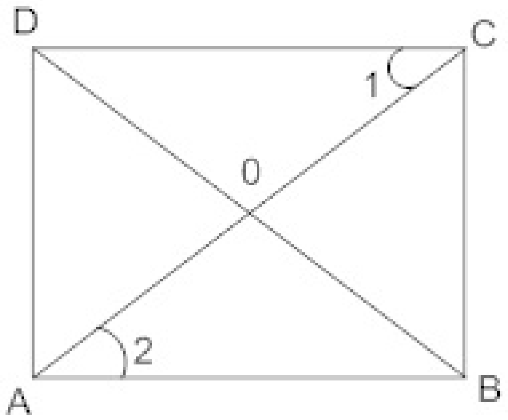
$\angle 1 = \angle 2$ [Proved]

AD = BC [Given]

$\therefore \triangle ABC \cong \triangle BAD$ [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.]

7. Prove that if the diagonals of a quadrilateral are equal and bisect each other at right angles then it is a square.



Ans. Given in a quadrilateral ABCD, AC = BD, AO = OC and BO = OD and $\angle AOB = 90^0$

To prove: ABCD is a square.

Proof: $In\ \Delta AOB\ and\ \Delta COD$

OA=OC

OB=OD [given]

And

$\angle AOB = \angle COD$ [vertically opposite angles]

$\therefore \Delta AOB \cong \Delta COD\ \ [By\ SAS]$

$\therefore AB = CD\ \ [By\ CPCT]$

$\angle 1 = \angle 2\ \ [By\ CPCT]$ But these are alternate angles $\therefore\ AB \parallel CD$

ABCD is a parallelogram whose diagonals bisects each other at right angles

$\therefore ABCD\ is\ a$ rhombus

Again in $\Delta ABD\ and\ \Delta BCA$

AB=BC [Sides of a rhombus]

AD=AB [Sides of a rhombus]

And BD=CA [Given]

$\therefore \Delta ABD \cong \Delta BCA$

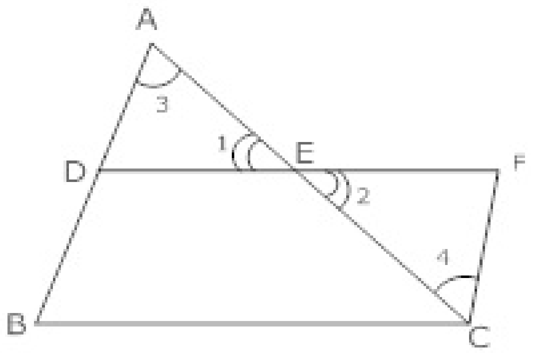
$\therefore \angle BAD = \angle CBA$ [By CPCT]

These are alternate angles of these same side of transversal

$\therefore \angle BAD + \angle CBA = 180^0$ or $\angle BAD = \angle CBA = 90^0$

Hence ABCD is a square.

8. Prove that in a triangle, the line segment joining the mid points of any two sides is parallel to the third side.



Ans. Given: A ΔABC in which D and E are mid-points of the side AB and AC respectively

To Prove: $DE \parallel BC$

Construction: Draw $CF \parallel BA$

Proof: In ΔADE and ΔCFE

$\angle 1 = \angle 2$ [Vertically opposite angles]

AE=CE [Given]

And $\angle 3 = \angle 4$ [Alternate interior angles]

$\therefore \Delta ADE \cong \Delta CFE$ [By ASA]

$\therefore$ DE=FE [By C.P.C.T]

But DA = DB

$\therefore$ DB = FC

Now DB $\|$ FC

$\therefore$ DBCF is a parallelogram

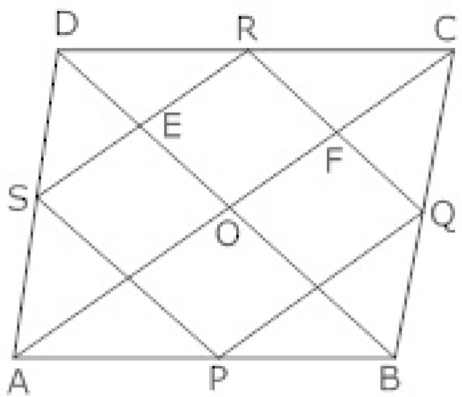
$\therefore$ DE $\|$ BC

Also DE = EF = $\frac{1}{2}$ BC

9. ABCD is a rhombus and P, Q, R, and S are the mid-Points of the sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS is a rectangle.

Ans. Join AC and BD which intersect at O let BD intersect RS at E and AC intersect RQ at F

IN $\triangle$ABD P and S are mid-points of sides AB and AD.



$\therefore$ PS||BD and PS=$\frac{1}{2}BD$

Similarly, RQ||DB and RQ=$\frac{1}{2}$ BD

$\therefore$ RS||BD||RQ and PS = $\frac{1}{2}BD = RQ$

PS=RQ and PS||RQ

$\therefore$ PQRS is a parallelogram

Now RF||EO and RE||FO

$\therefore$ OFRE is also a parallelogram.

Again, we know that diagonals of a rhombus bisect each other at right angles.

$\because \angle EOF = 90°$

$\because \angle EOF = \angle ERF$ [opposite angles of a parallelogram]

$\therefore \angle ERF = 90°$

$\therefore$ Each angle of the parallelogram PQRS is $90°$

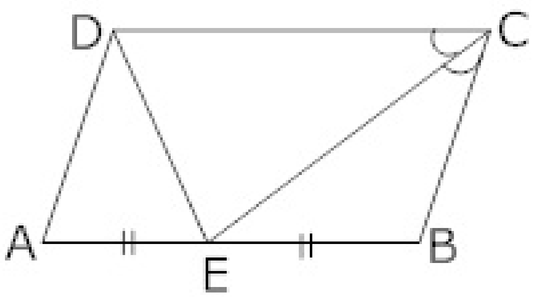
Hence PQRS is a rectangle.

10. In the given Fig ABCD is a parallelogram E is mid-point of AB and CE bisects $\angle BCD$ Prove that:

(i) AE = AD

(ii) DE bisects $\angle ADC$

(iii) $\angle DEC = 90°$



Ans. ABCD is a parallelogram

$\therefore AB \parallel CD$ And EC cuts them

$\Rightarrow \angle BEC = \angle ECD$ [Alternate interior angle]

$\Rightarrow \angle BEC = \angle ECB$ $[\angle ECD = \angle ECB]$

$\Rightarrow EB = BC$

$\Rightarrow AE = AD$

(i) Now AE=AD

$\Rightarrow \angle ADE = \angle AED$

$\Rightarrow \angle ADE = \angle EAC$ [$\because \angle AED = \angle EDC$ Alternate interior angles]

(ii) $\therefore$ DE bisects $\angle ADC$

(iii) Now $\angle ADC + \angle BCD = 180°$

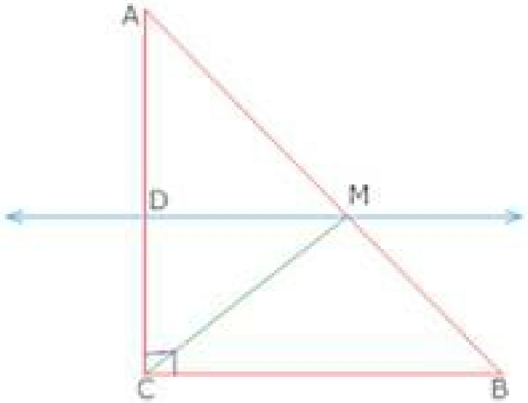
$\Rightarrow \frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = 90°$

$\Rightarrow \angle EDC + \angle DCE = 90°$

But, the sum of all the angles of the triangle is $180°$

$\Rightarrow 90° + \angle DEC = 180°$
$\Rightarrow \angle DEC = 90°$

11. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. show that
(i) D is mid-point of AC
(ii) MD $\perp$ AC
(iii) CM = MA = $\frac{1}{2}$ AB



Ans. Given ABC is a $\triangle$ right angle at C

(i) M is mid-point of AB

And MD||BC

$\therefore$ D is mid-Point of AC [a line through midpoint of one side of a $\triangle$ parallel to another side bisect the third side.

(ii). $\because$ MD||BC

$\angle ADM = \angle DCB$ [Corresponding angles]

$\angle ADM = 90°$

(iii) In $\triangle$ ADM and $\triangle$ CDM

AD=DC [$\because$ D is mid-point of AC]

DM=DM [Common]

$\therefore$ $\triangle$ADM $\cong$ $\triangle$CDM [By SAS]

$\therefore$ AM=CM [By C.P.C.T]

AM=CM=MB [$\because$ M is mid-point of AB]

$\therefore$ CM=MA= $\frac{1}{2}$ AB.