8 Percentage

Percentages are useful for comparing information where the sample sizes or totals are different. By converting different data to percentages you can readily compare them.

Why would a reporter need to understand percentages, fractions or statistics right? Well it just so happens that basic math knowledge is very important. In fact, good math skills can make all the difference between a successful and not-so-successful career. Journalists often depend on other sources for data, but it is important to understand conversions as well.

Percentages: The most useful statistics ever invented': William Buchanan. No meaningful discussion on percentage can start anywhere other than this great quote. But before we get down to understanding why the quote is so apt, let's look around for percentages which are part of our daily life:

- The marks obtained in an exam but awarded as percent or the 'final average marks at the end of an academic year'.
- The interest rate charged by banks on housing loan or a vehicle loan.
- The interest rate on the Fixed Deposits you parents keep in a bank as part of their savings plan.
- The VAT charged at shops on purchase at a shop or restaurants.
- The 'rate of inflation' announced by government every fortnight.

Interestingly: We all have at least a vague idea of what the aforementioned five uses of percentage mean (e.g. a higher interest rate on Fixed Deposit earns a higher amount of interest). But why we must have a far more than 'vague idea' about percentage? Simply because it's one of the best tools to express certain forms of number (and it's used very extensively).

The next obvious question is – why do we use percent and why it's so uniquely a powerful form of numbers? Let's recollect a couple of everyday life experiences to find answer to the above mentioned question:

- In math, you secured 35 marks in your SA-1 (Summative Assessment 1) and your friend in another school secured 40 marks in SA-1. Your friend has been telling everyone how he performed better than you and it annoys you to no end because you (somehow) know the truth he is not academically better than you; but you fail to understand the 'mystery' of his higher marks. You talk to your parents about it and they advised you to find out the maximum marks of his SA-1. The truth immediately tumbled out – his maximum mark was 100 while in your case it was only 50! And here is why percentage is a very powerful way to express numbers.

  If the marks were given out as 'percent' there would never have been a confusion – you had secured 70% while your friend secured only 40% – far lower than you (don't worry about not getting the '%' idea fully as yet, you will soon learn all about percent).
- A big retail shop has over 6000 different items. Come the 'sale period' it has to broadcast all the prospective buyers (i.e. people who may buy) the amount of discount it's offering on all the different items on sale; let's assume that 1800 different items of the 6000 are put on 'sale'. The most obvious way to tell the buyers about the amount of discount on each of the 1800 items is to make a list of the 1800 items and list the MRP and discounted price against each. But this is very impractical in several ways – how would the shop advertise for 1800 items (it will also be very expensive to advertise), who will spend time to go through the long list to find the discounted price of items one intends to buy, and how will the price tags of each of the item will be re-written (and more such issues).

  Once again, percentage is a very powerful way to express the discount numbers – the simplest way to share the amount of discount being offered is to announce a percentage figure for discount on the items! And this is what big retails shops do – they advertise a number in '%' form (e.g. '30% sale') and the exact amount of discount on individual items gets automatically defined – there is just one figure for all the items on sale! A very simple way of handling changes in descriptions of large numbers of items.
- In fact, there are many situations where the only practical way to communicate a number is to express it in the form of percentage. For example, growth in profit and revenues of companies, performance bonus to executives in a company, government taxes.

More importantly, percentages can be compared more easily than fractions. How easy is to compare 5/17 and 11/19? And how easy is it to compare 29.4% (5/17) and 57.8% (11/19)? Obviously, comparison of numbers in percentage form is very easy.

Having discovered that we live in a world of numbers in percent form, we must explore the mathematics behind percentage.

Results on Depreciation: Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

- Value of the machine after n year $= P\left(1-\frac{R}{100}\right)^{n}$
- Value of the machine n years ago $= \frac{P}{\left(1+\frac{R}{100}\right)^{n}}$
- If A is R% more than B, then B is less than A by $\left[\frac{R}{(100+R)}\times 100\right]\%$
- If A is R% less than B, then B is more than A by $\left[\frac{R}{(100-R)}\times 100\right]\%$

Exploring percentage: A percentage is a part of a whole. It can take on values between 0 (none of the whole) and 100 (all of the whole). The whole is called the base. The base must always be reported whenever a percentage is to be determined and that is done by adding the '%' sign whenever we have to express a number as percentage.

The unique thing about percentage is that every number expressed as percentage 'out of 100'. Thus, if it was 420 is boys out of a total of 600 students, this is how we write:

- As a fraction – 420/600 (boys/total students)
- As a ratio – 420 : 600 (420 boys in 600 students)
- As a percentage – 70% (70 boys in 100 students)

All the three forms of numbers mentioned above are equal and represent the same reality of 420 boys in 600 students. Percentage transforms the total to 100 (in the above case, 600 is made into 100).

Percent must bear some relationship with other forms of number representing 'part of a whole' i.e. fraction and ratio/proportion. We may actually start our discovery of percent by comparing all the 'parts of a whole'.

Concepts of True Discount: Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount of R. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of RS. 156 due 4 years hence. We say that:
Sum due = Rs. 156 due 4 years hence;
Present Worth (P.W.) = Rs. 100
True Discount (T.D.) = Rs. (156 – 100) = Rs. 56 = (Sum due) – (P.W.)
We define: T.D. = Interest on P.W.; Amount = (P.W.) + (T.D.)
Interest is reckoned on P.W. whereas true discount is reckoned on the amount.

How does a percentage differ from a fraction and a ratio?
Fractions and ratios also represent parts of a whole, but both take on values between 0 (none of the whole) and 1 (all of the whole), rather than between 0 and 100. To convert a number given in the form of a fraction or a ratio to number in the form of a percentage you multiply by 100 and add a % sign because we have to make them represent values from 0 to 100; the whole is 100 in percentage rather than 1.
The following examples illustrate the percents and their fractional values:

Example 1. A student gets 60 percent marks in Arithmetic means that he obtained 60 marks out every hundred of full marks. That is, if the full marks be 500, he gets $60+60+60+60=300$ marks in mathematics.
Solution: The total marks obtained by the student can be calculated in other ways, like, 60% of $500=\frac{60}{100}\times 500=300$
The above calculations can be made easier by reducing the fractional value to its prime. As, in the above case; $60\%=\frac{60}{100}=\frac{3}{5}$
If we remember that $60\%=\frac{3}{5}$, our calculation becomes easier.
In that case, the total marks obtained by the student $=\frac{3}{5}\times 500=300$

Example 2. A man invests 5% of his income into shares.
Solution:

$\Rightarrow$ It means: he invests 5 out of every 100 of his income into shares.

$\Rightarrow$ 2nd means: he invests $\frac{5}{100}$ of his income into shares.

$\Rightarrow$ 3rd means: she invests $\frac{1}{20}$th of his income into shares.

Now, if his income is Rs 1050, how does he invest in shares?

Your quick answer should be $\frac{1050}{20} = \text{Rs } 52.5$

We suggest you not to move with the fraction contain 100, if possible.

Example 3. The price of a sweater went up 20% since last year. If last year's price was x, what is this year's price in terms of x?

Solution: Last year's price $= 100\ \%$ of x

This year's price is 100% of x plus 20% of x.

$\Rightarrow \quad x + 20\%$

$\Rightarrow \quad x = x + 0.2x = 1.2x$

Example 4. One year ago, an average restaurant meal cost Rs 120. Today, the average restaurant meal costs Rs 150. By what percent has the cost of the meal increased?

Solution: You can figure percent increase by taking the difference in prices first and then expressing it as a percentage of the original price: Rs 150 – Rs 120 = Rs 30 difference

What percentage of the original price is Rs 30?

$\Rightarrow \quad \frac{30}{120} = \frac{x}{100}$

$\Rightarrow \quad 12x = 300$

$\Rightarrow \quad x = 25$

The cost increased by 25%.

Or you can figure what percent the new price is of the old price: 15 is what percent of 12?

$\Rightarrow \quad 15 = \left(\frac{x}{100}\right)12$

$\Rightarrow \quad \frac{15}{12} = \frac{x}{100} \quad \Rightarrow x = 125$

Therefore, 15 is 125 percent of 12.

This tells you what percent the current price ($15) is of the old price ($12). But the question asks for the percent increase, so you have to subtract 100 percent from 125 percent.

$125\% - 100\% = 25\%$ increase

Percentage as a Ratio: A percent can be expressed as a ratio with its second term 100 and first term equal to the given percent.

For example, $8\% = \frac{8}{100} = 8:100;\ 36\% = \frac{36}{100} = \frac{9}{25} = 9:25$

Percent in Decimal Form: To convert a given percent in decimal form, we express it as a fraction with denominator as 100 and then the fraction is written in decimal form.

For example, $65\% = \frac{65}{100} = 0.65;\ 7.4\% = \frac{7.4}{100} = 0.074$

Example 5. What percent is Rs. 50 of Rs. 250?

Solution: Out of Rs 250, Rs 50 is written as $\frac{50}{250}$

Now, converging into percentage

We have, $\left(\frac{50}{250} \times 100\right) = 20\%$

$\Rightarrow$ Rs 50 is 20% of Rs 250.

Multiple Choice Questions

1. 4% of $400 - 2\%$ of $800 = ?$

a. 2 **b.** –4 **c.** 0 **d.** 16

2. 40 % of $70 = 4 \times ?$

a. 28 **b.** 280 **c.** 7 **d.** 70

3. 0.2 % if $? = 0.03$

a. 20 **b.** 2.5 **c.** 15 **d.** 1.5

4. 0.025 in terms of rate percent is

a. $\frac{1}{4}\%$ **b.** 25% **c.** 2.5% **d.** $37\frac{1}{2}\%$

5. What is 25% of 25% equal to?

a. 6.25 **b.** 0.625

c. 0.0625 **d.** 0.00625

6. What percent of 7.2 kg is 18 gms?

a. 25% **b.** 2.5% **c.** 0.25% **d.** 0.025%

7. What percent of $\frac{2}{7}$ is $\frac{1}{35}$?

a. 25% **b.** 2.5%

c. 1000% **d.** 10%.

8. $\left(0.756 \times \frac{3}{4}\right)$ is equivalent to:

a. 18.9% **b.** 37.8% **c.** 56.7% **d.** 75%

9. $\frac{8\% \text{ of } 80}{8\% \text{ of } 40} = \frac{?}{8}$

a. 12 **b.** 16 **c.** 19 **d.** 22

10. $\frac{7}{8}$ is what % of $\frac{3}{4}$?

a. $116\frac{1}{3}\%$ **b.** 116% **c.** $116\frac{3}{2}\%$ **d.** $116\frac{2}{3}\%$

11. $33\frac{1}{3}\%$ of a number is 33. Find the number.

a. 9 **b.** 90

c. 99 **d.** 0.9

12. In a school the ratio between boys and girls is 3 : 4. Find the percentage of boys in the school number.

a. $42\frac{6}{7}\%$ **b.** 75 %

c. 50% **d.** 80%

13. Rohan's monthly income is Rs. 12000. If this monthly income is increased by 10% find the total income after 2 months?

a. 14800 **b.** 14520

c. 14502 **d.** 14500

14. A student has to score 60% to pass exam. He scores 225 marks and failed by 15%. Find the maximum marks of exam.

a. 400 **b.** 500

c. 600 **d.** Can't be determined

15. A student scores 25% and failed by 30 marks while another student who scores 60% get 40 marks more than minimum required marks to pass. Find the maximum marks in the exam?

a. 150 **b.** 175 **c.** 200 **d.** 225

16. In an election a candidate who gets 84% of the total votes and wins by 476 votes. What is the total number of votes polled?

a. 672 **b.** 700

c. 749 **d.** 848

17. The total population of a village is 5000. The number of males and females increases by 10% and 15% respectively and consequently the population of the village becomes 5600. What was the number of males in the village?

a. 2000 **b.** 2500

c. 30000 **d.** 4000

18. Two numbers are less than a third number by 30% and 37% respectively. How much percent as a percentage of the second number less than the first?

a. 3 % **b.** 4 %

c. 7% **d.** 10%

19. 40% of the population of a town are men and 35% are women. If the number of children are 20000, then the number of men will be:

a. 3200 **b.** 80000

c. 32000 **d.** 3,20,000

20. If the price of sugar is increased by 7%, then by how much percent should a housewife reduce her consumption of sugar, to have no extra expenditure?

a. $6\frac{58}{107}\%$ **b.** $1\frac{7}{100}\%$

c. 7% **d.** $2\frac{58}{107}\%$

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	c	c	c	c	c	d	c	b	d
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	a	b	b	c	b	c	d	c	a

SOLUTIONS

1. **(c)** $(4\% \text{ of } 400) - (2\% \text{ of } 800)$

$$= \left(\frac{4}{100} \times 400\right) - \left(\frac{2}{100} \times 800\right)$$

$$= 16 - 16 = 0.$$

2. **(c)** Let $40\% \text{ of } 70 = 4 \times x$

Then, $\frac{40}{100} \times 70 = 4 \times x$

$$\Rightarrow \quad x = \left(\frac{40}{100} \times 70 \times \frac{1}{4}\right) = 7$$

3. **(c)** $0.2\% \text{ of } x = 0.03$

$$\Rightarrow \quad \frac{0.2}{100} \times x = 0.03$$

$$\therefore \quad x = \frac{0.03 \times 100}{0.20} = 15$$

4. **(c)** $0.025 = \frac{25}{1000} = \frac{1}{40} = \left(\frac{1}{40} \times 100\right)\% = 2.5\%$

5. **(c)** $25\% \text{ of } 25\% = \left(\frac{25}{100} \times \frac{25}{100}\right)$

$$= \frac{625}{10000} = 0.0625$$

6. **(c)** Required percentage

$$= \left(\frac{18}{7.2 \times 1000} \times 100\right)\%$$

$$= 0.25\%$$

7. **(d)** Let x % of $\frac{2}{7}=\frac{1}{35}$

Then $\frac{x}{100}\times\frac{2}{7}=\frac{1}{35}$

$\Rightarrow \quad x=\frac{1}{35}\times\frac{100\times7}{2}=10$

8. **(c)** $0.756\times\frac{3}{4}=\left(\frac{756}{1000}\times\frac{3}{4}\right)$

$=\left(\frac{756\times3}{1000\times4}\times100\right)\%=56.7\%$

9. **(b)** $\frac{8\%\text{ of }80}{8\%\text{ of }40}=\frac{x}{8}$

or $\frac{\frac{8}{100}\times80}{\frac{8}{100}\times40}=\frac{x}{8}$ or $x=16$

10. **(d)** $\frac{\frac{7}{8}}{\frac{3}{4}}\times100$

$=\frac{7}{8}\times\frac{4}{3}\times100=\frac{7}{6}\times100$

$=116\frac{2}{3}\%$

11. **(c)** 99

12 **(a)** $\frac{3}{7}\times100=\frac{300}{7}$

$=42\frac{6}{7}\%$

13. **(b)** $\frac{1200}{100}\times110\times\frac{110}{100}$

14. **(b)** $60\%-15\%=225$

$\Rightarrow \quad 45\%=225$

$\Rightarrow \quad 100\%=\frac{225}{45}\times100=500$ marks

15. **(c)** $25\%-30=60\%+40$

$\Rightarrow \quad 35\%=70$

$\Rightarrow \quad 100\%=\frac{70}{35}\times100=200$ marks

16. **(b)** $84\%-(100\%-84\%)$

$=84\%-16\%$

$=68\%=476$ votes (extra)

$=100\%=\frac{476}{68}\times100=700$

17. **(c)** Total % increased population

$=\frac{5600-5000}{5000}\times100$

$=\frac{600}{5000}\times100=12\%$

By allegation method

$\therefore$ Male population before increment

$=\frac{5000}{5}\times3=3000$

18. **(d)** I : II : III $=70:63:100$

$\therefore$ Required % $=\frac{7}{70}\times100=10\%$

19. **(c)** $25\%=20,000$

$\Rightarrow \quad 40\%=\frac{20,000}{25}\times40=32000$ men

20. **(a)** Less % $=\frac{7}{107}\times100=\frac{700}{107}=6.54\%$

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