Q1: NTA Test 01 (Numerical)

If $f(x)$ is a polynomial satisfying $f(x)\, f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(2) > 1$, then $\lim\limits_{x \to 1} f(x)$ is

Q2: NTA Test 02 (Numerical)

If $\lim\limits_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ equals L, then the value of $(4L + 1)$ is

Q3: NTA Test 04 (Numerical)

$\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-\frac{x}{2}} - 3^{1-x}}$ is equal to

Q4: NTA Test 05 (Single Choice)

If p, q, r, s > 0, then $\lim\limits_{x \to \infty} \left(\frac{p^{\frac{1}{x}} + q^{\frac{1}{x}} + r^{\frac{1}{x}} + s^{\frac{1}{x}}}{4}\right)^{3x}$ is equal to

(A) pqrs

(B) $(\text{pqrs})^3$

(C) $(\text{pqrs})^{\frac{3}{2}}$

(D) $(\text{pqrs})^{\frac{3}{4}}$

Q5: NTA Test 06 (Single Choice)

The value of $\lim\limits_{x \to 0} \frac{x^{6000} - (\sin x)^{6000}}{x^2 (\sin x)^{6000}}$ is

(A) 1000

(B) 100

(C) 1100

(D) 1010

Q6: NTA Test 07 (Single Choice)

The value of $\lim\limits_{x \to \infty} \left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$ is equal to

(A) $e^{-1/3}$

(B) $e^{-2/3}$

(C) e^{-1}

(D) e^{-2}

Q7: NTA Test 08 (Single Choice)

If $f(x) = \begin{cases} \frac{\sqrt{g(x)} - 1}{\sqrt{x} - 1}; & x \neq 1 \\ 1; & x = 1 \end{cases}$ and $g'(1) = 2,\ g(1) = 1$, then $\lim\limits_{x \to 1} f(x)$ is equal to

(A) 1

(B) 3

(C) 2

(D) 4

Q8: NTA Test 09 (Single Choice)

The value of $\lim\limits_{x \to 0} \frac{\log(1 + x + x^2) + \log(1 - x + x^2)}{\sec x - \cos x}$ is equal to

(A) -1

(B) 1

(C) 0

(D) 2

Q9: NTA Test 10 (Single Choice)

$\lim\limits_{x \to 0} \frac{\ln(1+x)}{x^2} + \frac{x-1}{x} =$

(A) ∞

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) 1

Q10: NTA Test 11 (Single Choice)

$\lim_{x \to \frac{\pi}{2}} \frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is equal to

(A) $\frac{1}{8}$

(B) 0

(C) $\frac{1}{32}$

(D) ∞

Q11: NTA Test 12 (Single Choice)

If $\lim_{x \to 0} \frac{((a-n)nx-\tan x)\sin nx}{x^2} = 0$, where n is a non zero real number, then a is equal to

(A) 0

(B) $\frac{n+1}{n}$

(C) n

(D) $n + \frac{1}{n}$

Q12: NTA Test 13 (Single Choice)

The value of $\lim_{x \to \infty} \frac{2(x)^{1/2}+3(x)^{1/3}+4(x)^{1/4}+.....+(x)^{1/n}}{(2x-3)^{1/2}+(2x-3)^{1/3}+....+(2x-3)^{1/n}}$ is

(A) $\sqrt{2}$

(B) 2

(C) $\frac{1}{\sqrt{3}}$

(D) 0

Q13: NTA Test 14 (Single Choice)

The value of $\lim_{x \to 0} \frac{1+\sin x-\cos x+\ln(1-x)}{x \cdot \tan^2 x}$ is

(A) $-\frac{1}{2}$

(B) $-\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Q14: NTA Test 15 (Single Choice)

The value of $\lim_{n \to \infty} \frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$ is equal to

(A) $\frac{3}{5}$

(B) $-\frac{4}{7}$

(C) $-\frac{20}{7}$

(D) 0

Q15: NTA Test 17 (Single Choice)

The value of $\lim_{x \to \pi} \frac{\tan(\pi\cos^2 x)}{\sin^2 x}$ is equal to

(A) 1

(B) π

(C) $-\pi$

(D) $\frac{\pi}{2}$

Q16: NTA Test 18 (Single Choice)

The value of $\lim_{x \to 0} \frac{2^{1/x}+2^{3/x}}{3\left(2^{1/x}\right)+5\left(2^{3/x}\right)}$ is

(A) $1/3$

(B) $1/5$

(C) 1

(D) $1/4$

Q17: NTA Test 19 (Single Choice)

The value of $\lim_{x \to 0} \frac{\log(\sin 5x+\cos 5x)}{\tan 3x}$ is equal to

(A) $\frac{10}{3}$

(B) $\frac{20}{3}$

(C) $\frac{5}{6}$

(D) $\frac{5}{3}$

Q18: NTA Test 20 (Single Choice)

The value of $\lim_{x \to 0} \frac{\sin x}{3}\left[\frac{5}{x}\right]$ is equal to

(where, $[.]$ represents the greatest integer function)

(A) $\frac{1}{3}$
(B) 0
(C) $\frac{5}{3}$
(D) 1

Q19: NTA Test 21 (Single Choice)

The value of $\lim_{x \to 0} \frac{\tan^2 3x}{\sqrt{5}-\sqrt{4+\sec x}}$ is equal to

(A) $2\sqrt{5}$
(B) $-9\sqrt{5}$
(C) $9\sqrt{5}$
(D) $-36\sqrt{5}$

Q20: NTA Test 22 (Single Choice)

The value of $\lim_{x \to 1^-} \frac{\sqrt{\pi}-\sqrt{4\tan^{-1} x}}{\sqrt{1-x}}$ is equal to

(A) $2\sqrt{\pi}$
(B) $\frac{1}{2\sqrt{\pi}}$
(C) $4\sqrt{\pi}$
(D) 0

Q21: NTA Test 23 (Single Choice)

The value of $\lim_{x \to \infty} \frac{(\ln x)^2}{2+3x^2}$ is equal to

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 1
(D) 0

Q22: NTA Test 24 (Single Choice)

The value of $\lim_{x \to 0} \frac{x\cot(4x)}{\tan^2(3x)\cot^2(6x)}$ is equal to

(A) 0
(B) 4
(C) $\frac{2}{9}$
(D) 1

Q23: NTA Test 26 (Single Choice)

The value of $\lim_{x \to 0} \frac{e^{-\frac{x^2}{2}}-\cos x}{x^3 \tan x}$ is equal to

(A) $\frac{1}{4}$
(B) $\frac{1}{8}$
(C) $\frac{1}{12}$
(D) $\frac{1}{16}$

Q24: NTA Test 27 (Numerical)

The value of $\lim_{x \to 1^+} \frac{\int_1^x |t-1|\, dt}{\sin(x-1)}$ is equal to

Q25: NTA Test 28 (Single Choice)

If $\lim_{x \to 0}\left(1+px+qx^2\right)^{\operatorname{cosec} x}=e^5$, then

(A) $p=5, q \in R$
(B) $p=5, q>0$
(C) $q=5, p \in R$
(D) $q=5, p=0$

Q26: NTA Test 29 (Single Choice)

The value of $\lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

(A) 1
(B) 2

(C) $-\frac{1}{4}$

(D) $\frac{1}{2}$

Q27: NTA Test 32 (Single Choice)

If $\lim\limits_{x \to 0} \frac{\sin 2x - a \sin x}{x^3}$ exists finitely, then the value of a is

(A) 0

(B) 2

(C) 1

(D) 4

Q28: NTA Test 33 (Numerical)

The value of $\lim\limits_{x \to 0} \frac{1}{x^{18}} \left(1 - \cos\left(\frac{x^3}{3}\right) - \cos\left(\frac{x^6}{6}\right) + \cos\left(\frac{x^3}{3}\right) \cdot \cos\left(\frac{x^6}{6}\right)\right)$ is λ^2, then the value of 900λ is equal to (here, $\lambda > 0$)

Q29: NTA Test 36 (Numerical)

If $\lim\limits_{x \to \infty} \frac{ae^x + b\cos x + c + dx}{x \sin^2 x} = 3$, then the value of $272 \frac{abd}{c^3}$ is equal to

Q30: NTA Test 37 (Single Choice)

The value of $\lim\limits_{x \to 0^-} \frac{4^{\left(2 + \frac{3}{x}\right)} + 5\left(2^{\frac{1}{x}}\right)}{2^{\left(1 + \frac{6}{x}\right)} + 6\left(2^{\frac{1}{x}}\right)}$ is equal to

(A) $\frac{5}{6}$

(B) 8

(C) 16

(D) $\frac{5}{2}$

Q31: NTA Test 38 (Single Choice)

The value of $\lim\limits_{x \to 0} \frac{\sec x - (\sec x)^{\sec x}}{1 - \sec x + \ln(\sec x)}$ is equal to

(A) 0

(B) 1

(C) 2

(D) -1

Q32: NTA Test 40 (Single Choice)

The value of $\lim\limits_{n \to \infty} \left(\cos x \cos\frac{x}{2} \cos\frac{x}{4} \ldots \cos\frac{x}{2^n}\right)$ is equal to

(A) $\frac{x}{\sin x}$

(B) $\frac{\sin x}{x}$

(C) $\frac{\sin 2x}{2x}$

(D) $\frac{2x}{\sin 2x}$

Q33: NTA Test 41 (Single Choice)

The value of $\lim\limits_{x \to \frac{\pi}{4}} (\sin 2x)^{\sec^2 2x}$ is equal to

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $e^{-\frac{1}{2}}$

(D) $e^{\frac{1}{2}}$

Q34: NTA Test 41 (Numerical)

If $\lim\limits_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$, then the value of ab is equal to

Q35: NTA Test 43 (Numerical)

If $L = \lim\limits_{x \to \frac{\pi}{4}} \frac{(1 - \tan x)(1 - \sin 2x)}{(1 + \tan x)(\pi - 4x)^3}$, then the value of $40L$ is equal to

Q36: NTA Test 44 (Single Choice)

The value of $\lim_{x \to 0} \frac{1-\cos^3(\sin x)}{\sin x \sin(\sin x)\cos(\sin x)}$
is equal to

(A) $\frac{3}{2}$

(B) 1

(C) 0

(D) 2

Q37: NTA Test 45 (Single Choice)

The value of $\lim_{x \to 1} \frac{x\tan\{x\}}{x-1}$ is equal to (where $\{x\}$ denotes the fractional part of x)

(A) -1

(B) 0

(C) 1

(D) Does not exist

Q38: NTA Test 46 (Single Choice)

The value of $\lim_{x \to 0}(\sec x + \tan x)^{\frac{1}{x}}$ is equal to

(A) e

(B) e^2

(C) e^{-1}

(D) 1

Q39: NTA Test 48 (Numerical)

The value of $\lim_{x \to \frac{\pi}{3}} \frac{2-\sqrt{3}\sin x - \cos x}{(3x-\pi)^2}$ is equal to the reciprocal of the number

Answer Keys

Q1: 2	**Q2:** 3	**Q3:** 36
Q4: (D)	**Q5:** (A)	**Q6:** (B)
Q7: (C)	**Q8:** (B)	**Q9:** (B)
Q10: (C)	**Q11:** (D)	**Q12:** (A)
Q13: (A)	**Q14:** (C)	**Q15:** (C)
Q16: (A)	**Q17:** (D)	**Q18:** (C)
Q19: (D)	**Q20:** (D)	**Q21:** (D)
Q22: (D)	**Q23:** (C)	**Q24:** 0
Q25: (A)	**Q26:** (B)	**Q27:** (B)
Q28: 25	**Q29:** 34	**Q30:** (A)
Q31: (C)	**Q32:** (C)	**Q33:** (C)
Q34: 3.75	**Q35:** 1.25	**Q36:** (A)
Q37: (D)	**Q38:** (A)	**Q39:** 9

Solutions

Q1: 2

As given $f(x) \,.\, f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right),\ f(2),$

$\Rightarrow f(x)f\left(\frac{1}{x}\right) - f(x) - f\left(\frac{1}{x}\right) = 0$

$\Rightarrow (f(x) - 1)\left(f\left(\frac{1}{x}\right) - 1\right) = 1$

$\Rightarrow h(x)h\left(\frac{1}{x}\right) = 1$ as it $f(x)$ is a polynomial,hence $h(x)$ will also be a polynomial

$\Rightarrow h(x) = \pm x^n$
$\Rightarrow f(x) = \pm x^n + 1$

Hence $f(x) = \pm x^n + 1$

Now $f(2) > 1$

$\Rightarrow f(x) = x^n + 1$

hence $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1}(x^n + 1) = 2$

Q2: 3

Let $L = \lim\limits_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3} \left(\frac{0}{0}\text{form}\right)$

$= \lim\limits_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2}}{3x^2}$ (L′ Hospital rule)

$= \frac{1}{3}\lim\limits_{x \to 0} \frac{1}{x^2}\left[\frac{1+x^2-\sqrt{1-x^2}}{(1+x^2)\sqrt{1-x^2}}\right]$

$= \frac{1}{3}\lim\limits_{x \to 0} \frac{1}{x^2}\left[\frac{(1+x^2)^2-(1-x^2)}{(1+x^2)\sqrt{1-x^2}} \cdot \frac{1}{(1+x^2)+\sqrt{1-x^2}}\right]$

$= \frac{1}{3}\lim\limits_{x \to 0} \frac{(3+x^2)}{(1+x^2)\sqrt{1-x^2}} \frac{1}{\left[(1+x^2)+\sqrt{1-x^2}\right]}$

$= \frac{1}{3}\left(\frac{3}{2}\right) = \frac{1}{2}$

Q3: 36

Let, $3^{\frac{x}{2}} = t,\ x \to 2 \Rightarrow t \to 3$

$\lim\limits_{t \to 3} \frac{t^2 \frac{27}{t^2} - 12}{\frac{1}{t} - \frac{3}{t^2}} = \lim\limits_{t \to 3} \frac{t^4+27-12t^2}{t-3}$

$\lim\limits_{t \to 3} \frac{(t^2-3)(t+3)(t-3)}{(t-3)} = 6 \times 6 = 36$

Q4: (D) $(\text{pqrs})^{\frac{3}{4}}$

Consider the given expression,

$\lim\limits_{x \to \infty} \left(\frac{p^{\frac{1}{x}} + q^{\frac{1}{x}} + r^{\frac{1}{x}} + s^{\frac{1}{x}}}{4}\right)^{3x}$ [where, p, q, r, s > 0]

Put, $\frac{1}{x} = z$, so that $x \to \infty \Rightarrow z \to 0$

$= \lim\limits_{z \to 0} \left(\frac{p^z + q^z + r^z + s^z}{4}\right)^{\frac{3}{z}}$

which is in the form 1^{∞}.

$$=e^{\lim_{z\to 0}\left(\frac{p^z+q^z+r^z+s^z}{4}-1\right)\frac{3}{z}}$$

$$=e^{\lim_{z\to 0}\left(\frac{p^z-1}{z}+\frac{q^z-1}{z}+\frac{r^z-1}{z}+\frac{s^z-1}{z}\right)\frac{3}{4}}$$

$$=e^{\frac{3}{4}\left(\log_e p+\log_e q+\log_e r+\log_e s\right)}$$

$$=e^{\frac{3}{4}\log_e(pqrs)}=(pqrs)^{\frac{3}{4}}$$

Q5: (A) 1000

$$\lim_{x\to 0}\frac{x^{6000}-(\sin x)^{6000}}{x^2\frac{(\sin x)^{6000}}{x^{6000}}.x^{6000}} \qquad \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{x^{6000}-(\sin x)^{6000}}{x^{6002}}$$

$$=-\lim_{x\to 0}\frac{(\sin x)^{6000}-x^{6000}}{x^{6002}}$$

$$=-\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^{6000}-1}{x^2}$$

$$=-\lim_{x\to 0}\frac{6000\left(\frac{\sin x}{x}\right)^{5999}\left(\frac{x\cos x-\sin x}{x^2}\right)}{2x} \text{ (By L-Hospital Rule)}$$

$$=-\frac{6000}{2}\lim_{x\to 0}\frac{x\cos x-\sin x}{x^3}$$

$$=-3000\lim_{x\to 0}\frac{\cos x-x\ \sin x-\cos x}{3x^2}$$

$$=1000$$

Q6: (B) $e^{-2/3}$

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$$

$$=\lim_{x\to\infty}\left[1+\frac{-6}{3x+2}\right]^{\frac{x+1}{3}}$$

$$=e^{\lim_{x\to\infty}\frac{-6}{3x+2}\times\frac{x+1}{3}} \qquad [\because\ 1^\infty\ form]$$

$$=e^{\lim_{x\to\infty}\frac{-2x-2}{3x+2}}=e^{-2/3} \text{ (Applying L' Hospital rule)}$$

Q7: (C) 2

$$\lim_{x\to 1}f(x)=\lim_{x\to 1}\frac{\frac{g'(x)}{2\sqrt{g(x)}}}{\frac{1}{2\sqrt{x}}}$$

$$=\lim_{x\to 1}\frac{g'(x).\sqrt{x}}{\sqrt{g(x)}}=\frac{g'(1).\sqrt{1}}{\sqrt{g(1)}}=2$$

Q8: (B) 1

$$\lim_{x\to 0}\frac{\log(1+x+x^2)+\log(1-x+x^2)}{\sec x-\cos x}$$

$$=\lim_{x\to 0}\frac{\log\left[(1+x^2)^2-x^2\right]}{(1-\cos^2 x)/\cos x}$$

$$=\lim_{x\to 0}\frac{\log(1+x^2+x^4)}{\sin \text{x} \tan x}$$

$$=\lim_{x\to 0}\frac{\log(1+x^2(1+x^2))}{x^2(1+x^2)}\cdot x^2\left(1+x^2\right)\cdot\frac{1}{\frac{\sin x}{x}\cdot\frac{\tan x}{x}\cdot x^2}$$

$$=1\left(\text{as }\lim_{x\to 0}\frac{\log(1+x)}{x}=1\right)$$

Q9: (B) $\frac{1}{2}$

$$\lim_{\text{x}\to 0}\frac{\ln(1+\text{x})+\text{x}^2-\text{x}}{\text{x}^2}$$

using expansion, we know,

$$\ln(1+\text{x})=\text{x}-\frac{\text{x}^2}{2}+\frac{\text{x}^3}{3}\ldots\ldots$$

$$\Rightarrow\frac{\lim_{x\to 0}x-\frac{x^2}{2}+\ldots.+x^2-x}{x^2}$$

$$=\frac{1}{2}$$

Q10: (C) $\frac{1}{32}$

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{x}{4}-\frac{x}{2}\right)(1-\sin x)}{(\pi-2x)^3}$$

Let, $x=\frac{\pi}{2}+y$

$$\Rightarrow\lim_{y\to 0}=\frac{\tan\left(-\frac{y}{2}\right)(1-\cos y)}{(-2y)^3}$$

$$=\lim_{y\to 0}\frac{-\tan\frac{y}{2}\cdot 2\sin^2\frac{y}{2}}{(-8)y^3}=\lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}\cdot\left[\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right]^2$$

$$=\lim_{y\to 0}\frac{1}{32}\times\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left(\lim_{y\to 0}\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2$$

$$\because\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\tan x}{x}=1$$

$$\Rightarrow\frac{1}{32}\times 1\times 1^2$$

$$=\frac{1}{32}$$

Q11: (D) $\text{n}+\frac{1}{\text{n}}$

$$\lim_{\text{x}\to 0}\frac{((\text{a}-\text{n})\text{nx}-\tan\text{x})\sin\text{nx}}{\text{x}^2}=0$$

$$\Rightarrow \quad \lim_{x\to 0}\left(\left(a-n\right)n-\frac{\tan x}{x}\right)\frac{\sin nx}{x}=0$$

$$\Rightarrow \quad \left(\left(a-n\right)n-1\right)n=0$$

$$\Rightarrow \quad \left(a-n\right)n-1=0 \qquad \left(\because n\neq 0\right)$$

$$\therefore \quad a=n+\frac{1}{n}$$

Q12: (A) $\sqrt{2}$

Given,

$$\lim_{x\to\infty}\frac{2x^{1/2}+3x^{1/3}+....+nx^{1/n}}{(2x-3)^{1/2}+(2x-3)^{1/3}+...+(2x-3)^{1/n}}$$

$$=\lim_{h\to 0}\frac{2\left(\frac{1}{h^{1/2}}\right)+3\left(\frac{1}{h^{1/3}}\right)+....+n\left(\frac{1}{h^{1/n}}\right)}{\frac{1}{h^{1/2}}(2-3h)^{1/2}+\frac{1}{h^{1/3}}(2-3h)^{1/3}+.....+\frac{1}{h^{1/n}}(2-3h)^{1/n}}$$

$\left[\text{On putting } x=\frac{1}{h} \text{ as } x\to\infty,\ h\to\ 0\right]$

$$=\lim_{h\to 0}\frac{2+3h^{\left(\frac{1}{2}-\frac{1}{3}\right)}+4h^{\left(\frac{1}{2}-\frac{1}{4}\right)}+....+nh^{\left(\frac{1}{2}-\frac{1}{n}\right)}}{(2-3h)^{1/2}+h^{\left(\frac{1}{2}-\frac{1}{3}\right)}(2-3h)^{1/3}+....+h^{\left(\frac{1}{2}-\frac{1}{n}\right)}(2-3h)^{1/n}}$$

$$=\frac{2+0+0+.......}{2^{1/2}+0+0+.......}=\sqrt{2}.$$

Q13: (A) $-\frac{1}{2}$

$$\lim_{x\to 0}\frac{1+\sin x-\cos x+\ell n(1-x)}{x\cdot\tan^2 x}$$

$$=\lim_{x\to 0}\frac{1+\sin x-\cos x+\ell n(1-x)}{x^3}\quad\left(\frac{0}{0}\right)\quad\left[\because \lim_{x\to 0}\frac{x}{\tan x}=1\right]$$

$$=\lim_{x\to 0}\frac{\cos x+\sin x-\frac{1}{1-x}}{3x^2}\quad\left(\frac{0}{0}\right)\quad\text{[using L'hospital rule]}$$

$$=\lim_{x\to 0}\frac{-\sin x+\cos\ x-\frac{1}{(1-x)^2}}{6x}\quad\left(\frac{0}{0}\right)\quad\text{[Using L'hospital rule again]}$$

$$=\lim_{x\to 0}\frac{-\cos x-\sin\ x-\frac{2}{(1-x)^3}}{6}\quad\text{[Using L'hospital rule again]}$$

$$=\frac{-1-2}{6}=-\frac{1}{2}$$

Q14: (C) $-\frac{20}{7}$

$$\lim_{n\to\infty}\frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$$

$$=\lim_{n\to\infty}\frac{5^n\left(6.\left(\frac{2}{5}\right)^n-20\right)}{5^n\left(5.\left(\frac{2}{5}\right)^n+7\right)}=-\frac{20}{7}\left(\because \lim_{n\to\infty}\left(\frac{2}{5}\right)^n=0\right)$$

Q15: (C) $-\pi$

Let, $x = \pi + t$

$$\lim_{t\to 0}\frac{\tan(\pi\cos^2(\pi+t))}{\sin^2(\pi+t)} \Rightarrow \lim_{t\to 0}\frac{\tan(\pi\cos^2 t)}{\sin^2 t}$$

$$\lim_{t\to 0}\frac{\tan(\pi-\pi\sin^2 t)}{\sin^2 t} \Rightarrow \lim_{t\to 0}\frac{-\tan(\pi\sin^2 t)}{\pi\sin^2 t}\cdot\pi = -\pi$$

Q16: (A) $1/3$

$$\lim_{x\to 0^-}\frac{2^{1/x}\left\{1+2^{2/x}\right\}}{2^{1/x}\left\{3+5\cdot 2^{2/x}\right\}} \Rightarrow \lim_{x\to 0^-}\frac{1+2^{1/x}}{3+5\left(2^{2/x}\right)}$$
$$=\frac{1+0}{3+0}=1/3$$

Q17: (D) $\frac{5}{3}$

$$\lim_{x\to 0}\frac{1}{2}\frac{\log(1+\sin 10x)}{\tan 3x} \quad (\text{as, } 1+\sin 10x = (\sin 5x+\cos 5x)^2)$$
$$=\lim_{x\to 0}\frac{1}{2}\frac{\log(1+\sin 10x)}{(\sin 10x)}\cdot\left(\frac{\sin 10x}{10x}\right)\cdot\left(\frac{3x}{\tan 3x}\right)\cdot\frac{10x}{3x}$$
$$=\frac{1}{2}\times 1\times 1\times\frac{10}{3}=\frac{5}{3}$$

Q18: (C) $\frac{5}{3}$

$$\frac{5}{x}-1<\left[\frac{5}{x}\right]\le\frac{5}{x}$$
$$\underbrace{\frac{\sin x}{3}\left(\frac{5}{x}-1\right)}_{h(x)}<\underbrace{\frac{\sin x}{3}\left[\frac{5}{x}\right]}_{f(x)}\le\underbrace{\frac{\sin x}{3}\left(\frac{5}{x}\right)}_{g(x)}$$

by sandwich theorem

$$\because \lim_{x\to 0}g(x)=\lim_{x\to 0}h(x)=\frac{5}{3}$$
$$\therefore \lim_{x\to 0}f(x)=\frac{5}{3}$$

Q19: (D) $-36\sqrt{5}$

After rationalization,

$$\lim_{x\to 0}\frac{\tan^2 3x(\sqrt{5}+\sqrt{4+\sec x})}{5-(4+\sec x)}$$
$$\Rightarrow \lim_{x\to 0}\left(\frac{\tan^2 3x}{9x^2}\right)\cdot\frac{9x^2}{(\cos x-1)}\left(\sqrt{5}+\sqrt{4+1}\right)$$
$$\Rightarrow 9(-2)\left(2\sqrt{5}\right)=-36\sqrt{5}$$

Q20: (D) 0

After rationalization

$$\lim_{x\to 1}\frac{\pi-4\tan^{-1}x}{\sqrt{1-x}(\sqrt{\pi}+\sqrt{4\tan^{-1}x})}=\frac{1}{2\sqrt{\pi}}\lim_{x\to 1^-}\frac{\pi-4\tan^{-1}x}{\sqrt{1-x}}$$

By L-hospital rule, we get,

$$\frac{1}{2\sqrt{\pi}}\lim_{x\to 1}\frac{\frac{-4}{1+x^2}}{\frac{-1}{2\sqrt{1-x}}}=\frac{4}{\sqrt{\pi}}\lim_{x\to 1}\frac{\sqrt{1-x}}{1+x^2}=0$$

Q21: (D) 0

By L-hospital rule,

$$\lim_{x\to\infty}\frac{2(\ln x).\frac{1}{x}}{6x}$$
$$\Rightarrow \lim_{x\to\infty}\frac{\ln x}{3x^2}$$
Again by L-hospital rule,
$$\lim_{x\to\infty}\frac{\frac{1}{x}}{6x} \Rightarrow \lim_{x\to\infty}\frac{1}{6x^2}=0$$

Q22: (D) 1

$$\lim_{x\to 0}\frac{9x^2}{(\tan 3x)^2}\frac{1}{9x}\cdot\frac{(\tan 6x)^2}{36\cdot x^2}\cdot\frac{4x}{(\tan 4x)}\frac{36x^2}{4x}$$
$$=1\times\frac{1}{9}\times 1\times 1\times\frac{36}{4}=1$$

Q23: (C) $\frac{1}{12}$

Using expansion,
$$\lim_{x\to 0}\frac{\left(1+\left(-\frac{x^2}{2}\right)+\frac{\left(\frac{x^3}{4}\right)}{2!}+....\right)-\left(1-\frac{x^2}{2!}+\frac{x^2}{4!}-....\right)}{x^3\left(x+\frac{x^3}{3}+....\right)}$$
$$\lim_{x\to 0}\frac{(1-1)+\left(-\frac{x^2}{2}+\frac{x^2}{2}\right)+\left(\frac{x^4}{8}-\frac{x^4}{24}\right)+....}{x^3\left(x+\frac{x^3}{3}+....\right)}$$
$$\Rightarrow\frac{1}{8}-\frac{1}{24}=\frac{1}{12}$$

Q24: 0

Given: $\lim_{x\to 1^+}\frac{\int_1^x (t-1)dt}{\sin(x-1)}$

Using L'Hospital's rule and leibuitzrule,

$$l=\lim_{x\to 1^+}\frac{\frac{d}{dx}\left(\int_1^x (t-1)dt\right)}{\frac{d}{dx}(\sin(x-1))}$$

$$\lim_{x\to 1^+}\frac{(x-1)}{\cos(x-1)}=\frac{0}{1}=0$$

Q25: (A) $p=5, q\in R$

$\because$ given limit is of the form 1^∞
$$\therefore\ e^{\lim\limits_{x\to 0}(\operatorname{cosec}x)\{1+px+qx^2-1\}}=e^5$$
$$\lim_{x\to 0}\left(\frac{px+qx^2}{\sin x}\right)=5$$
$$\lim_{x\to 0}\left(\frac{px+qx^2}{x}\right)\cdot\left(\frac{x}{\sin x}\right)=5$$
$$\lim_{x\to 0}(p+qx)(1)=5\Rightarrow p=5$$

Q26: (B) 2
$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x(3+\cos x)}{x\tan 4x}$$

$= \lim_{x\to0}\left(\frac{\sin x}{x}\right)^2\left(\frac{4x}{\tan 4x}\right)(3+\cos x)\times\frac{1}{2}$

$= 2$

Q27: (B) 2

$\lim_{x\to0}\frac{\sin x(2\cos x-a)}{x\cdot x^2} = \lim_{x\to0}\left(\frac{\sin x}{x}\right)\left(\frac{2\cos x-a}{x^2}\right)$

For this limit to exists finitely,

$\lim_{x\to0}\frac{2\cos x-a}{x^2} = \text{finite}$

$\therefore$ It must be $\frac{0}{0}$ form

$\therefore\ 2\cos(0)-a=0\Rightarrow a=2$

Q28: 25

$\lim_{x\to0}\left\{1-\cos\left(\frac{x^3}{3}\right)-\cos\left(\frac{x^6}{6}\right)\left(1-\cos\left(\frac{x^3}{3}\right)\right)\right\}\cdot\frac{1}{x^{18}}$

$\lim_{x\to0}\frac{\left(1-\cos\left(\frac{x^3}{3}\right)\right)\left(1-\cos\left(\frac{x^6}{6}\right)\right)}{x^{18}}$

$\lim_{x\to0}\frac{1-\cos\left(\frac{x^3}{3}\right)}{\left(\frac{x^3}{3}\right)^2}\left(\frac{x^6}{9}\right)\cdot\frac{1-\cos\left(\frac{x^6}{6}\right)}{\left(\frac{x^6}{6}\right)^2}\cdot\left(\frac{x^6}{6}\right)^2\cdot\frac{1}{x^{18}}$

$\lim_{x\to0}\frac{1}{2}\times\frac{x^6}{9}\cdot\frac{1}{2}\cdot\frac{x^{12}}{36}\cdot\frac{1}{x^{18}}=\frac{1}{(36)^2}$

$\therefore\lambda=\frac{1}{36}$

Q29: 34

Using expansions, we get,

$\lim_{x\to0}\frac{a\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+..\right)+b\left(1-\frac{x^2}{2!}+...\right)+c+dx}{x\left(x-\frac{x^3}{3!}+.....\right)^2}=3$

$\lim_{x\to0}\frac{(a+b+c)+(a+d)x+\left(\frac{a-b}{2}\right)x^2+\frac{a}{6}x^3+\ldots}{x^3\left(1-\frac{x^2}{3!}+.....\right)^2}=3$

$\because$ in the denominator lowest power of x is 3

For the limit to be finite, the numerator should also have the least power of x as 3

$\therefore a+b+c=0\ \ \ldots(1)$

$a+d=0\ldots(2)$

$\frac{a-b}{2}=0\ldots(3)$

Now, $\frac{\left(\frac{a}{6}\right)}{1}=3\Rightarrow a=18$

From (1), (2), (3), we get,

$a=18,\ b=18,\ c=-36,\ d=-18$

$\frac{abd}{c^3}=\frac{-(18)^3}{-8(18)^3}=\frac{1}{8}$

Q30: (A) $\frac{5}{6}$

$\lim_{x\to0^-}\frac{2^{\left(4+\frac{6}{x}\right)}+5.2^{\frac{1}{x}}}{2^{\left(1+\frac{6}{x}\right)}+6.2^{\frac{1}{x}}}=\lim_{x\to0^-}\frac{2^{\frac{1}{x}}\left\{16\left(2^{\frac{5}{x}}\right)+5\right\}}{2^{\frac{1}{x}}\left\{2\left(2^{\frac{5}{x}}\right)+6\right\}}$

$\lim_{x\to0^-}\frac{16\left(2^{\frac{5}{x}}\right)+5}{2\left(2^{\frac{5}{x}}\right)+6}=\frac{16(0)+5}{2(0)+6}=\frac{5}{6}$

Q31: (C) 2

Let, $\sec x = t$

$\lim\limits_{t\to1}\frac{t-t^t}{1-t+\ln(t)} = \lim\limits_{t\to1}\frac{1-t^t(1+\ln t)}{-1+\frac{1}{t}}$ {by L-hospital rule}

$= \lim\limits_{t\to1}\frac{-t^t\left(\frac{1}{t}\right)-t^t(1+\ln t)^2}{-1/t^2}$ {again by L-hospital rule}

$= \frac{-1-1}{-1} = 2$

Q32: (C) $\frac{\sin 2x}{2x}$

Required limit $= \lim\limits_{n\to\infty}\cos\frac{x}{2^0}\cos\frac{x}{2^1}\cos\frac{x}{2^2}\ldots\cos\frac{x}{2^n}$

$= \lim\limits_{n\to\infty}\frac{1}{2\sin\frac{x}{2^n}}\left\{\cos x\ldots\cos\frac{x}{2^{n-1}}\left(2\sin\frac{x}{2^n}\cos\frac{x}{2^n}\right)\right\}$

$= \lim\limits_{n\to\infty}\frac{1}{2^2\sin\frac{x}{2^n}}\left\{\cos x\ldots\left(2\cos\frac{x}{2^{n-1}}\sin\frac{x}{2^{n-1}}\right)\right\}$

$= \lim\limits_{n\to\infty}\frac{1}{2^{n+1}\sin\frac{x}{2^n}}(2\cos x\sin x)$

$= \lim\limits_{n\to\infty}\frac{\sin 2x}{2^{n+1}\sin\left(\frac{x}{2^n}\right)}$

$= \frac{\sin 2x}{2x}\lim\limits_{n\to\infty}\left(\frac{\left(\frac{x}{2^n}\right)}{\sin\left(\frac{x}{2^n}\right)}\right) = \frac{\sin 2x}{2x}$

Q33: (C) $e^{-\frac{1}{2}}$

$\lim\limits_{x\to\frac{\pi}{4}}(\sin 2x)^{\sec^2 2x}$

$= e^{\lim\limits_{x\to\frac{\pi}{4}}\sec^2 2x(\sin 2x-1)}$

$= e^{\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin 2x-1}{\cos^2 2x}}$

Applying L'Hospital Rule, we get,

$= e^{\lim\limits_{x\to\frac{\pi}{4}}\frac{2\cos 2x}{2\cos 2x(-2\sin 2x)}}$

$= e^{\lim\limits_{x\to\frac{\pi}{4}}\frac{-1}{2\sin 2x}} = e^{-\frac{1}{2}}$

Q34: 3.75

Using the expansion, we get,

$\lim\limits_{x\to0}\frac{x+ax\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\ldots\right)-b\left(x-\frac{x^3}{3!}+\ldots\ldots\right)}{x^3} = 1$

$\lim\limits_{x\to0}\frac{(1+a-b)x+\left(\frac{-a}{2}+\frac{b}{6}\right)x^3+\ldots..}{x^3} = 1$

For limit to exist numerator & denominator must be of the same degree

$\therefore 1+a-b = 0 \ldots (1)$

Also, $\frac{-a}{2}+\frac{b}{6} = 1 \Rightarrow 3a-b+6 = 0 \ldots (2)$

By equations (1) & (2), we get,

$a = -\frac{5}{2}$ and $b = -\frac{3}{2}$

$\Rightarrow ab = \frac{15}{4} = 3.75$

Q35: 1.25

$L = \lim\limits_{x\to\frac{\pi}{4}}\frac{\tan\left(\frac{\pi}{4}-x\right)(1-\sin 2x)}{4\left(\frac{\pi}{4}-x\right)(\pi-4x)^2}$

$= \lim_{x \to \frac{\pi}{4}} \frac{1}{4} \frac{(1-\sin 2x)}{(\pi-4x)^2}$

$= \lim_{x \to \frac{\pi}{4}} \frac{1}{4} \frac{(-\cos 2x)\cdot 2}{2(\pi-4x)(-4)}$ (applying L-Hospital rule)

$= \lim_{x \to \frac{\pi}{4}} \frac{1}{16} \frac{\cos(2x)}{(\pi-4x)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{1}{16} \frac{(-\sin 2x)\times 2}{-4}$ (again applying L-Hospital rule)

$= \frac{1}{16}\left(\frac{-1}{-4}\right) \times 2$

$\Rightarrow L = \frac{1}{32}$

Q36: (A) $\frac{3}{2}$

Let, $\sin x = t$

$\Rightarrow \lim_{t \to 0} \frac{1-\cos^3 t}{t \sin t \cos t}$

$\Rightarrow \lim_{t \to 0} \frac{(1-\cos t)}{t^2} \times \left(\frac{t}{\sin t}\right) \times \frac{(1+\cos t+\cos^2 t)}{\cos t}$

$\Rightarrow \frac{1}{2} \times 1 \times \frac{(1+1+1)}{1} = \frac{3}{2}$

Q37: (D) Does not exist

$\text{LHL} = \lim_{x \to 1^-} \frac{x \tan\{x\}}{x-1}$

Let, $x = 1 - h,\ \text{ as } x \to 1^-,\ h \to 0^+$

$\Rightarrow \text{LHL} = \lim_{h \to 0^+} \frac{(1-h)\tan\{1-h\}}{-h}$

$\therefore \text{LHL} = \lim_{h \to 0^+} \frac{(1-h)}{-h} \tan(1) = -\infty$

Now, $\text{RHL} = \lim_{x \to 1^+} \frac{x \tan\{x\}}{x-1}$

Let, $x = 1 + h,\ \text{as } x \to 1^+,\ h \to 0^+$

$\Rightarrow \text{RHL} = \lim_{h \to 0^+} \frac{(1+h)\tan\{1+h\}}{h}$

$\Rightarrow \text{RHL} = \lim_{h \to 0^+} \frac{(1+h)\tan h}{h} = \lim_{h \to 0^+} (1+h)$

$\therefore \text{RHL} = (1+0) = 1$

Since, $\text{LHL} \neq \text{RHL}$

$\therefore$ the limit of the function does not exist at $x = 1$

Q38: (A) e

$\lim_{x \to 0} (\sec x + \tan x)^{\frac{1}{x}}$ (1^∞ form)

$= e^{\lim_{x \to 0} \frac{1}{x}(\sec x + \tan x - 1)}$

$= e^{\lim_{x \to 0}\left(\frac{1+\sin x - \cos x}{x \cos x}\right)}$

$= e^{\lim_{x \to 0}\left(\frac{\cos x + \sin x}{-x \sin x + \cos x}\right)}$ (by L'Hospital rule)

$= e^{\frac{1+0}{0+1}} = e$

Q39: 9

$\lim_{x \to \frac{\pi}{3}} \frac{2-2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)}{(3x-\pi)^2}$

$\Rightarrow \lim_{x \to \frac{\pi}{3}} \frac{2-2\cos\left(x-\frac{\pi}{3}\right)}{(3x-\pi)^2}$

Let, $x - \frac{\pi}{3} = t$

$$\Rightarrow \lim_{t \to 0} \frac{2-2\cos t}{(3t)^2} = \lim_{t \to 0} \frac{2(1-\cos t)}{9t^2} = \frac{2}{9} \cdot \frac{1}{2} = \frac{1}{9}$$