DAY TWO

Kinematics

Learning & Revision for the Day

- Frame of Reference
- Motion in a Straight Line
- Uniform and Non-uniform Motion
- Uniformly Accelerated Motion
- Elementary Concept of Differentiation and Integration for Describing Motion
- Graphs

Frame of Reference

The frame of reference is a suitable coordinate system involving space and time used as a reference to study the motion of different bodies. The most common reference frame is the cartesian frame of reference involving (x, y, z and t).

(i) **Inertial Frame of Reference** A frame of reference which is either at rest or moving with constant velocity is known as inertial frame of reference. Inertial frame of reference is one in which Newton's first law of motion holds good.

(ii) **Non-Inertial Frame of Reference** A frame of reference moving with some acceleration is known as non-inertial frame of reference. Non-inertial frame of reference in one which Newton's law of motion does not hold good.

Motion in a Straight Line

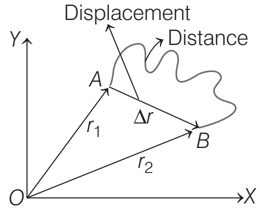
The motion of a point object in a straight line is one dimensional motion. During such a motion the point object occupies definite position on the path at each instant of time. Different terms used to described motion are defined below:

Distance and Displacement

- **Distance** is the total length of the path travelled by a particle in a given interval of time. It is a scalar quantity and its SI unit is metre (m).
- **Displacement** is shortest distance between initial and final positions of a moving object. It is a vector quantity and its SI unit is metre.

  From the given figure, mathematically it is expressed as,

  $$\Delta r = r_2 - r_1$$



- Displacement of motion may be zero or negative but path length or distance can never be negative.
- For motion between two points displacement is single valued while distance depends on actual path and so can have many values.
- Magnitude of displacement can never be greater than distance. However, it can be equal, if the motion is along a straight line without any change in direction.

Speed and Velocity

- **Speed** is defined as the total path length (or actual distance covered) by time taken by object.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time taken}}$$

It is scalar quantity. Its SI unit is m/s.

- Average Speed, $v_{av} = \dfrac{\text{Total distance travelled}}{\Delta t}$
- When a body travels equal distance with speeds v_1 and v_2, the average speed (v) is the harmonic mean of the two speeds.

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

- When a body travels for equal time with speeds v_1 and v_2, the average speed v is the arithmetic mean of the two speeds.

$$v_{av} = \frac{v_1 + v_2}{2}$$

- **Velocity** is defined as ratio of displacement and corresponding time interval taken by an object.

i.e. velocity $= \dfrac{\text{Displacement}}{\text{time interval}}$

- Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time taken}} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{\Delta x}{\Delta t}$

Here, x_2 and x_1 are the positions of a particle at the time t_2 and t_1 respectively, with respect to a given frame of reference.

- For a moving body speed can never be negative or zero while velocity can be negative and zero.
- The **instantaneous speed** is average speed for infinitesimal small time interval (i.e. $\Delta t \to 0$)

i.e. Instantaneous speed, $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

- The instantaneous velocity (or simply velocity) v of a moving particle is $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

It (at a particular time) can be calculated as the slope (at that particular time) of the graph of x *versus* t.

Uniform and Non-uniform Motion

- An object is said to be in uniform motion if its velocity is uniform i.e. it undergoes equal displacement in equal may be intervals of time, howsoever small these interval.
- An object is said to be in non-uniform motion if its undergoes equal displacement in unequal intervals of time., howsoever small these intervals may be.

Acceleration

Acceleration of an object is defined as rate of change of velocity. It is a vector quantity having unit m/s^2 or ms^{-2}. It can be positive, zero or negative.

Average and Instantaneous Acceleration If velocity of a particle at instant t is v_1 and at instant t_2 is v_2, then

- Average acceleration, $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\Delta v}{\Delta t}$
- Instantaneous acceleration, $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Uniformly Accelerated Motion

- A motion, in which change in velocity in each unit of time is constant, is called an uniformly accelerated motion. So, for an uniformly accelerated motion, acceleration is constant.
- For uniformly accelerated motion are given below

Equations of motion, $v = u + at$ ...(i)

$$s = ut + \frac{1}{2}at^2 \quad \text{...(ii)}$$

and $v^2 = u^2 + 2as$ ...(iii)

where, u = initial velocity, v = velocity at time t and s = displacement of particle at time t.

- Equation of uniformaly accelerated motion under gravity are

(i) $v = u - gt$ (ii) $h = ut - \dfrac{1}{2}gt^2$ (iii) $v^2 = u^2 - 2gh$

Elementary Concept of Differentiation and Integration for Describing Motion

- At an instant t, the body is at point $P(x, y, z)$.

Thus, velocity along X-axis, $v_x = \dfrac{dx}{dt}$

Acceleration along X-axis is $a_x = \dfrac{dv_x}{dt}$

Velocity along Y-axis is $v_y = \dfrac{dy}{dt}$

Acceleration along Y-axis is $a_y = \dfrac{dv_y}{dt}$

Similarly, $v_z = \dfrac{dz}{dt}$ and $a_z = \dfrac{dv_z}{dt}$

- **For a accelerating body**

(i) If a_x variable, $x = \int v_x\, dt$, $\int dv_x = \int a_x\, dt$

(ii) If a_y is variable, $y = \int v_y\, dt$, $\int dv_y = \int a_y\, dt$

(iii) If a_z is variable, $z = \int v_z\, dt$, $\int dv_z = \int a_z\, dt$

Also, distance travelled by a particle is $s = \int |v|\, dt$

(i) x-component of displacement is $\Delta x = \int v_x\, dt$

(ii) y-component of displacement is $\Delta y = \int v_y\, dt$

(iii) z-component of displacement is $\Delta z = \int v_z\, dt$

Graphs

During motion of the particle, its parameters of kinematical analysis changes with time. These can be represented on the graph, which are given as follows:

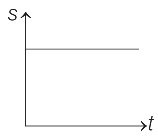
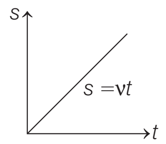
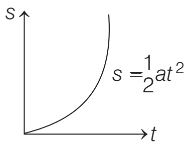
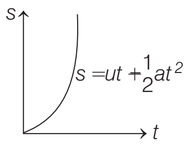
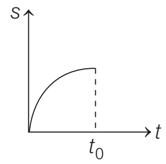
Position-Time Graph

(i) Position-time graph gives instantaneous value of displacement at any instant.

(ii) The slope of tangent drawn to the graph at any instant of time gives the instantaneous velocity at that instant.

(iii) The s-t graph cannot make sharp turns.

Different Cases of Position-Time Graph

Different Cases	s- t Graph	The main Features of Graph
At rest		Slope = $v = 0$
Uniform motion		Slope = constant, v = constant $a = 0$
Uniformly accelerated motion with $u = 0, s = 0$ at $t = 0$		$u = 0$, i.e. Slope of s-t graph at $t = 0$, should be zero.
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Slope of s-t graph gradually goes on increasing
Uniformly retarded motion		θ is decreasing so, v is decreasing, a is negative

Velocity-Time Graph

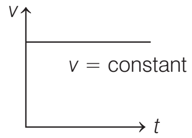
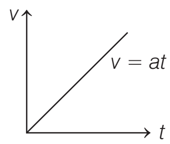
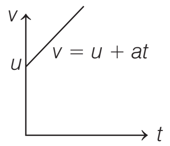
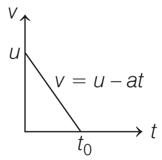
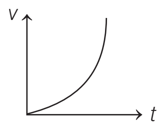
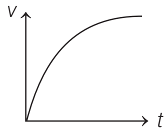
(i) Velocity-time graph gives the instantaneous value of velocity at any instant.

(ii) The slope of tangent drawn on graph gives instantaneous acceleration.

(iii) Area under v-t graph with time axis gives the value of displacement covered in given time.

(iv) The v-t curve cannot take sharp turns.

Different Cases in Velocity-Time Graph

Different Cases	v- t Graph	The main Features of Graph
Uniform motion		(i) $\theta = 0^\circ$ (ii) v = constant (iii) Slope of v-t graph = $a = 0$
Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$		So slope of v-t graph is constant $u = 0$ i.e. so, a = constant $u = 0$ i.e. $v = 0$ at $t = 0$
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Positive constant acceleration because θ is constant and $<90^\circ$ but the initial velocity of the particle is positive
Uniformly decelerated motion		Slope of v-t graphs = $-a$ (retardation)
Non-uniformly accelerated motion		Slope of v-t graph increases with time. θ is increasing, so, acceleration is increasing
Non-uniformly decelerating motion		θ is decreasing, so acceleration decreasing

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 An aeroplane flies 400 m from North and then flies 300 m South and then flies 1200 m upwards, then net displacement is

(a) 1200 m (b) 1300 m (c) 1400 m (d) 1500 m

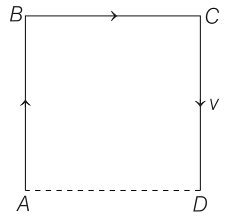
2 The correct statement from the following is

(a) A body having zero velocity will not necessarily have zero acceleration
(b) A body having zero velocity will necessarily have zero acceleration
(c) A body having uniform speed can have only uniform acceleration
(d) A body having non-uniform velocity will have zero acceleration

3 A vehicle travels half the distance L with speed v_1 and the other half with speed v_2, then its average speed is

(a) $\frac{v_1+v_2}{2}$ (b) $\frac{2v_1+v_2}{v_1+v_2}$ (c) $\frac{2v_1v_2}{v_1+v_2}$ (d) $\frac{2(v_1+v_2)}{v_1v_2}$

4 A particle moves along the sides AB, BC, CD of a square of side 25 m with a velocity of 15 m/s. Its average velocity is

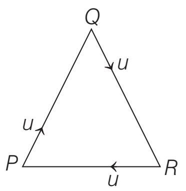


(a) 5 m/s (b) 7.5 m/s (c) 10 m/s (d) 15 m/s

5 A body sliding down on a smooth inclined plane slides down 1/4th of plane's length in 2 s. It will slide down the complete plane in

(a) 4 s (b) 5 s (c) 2 s (d) 3 s

6 Three particles P, Q and R are situated at corners of an equilateral triangle of side length (d). At $t = 0$, they started to move such that P is moving towards Q, Q is moving towards R and R is moving towards P at every instant. After how much time (in second) will they meet each other?



(a) $\frac{d}{u}$ (b) $\frac{2d}{3u}$ (c) $\frac{2d}{\sqrt{(3)}u}$ (d) $\frac{d}{\sqrt{(3)}\,u}$

7 A body is thrown vertically upwards in air, when air resistance is taken into account, the time of ascent is t_1 and time of descent is t_2, then which of the following is true?

(a) $t_1 = t_2$ (b) $t_1 < t_2$
(c) $t_1 > t_2$ (d) $t_1 \geq t_2$

8 A stone falls freely from rest and the total distance covered by it in the last second of its motion equals the distance covered by it in the first three seconds of its motion. The stone remains in the air for

(a) 6 s (b) 5 s (c) 7 s (d) 4 s

9 The motor of an electric train can give it an acceleration of 1 ms^{-2} and brakes can give a negative acceleration of 3 ms^{-2}. The shortest time in which the train can make a trip between the two stations 1215 m apart is

(a) 113.6 s (b) 56.9 s
(c) 60 s (d) 55 s

10 A train is moving along a straight path with a uniform acceleration. Its engine passes a pole with a velocity of 60 kmh^{-1} and the end (guard's van) passes across the same pole with a velocity of 80 kmh^{-1}. The middle point of the train will pass the same pole with a velocity

(a) 70 kmh^{-1} (b) 70.7 kmh^{-1}
(c) 65 kmh^{-1} (d) 75 kmh^{-1}

11 The acceleration experienced by a moving boat after its engine is cut-off, is given by $a = -kv^3$, where k is a constant. If v_0 is the magnitude of velocity at cut-off, then the magnitude of the velocity at time t after the cut-off is

(a) $\frac{v_0}{2ktv_0^2}$ (b) $\frac{v_0}{1+2ktv_0^2}$
(c) $\frac{v_0}{\sqrt{1-2kv_0^2}}$ (d) $\frac{v_0}{\sqrt{1+2ktv_0^2}}$

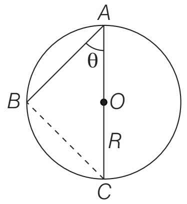
12 A body moving with an uniform acceleration describes 12 m in the 3rd second of its motion and 20 m in the 5th second. Find the velocity after the 10th second.

(a) 40 ms^{-1} (b) 42 ms^{-1}
(c) 52 ms^{-1} (d) 4 ms^{-1}

13 A train accelerating uniformly from rest attains a maximum speed of 40 ms^{-1} in 20 s. It travels at this speed for 20 s and is brought to rest with an uniform retardation in the next 40 s. What is the average velocity during this period?

(a) $\frac{80}{3}$ ms^{-1} (b) 25 ms^{-1}
(c) 40 ms^{-1} (d) 30 ms^{-1}

14 A frictionless wire AB is fixed on a sphere of radius R. A very small spherical ball slips on this wire. The time taken by this ball to slip from A to B is



(a) $\frac{2\sqrt{gR}}{g\cos\theta}$ (b) $2\sqrt{gR}\,\frac{\cos\theta}{g}$ (c) $2\sqrt{\frac{R}{g}}$ (d) $\frac{gR}{\sqrt{g}\,\cos\theta}$

15 A balloon is going upwards with velocity 12 ms^{-1}. It releases a packet when it is at a height of 65 m from the ground. How much time the packet will take to reach the ground if $g = 10\text{ ms}^{-2}$?

(a) 5 s (b) 6 s (c) 7 s (d) 8 s

16 A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m length of window some distance below from the top of building. With what speed does the ball pass the top of window?

(a) 6 ms^{-1} (b) 12 ms^{-1} (c) 7 ms^{-1} (d) 3.5 ms^{-1}

17 A body starts from the origin and moves along the axis such that the velocity at any instant is given by $v = 4t^3 - 2t$ where, t is in second and the velocity in ms^{-1}. Find the acceleration of the particle when it is at a distance of 2 m from the origin.

(a) 28 ms^{-2} (b) 22 ms^{-2} (c) 12 ms^{-2} (d) 10 ms^{-2}

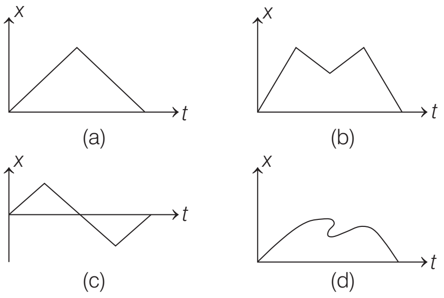
18 A point initially at rest moves along the x-axis. Its acceleration varies with time as $a = (5t + 6)\text{ ms}^{-2}$. If it starts from the origin, the distance covered by it in 2s is

(a) 18.66 m (b) 14.33 m (c) 12.18 m (d) 6.66 m

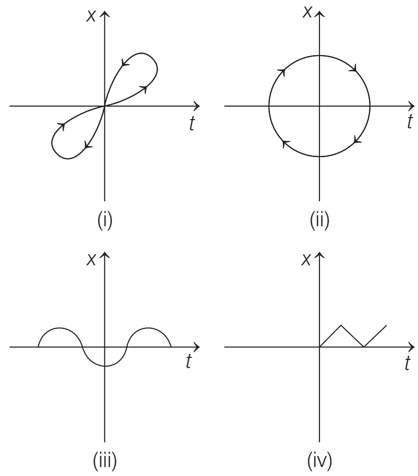
19 A rod of length l leans by its upper end against a smooth vertical wall, while its other end leans against the floor. The end that leans against the wall moves uniformly downwards. Then,

(a) the other end also move uniformly
(b) the speed of other end goes on increasing
(c) the speed of other end goes on decreasing
(d) the speed of other end first decreases and then increases

20 Which of the following distance-time graphs is not possible?

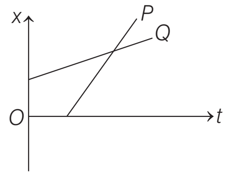


21 Look at the graphs (i) to (iv) in figure carefully and choose, which of these can possibly represent one-dimensional motion of particle?



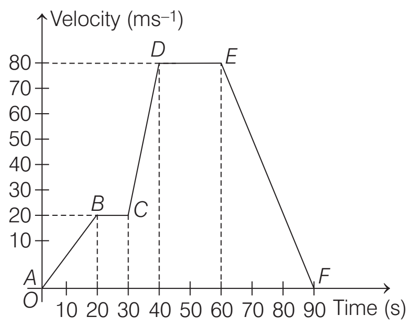
(a) Both (i) and (ii)
(b) Only (iv)
(c) Only (iii)
(d) Both (iii) and (iv)

22 Figure shows the time-displacement curve of the particles P and Q. Which of the following statement is correct?



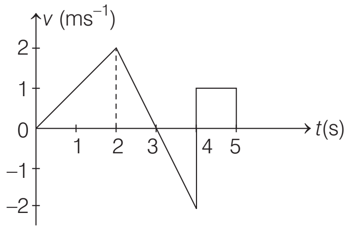
(a) Both P and Q move with uniform equal speed
(b) P is accelerated and Q moves with uniform speed but the speed of P is more than the speed of Q
(c) Both P and Q moves with uniform speeds but the speed of P is more than the speed of Q
(d) Both P and Q moves with uniform speeds but the speed of Q is more than the speed of P

23 The velocity *versus* time curve of a moving point is shown in the figure below. The maximum acceleration is



(a) 1ms^{-2} (b) 6ms^{-2}
(c) 2 ms^{-2} (d) 1.5ms^{-2}

24 The velocity-time graph of a body in a straight line is as shown in figure.



The displacement of the body in five seconds is
(a) 2 m (b) 3 m (c) 4 m (d) 5 m

25 All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

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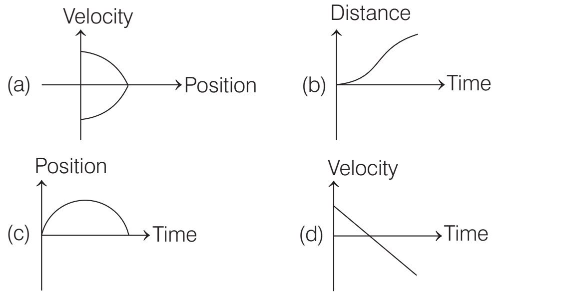

26 A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *versus* time?

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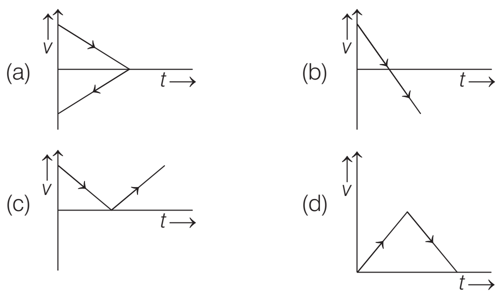


27 When two bodies move uniformly towards each other the distance between them decreases by 8 ms^{-1}. If both bodies move in the same direction with different speeds, the distance between them increases by 2 ms^{-1}. The speeds of two bodies will be
(a) 4 ms^{-1} and 3 ms^{-1}
(b) 4 ms^{-1} and 2 ms^{-1}
(c) 5 ms^{-1} and 3 ms^{-1}
(d) 7 ms^{-1} and 3 ms^{-1}

Direction (Q. Nos. 28-30) *Each of these questions contains two statements : Statement* I (*Assertion*) *and Statement* II (*Reason*). *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below.*

(a) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true, Statement II is not the correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

28 **Statement I** A particle moving with a constant velocity, changes its direction uniformly.

Statement II In a uniform motion, the acceleration is zero.

29 **Statement I** Two objects moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ in the opposite directions, have their relative velocity along the direction of the one with a larger velocity.

Statement II The relative velocity between two bodies moving with velocity $\mathbf{v}_1$ and $\mathbf{v}_2$ in same direction is given by $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$

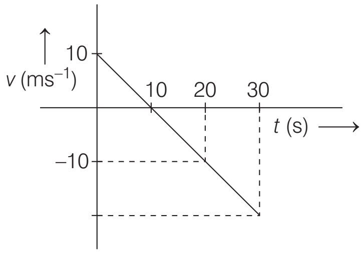
30 **Statement I** Acceleration of a moving particle can be change without changing direction of velocity.

Statement II If the direction of velocity changes, so the direction of acceleration also changes.

DAY PRACTICE SESSION 2

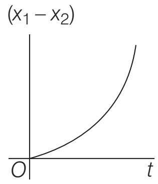
PROGRESSIVE QUESTIONS EXERCISE

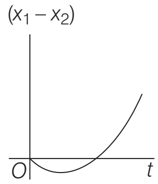
1 The velocity-time plot for a particle moving on a straight line is as shown in figure, then

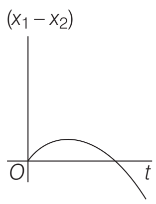


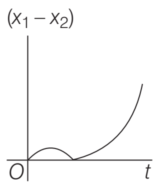
(a) the particle has a constant acceleration
(b) the particle has never turned around
(c) the average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s
(d) Both (a) and (c) are correct

2 A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

(a)


(b)


(c)


(d)


3 The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is

(a) $v_0 + 2g + 3f$
(b) $v_0 + \frac{g}{2} + \frac{f}{3}$
(c) $v_0 + g + f$
(d) $v_0 + \frac{g}{2} + f$

4 A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

5 A stone is dropped from a certain height and reaches the ground in 5 s. If the stone is stopped after 3 s of its fall and then allowed to fall again, then the time taken by the stone to reach the ground after covering the remaining distance is

(a) 2 s (b) 3 s
(c) 4 s (d) None of these

6 A point moves with a uniform acceleration and v_1, v_2, v_3 denote the average velocities in three successive intervals of time t_1, t_2, t_3. Which of the following relations is correct?

(a) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
(b) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
(c) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$
(d) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

7 From the top of a tower of height 50 m, a ball is thrown vertically upwards with a certain velocity. It hits the ground 10 s after it is thrown up. How much time does it take to cover a distance AB where A and B are two points 20 m and 40 m below the edge of the tower? (take, $g = 10\ ms^{-2}$)

(a) 2.0 s (b) 1 s (c) 0.5 s (d) 0.4 s

8 Car A is moving with a speed of 36 kmh^{-1} on a two lane road. Two cars B and C, each moving with a speed of 54 kmh^{-1} in opposite directions on the other lane are approaching car A. At certain instant of time, when the distance $AB = AC = 1$ km, the driver of car B decides to overtake A before C does. What must be the minimum acceleration of car B, so as to avoid an accident ?

(a) 1 ms^{-2} (b) 4 ms^{-2} (c) 2 ms^{-2} (d) 3 ms^{-2}

9 The displacement x of a particle varies with time, according to the relation $x = \frac{a}{b}(1 - e^{-bt})$. Then

(a) the particle can not reach a point at a distance x from its starting position, if $x > a/b$
(b) at $t = 1/b$, the displacement of the particle is nearly $(2/3)(a/b)$
(c) the velocity and acceleration of the particle at $t = 0$ are a and $-ab$ respectively
(d) the particle will come back its starting point as $t \to \infty$

10 From the top of a tower, a stone is thrown up which reaches the ground in time t_1. A second stone thrown down, with the same speed, reaches the ground in time t_2. A third stone released from rest, from the same location, reaches the ground in a time t_3. Then,

(a) $\frac{1}{t_3} = \frac{1}{t_2} - \frac{1}{t_1}$ (b) $t_3^2 = t_1^2 - t_2^2$

(c) $t_3 = \frac{t_1 + t_2}{2}$ (d) $t_3 = \sqrt{t_1 t_2}$

11 A bullet moving with a velocity of $100\,\text{ms}^{-1}$ can just penetrate two plancks of equal thickness. The number of such plancks penetrated by the same bullet, when the velocity is doubled, will be

(a) 4 (b) 6 (c) 8 (d) 10

12 The acceleration in ms^{-2} of a particle is given by, $a = 3t^2 + 2t + 2$ where, t is time. If the particle starts out with a velocity $v = 2\,\text{ms}^{-1}$ at $t = 0$, then the velocity at the end of 2 s is

(a) $36\,\text{ms}^{-1}$ (b) $18\,\text{ms}^{-1}$ (c) $12\,\text{ms}^{-1}$ (d) $27\,\text{ms}^{-1}$

13 A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is 15 s, then

(a) $s = ft$ (b) $s = \frac{1}{6}ft^2$

(c) $s = \frac{1}{72}ft^2$ (d) $s = \frac{1}{4}ft^2$

14 The displacement of a particle is given by $x = (t-2)^2$ where, x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is

(a) 4 m (b) 8 m (c) 12 m (d) 16 m

15 A metro train starts from rest and in five seconds achieves $108\,\text{kmh}^{-1}$. After that it moves with constant velocity and comes to rest after travelling 45 m with uniform retardation. If total distance travelled is 395 m, find total time of travelling.

(a) 12.2 s (b) 15.3 s
(c) 9 s (d) 17.2 s

16 From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is **→ 2014 JEE Main**

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2$
(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)^2u^2$

17 An object, moving with a speed of $6.25\,\text{ms}^{-1}$, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where, v is the instantaneous speed. The time taken by the object, to come to rest, would be

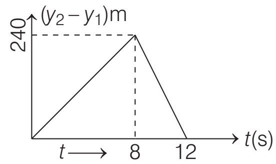
(a) 2 s (b) 4 s (c) 8 s (d) 1 s

18 A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $\frac{T}{3}$ s?

(a) $\frac{h}{9}$ m from the ground (b) $\frac{7h}{9}$ m from the ground

(c) $\frac{8h}{9}$ m from the ground (d) $\frac{17h}{18}$ m from the ground

19 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed at $10\,\text{ms}^{-1}$ and $40\,\text{ms}^{-1}$, respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\,\text{ms}^{-2}$. The figures are schematic and not drawn to scale **→ JEE Main 2015**

(a)


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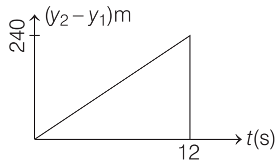

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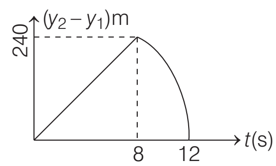
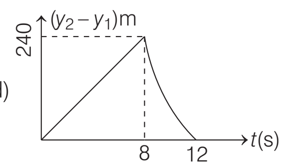


(d)


ANSWERS

SESSION 1	**1** (a)	**2** (a)	**3** (c)	**4** (a)	**5** (a)	**6** (b)	**7** (b)	**8** (b)	**9** (b)	**10** (b)
	11 (d)	**12** (b)	**13** (b)	**14** (c)	**15** (a)	**16** (d)	**17** (b)	**18** (a)	**19** (c)	**20** (c)
	21 (d)	**22** (c)	**23** (b)	**24** (b)	**25** (b)	**26** (b)	**27** (c)	**28** (d)	**29** (d)	**30** (c)
SESSION 2	**1** (d)	**2** (b)	**3** (b)	**4** (a)	**5** (c)	**6** (b)	**7** (d)	**8** (a)	**9** (b)	**10** (d)
	11 (c)	**12** (b)	**13** (c)	**14** (b)	**15** (d)	**16** (c)	**17** (a)	**18** (c)	**19** (c)	

Hints and Explanations

SESSION 1

1 Displacement along North

$= 400 - 300 = 100\,\text{m}$

Upward displacement $= 1200\,\text{m}$

$\therefore$ Net displacement

$= \sqrt{(100)^2 + (1200)^2}$

$= 1204.15\,\text{m} \approx 1200\,\text{m}$

2 When a body is projected vertically upwards, at the highest point of its motion, the velocity of the body becomes zero but acceleration is not zero.

3 Time taken to travel first half distance,

$$t_1 = \frac{L/2}{v_1} = \frac{L}{2v_1}$$

Time taken to travel second half distance,

$$t_2 = \frac{L/2}{v_2} = \frac{L}{2v_2}$$

Total time $= t_1 + t_2$

$$= \frac{L}{2v_1} + \frac{L}{2v_2}$$

$$= \frac{L}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$$

$$\therefore \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{L}{\frac{L}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]}$$

$$= \frac{2v_1v_2}{v_1 + v_2}$$

4 Since, average velocity

$$= \frac{\text{total displacement}}{\text{total time taken}}$$

$$= \frac{25}{\left(\frac{75}{15}\right)}$$

$$= \frac{25 \times 15}{75} = 5\,\text{m/s}$$

5 As, $u = 0$ and a is a constant

$$\frac{l}{4} = \frac{1}{2}a(2)^2 \qquad \ldots\text{(i)}$$

$$l = \frac{1}{2}at^2 \qquad \ldots\text{(ii)}$$

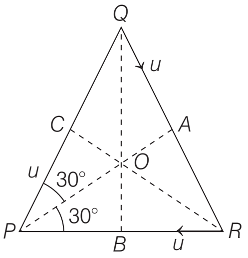
On dividing Eq. (ii) by Eq. (i), we get

$$\frac{l}{l/4} = \frac{t^2}{(2)^2},$$

$$t = 4\,\text{s}$$

6 The person at P will travel a distance PO, with velocity along $PO = u\cos 30°$

Here,



$$PO = PB\sec 30°$$

$$= \frac{d}{2} \times \frac{2}{\sqrt{3}} = \frac{d}{\sqrt{3}}$$

$\therefore$ Time of meeting,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{d/\sqrt{3}}{u\cos 30°}$$

$$= \frac{d/\sqrt{3}}{u\sqrt{3}/2}$$

$$= \frac{2d}{3u}\,\text{s}$$

7 When a body is thrown up, its velocity goes on decreasing as air resistance is small. When a body falls down, its velocity goes on increasing as air resistance is large, t_2 increases.

8 As, $s_n = u + \frac{g}{2}(2n - 1)$

$$= 0 + \frac{g}{2}(2n - 1)$$

Distance travelled in the first three second

From $s = ut + \frac{1}{2}at^2$

$$s_3 = 0 \times 3 + \frac{1}{2} \times g \times 3^2 = \frac{9}{2}g$$

As, $S_n = s_3$

$$\frac{g}{2}(2n - 1) = \frac{9}{2}g$$

$$2n - 1 = 9$$

$$n = 5\text{s}$$

9 Let s_1 be the distance travelled by the train moving with acceleration 1ms^{-2} for time t_1 and s_2 be the distance travelled by the train moving with retardation 3 ms^{-2} for time t_2. If v is the velocity of the train after time t_1, then

$$v = 1 \times t_1 \qquad \ldots\text{(i)}$$

$$s_1 = \frac{1}{2} \times 1 \times t_1^2 = \frac{t_1^2}{2} \qquad \ldots\text{(ii)}$$

Also, $v = 3t_2 \qquad \ldots$(iii)

and $s_2 = vt_2 - \frac{1}{2} \times 3 \times t_2^2$

$$= t_1t_2 - \frac{3}{2}t_2^2$$

From Eqs. (i) and (iii), we get

$$t_1 = 3t_2 \quad \text{or} \quad t_2 = \frac{t_1}{3}$$

$$s_1 + s_2 = \frac{t_1^2}{2} + t_1 \times \frac{t_1}{3} - \frac{3}{2} \times \frac{t_1^2}{9} = \frac{2}{3}t_1^2$$

$$1215 = \frac{2}{3}t_1^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{3 \times 1215}{2}}$$

$$= 42.69\,\text{s}$$

$$\text{Total time} = t_1 + t_2 = t_1 + \frac{t_1}{3} = 56.9\,\text{s}$$

10 From $v^2 - u^2 = 2as$

$$\Rightarrow \frac{80^2 - 60^2}{2a} = s$$

$$\Rightarrow \quad s = \frac{6400 - 3600}{2a}$$

$$= \frac{1400}{a}$$

The middle point of the train has to cover a distance

$$\frac{s}{2} = \frac{700}{a}$$

From $v^2 - u^2 = 2as$

$$v^2 - 60^2 = 2a \times \frac{700}{a} = 1400$$

$$v^2 = 1400 + 3600$$

$$v = \sqrt{5000}$$

$$= 70.7\ \text{kmh}^{-1}$$

11 Given, acceleration $a = -kv^3$

Initial velocity at cut-off, $v_1 = v_0$

Initial time of cut-off, $t = 0$ and final time after cut-off, $t_2 = t$

Again, $a = \frac{dv}{dt} = -kv^3$ or $\frac{dv}{v^3} = -kdt$

Integrating both sides, with in the condition of motion.

$$\int_{v_0}^{v} \frac{dv}{v^3} = -\int_0^t k\,dt$$

or $$\left[-\frac{1}{2v^2}\right] = -[kt]_0^t$$

or $$\frac{1}{2v^2} - \frac{1}{2v_0^2} = kt$$

or $$v = \frac{v_0}{\sqrt{1 + 2kt\,v_0^2}}$$

12 Using, $s_n = u + \frac{a}{2}(2n-1)$

$$12 = u + \frac{a}{2}(2 \times 3 - 1) \quad \ldots\text{(i)}$$

$$20 = u + \frac{a}{2}(2 \times 5 - 1) \quad \ldots\text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$8 = \frac{a}{2}(10-6) = 2a$$

$$a = 4\,\text{ms}^{-2}$$

From Eq. (i), $12 = u + \frac{4}{2} \times 5$

$$u = 2\,\text{ms}^{-1}$$

From $v = u + at = 2 + 4 \times 10$

$$= 42\,\text{ms}^{-1}$$

13 As, $v = u + at_1 \quad \ldots\text{(i)}$

$$40 = 0 + a \times 20$$

$$a = 2\,\text{ms}^{-2}$$

Now, $v^2 - u^2 = 2as$

$$40^2 - 0 = 2 \times 2 \times s_1$$

$$s_1 = 400\,\text{m}$$

$$s_2 = v \times t_2 \quad \ldots\text{(ii)}$$

$$= 40 \times 20 = 800\,\text{m}$$

and $v = u + at \quad \ldots\text{(iii)}$

$$0 = 40 + a \times 40,$$

$$a = -1\,\text{ms}^{-2}$$

Also, $v^2 - u^2 = 2as$

$$0^2 - 40^2 = 2(-1)\,s_3$$

$$s_3 = 800\,\text{m}$$

$\therefore$ Total distance travelled

$$= s_1 + s_2 + s_3$$

$$= 400 + 800 + 800$$

$$= 2000\,\text{m}$$

and total time taken $= 20 + 20 + 40$

$$= 80\,\text{s}$$

$\therefore$ Average velocity $= \frac{2000}{80} = 25\,\text{ms}^{-1}$

14 Acceleration of the body down the plane $= g\cos\theta$

Distance travelled by ball in time t second is

$$AB = \frac{1}{2}(g\cos\theta)\,t^2 \quad \ldots\text{(i)}$$

From $\Delta\, ABC$,

$$AB = 2R\cos\theta \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$2R\cos\theta = \frac{1}{2}\,g\cos\theta t^2$$

$$t^2 = \frac{4R}{g}$$

or $t = 2\sqrt{\frac{R}{g}}$

15 $a = +g = 10\,\text{ms}^{-2}$,

$s = 65\,\text{m}, t = ?$

As, $s = ut + \frac{1}{2}at^2$

$\Rightarrow 65 = -12t + 5t^2$

$5t^2 - 12t - 65 = 0$

This gives,

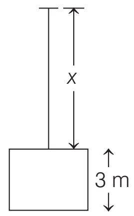
$$t = \frac{12 \pm \sqrt{144 + 1300}}{10} = \frac{12 \pm 38}{10} = 5\,\text{s}$$

16 From $s = ut + \frac{1}{2}at^2$,

$$x = 0 + \frac{1}{2} \times 10t^2 = 5t^2 \quad \ldots\text{(i)}$$

Also, $x + 3 = 0 + \frac{1}{2} \times 10(t + 0.5)^2$

$$= 5\left(t^2 + \frac{1}{4} + t\right) \quad \ldots\text{(ii)}$$



Subtract Eq. (i) from Eq. (ii), we get

$$3 = 5\left(\frac{1}{4} + t\right) = \frac{5}{4} + 5t$$

$$3 - \frac{5}{4} = 5t$$

$$\frac{7}{4} = 5t \text{ or } t = \frac{7}{20}\,\text{s}$$

From $v = u + at$,

$$v = 0 + 10 \times \frac{7}{20} = 3.5\,\text{ms}^{-1}$$

17 $v = 4t^3 - 2t \quad \ldots\text{(i)}$

$$\frac{dx}{dt} = 4t^3 - 2t$$

On integration, we get,

$$x = 2 = t^4 - t^2$$

Let $t^2 = \alpha$

$\therefore \quad 2 = \alpha^2 - \alpha \quad \ldots\text{(ii)}$

Let $t^2 = \alpha$

$$\alpha^2 - \alpha - 2 = 0$$

$$(\alpha - 2)(\alpha + 1) = 0$$

$\therefore \quad \alpha = 2, \ \alpha = -1$, which is not possible

$$t^2 = \alpha = 2 \text{ or } t = \sqrt{2},$$

Differentiating Eq. (i) w.r.t. t,

$$\frac{dv}{dt} = 12t^2 - 2$$

$$a = 12 \times 2 - 2 = 22\,\text{ms}^{-2}$$

18 Acceleration, $a = \frac{dv}{dt} = 5t + 6$

On integrating, we get

$$v = \frac{5}{2}t^2 + 6t = \frac{dx}{dt}$$

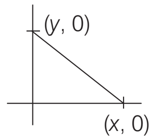
Integrating again,

$$x = \frac{5}{6}t^3 + \frac{6}{2}t^2$$

At $t = 2\text{s}$,

$$x = \frac{5}{6} \times 8 + 3 \times 4 = 18.66\,\text{m}$$

19 If $(x, 0)$ and $(y, 0)$ are the coordinates of the end points of the rod at a given location, then $x^2 + y^2 = l^2$



Differentiating it w.r.t. t, we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -y\frac{dy/dt}{x}$$

and $v_x = -\frac{y}{x}v_y$

As, y decreases, x increases, so v_x decreases.

v_x becomes zero when y is zero.

20 The distance travelled can never be negative in one dimensional motion.

21 In one dimensional motion, there is a single value of displacement at one particular time.

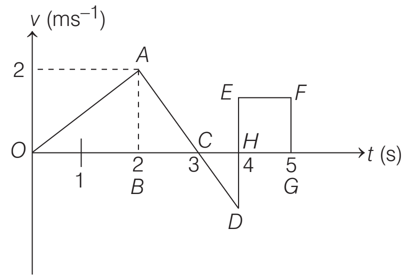
22 As x-t graph is a straight line in either case, velocity of both is uniform. As the slope of x-t graph for P is greater, therefore, velocity of P is greater than that of Q.

23 Maximum acceleration is represented by the maximum slope of the velocity-time graph. Thus, it is the portion CD of the graph, which has a slope $= \frac{80-20}{40-30}$

$$= 6\,\text{ms}^{-2}.$$

24 Displacement is the algebraic sum of area under velocity-time graph.

As, displacement = area of triangles + area of rectangle



$\Delta OAB + \Delta ABC + \Delta CDH + HEFG$

$$= \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 2 + \frac{1}{2} \times 1 \times (-2) + 1 \times 1$$

$$= 2 + 1 - 1 + 1 = 3\,\text{m}$$

25 If velocity *versus* time graph is a straight line with negative slope, then acceleration is constant and negative. With a negative slope distance-time graph will be parabolic $\left(s = ut - \frac{1}{2}at^2\right)$.

So, option (b) will be incorrect.

26 Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

27 **Case I**

Relative velocity is $v_1 + v_2 = 8$

Case II

Relative velocity is $v_1 - v_2 = 2$

On solving,

$v_1 = 5\,\text{ms}^{-1}, v_2 = 3\,\text{ms}^{-1}$

28 When a particle moves with constant velocity, then acceleration of particle is zero and hence particle is not able to change the direction. Hence, statement I is false while statement II is true. Hence, correct answer is (d).

29 When two objects moving in opposite direction, then their relative velocity becomes $(v_1 + v_2)$, hence statement I is false. When moves in same direction, then relative velocity $v = (v_1 - v_2)$, hence statement II is true. Hence, correct answer is (d).

30 Without changing direction of velocity, it is possible to change the acceleration of a moving particle, hence statement I is true, while statement II is false. Hence, correct answer is (c).

SESSION 2

1 The slope of velocity-time graph gives acceleration. Since, the given graph is a straight line and slope of graph is constant. Hence acceleration is constant. Thus, (a) is correct. The area of v-t graph between 0 to 10 s is same as between 10 s to 20 s.

2 As, $x_1(t) = \frac{1}{2}at^2$ and $x_2(t) = vt$

$\therefore \quad x_1 - x_2 = \frac{1}{2}at^2 - vt$ (parabola)

Clearly, graph (b) represents it correctly.

3 As, $v = v_0 + gt + ft^2$ or

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$\Rightarrow dx = (v_0 + gt + ft^2)\,dt$

So, $\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$

$$\Rightarrow \quad x = v_0 + \frac{g}{2} + \frac{f}{3}$$

4 Given, $v = \alpha\sqrt{x}$

or $\frac{dx}{dt} = \alpha\sqrt{x}$ $\quad \left(\because v = \frac{dx}{dt}\right)$

or $\frac{dx}{\sqrt{x}} = \alpha\, dt$

On integration,

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$$

[$\because$ at $t = 0$, $x = 0$ and let at any time t, particle is at x]

$$\Rightarrow \quad \left.\frac{x^{1/2}}{1/2}\right|_0^x = \alpha t$$

or $\quad x^{1/2} = \frac{\alpha}{2}t$

or $\quad x = \frac{\alpha^2}{4} \times t^2$

$\therefore \quad x \propto t^2$

5 From $s = ut + \frac{1}{2}at^2$,

$$s = 0 + \frac{1}{2} \times 10 \times 5^2 = 125\,\text{m}$$

Distance covered in 3 s,

$$= 0 + \frac{1}{2} \times 10 \times 3^2 = 45\,\text{m}$$

Distance to be covered $= 125 - 45 = 80$ m

From $\quad s = ut + \frac{1}{2}at^2$

$$80 = 0 + \frac{1}{2} \times 10t^2$$

$$\Rightarrow \quad t^2 = \frac{80}{5} = 16$$

$\therefore \quad t = 4$ s

6 Suppose velocity at O = zero

As average velocity in interval t_1 is v_1,

$\therefore$ Velocity at $A = v_1$

As average velocity in interval t_2 is v_2,

$\therefore$ Velocity at $B = (v_2 - v_1)$

As average velocity in interval t_3 is v_3,

Velocity at $C = (v_3 - v_2 + v_1)$

Using $\quad v = u + at$

$$v_1 = 0 + at_1 \quad \ldots\text{(i)}$$

$$(v_2 - v_1) = 0 + a(t_1 + t_2) \quad \ldots\text{(ii)}$$

$$(v_3 - v_2 + v_1) = 0 + a(t_1 + t_2 + t_3) \quad \ldots\text{(iii)}$$

Subtract Eq. (i) from Eq. (iii), we get

$$(v_3 - v_2) = a\,(t_2 + t_3) \quad \ldots\text{(iv)}$$

Divide Eq. (ii) by Eq. (iv), we get

$$\frac{(v_2 - v_1)}{(v_3 - v_2)} = \frac{a(t_1 + t_2)}{a(t_2 + t_3)}$$

$$\frac{(v_1 - v_2)}{(v_2 - v_3)} = \frac{t_1 + t_2}{t_2 + t_3}$$

7 Given, $v = -u$, $a = g = 10\,\text{ms}^{-2}$,

$s = 50\,\text{m}, t = 10\,\text{s}$

As, $\quad s = ut + \frac{1}{2}at^2$,

$\Rightarrow \quad 50 = -u \times 10 + \frac{1}{2} \times 10 \times 10^2$

On solving,

$$u = 45\,\text{ms}^{-1}$$

If t_1 and t_2 are the timings taken by the ball to reach the points A and B respectively, then

$$20 = -45t_1 + \frac{1}{2} \times 10 \times t_1^2$$

$$40 = -45t_2 + \frac{1}{2} \times 10 \times t_2^2$$

On solving, we get

$t_1 = 9.4$ s and $t_2 = 9.8$ s

Time taken to cover the distance AB,

$$= (t_2 - t_1)$$

$$= 9.8 - 9.4 = 0.4\,\text{s}$$

8 Let us suppose that the cars A and B are moving in the positive x-direction. Then, car C is moving in the negative x-direction.

Therefore,

$v_A = 36\,\text{kmh}^{-1} = 10\,\text{ms}^{-1}$

$v_B = 54\,\text{kmh}^{-1} = 15\,\text{ms}^{-1}$

and $v_C = -54\,\text{kmh}^{-1} = -15\,\text{ms}^{-1}$

Thus, the relative speed of B with respect to A is,

$$v_{BA} = v_B - v_A = 15 - 10 = 5\,\text{ms}^{-1}$$

and the relative speed of C with respect to A is,

$$v_{CA} = v_C - v_A = -15 - 10 = -25\,\text{ms}^{-1}$$

At time $t = 0$, the distance between A and B = distance between A and $C = 1\,\text{km} = 1000$ m.

The car C covers a distance $AC = 1000$ m and reaches car A at a time t given by

$$t = \frac{AC}{|v_{CA}|} = \frac{1000\,\text{m}}{25\,\text{ms}^{-1}} = 40\,\text{s}$$

Car B will overtake car A just before car C does and the accident can be avoided if it acquires a minimum acceleration a such that it covers a distance, $s = AB = 1000$ m in time $t = 40$ s travelling with a relative speed of

$$u = v_{BA} = 5\,\text{ms}^{-1}.$$

This gives, from

$$s = ut + \frac{1}{2}at^2, a = 1\,\text{ms}^{-2}$$

9 Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}(1 - e^{-bt})\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}(ae^{-bt}) = -\,abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}(1 - e^{-1}) \approx \frac{a}{b}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\frac{a}{b}$$

$$\left(\because e^{-1} \approx \frac{1}{3}\right)$$

Thus, choice (b) is correct. At $t = 0$, the value v and α are $v = ae^{-0} = a$

and $\alpha = -\,ab\,e^{-0} = ab$

The displacement x is maximum, when

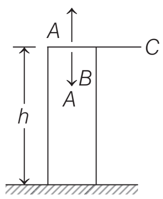
$$t \to \infty,$$

i.e. $$x_{\max} = \frac{a}{b}(1 - e^{-\infty}) = \frac{a}{b}$$

10 We know that, $h = ut + \frac{1}{2}gt^2$

$$\Rightarrow \quad h = -\,ut + \frac{1}{2}gt^2$$

and $$t = \frac{2u}{g} = t_1 - t_2 \qquad \ldots\text{(i)}$$



***Case* I** $h = -\,ut_1 + \frac{1}{2}gt_1^2$ ...(ii)

***Case* II** $h = +\,ut_2 + \frac{1}{2}gt_2^2$ ...(iii)

***Case* III** $h = \frac{1}{2}gt_3^2$ ...(iv)

This gives,

$$\frac{2h}{g} = \frac{2u}{g}t_2 + t_2^2 \qquad \ldots\text{(v)}$$

Solving these, give us

$$t_3^2 = (t_1 - t_2)t_2 + t_2^2$$

$$\Rightarrow \quad t_3 = \sqrt{t_1 t_2}$$

11 Given that the initial velocity of the bullet in the first case is $u_1 = 100\ \text{ms}^{-1}$.

Initial number of plancks, $n_1 = 2$

Initial stopping distance

$$= s_1 = n_1\,x = 2x,$$

with x as the thickness of one planck.

Similarly, Initial velocity of the bullet in second case,

$$u_2 = 2 \times 100 = 200\ \text{ms}^{-1}$$

We know that the relation for the stopping distance s is

$$v^2 = u^2 + 2as$$

Since, $v = 0,$

So, $2as = -\,u^2$

As, $s \propto u^2$

Hence, $\frac{s_1}{s_2} = \left(\frac{u_1}{u_2}\right)^2 = \left(\frac{100}{200}\right)^2 = \frac{1}{4}$

Thus, $s_2 = 4s_1 = 8x$

Hence, the number of plancks

$$= n_2 = \frac{s_2}{x} = 8$$

12 Given, $a = \frac{dv}{dt} = 3t^2 + 2t + 2$

$$\Rightarrow \quad dv = (3t^2 + 2t + 2)\,dt$$

On integrating, this gives

$$\int_u^v dv = \int_0^2 (3t^2 + 2t + 2)\,dt$$

$$\Rightarrow \quad v - u = \left[\frac{3t^3}{3} + \frac{2t^2}{2} + 2t\right]_0^2$$

$$\Rightarrow \quad v = u + [t^3 + t^2 + 2t]_0^2$$

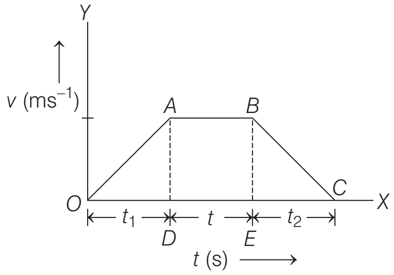
$$v = 2 + [2^3 + 2^2 + 2 \times 2]$$

$$= 2 + 16$$

$$= 18\ \text{ms}^{-1}$$

13 The velocity-time graph for the given situation can be drawn as below.

Magnitudes of slope of $OA = f$



and slope of $$BC = \frac{f}{2}$$

$$v = ft_1 = \frac{f}{2}t_2$$

$$t_2 = 2t_1$$

In graph area of $\Delta\,OAD$ gives distance,

$$s = \frac{1}{2}ft_1^2 \qquad \ldots\text{(i)}$$

Area of rectangle $ABED$ gives distance travelled in time t

$$s_2 = (ft_1)t$$

Distance travelled in time t_2,

$$s_3 = \frac{1}{2}\frac{f}{2}(2t_1)^2$$

Thus, $s_1 + s_2 + s_3 = 15\,s$

$$\Rightarrow \quad s + (ft_1)t + ft_1^2 = 15\,s$$

or $$s + (ft_1)t + 2s = 15\,s$$

$$\left(s = \frac{1}{2}ft_1^2\right)$$

or $$(ft_1) = 12s \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{12s}{s} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1}$$

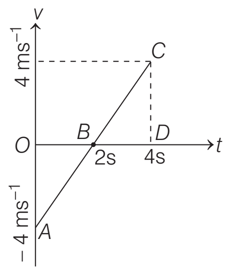
or $$t_1 = \frac{t}{6}$$

From Eq. (i), we get

$$s = \frac{1}{2}f(t_1)^2$$

$$= \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{1}{72}ft^2$$

14 Here, $x = (t - 2)^2$



Velocity, $v = \frac{dx}{dt} = 2(t - 2)\,\text{ms}^{-1}$

Acceleration, $a = \frac{dv}{dt} = 2\,\text{ms}^{-2}$

(i.e. uniform)

When $t = 0, v = -4\,\text{ms}^{-1}$,

$t = 2\,\text{s},\ v = 0,\ t = 4\text{s},\ v = 4\,\text{ms}^{-1}$

Velocity (v) - time (t) graph of this motion is as shown in figure.

Distance travelled

= Area AOB + Area BCD

$$= \frac{4 \times 2}{2} + \frac{4 \times 2}{2} = 8\ \text{m}$$

15 Given, $v = 108\ \text{kmh}^{-1} = 30\ \text{ms}^{-1}$

From first equation of motion

$$v = u + at$$

$\therefore$ $30 = 0 + a \times 5$ $(\because u = 0)$

or $a = 6\ \text{ms}^{-2}$

So, distance travelled by metro train in 5 s

$$s_1 = \frac{1}{2}at^2 = \frac{1}{2} \times (6) \times (5)^2 = 75\ \text{m}$$

Distance travelled before coming to rest $= 45\,\text{m}$

So, from third equation of motion

$$0^2 = (30)^2 - 2a' \times 45$$

or $$a' = \frac{30 \times 30}{2 \times 45} = 10\ \text{ms}^{-2}$$

Time taken in travelling 45 m is

$$t_3 = \frac{30}{10} = 3\text{ s}$$

Now, total distance = 395 m

i.e. $\quad 75 + s' + 45 = 395$ m

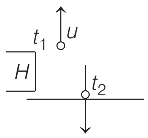
or $\quad s' = 395 - (75 + 45) = 275$ m

$\therefore \quad t_2 = \frac{275}{30} = 9.2$ s

Hence, total time taken in whole journey $= t_1 + t_2 + t_3$

$= 5 + 9.2 + 3 = 17.2$ s

16 Time taken to reach the maximum height, $t_1 = \frac{u}{g}$



If t_2 is the time taken to hit the ground,

i.e. $\quad -H = ut_2 - \frac{1}{2}gt_2^2$

But $\quad t_2 = nt_1$ [Given]

So, $\quad -H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2}$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g}$$

$$H = \frac{1}{2}\frac{n^2u^2}{g} - \frac{nu^2}{g} = \frac{n^2u^2 - 2nu^2}{2g}$$

$$2gH = n^2u^2 - 2nu^2$$

$$2gH = nu^2(n-2)$$

17 Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$\Rightarrow \quad \frac{dv}{\sqrt{v}} = -2.5dt$

$\Rightarrow \quad \int_{6.25}^{0} v^{-1/2}\, dv = -2.5\int_0^t dt$

$\Rightarrow \quad -2.5\,[t]_0^t = [2\, v^{1/2}]_{6.25}^0$

$= 2(-\sqrt{6.25}) = 2 \times 2.5$

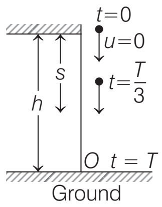
$\Rightarrow \quad t = 2$ s

18 From equation law of motion gives,

$$s = ut + \frac{1}{2}gT^2$$

or $\quad h = 0 + \frac{1}{2}gT^2 \qquad (\because u = 0)$

$\Rightarrow \quad T = \sqrt{\left(\frac{2h}{g}\right)}$



At, $\quad t = \frac{T}{3}$ s,

$$s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$$

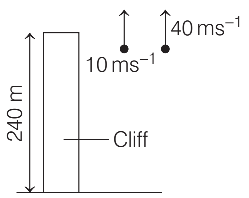
or $\quad s = \frac{1}{2}g \cdot \frac{T^2}{9}$

$\Rightarrow \quad s = \frac{g}{18} \times \frac{2h}{g} \qquad \left(\therefore T = \sqrt{\frac{2h}{g}}\right)$

or $\quad s = \frac{h}{9}$ m

Hence, the position of ball from the ground $= h - \frac{h}{9} = \frac{8h}{9}$ m

19 Central idea concept of relative motion can be applied to predict the nature of motion of one particle with respect to the other.



Consider the stones thrown up simultaneously as shown in the diagram below. As motion of the second particle with respect to the first we have relative acceleration

$$|a_{21}| = |a_2 - a_1| = g - g = 0.$$

Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time is given by

$$-240 = 10t - \frac{1}{2} \times 10 \times t^2$$

or $\quad t^2 - 2t - 48 = 0$

or $\quad t^2 - 8t + 6t - 48 = 0$

or $\quad t = 8, -6$

[As, $t = -6$s is not possible]

i.e., $\quad t = 8$ s

Thus, distance covered by second particle with respect to first particle in 8 s is

$$s_{12} = (v_{21})t = (40 - 10)\,(8\text{s})$$

$= 30 \times 8$

$= 240$ m

Similarly, time taken by second particle to strike the ground is given by

$$-240 = 40t - \frac{1}{2} \times 10 \times t$$

or $\quad -240 = 40t - 5t^2$

or $\quad 5t^2 - 40t - 240 = 0$

or $\quad t^2 - 8t - 48 = 0$

$\quad t^2 - 12t + 4t - 48 = 0$

or $\quad t(t - 12) + 4(t - 12) = 0$

or $\quad t = 12, -4$

(As, $t = -4$ s is not possible)

i.e. $\quad t = 12$ s

Thus, after 8 s, magnitude of relative velocity will increase upto 12 s when second particle strikes the ground. Hence, graph (c) is the correct description.