9 Statistics and Linear Programming

QUICK LOOK

Measures of Central Tendency

Mean: Arithmetic mean or mean is what is usually thought of when talking about averages. If you want to know the arithmetic mean of a list of values, the formula is:

$$\frac{\text{The sum of a list of values}}{\text{The number of values in the list}}$$

For example, if there are three children, aged 6, 7, and 11, the arithmetic mean of their ages is: $\frac{6+7+11}{3}=\frac{24}{3}$ or 8 years.

The mean of a number of observation is the sum of the values of all the observations divided by the total number of observations. It is denoted by the symbol $\bar{x}$, read as x bar.

Properties of Mean

- If a constant real number 'a' is added to each of the observation than new mean will be $\bar{x}+a$.
- If a constant real number 'a' is subtracted from each of the observation then new mean will be $\bar{x}-a$.
- If a constant real number 'a' is multiplied with each of the observation then new mean will be $\bar{x}$.
- If each of the observation is dived by a constant no 'a' then new mean will be $\frac{\bar{x}}{a}$.

Mean of Ungrouped Data: If $x_1, x_2, x_3, \ldots, x_n$ are then n values (or observations) then A.M. (Arithmetic mean) is

$$\bar{x}=\frac{x_1+x_2+\ldots+x_n}{n}=\frac{\sum_{i-1}^{n} x_i}{n} \quad n\bar{x}=\text{Sum of observation }=\sum_{i-1}^{n} x_i$$

i.e. product of means & no. of items given sum of observation.

- Method for Mean of ungrouped frequency distribution.

x_i	f_i	$f_i x_i$
x_1	f_1	$f_1 x_1$
x_2	f_2	$f_2 x_2$
x_3	f_3	$f_3 x_3$
.	.	.
.	.	.
.	.	.
x_n	f_n	$f_n x_n$
	$\sum f_i =$	$\sum f_i x_i =$

- Method for Mean of grouped frequency distribution. Then mean $\bar{x}=\frac{\sum f_i x_i}{\sum f_i}$
- Combined Mean: $\bar{x}=\frac{n_1\bar{x}_1+n_2+\bar{x}_2+\ldots}{n_1+n_2+\ldots}$
- Uses of Arithmetic Mean
- It is used for calculating average marks obtained by a student.
- It is extensively used in practical statistics.
- It is used to obtain estimates.
- It is used by businessman to find out profit per unit article, output per machine, average monthly income and expenditure etc.
- Deviation Method : (Assumed Mean Method)

  $\bar{x}=A+\frac{\sum f_i d_i}{\sum f_i}$ where, $A=$ Assumed mean

  $d_i=$ Deviation from mean (x_i-A)

Median: The median is the middle value of a list when the numbers are in order. To find the median, place the values in ascending (or descending) order and select the middle value. For instance, what is the median of the following values?
200, 2, 667, 19, 4, 309, 44, 6, 1

- Place the values in ascending order : 1, 2, 4, 6, 19, 44, 200, 309, 667
- Select the value in the middle.
- There are nine values listed. The middle value is the fifth.
- The median of these values is 19.

The Median of a List with an Even Number of Values: When the number of values in a list is an even number, the median is the average (arithmetic mean) of the two middle values, when the numbers are placed in order. For example, the median of 3, 7, 10, 20 is $\frac{7+10}{2}=8.5$

Median of a distribution is the value of the variable which divides the distribution into two equal parts.

Median or Ungrouped data: Arrange the data in ascending order. Count the no. of observations (Let there be 'n' observations)

If n is odd then median $=$ value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

If n is even the median $=$ value of mean of $\left(\frac{n}{2}\right)^{th}$ observation and $\left(\frac{n}{2}+1\right)^{th}$ observation.

Merits of Median

- It is rigidly defined, easily, understood and calculate.
- It is not all affected by extreme values.
- It can be located graphically, even if the class - intervals are unequal.
- It can be determined even by inspection is some cases.

Demerits of Median

- In case of even numbers of observations median cannot be determined exactly.
- It is not based on all the observations and not subject to algebraic treatment.
- It is much affected by fluctuations of sampling.

Uses of Median

- Median is the only average to be used while dealing with qualitative data which cannot be measured quantitatively but can be arranged in ascending or descending order of magnitude.
- It is used for determining the typical value in problems concerning wages, distribution of wealth etc.

Mode: The mode of a list of values is the value or values that appear the greatest number of times. Consider the following list: 1, 5, 5, 7, 89, 4, 100, 276, 89, 4, 89, 1, 8

- The number 89 appears three times, which is more times than any other number appears.
- The mode of this list is 89.

Multiple Modes: It is possible to have more than one mode in a list of numbers: 1, 5, 5, 7, 276, 4, 10004, 89, 4, 276, 1, 8. In the list above, there are four modes: 1, 4, 5 and 276.

- **Mode or ungrouped data (By inspection only):** Arrange the data in an array and then count the frequencies of each variate. The variate having maximum frequency is the mode.

Mode of Continuous Frequency Distribution

Mode $=l+\frac{f_1+f_0}{2f_1-f_0-f_2}\times h$

Where $l=$ lower limit of the modal class

$f_1=$ frequency of the class i.e. the largest frequency.

$f_0=$ frequency of the class preceding the modal class.

$f_2=$ frequency of the class succeeding the modal class.

$h=$ width of the modal class

Uses of Mode: Mode is the average to be used to find the ideal size, e.g., in business forecasting, in manufacture of ready-made garments, shoes etc.

Coefficient of the Range: If l and h are the lowest and highest scores in a distribution then the coefficient of the Range $=\frac{h-l}{h+l}$

Merits of Mode

- It can be easily understood and is easy to calculate.
- It is not affected by extreme values and can be found by inspection is some cases.
- It can be measured even if open - end classes and can be represented graphically.

Demerits of Mode

- It is ill - fined. It is not always possible to find a clearly defined mode.
- It is not based upon all the observation.
- It is not capable of further mathematical treatment. it is after indeterminate.
- It is affected to a greater extent by fluctuations of sampling.

Uses of Mode: Mode is the average to be used to find the ideal size, e.g., in business forecasting, in manufacture of ready-made garments, shoes etc.

- Empirical Relation between Mode, Median & Mean :
  Mode = 3 Median – 2 Mean

Correlation

- **Univariate and Bivariate distribution:** "If it is proved true that in a large number of instances two variables tend always to fluctuate in the same or in opposite directions, we consider that the fact is established and that a relationship exists. This relationship is called correlation."
- **Univariate distribution:** These are the distributions in which there is only one variable such as the heights of the students of a class.
- **Bivariate distribution:** Distribution involving two discrete variable is called a bivariate distribution. For example, the heights and the weights of the students of a class in a school.
- **Bivariate frequency distribution:** Let x and y be two

variables. Suppose x takes the values $x_1, x_2, ..., x_n$ and y takes the values $y_1, y_2, ..., y_n$, then we record our observations in the form of ordered pairs (x_1, y_1), where $1 \le i \le n, 1 \le j \le n$. If a certain pair occurs f_{ij} times, we say that its frequency is f_{ij}.

- The function which assigns the frequencies f_{ij}'s to the pairs (x_i, y_j) is known as a bivariate frequency distribution.

Covariance: Let $(x_i, y_i); i = 1, 2, ..., n$ be a bivariate distribution, where $x_1, x_2, ..., x_n$ are the values of variable x and $y_1, y_2, ..., y_n$ those of y. Then the covariance $Cov(x, y)$ between x and y is given by $Cov(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$

or $Cov(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i y_i - \bar{x}\,\bar{y})$, where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ are means of variables x and y respectively.

Covariance is not affected by the change of origin, but it is affected by the change of scale.

Correlation: The relationship between two variables such that a change in one variable results in a positive or negative change in the other variable is known as correlation.

Types of correlation

- **Perfect correlation:** If the two variables vary in such a manner that their ratio is always constant, then the correlation is said to be perfect.
- **Positive or direct correlation:** If an increase or decrease in one variable corresponds to an increase or decrease in the other, the correlation is said to be positive.
- **Negative or indirect correlation:** If an increase or decrease in one variable corresponds to a decrease or increase in the other, the correlation is said to be negative.
- **Karl Pearson's coefficient of correlation:** The correlation coefficient $r(x, y)$, between two variable x and y is given by, $r(x,y) = \frac{Cov(x,y)}{\sqrt{Var(x)}\sqrt{Var(y)}}$ or $\frac{Cov(x,y)}{\sigma_x \sigma_y}$

$$r(x,y) = \frac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sqrt{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}\sqrt{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}}$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}} = \frac{\Sigma\, dxdy}{\sqrt{\Sigma\, dx^2}\sqrt{\Sigma\, dy^2}}.$$

- **Modified formula**

$$r = \frac{\sum dxdy - \frac{\sum dx . \sum dy}{n}}{\sqrt{\left\{\sum dx^2 - \frac{\left(\sum dx\right)^2}{n}\right\}\left\{\sum dy^2 - \frac{\left(\sum dy\right)^2}{n}\right\}}}$$

Where $dx = x - \bar{x}; dy = y - \bar{y}$

Also, $r_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{Cov(x,y)}{\sqrt{Var(x).Var(y)}}$.

- **Step deviation method**

Let A and B are assumed mean of x_i and y_i respectively, then $r(x,y) = \dfrac{\sum u_i v_i - \frac{1}{n}\sum u_i . \sum v_i}{\sqrt{\sum u_i^2 - \frac{1}{n}\left(\sum u_i\right)^2}\sqrt{\sum v_i^2 - \frac{1}{n}\left(\sum v_i\right)^2}}$

where $u_i = x_i - A, v_i = y_i - B$.

Rank Correlation: Let us suppose that a group of n individuals is arranged in order of merit or proficiency in possession of two characteristics A and B. These rank in two characteristics will, in general, be different. For example, if we consider the relation between intelligence and beauty, it is not necessary that a beautiful individual is intelligent also. Rank Correlation: $\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$, which is the Spearman's formulae for rank correlation coefficient. Where $\sum d^2$ = sum of the squares of the difference of two ranks and n is the number of pairs of observations. We always have,

$\sum d_i = \sum(x_i - y_i) = \sum x_i - \sum y_i = n(\bar{x}) - n(\bar{y}) = 0, (\because \bar{x} = \bar{y})$

If all d's are zero, then $r = 1$, which shows that there is perfect rank correlation between the variable and which is maximum value of r. If however some values of x_i are equal, then the coefficient of rank correlation is given by

$$r = 1 - \frac{6\left[\sum d^2 + \left(\frac{1}{12}\right)(m^3 - m)\right]}{n(n^2-1)}$$

where m is the number of times a particular x_i is repeated.

Positive and Negative Rank Correlation Coefficients

- Let r be the rank correlation coefficient then, if $r > 0$, it means that if the rank of one characteristic is high, then that of the other is also high or if the rank of one characteristic is low, then that of the other is also low.

- $r = 1$, it means that there is perfect correlation in the two characteristics *i.e.,*every individual is getting the same ranks in the two characteristics.
- $r < 1$, it means that if the rank of one characteristics is high, then that of the other is low or if the rank of one characteristics is low, then that of the other is high.
- $r = -1$, it means that there is perfect negative correlation in the two characteristics *i.e,* an individual getting highest rank in one characteristic is getting the lowest rank in the second characteristic. Here the rank, in the two characteristics in a group of n individuals are of the type $(1, n), (2, n-1), \ldots\ldots, (n, 1)$.
- $r = 0$, it means that no relation can be established between the two characteristics.

Standard Error and Probable Error

- **Standard error of prediction:** The deviation of the predicted value from the observed value is known as the standard error prediction and is defined as $S_y = \sqrt{\left\{\frac{\sum (y - y_p)^2}{n}\right\}}$. where y is actual value and y_p is predicted value. In relation to coefficient of correlation, it is given by

  Standard error of estimate of x is $S_x = \sigma_x \sqrt{1 - r^2}$

  Standard error of estimate of y is $S_y = \sigma_y \sqrt{1 - r^2}$.
- **Relation between probable error and standard error:** If r is the correlation coefficient in a sample of n pairs of observations, then its standard error S.E. $(r) = \frac{1 - r^2}{\sqrt{n}}$ and probable error P.E. $(r) = 0.6745$ (S.E.) $= 0.6745 \left(\frac{1 - r^2}{\sqrt{n}}\right)$.

  The probable error or the standard error are used for interpreting the coefficient of correlation.
- If $r <$ P.E.(r), there is no evidence of correlation.
- If $r > 6$P.E.(r), the existence of correlation is certain. The square of the coefficient of correlation for a bivariate distribution is known as the "Coefficient of determination".

Regression

- **Linear regression :**If a relation between two variates x and y exists, then the dots of the scatter diagram will more or less be concentrated around a curve which is called the curve of regression. If this curve be a straight line, then it is known as line of regression and the regression is called linear regression.

  Line of regression: The line of regression is the straight line which in the least square sense gives the best fit to the given frequency.

Equations of Lines of Regression

- **Regression Line of *y* on *x* :** If value of x is known, then value of y can be found as

$$y - \overline{y} = \frac{Cov(x, y)}{\sigma_x^2}(x - \overline{x})$$

or $y - \overline{y} = r\frac{\sigma_y}{\sigma_x}(x - \overline{x})$

- **Regression Line of *x* on *y* :** It estimates x for the given value of y as

$$x - \overline{x} = \frac{Cov(x, y)}{\sigma_y^2}(y - \overline{y})$$

or $x - \overline{x} = r\frac{\sigma_x}{\sigma_y}(y - \overline{y})$

Regression Coefficient

- Regression coefficient of y on x is $b_{yx} = \frac{r\sigma_y}{\sigma_x} = \frac{Cov(x, y)}{\sigma_x^2}$
- Regression coefficient of x on y is $b_{xy} = \frac{r\sigma_x}{\sigma_y} = \frac{Cov(x, y)}{\sigma_y^2}$.

Angle between two lines of regression: Equation of the two lines of regression are

$$y - \overline{y} = b_{yx}(x - \overline{x})$$

and $x - \overline{x} = b_{xy}(y - \overline{y})$.

We have, $m_1 =$ slope of the line of regression of y on

$$x = b_{yx} = r.\frac{\sigma_y}{\sigma_x}$$

$m_2 =$ Slope of line of regression of x on $y = \frac{1}{b_{xy}} = \frac{\sigma_y}{r.\sigma_x}$.

$$\therefore \quad \tan\theta = \pm\frac{m_2 - m_1}{1 + m_1 m_2} = \pm\frac{\frac{\sigma_y}{r\sigma_x} - \frac{r\sigma_y}{\sigma_x}}{1 + \frac{r\sigma_y}{\sigma_x}.\frac{\sigma_y}{r\sigma_x}}$$

$$= \pm\frac{(\sigma_y - r^2\sigma_y)\sigma_x}{r\sigma_x^2 + r\sigma_y^2} = \pm\frac{(1 - r^2)\sigma_x\sigma_y}{r(\sigma_x^2 + \sigma_y^2)}$$

Here the positive sign gives the acute angle θ, because $r^2 \le 1$ and σ_x, σ_y are positive.

$$\therefore \quad \tan\theta = \frac{1-r^2}{r}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2} \qquad \ldots(i)$$

If $r = 0$, from (i) we conclude $\tan\theta = \infty$ or $\theta = \frac{\pi}{2}$ *i.e.,* two regression lines are at right angles. If $r = \pm 1$, $\tan\theta = 0$ *i.e.,* $\theta = 0$, since θ is acute *i.e.,* two regression lines coincide.

Important Points about Regression Coefficients b_{xy} and b_{yx}

- $r = \sqrt{b_{yx}.b_{xy}}$ *i.e.,* the coefficient of correlation is the geometric mean of the coefficient of regression.
- If $b_{yx} > 1$, then $b_{xy} < 1$ *i.e.,* if one of the regression coefficient is greater than unity, the other will be less than unity.
- If the correlation between the variable is not perfect, then the regression lines intersect at $(\bar{x}, \bar{y})$.
- b_{yx} is called the slope of regression line y on x and $\frac{1}{b_{xy}}$ is called the slope of regression line x on y.
- $b_{yx} + b_{xy} > 2\sqrt{b_{yx}b_{xy}}$ or $b_{yx} + b_{xy} > 2r$ *i.e.,* the arithmetic mean of the regression coefficients is greater than the correlation coefficient.
- Regression coefficients are independent of change of origin but not of scale.
- The product of lines of regression's gradients is given by $\frac{\sigma_y^2}{\sigma_x^2}$.
- If both the lines of regression coincide, then correlation will be perfect linear.
- If both b_{yx} and b_{xy} are positive, the r will be positive and if both b_{yx} and b_{xy} are negative, the r will be negative.

Linear Inequations

- **Graph of Linear Inequations**
- **Linear Inequation in One Variable:** $ax + b > 0, ax + b < 0$, $cy + d > 0$ etc. are called linear inequations in one variable. Graph of these inequations can be drawn as follows:

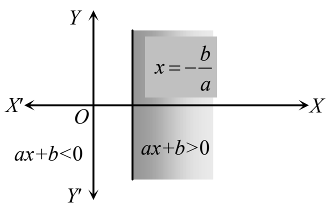


Figure: 9.1

The graph of $ax + b > 0$ and $ax + b < 0$ are obtained by dividing xy-plane in two semi-planes by the line $x = -\frac{b}{a}$ (which is parallel to y-axis). Similarly for $cy + d > 0$ and $cy + d < 0$..

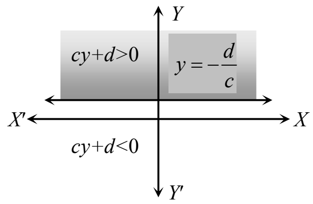


Figure: 9.2

Linear Inequation in Two Variables: General form of these inequations are $ax + by > c, ax + by < c$. If any ordered pair (x_1, y_1) satisfies some inequations, then it is said to be a solution of the inequations.

The graph of these inequations is given below (for $c > 0$):

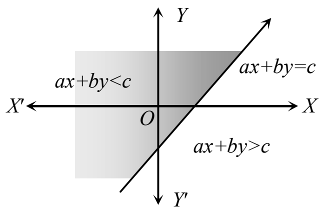


Figure: 9.3

To Draw the Graph of an In-equation, following Procedure is followed

- Write the equation $ax+by=c$ in place of $ax+by<c$ and $ax + by > c$.
- Make a table for the solutions of $ax + by = c$.
- Now draw a line with the help of these points. This is the graph of the line $ax + by = c$.
- If the in-equation is > or <, then the points lying on this line is not considered and line is drawn dotted or discontinuous.
- If the in-euqation is $\geq$ or $\leq$, then the points lying on the line is considered and line is drawn bold or continuous.
- This line divides the plane XOY in two region.

To find the region that satisfies the in-equation, we apply the following rules:

- Take an arbitrary point which will be in either region.
- If it satisfies the given in-equation, then the required region will be the region in which the arbitrary point is located.
- If it does not satisfy the in-equation, then the other region is the required region.
- Draw the lines in the required region or make it shaded.

- **Simultaneous Linear In-equations in Two Variables:** Since the solution set of a system of simultaneous linear in-equations is the set of all points in two dimensional space which satisfy all the in-equations simultaneously. Therefore to find the solution set we find the region of the plane common to all the portions comprising the solution set of given in-equations. In case there is no region common to all the solutions of the given in-equations, we say that the solution set is void or empty.

- **Feasible Region:** The limited (bounded) region of the graph made by two in-equations is called feasible region. All the points in feasible region constitute the solution of a system of in-equations. The feasible solution of a L.P.P. belongs to only first quadrant. If feasible region is empty then there is no solution for the problem.

Terms of Linear Programming

The term programming means planning and refers to a process of determining a particular program.

- **Objective Function:** The linear function which is to be optimized (maximized or minimized) is called objective function of the *L.P.P.*
- **Constraints or Restrictions:** The conditions of the problem expressed as simultaneous equations or inequalities are called constraints or restrictions.
- **Non-negative Constraints:** Variables applied in the objective function of a linear programming problem are always non-negative. The ineqaulities which represent such constraints are called non-negative constraints.
- **Basic Variables:** The m variables associated with columns of the $m \times n$ non-singular matrix which may be different from zero, are called basic variables.
- **Basic Solution:** A solution in which the vectors associated to m variables are linear and the remaining $(n-m)$ variables are zero, is called a basic solution. A basic solution is called a degenerate basic solution, if at least one of the basic variables is zero and basic solution is called non-degenerate, if none of the basic variables is zero.
- **Feasible Solution:** The set of values of the variables which satisfies the set of constraints of linear programming problem (*L.P.P*) is called a feasible solution of the *L.P.P.*
- **Optimal Solution:** A feasible solution for which the objective function is minimum or maximum is called optimal solution.
- **Iso-profit Line:** The line drawn in geometrical area of feasible region of *L.P.P.* for which the objective function remains constant at all the points lying on the line, is called iso-profit line. If the objective function is to be minimized then these lines are called iso-cost lines.
- **Convex Set:** In linear programming problems feasible solution is generally a polygon in first quadrant. This polygon is convex. It means if two points of polygon are connecting by a line, then the line must be inside to polygon. For example,

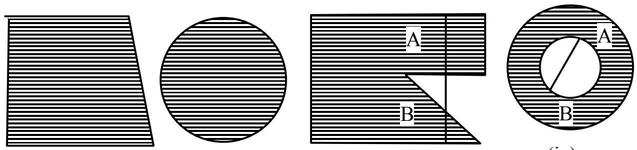


Figure 9.4: (i) and (ii) are convex set while (iii) and (iv) are not convex set

Mathemiatical Formulation and Graphical Solution of a Linear Programming Problem

Mathemiatical Formulation

There are mainly four steps in the mathematical formulation of a linear programming problem, as mathematical model. We will discuss formulation of those problems which involve only two variables.

- Identify the decision variables and assign symbols x and y to them. These decision variables are those quantities whose values we wish to determine.
- Identify the set of constraints and express them as linear equations/inequations in terms of the decision variables. These constraints are the given conditions.
- Identify the objective function and express it as a linear function of decision variables. It might take the form of maximizing profit or production or minimizing cost.
- Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation. There are techniques of solving an *L.P.P.* by graphical method.

Corner Point Method

Working Rule

- Formulate mathematically the *L.P.P.*
- Draw graph for every constraint.
- Find the feasible solution region.
- Find the coordinates of the vertices of feasible solution region.
- Calculate the value of objective function at these vertices.
- Optimal value (minimum or maximum) is the required solution.
- If there is no possibility to determine the point at which the suitable solution found, then the solution of problem is unbounded.
- If feasible region is empty, then there is no solution for the problem.
- Nearer to the origin, the objective function is minimum and that of further from the origin, the objective function is maximum.

Iso-profit or Iso-cost Method: Various steps of the method are as follows:

- Find the feasible region of the *L.P.P.*
- Assign a constant value Z_1 to Z and draw the corresponding line of the objective function.
- Assign another value Z_2 to Z and draw the corresponding line of the objective function.
- If $Z_1 < Z_2$, ($Z_1 > Z_2$), then in case of maximization (minimization) move the line P_1Q_1 corresponding to Z_1 to the line P_2Q_2 corresponding to Z_2 parallel to itself as far as possible, until the farthest point within the feasible region is touched by this line. The coordinates of the point give maximum (minimum) value of the objective function.
- The problem with more equations/inequations can be handled easily by this method.
- In case of unbounded region, it either finds an optimal solution or declares an unbounded solution. Unbounded solutions are not considered optimal solution. In real world problems, unlimited profit or loss is not possible.

Feasible Region

- **Bounded Region:** The region surrounded by the inequalities $ax + by \le m$ and $cx + dy \le n$ in first quadrant is called bounded region. It is of the form of triangle or quadrilateral. Change these inequalities into equation, then by putting $x = 0$ and $y = 0$, we get the solution also by solving the equation in which there may be the vertices of bounded region. The maximum value of objective function lies at one vertex in limited region.

- **Unbounded Region:** The region surrounded by the inequations $ax + by \ge m$ and $cx + dy \ge n$ in first quadrant, is called unbounded region.Change the inequation in equations and solve for $x = 0$ and $y = 0$. Thus we get the vertices of feasible region. The minimum value of objective function lies at one vertex in unbounded region but there is no existence of maximum value.

- **Problems having Infeasible Solutions**: In some of the linear programming problems, constraints are inconsistent *i.e.* there does not exist any point which satisfies all the constraints. Such type of linear programming problems are said to have infeasible solution.

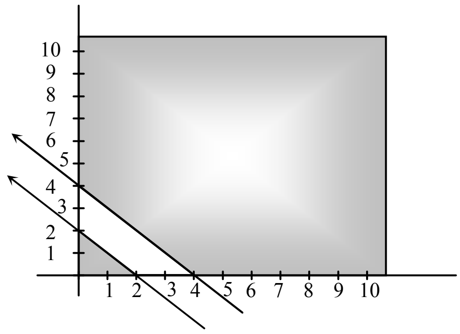


Figure: 9.5

For Example: Maximize $Z = 5x + 2y$ subject to the constraints $x + y \le 2,\ 3x + 3y \ge 2,\ x, y \ge 0$.

The above problem is illustrated graphically in the fig.

There is no point satisfying the set of above constraints. Thus, the problem is having an infeasible solution.

Some Important Points about L.P.P

- If the constraints in a linear programming problem are changed, the problem is to be re-evaluated.
- The optimal value of the objective function is attained at the point, given by corner points of the feasible region.
- If a *L.P.P.* admits two optimal solutions, it has an infinite number of optimal solutions.
- If there is no possibility to determine the point at which the suitable solution can be found, then the solution of problem is unbounded.
- The maximum value of objective function lies at one vertex in limited region.

Advantages and Limitations of L.P.P

- **Advantages:** Linear programming is used to minimize the cost of production for maximum output. In short, with the help of linear programming models, a decision maker can most efficiently and effectively employ his production factor and limited resources to get maximum profit at minimum cost.

Limitations

- The linear programming can be applied only when the objective function and all the constraints can be expressed in terms of linear equations/inequations.
- Linear programming techniques provide solutions only when all the elements related to a problem can be quantified.
- The coefficients in the objective function and in the constraints must be known with certainty and should remain unchanged during the period of study.
- Linear programming technique may give fractional valued answer which is not desirable in some problems.

MULTIPLE CHOICE QUESTIONS

Mean

1. If the mean of 3, 4, x, 7, 10 is 6, then the value of x is:

a. 4 **b.** 5 **c.** 6 **d.** 7

2. d_i is the deviation of a class mark y_i from 'a' the assumed mean and f_i is the frequency, if $M_g = x + \frac{1}{\Sigma f_i}(\Sigma f_i d_i)$, then x is:

a. Lower limit **b.** Assumed mean **c.** Number of observations **d.** Class size

3. Consider the frequency distribution of the given numbers

Value :	1	2	3	4
Frequency :	5	4	6	f

If the mean is known to be 3, then the value of f is:

a. 3 **b.** 7 **c.** 10 **d.** 14

4. If the arithmetic mean of the numbers $x_1, x_2, x_3, .., x_n$ is $\bar{x}$, then the arithmetic mean of numbers $ax_1+b, ax_2+b, ax_3+b, ..., ax_n+b$, where a, b are two constants would be

a. $\bar{x}$ **b.** $n\, a\bar{x} + nb$ **c.** $a\bar{x}$ **d.** $a\bar{x} + b$

5. The G.M. of the numbers $3, 3^2, 3^3, ..., 3^n$ is:

a. $3^{2/n}$ **b.** $3^{(n-1)/2}$ **c.** $3^{n/2}$ **d.** $3^{(n+1)/2}$

6. The harmonic mean of 3, 7, 8, 10, 14 is:

a. $\frac{3+7+8+10+14}{5}$ **b.** $\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{14}$ **c.** $\frac{\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{14}}{4}$ **d.** $\frac{5}{\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{14}}$

7. The mean age of a combined group of men and women is 30 years. If the means of the age of men and women are respectively 32 and 27, then the percentage of women in the group is:

a. 30 **b.** 40 **c.** 50 **d.** 60

8. If the mean of the distribution is 2.6, then the value of y is:

Variate x	1	2	3	4	5
Frequency f of x	4	5	y	1	2

a. 24 **b.** 13 **c.** 8 **d.** 3

9. If the mean of the set of numbers $x_1, x_2, x_3, ..., x_n$ is $\bar{x}$, then the mean of the numbers $x_i + 2i$, $1 \le i \le n$ is:

a. $\bar{x} + 2n$ **b.** $\bar{x} + n + 1$ **c.** $\bar{x} + 2$ **d.** $\bar{x} + n$

10. The weighted mean of first n natural numbers whose weights are equal to the squares of corresponding numbers is:

a. $\frac{n+1}{2}$ **b.** $\frac{3n(n+1)}{2(2n+1)}$ **c.** $\frac{(n+1)(2n+1)}{6}$ **d.** $\frac{n(n+1)}{2}$

11. A student obtain 75%, 80% and 85% in three subjects. If the marks of another subject are added, then his average cannot be less than:

a. 60% **b.** 65% **c.** 80% **d.** 90%

12. The A.M. of a 50 set of numbers is 38. If two numbers of the set, namely 55 and 45 are discarded, the A.M. of the remaining set of numbers is:

a. 38.5 **b.** 37.5 **c.** 36.5 **d.** 36

13. The average weight of students in a class of 35 students is 40 *kg*. If the weight of the teacher be included, the average rises by $\frac{1}{2}$ *kg*; the weight of the teacher is:

a. 40.5 *kg* **b.** 50 *kg* **c.** 41 *kg* **d.** 58 *kg*

14. Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is:

a. 48 **b.** $82\frac{1}{2}$ **c.** 50 **d.** 80

15. The mean of 5 numbers is 18. If one number is excluded, their mean becomes 16. Then the excluded number is:

a. 18 **b.** 25 **c.** 26 **d.** 30

16. Let $x_1, x_2, ..., x_n$ be n observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then a possible value of n among the following is:

a. 9 **b.** 12 **c.** 15 **d.** 18

Median and Mode

17. The median of 10, 14, 11, 9, 8, 12, 6 is:

a. 10 **b.** 12
c. 14 **d.** 11

18. For a symmetrical distribution $Q_1 = 25$ and $Q_3 = 45$, the median is:

a. 20 **b.** 25
c. 35 **d.** None of these

19. The upper quartile for the following distribution

Size of items	1	2	3	4	5	6	7
Frequency	2	4	5	8	7	3	2

is given by the size of:

a. $\left(\frac{31+1}{4}\right)$th item **b.** $\left[2\left(\frac{31+1}{4}\right)\right]$th item
c. $\left[3\left(\frac{31+1}{4}\right)\right]$th item **d.** $\left[4\left(\frac{31+1}{4}\right)\right]$th item

20. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is:

a. 6 **b.** 7
c. 8 **d.** 10

21. The mode of the distribution:

Marks	4	5	6	7	8
No. of students	6	7	10	8	3

a. 5 **b.** 6
c. 8 **d.** 10

Correlation and Regression

22. If $Var(x) = 8.25$, $Var(y) = 33.96$ and $Cov\,(x,y) = 10.2$, then the correlation coefficient is:

a. 0.89 **b.** – 0.98
c. 0.61 **d.** – 0.16

23. Karl Pearson's coefficient of correlation between the heights (in inches) of teachers and students corresponding to the given data is:

Height of teachers x :	66	67	68	69	70
Height of students y :	68	66	69	72	70

a. $\frac{1}{\sqrt{2}}$ **b.** $\sqrt{2}$ **c.** $-\frac{1}{\sqrt{2}}$ **d.** 0

24. For the given data, the calculation corresponding to all values of pairs (x, y) is following $\Sigma(x-\overline{x})^2 = 36$, $\Sigma(y-\overline{y})^2 = 25$, $\Sigma(x-\overline{x})(y-\overline{y}) = 20$, Then the Karl Pearson's correlation coefficient is:

a. 0.2 **b.** 0.5 **c.** 0.66 **d.** 0.33

25. Karl Pearson's coefficient of correlation between x and y for the following data is:

x :	3	4	8	9	6	2	1
y :	5	3	7	7	6	9	2

a. 0.480 **b.** – 0.480
c. 0.408 **d.** – 0.408

26. The coefficients of correlation between the heights (in inches) of fathers and sons from the following data will be:

Heights of fathers(x) :	65	66	67	68	69	70	71
Heights of sons (y) :	67	68	66	69	72	72	69

a. 0.60 **b.** – 0.60
c. – 0.67 **d.** 0.67

27. For the following data $\Sigma(x-\overline{x})^2 = 25, \Sigma(y-\overline{y})^2 = 256$, $\Sigma(x-\overline{x})(y-\overline{y}) = 80$ The Pearson's coefficient of correlation is:

a. 0.2 **b.** 0.01
c. 1 **d.** 0.1

28. If the covariance between x and y is 10 and the variance of x and y are 16 and 9 respectively, then the coefficient of correlation between x and y is:

a. 0.61 **b.** 0.79
c. 0.83 **d.** 0.93

29. If the regression equations of the variables x and y be $x = 19.13 - 0.83y$ and $y = 11.64 - 0.50x$, then the correlation coefficient between x and y is:

a. 0.66 **b.** – 0.64
c. 0.001 **d.** – 0.001

30. If the lines of regression be $x - y = 0$ and $4x - y - 3 = 0$, and $\sigma_x^2 = 1$, then the coefficient of correlation is

a. – 0.5 **b.** 0.5
c. 1.0 **d.** – 1.0

31. If the values of regression coefficients are – 0.33 and – 1.33, then the value of coefficient of correlation between the two variables, is

a. 0.2 **b.** – 0.66
c. 0.4 **d.** – 0.4

32. If regression coefficient of y on x is $\frac{8}{5}$ and that of x on y is $\frac{2}{5}$ and the acute angle between the two lines is α, then the value of $\tan\alpha$ is:

a. 9/25 **b.** $9/2\sqrt{5}$
c. 3/25 **d.** 9/50

33. The two lines of regression are given by $3x+2y=26$ and $6x+y=31$. The coefficient of correlation between x and y is

a. $-\frac{1}{3}$ **b.** $\frac{1}{3}$ **c.** $-\frac{1}{2}$ **d.** $\frac{1}{2}$

34. The two regression lines are $2x-9y+6=0$ and $x-2y+1=0$. What is the correlation coefficient between x and y

a. –2/3 **b.** 2/3
c. 4/9 **d.** None of these

35. The relationship between the correlation coefficient r and the regression coefficients b_{xy} and b_{yx} is:

a. $r=\frac{1}{2}(b_{xy}+b_{yx})$ **b.** $r=\sqrt{b_{xy}.b_{yx}}$
c. $r=(b_{xy}b_{yx})^2$ **d.** $r=b_{xy}+b_{yx}$

Average or Mean

36. The harmonic mean of 4, 8, 16 is

a. 6.4 **b.** 6.7
c. 6.85 **d.** 7.8

37. The average of n numbers x_1, x_2, x_3,…, x_n is M. If x_n is replaced by x', then new average is

a. $M-x_n+x'$ **b.** $\frac{nM-x_n+x'}{n}$
c. $\frac{(n-1)M+x'}{n}$ **d.** $\frac{M-x_n+x'}{n}$

Skewness

38. If μ is the mean of distribution (y_i, f_i), then $\sum f_i(y_i-\mu)=?$

a. M.D. **b.** S.D.
c. 0 **d.** Relative frequency

39. In an experiment with 15 observations on x, the following results were available $\sum x^2=2830, \sum x=170$. On observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is:

a. 78.00 **b.** 188.66
c. 177.33 **d.** 8.33

40. The S.D. of a variate x is σ. The S.D. of the variate $\frac{ax+b}{c}$ where a, b, c are constant, is:

a. $\left(\frac{a}{c}\right)\sigma$ **b.** $\left|\frac{a}{c}\right|\sigma$
c. $\left(\frac{a^2}{c^2}\right)\sigma$ **d.** None of these

Covariance

41. Covariance (x, y) between x and y, if $\sum x=15, \sum y=40$, $\sum x.y=110$, $n=5$ is

a. 22 **b.** 2
c. – 2 **d.** None of these

Rank Correlation

42. Let $x_1, x_2, x_3, ..., x_n$ be the rank of n individuals according to character A and $y_1, y_2, ..., y_n$ the ranks of same individuals according to other character B such that $x_i+y_i=n+1$ for $i=1,2,3,...,n$ .Then the coefficient of rank correlation between the characters A and B is

a. 1 **b.** 0
c. – 1 **d.** None of these

Angle between Two Lines of Regression and Regression Coefficients

43. The two lines of regression are $2x-7y+6=0$ and $7x-2y+1=0$. The correlation coefficient between x and y is

a. – 2/7 **b.** 2/7
c. 4/49 **d.** None of these

44. If two random variables x and y, are connected by relationship $2x+y=3$, then $r_{xy}=$

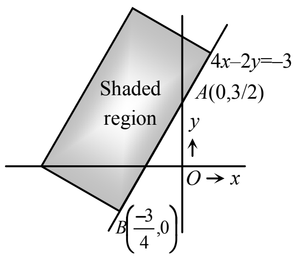
a. 1 **b.** – 1
c. – 2 **d.** 3

Linear In-equations

45. In-equations $3x-y\geq 3$ and $4x-y>4$

a. Have solution for positive x and y
b. Have no solution for positive x and y
c. Have solution for all x
d. Have solution for all y

46. Shaded region is represented by



a. $4x-2y\leq 3$ **b.** $4x-2y\leq -3$
c. $4x-2y\geq 3$ **d.** $4x-2y\geq -3$

47. For the constraint of a linear optimizing function $z=x_1+x_2$, given by $x_1+x_2 \le 1, 3x_1+x_2 \ge 3$ and $x_1, x_2 \ge 0$?

a. There are two feasible regions
b. There are infinite feasible regions
c. There is no feasible region
d. None of these

48. The true statement for the graph of in-equations $3x+2y \le 6$ and $6x+4y \ge 20$, is:

a. Both graphs are disjoint
b. Both do not contain origin
c. Both contain point (1, 1)
d. None of these

49. The vertices of a feasible region of the above question are

a. (0, 18), (36, 0) **b.** (0, 18), (10, 13)
c. (10, 13), (8, 14) **d.** (10, 13), (8, 14), (12, 12)

50. The maximum value of objective function in the above question is:

a. 100 **b.** 92
c. 95 **d.** 94

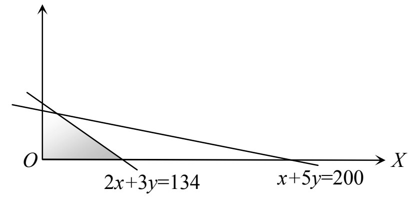
51. For the L.P. problem Min $z=-x_1+2x_2$ such that $-x_1+3x_2 \le 0,\ x_1+x_2 \le 6, x_1-x_2 \le 2$ and $x_1, x_2 \ge 0$, $x_1=?$

a. 2 **b.** 8
c. 10 **d.** 12

52. For the L.P. problem Mi $z=2x+y$ subject to $5x+10y \le 50$, $x+y \ge 1,\ y \le 4$ and $x, y \ge 0$, $z=?$

a. 0 **b.** 1
c. 2 **d.** 1/2

53. The minimum value of objective function $c=2x+2y$ in the given feasible region, is



a. 134 **b.** 40
c. 38 **d.** 80

Measures of dispersion

54. The S.D. of 5 scores 1, 2, 3, 4, 5 is:

a. $\frac{2}{5}$ **b.** $\frac{3}{5}$
c. $\sqrt{2}$ **d.** $\sqrt{3}$

55. The mean deviation of the numbers 3, 4, 5, 6, 7 is:

a. 0 **b.** 1.2
c. 5 **d.** 25

56. The variance of the first *n* natural numbers is:

a. $\frac{n^2-1}{12}$ **b.** $\frac{n^2-1}{6}$
c. $\frac{n^2+1}{6}$ **d.** $\frac{n^2+1}{12}$

57. If Q.D. is 16, the most likely value of S.D. will be:

a. 24 **b.** 42
c. 10 **d.** None of these

58. The range of following set of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is:

a. 11 **b.** 7
c. 5.5 **d.** 6

59. For a given distribution of marks mean is 35.16 and its standard deviation is 19.76. The co-efficient of variation is

a. $\frac{35.16}{19.76}$ **b.** $\frac{19.76}{35.16}$
c. $\frac{35.16}{19.76}\times 100$ **d.** $\frac{19.76}{35.16}\times 100$

60. The mean deviation from the mean for the set of observations –1, 0, 4 is:

a. $\sqrt{\frac{14}{3}}$ **b.** 2
c. $\frac{2}{3}$ **d.** None of these

61. The standard deviation of 25 numbers is 40. If each of the numbers is increased by 5, then the new standard deviation will be:

a. 40 **b.** 45
c. $40+\frac{21}{25}$ **d.** None of these

62. The quartile deviation for the following data is

x :	2	3	4	5	6
f :	3	4	8	4	1

a. 0 **b.** $\frac{1}{4}$
c. $\frac{1}{2}$ **d.** 1

63. The means of five observations is 4 and their variance is 5.2. If three of these observations are 1, 2 and 6, then the other two are

a. 2 and 9 **b.** 3 and 8
c. 4 and 7 **d.** 5 and 6

64. For (2*n*+1) observations $x_1, -x_1$, $x_2, -x_2, ..x_n, -x_n$ and 0 where *x*'s are all distinct. Let S.D. and M.D. denote the standard deviation and median respectively. Then which of the following is always true:

a. S.D. < M.D.
b. S.D. > M.D.
c. S.D. = M.D.
d. Nothing can be said in general about the relationship of S.D. and M.D.

65. The variance of α, β and γ is 9, then variance of 5α, 5β and 5γ is:

a. 45 **b.** 9/5
c. 5/9 **d.** 225

66. What is the standard deviation of the following series?

Measurements	0-10	10-20	20-30	30-40
Frequency	1	3	4	2

a. 81 **b.** 7.6
c. 9 **d.** 2.26

67. The quartile deviation of daily wages (in Rs.) of 7 persons given below 12, 7, 15, 10, 17, 19, 25 is:

a. 14.5 **b.** 5
c. 9 **d.** 4.5

68. Karl-Pearson's coefficient of skewness of a distribution is 0.32. Its S.D. is 6.5 and mean 39.6. Then the median of the distribution is given by:

a. 28.61 **b.** 38.81
c. 29.13 **d.** 28.31

Feasible Region

69. Maximize $z = 3x + 2y$, subject to $x + y \geq 1$, $y - 5x \leq 0$, $x - y \geq -1$, $x + y \leq 6, x \leq 3$ and $x, y \geq 0$

a. $x = 3$ **b.** $y = 3$
c. $z = 15$ **d.** All the above

70. Maximum value of $4x + 5y$ subject to the constraints $x + y \leq 20$, $x + 2y \leq 35, x - 3y \leq 12$ is

a. 84 **b.** 95
c. 100 **d.** 96

71. The maximum value of $\mu = 3x + 4y$ subjected to the conditions $x + y \leq 40$, $x + 2y \leq 60; x, y \geq 0$ is

a. 130 **b.** 120
c. 40 **d.** 140

NCERT EXEMPLAR PROBLEMS

More than One Answer

72. $z = ax + by, a, b$ being positive, under constraints $y \geq 1$, $x - 4y + 8 \geq 0$, $x, y \geq 0$ has:

a. Finite maximum
b. Finite minimum
c. An unbounded minimum solution
d. An unbounded maximum solution

73. *A*, *B*, *C*, *D* are non-zero constants, such that
A. both *A* and *C* are negative.
B. *A* and *C* are of opposite sign.
If coefficient of correlation between *x* and *y* is *r*, then that between $AX + B$ and $CY + D$ is

a. r **b.** $-r$
c. $\frac{A}{C}r$ **d.** $-\frac{A}{C}r$

74. Karl-Pearson's coefficient of skewness of a distribution is 0.32. Its S.D. is 6.5 and mean 39.6. Then the median of the distribution is given by:

a. 28.61 **b.** 38.81
c. 29.13 **d.** 28.31

75. Let *r* be the range and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$ be the S.D. of a set of observations $x_1, x_2, ...x_n$, then:

a. $S \leq r\sqrt{\frac{n}{n-1}}$ **b.** $S = r\sqrt{\frac{n}{n-1}}$
c. $S \geq r\sqrt{\frac{n}{n-1}}$ **d.** None of these

76. In a series of 2*n* observations, half of them equal to *a* and remaining half equal to –*a*. If the standard deviation of the observations is 3, then |*a*| equals?

a. $\frac{\sqrt{2}}{n}$ **b.** $\sqrt{2}$
c. 3 **d.** $\frac{1}{n}$

77. The S.D. of a variate *x* is σ. The S.D. of the variate $\frac{ax+b}{c}$ where *a*, *b*, *c* are constant, is:

a. $\left(\frac{a}{c}\right)\sigma$ **b.** $\left|\frac{a}{c}\right|\sigma$
c. $\left(\frac{a^2}{c^2}\right)\sigma$ **d.** None of these

78. Given that the regression coefficients are – 1.5 and – 0.5, the value of the correlation coefficient is:

a. 0.75 **b.** 0.7

c. – 0.87 **d.** – 0.5

79. Angle between two lines of regression is given by:

a. $\tan^{-1}\left(\dfrac{b_{yx}+\dfrac{1}{b_{xy}}}{1-\dfrac{b_{xy}}{b_{yx}}}\right)$ **b.** $\tan^{-1}\left(\dfrac{b_{yx}-b_{xy}-1}{b_{yx}+b_{xy}}\right)$

c. $\tan^{-1}\left(\dfrac{b_{xy}-\dfrac{1}{b_{yx}}}{1+\dfrac{b_{xy}}{b_{yx}}}\right)$ **d.** $\tan^{-1}\left(\dfrac{b_{yx}-b_{xy}}{1+b_{yx}.b_{xy}}\right)$

80. The solution of set of constraints $x+2y\geq 11$, $3x+4y\leq 30,\ 2x+5y\leq 30,\ x\geq 0,\ y\geq 0$ includes the point:

a. (2, 3) **b.** (3, 2)

c. (3, 4) **d.** (4, 3)

81. The graph of $x\leq 2$ and $y\geq 2$ will be situated in the:

a. First and second quadrant

b. Second and third quadrant

c. First and third quadrant

d. Third and fourth quadrant

Assertion and Reason

Note: Read the Assertion (A) and Reason (R) carefully to mark the correct option out of the options given below:

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of the assertion.

c. If assertion is true but reason is false.

d. If the assertion and reason both are false.

e. If assertion is false but reason is true.

82. **Assertion:** The variance of first n even natural numbers is $=\dfrac{(n+1)(n-1)}{3}=\dfrac{n^2-1}{3}$?

Reason: Arithmetic mean and the variance are same

83. Let $x_1,x_2,...,x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance?

Assertion: Variance of $2x_1,2x_2,...,2x_n$ is $4\sigma^2$.

Reason: Arithmetic mean $2x_1,2x_2,...,2x_n$ is $\bar{x}$.

Comprehension Based

Paragraph –I

Study the following table carefully and answer the questions given below it.

Number of boys of standard xi participating in different games

Class → Games ↓	XI A	XI B	XI C	XI D	XI E	Total
Chess	8	8	8	4	4	32
Badminton	8	12	8	12	12	52
Table Tennis	12	16	12	8	12	60
Hockey	8	4	8	4	8	32
Football	8	8	12	12	12	52
Total Number of Boys	44	48	48	40	48	228

Every student of each section of standard xi participates in a game.

In each class, the number of girls participating in each game is 25% of the number of boys participating in that game.

Each student participates in one and only game.

84. All the boys of class XI D passed at the annual examination but a few girls failed If all the boys and girls who passed and entered class XII D and if in class XIII D, the ratio of boys to girls is 5:1, what would be the number of girls who failed in XI D?

a. 8 **b.** 5

c. 2 **d.** 1

85. Girls playing which of the following games need to be combined to yield a ratio of boys to girls of 4:1, if all boys playing Chess and Badminton are combined?

a. Chess and Hockey

b. Hockey and Football

c. Tables Tennis and Hockey

d. Badminton and Table Tennis

86. What should be the total number of students in the school if all the boys of XII A together with all the girls of XI B and XI C were to be equal to 25% of the total number of students?

a. 272 **b.** 560

c. 656 **d.** 340

87. Boys of which of the following classes need to be combined to equal to four times the number of girls in XI B and XI C ?

a. XI D and XI E **b.** XI A and XI B

c. XI A and XI E **d.** None of these

88. If boys if XI E participates in Chess together with girls of XI V and XI C participating in Table Tennis and Hockey respectively are selected for a course at the college of sports, what percent of the students will get this advantage approximately?

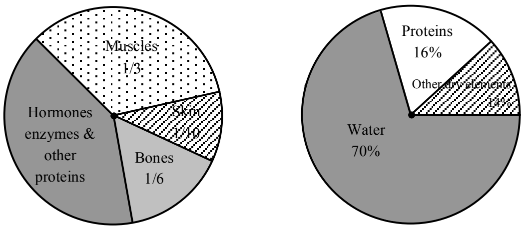
a. 13.5 **b.** 10.52
c. 3.51 **d.** 4.38

89. If for social work, every boy of XI D and XI C is paired with a girl of the same class, what percentage of the boys of these two classes can participate in Social work?

a. 60 **b.** 75
c. 66 **d.** 88

Paragraph –II

Study the following Pie-diagrams carefully and answer the questions given below it.



90. In the human body, what part is made of neither bones nor skin?

a. $\frac{2}{5}$ **b.** $\frac{3}{5}$
c. $\frac{1}{40}$ **d.** None of these

91. What is the ratio of the distribution of proteins in the muscles to that distribution of proteins in the bones?

a. 1 : 18 **b.** 18 : 1
c. 2 : 1 **d.** 1 : 2

92. What will be the quantity of water in the body of a person weighing 50 kg?

a. 20 kg **b.** 35 kg
c. 71.42 kg **d.** 120 kg

93. What percent of the weight of human body is equivalent to the weight of the skin in human body?

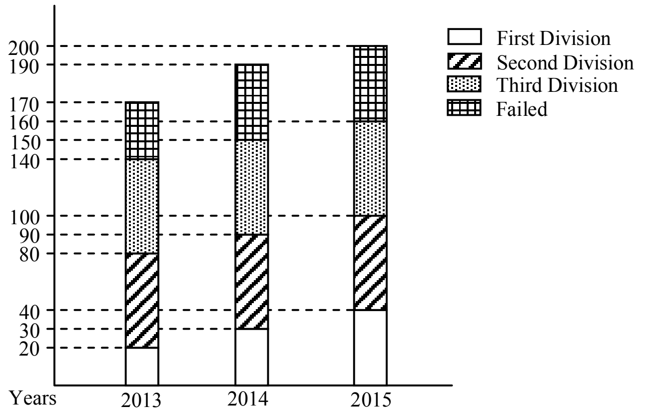
a. .016 **b.** 1.6
c. 0.16 **d.** Data inadequate

94. To show the distribution of proteins and other dry elements in the human body, they are of the circle should subtend at the centre an angle of :

a. 50° **b.** 126°
c. 108° **d.** 252°

Match the Column

The following sub-divided bar diagram depicts the result of B.Com. Students of a college of a three years.



Read the diagram carefully and answer the questions given below it.

95. Observe the following columns:

Column I	Column II
(A) How many percent passed in first division in 2013?	**1.** 11.76%
(B) What was the pass percentage in 2013?	**2.** 80.3%
(C) What is the percentage of students in 2015 over 2013?	**3.** 82.3%
(D) What is the aggregate pass percentage during three years?	**4.** 117.6%

a. A→1; B→3; C→4; D→2
b. A→1; B→3; C→2-4; D→2
c. A→2; B→4-5; C→3,1; D→2
d. A→2; B→1-4; C→3; D→2

Integer

96. If the mean of the distribution is 2.6, then the value of y is:

Variate x	1	2	3	4	5
Frequency f of x	4	5	y	1	2

97. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class are 72, then what are the average marks of the girls?

98. The mode of the distribution:

Marks	4	5	6	7	8
No. of students	6	7	10	8	3

99. What is the standard deviation of the following series?

Measurements	0-10	10-20	20-30	30-40
Frequency	1	3	4	2

100. If $Q.D.$ is 16, the most likely value of $S.D.$ will be?

ANSWER

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	b	d	d	d	d	b	c	b	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
b	b	d	c	c	d	a	c	c	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
b	c	a	c	c	d	c	c	b	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
b	d	c	b	b	c	b	c	a	b
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
c	c	b	b	a	b	c	a	c	c
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
a	b	d	c	b	a	a	b	d	b
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
a	d	c	b	d	c	d	b	d	b
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
d	b,d	a,b	b	a	c	b	c	c	c
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**	**88.**	**89.**	**90.**
a	b	c	c	b	a	d	c	b	d
91.	**92.**	**93.**	**94.**	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
c	b	b	c	a	8	65	6	9	24

SOLUTION

Multiple Choice Questions

1. **(c)** $6 = \frac{3+4+x+7+10}{5}$

$\Rightarrow \quad 30 = 24 + x$

$\Rightarrow \quad x = 6$.

2. **(b)** $M_g = a + \frac{1}{\Sigma f_i}(\Sigma f_i d_i)$

$\therefore \quad x = a$ *i.e.*, x = assumed mean.

3. **(d)** Mean = $\frac{1\times5+2\times4+3\times6+4\times f}{5+4+6+f}$

i.e., $3 = \frac{5+8+18+4f}{15+f}$

$\Rightarrow \quad 45+3f = 31+4f$

$\Rightarrow \quad 45-31 = f$

$\Rightarrow \quad f = 14$.

4. **(d)** Required mean $= \frac{(ax_1+b)+(ax_2+b)+.....+(ax_n+b)}{n}$

$= \frac{a(x_1+x_2+.....+x_n)+nb}{n} = a\overline{x}+b,$

$\left(\because \frac{x_1+x_2+.....+x_n}{n} = \overline{x}\right)$.

5. **(d)** G.M. $= (3.3^2.3^3.......3^n)^{1/n}$

$= (3^{1+2+......n})^{1/n} = \left(3^{\frac{n(n+1)}{2}}\right)^{1/n} = 3^{\frac{n+1}{2}}$.

6. **(d)** H.M. $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_n}} = \frac{5}{\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{14}}$.

7. **(b)** The formula for combined mean is $\overline{x} = \frac{n_1\overline{x}_1+n_2\overline{x}_2}{n_1+n_2}$

Given, $\overline{x} = 30$, $\overline{x}_1 = 32$, $\overline{x}_2 = 27$

Let $n_1+n_2 = 100$ and n_1 denotes men, n_2 denotes women for this $n_2 = 100-n_1$

$30 = \frac{32n_1+(100-n_1)27}{100} \Rightarrow 30 = \frac{32n_1+2700-27n_1}{100}$

$\Rightarrow \quad 3000-2700 = 32n_1-27n_1 \Rightarrow 300 = 5n_1 \Rightarrow n_1 = 60$

So, $n_2 = 40$; Hence, the percentage of women in the group is 40.

8. **(c)** We know that, Mean $= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

i.e., $2.6 = \frac{1\times4+2\times5+3\times y+4\times1+5\times2}{4+5+y+1+2}$

or $31.2+2.6y = 28+3y$ or $0.4y = 3.2 \Rightarrow y = 8$.

9. **(b)** We know that $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ *i.e.*, $\sum_{i=1}^{n} x_i = n\overline{x}$

$\therefore \quad \frac{\sum_{i=1}^{n}(x_i+2i)}{n} = \frac{\sum_{i=1}^{n} x_i + 2\sum_{i=1}^{n} i}{n} = \frac{n\overline{x}+2(1+2+...n)}{n}$

$= \frac{n\overline{x}+2\frac{n(n+1)}{2}}{n} = \overline{x}+n+1$.

10. **(b)** Weighted mean $= \frac{1.1^2+2.2^2+......+n.n^2}{1^2+2^2+......+n^2}$

$= \frac{\Sigma n^3}{\Sigma n^2} = \frac{\frac{n(n+1)}{2}\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6}} = \frac{3n(n+1)}{2(2n+1)}$.

11. **(b)** Marks obtained from 3 subjects out of 300 = 75 + 80 + 85 = 240

If the marks of another subjected is added, then the marks will be $\geq$ 240 out of 400

$\therefore$ Minimum average marks $= \frac{240}{4} = 60\%$,

[When marks in the fourth subject = 0].

12. **(b)** Given, $\frac{\Sigma x_i}{50} = 38, \therefore \Sigma x_i = 1900$

New value of $\Sigma x_i = 1900 - 55 - 45 = 1800$, $n = 48$

$\therefore$ New mean $= \frac{1800}{48} = 37.5$.

13. **(d)** Let the weight of the teacher is $w\ kg$, then

$40 + \frac{1}{2} = \frac{35 \times 40 + w}{35 + 1}$

$\Rightarrow$ $36 \times 40 + 36 \times \frac{1}{2} = 35 \times 40 + w \Rightarrow w = 58$

$\therefore$ Weight of the teacher = 58*kg*.

14. **(c)** Sum of 100 items = 49 × 100 = 4900

Sum of items added = 60 + 70 + 80 = 210

Sum of items replaced = 40 + 20 + 50 = 110

New sum = 4900 + 210 – 110 = 5000

$\therefore$ Correct mean $= \frac{5000}{100} = 50$.

15. **(c)** Sum of total number = 18 × 5 = 90

After one number excluded

Sum of total number = 16 × 4 = 64

Then, excluded number is 90 – 64 = 26.

16. **(d)** Since, root mean square ≥ arithmetic mean

$\therefore$ $\sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}} \geq \frac{\sum_{i=1}^{n} x_i}{n} = \sqrt{\frac{400}{n}} \geq \frac{80}{n} \Rightarrow n \geq 16$

Hence, possible value of n = 18.

17. **(a)** Arrange the items in ascending order *i.e.*, 6, 8, 9, 10, 11, 12, 14.

If n is odd then, Median = value of $\left(\frac{n+1}{2}\right)^{th}$ term

$\therefore$ Median $= \left(\frac{7+1}{2}\right)^{th}$ term $= 4^{th}$ term $= 10$.

18. **(c)** As the distribution is symmetrical, therefore,

Q_2 (Median) $= \frac{Q_1 + Q_3}{2} = \frac{25 + 45}{2} = 35$.

19. **(c)** Upper Quartile = Size of $\left[3\frac{(n+1)}{4}\right]^{th}$ item

= Size of $\left[3\left(\frac{31+1}{4}\right)\right]^{th}$ item, $[\because \Sigma f = 31]$.

20. **(c)** Mode of the data is 8 as it is repeated maximum number of times.

21. **(b)** Since frequency is maximum for 6.

$\therefore$ Mode = 6.

22. **(c)** $r_{xy} = \frac{Cov(x,y)}{\sqrt{Var(x).Var(y)}} = \frac{10.2}{\sqrt{(8.25)(33.96)}} = 0.61$.

23. **(a)** Here $\Sigma(x - \bar{x})(y - \bar{y}) = 10, \Sigma(x - \bar{x})^2 = 10$

and $\Sigma(y - \bar{y})^2 = 20$

Hence the coefficient of correlation is given by

$r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2 \Sigma(y - \bar{y})^2}} = \frac{10}{\sqrt{10 \times 20}} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$.

24. **(c)** $r_{xy} = \frac{20}{\sqrt{(36)(25)}} = \frac{2}{3} = 0.66$.

25. **(c)**

x	y	xy	x^2	y^2
3	5	15	09	25
4	3	12	16	09
8	7	56	64	49
9	7	63	81	49
6	6	36	36	36
2	9	18	04	81
1	2	02	01	04
$\Sigma x = 33$	$\Sigma y = 39$	$\Sigma xy = 202$	$\Sigma x^2 = 211$	$\Sigma y^2 = 253$

Now, $r = \frac{\Sigma xy - \frac{\Sigma x . \Sigma y}{n}}{\sqrt{\left\{\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right\}\left\{\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right\}}}$

$\Rightarrow$ $r = \frac{202 - \frac{33 \times 39}{7}}{\sqrt{\left\{211 - \frac{(33)^2}{7}\right\}\left\{253 - \frac{(39)^2}{7}\right\}}}$

$\Rightarrow$ $r = 0.408$.

26. **(d)** Here, $\bar{x} = 68, \bar{y} = 69$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
– 3	– 2	6	9	4
– 2	– 1	2	4	1
– 1	– 3	3	1	9
0	0	0	0	0
1	3	3	1	9
2	3	6	4	9
3	0	0	9	0

Hence, $r_{xy} = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2 \Sigma(y - \bar{y})^2}} = \frac{20}{\sqrt{(28)(32)}} = 0.67$.

27. **(c)** $r = \frac{80}{\sqrt{25 \times 256}} = 1$.

28. **(c)** $Cov(x, y) = 10$, $Var(x) = 16$, $Var(y) = 9$

$\therefore \quad r = \frac{Cov(x, y)}{\sqrt{Var(x).Var(y)}} = \frac{10}{\sqrt{16 \times 9}} = \frac{10}{12} = 0.83$.

29. **(b)** The given lines of regression are
$x = -0.83y + 19.13$

and $y = -0.50x + 11.64$

$\Rightarrow \quad r = -\sqrt{0.83 \times 0.50} = -0.64$,
(because both gradients are negative).

30. **(b)** $y = x$ and $x = \frac{1}{4}y + \frac{3}{4}$.

Hence, $r = \sqrt{1.\frac{1}{4}} = 0.5$.

31. **(b)** $r = \sqrt{(-0.33)(-1.33)} = -0.66$.

32. **(d)** Here $b_{yx} = \frac{8}{5}$ and $b_{xy} = \frac{2}{5}$

Hence $m_1 = \frac{8}{5}$ and $m_2 = \frac{1}{b_{xy}} = \frac{5}{2}$

$\Rightarrow \quad \tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) = \pm\frac{\frac{8}{5} - \frac{5}{2}}{1 + \frac{8}{5} \times \frac{5}{2}} = \frac{9}{50}$.

33. **(c)** Here, $3x + 2y = 26 \Rightarrow y = -\frac{3}{2}x + 13$

and $6x + y = 31 \Rightarrow x = -\frac{1}{6}y + \frac{31}{6}$

$\therefore \quad r = \sqrt{\left(\frac{-3}{2}\right)\left(-\frac{1}{6}\right)} = -\frac{1}{2}$.

34. **(b)** Given regression lines are, $2x - 9y + 6 = 0$
and $x - 2y + 1 = 0$

$\therefore \quad b_{yx}$ = Slope of line of regression of y on x

$\Rightarrow \quad b_{xy} = \frac{2}{9}$.

$\frac{1}{b_{xy}}$ = Slope of line of regression of x on y $= \frac{1}{2}$

$\Rightarrow \quad b_{xy} = 2$.

Hence, $r^2 = b_{yx}.b_{xy}$

$\Rightarrow \quad r = +\frac{2}{3}$.

35. **(b)** Regression coefficients of y on x $= b_{yx} = r.\frac{\sigma_y}{\sigma_x}$

Regression coefficients of x on y $= b_{xy} = r.\frac{\sigma_x}{\sigma_y}$

Then, $b_{yx}.b_{xy} = r.\frac{\sigma_y}{\sigma_x} \times r.\frac{\sigma_x}{\sigma_y}$

$r^2 = b_{yx}.b_{xy} \Rightarrow r = \sqrt{b_{yx}.b_{xy}}$.

36. **(c)** H.M. of 4, 8, 16

$= \frac{3}{\frac{1}{4} + \frac{1}{8} + \frac{1}{16}} = \frac{48}{7} = 6.85$

37. **(b)** $M = \frac{x_1 + x_2 + x_3 ......x_n}{n}$

i.e. $nM = x_1 + x_2 + x_3 + ...x_{n-1} + x_n$

$nM - x_n = x_1 + x_2 + x_3 + ...x_{n-1}$

$\frac{nM - x_n + x'}{n} = \frac{x_1 + x_2 + x_3 + ... + x_{n-1} + x'}{n}$

$\therefore \quad$ New average $= \frac{nM - x_n + x'}{n}$

38. **(c)** We have, $\Sigma f_i(y_i - \mu) = \Sigma f_i y_i - \mu \Sigma f_i$

$= \mu \Sigma f_i - \mu \Sigma f_i = 0 \left[\because \mu = \frac{\Sigma f_i y_i}{\Sigma f_i}\right]$

39. **(a)** $\Sigma x = 170,\ \Sigma x^2 = 2830$

Increase in $\Sigma x = 10$, then $\Sigma x' = 170 + 10 = 180$

Increase in $\Sigma x^2 = 900 - 400 = 500$,

then $\Sigma x' = 2830 + 500 = 3330$

Variance $= \frac{1}{n}\Sigma x'^2 - \left(\frac{\Sigma x'}{n}\right)^2 = \frac{3330}{15} - \left(\frac{180}{15}\right)^2$

$= 222 - 144 = 78$

40. **(b)** Let $y = \frac{ax + b}{c}$ *i.e.*, $y = \frac{a}{c}x + \frac{b}{c}$

i.e. $y = Ax + B$, where $A = \frac{a}{c}$, $B = \frac{b}{c}$

$\therefore \quad \bar{y} = A\bar{x} + B$

$\therefore \quad y - \bar{y} = A(x - \bar{x})$

$\Rightarrow \quad (y - \bar{y})^2 = A^2(x - \bar{x})^2$

$\Rightarrow \quad \Sigma(y - \bar{y})^2 = A^2\,\Sigma(x - \bar{x})^2$

$\Rightarrow \quad n.\sigma_y^2 = A^2.n\sigma_x^2$

$\Rightarrow \quad \sigma_y^2 = A^2\sigma_x^2$

$\Rightarrow \quad \sigma_y = |A|\,\sigma_x$

$\Rightarrow \quad \sigma_y = \left|\frac{a}{c}\right|\sigma_x$

Thus, new S.D. $= \left|\frac{a}{c}\right|\sigma.$

41. **(c)** Given, $\sum x = 15, \sum y = 40$

$\sum x.y = 110,\ n = 15$

We know that,

$$Cov(x, y) = \frac{1}{n}\sum_{i=1}^{n} x_i . y_i - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)$$

$$= \frac{1}{n}\sum x.y - \left(\frac{1}{n}\sum x\right)\left(\frac{1}{n}\sum y\right)$$

$$= \frac{1}{5}(110) - \left(\frac{15}{5}\right)\left(\frac{40}{5}\right) = 22 - 3\times 8 = -2.$$

42. **(c)** $x_i + y_i = n+1$ for all $i = 1, 2, 3, \ldots\ldots, n$

Let $x_i - y_i = d_i$. Then, $2x_i = n + 1 + d_i$

$\Rightarrow \quad d_i = 2x_i - (n+1)$

$$\therefore \quad \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n}[2x_i - (n+1)]^2$$

$$= \sum_{i=1}^{n}[4x_i^2 + (n+1)^2 - 4x_i(n+1)]$$

$$\sum_{i=1}^{n} d_i^2 = 4\sum_{i=1}^{n} x_i^2 + (n)(n+1)^2 - 4(n+1)\sum_{i=1}^{n} x_i$$

$$= 4\frac{n(n+1)(2n+1)}{6} + (n)(n+1)^2 - 4(n+1)\frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} d_i^2 = \frac{n(n^2-1)}{3}.$$

$$\therefore \quad = \frac{n\bar{x} + 2\frac{n(n+1)}{2}}{n} = \bar{x} + n + 1 \ \ i.e.,\ r = -1.$$

43. **(b)** The two lines of regression are

$2x - 7y + 6 = 0 \ \ldots (i)$ and $7x - 2y + 1 = 0 \ \ldots .(ii)$

If (*i*) is regression equation of *y* on *x*, then (*ii*) is regression equation of *x* on *y*.

We write these as $y = \frac{2}{7}x + \frac{6}{7}$ and $x = \frac{2}{7}y - \frac{1}{7}$

$\therefore \quad b_{yx} = \frac{2}{7},\ b_{xy} = \frac{2}{7};$

$\therefore \quad b_{yx}.b_{xy} = \frac{4}{49} < 1$, So our choice is valid.

$\therefore \quad r^2 = \frac{4}{49} \Rightarrow r = \frac{2}{7}. \quad [\because\ b_{yx} > 0, b_{xy} > 0\,]$

44. **(b)** Since $2x + y = 3$

$\therefore \quad 2\bar{x} + \bar{y} = 3;$

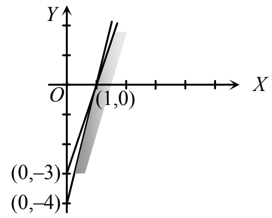
$\therefore \quad y - \bar{y} = -2(x - \bar{x}).$ So, $b_{yx} = -2$

Also $x - \bar{x} = -\frac{1}{2}(y - \bar{y}),\ \ \therefore\ b_{xy} = -\frac{1}{2}$

$\therefore \quad r_{xy}^2 = b_{yx}.b_{xy} = (-2)\left(-\frac{1}{2}\right) = 1$

$\Rightarrow \quad r_{xy} = -1. \quad (\because$ both b_{yx}, b_{xy} are –*ive*)

45. **(a)** Following figure will be obtained on drawing the graphs of given in-equations:



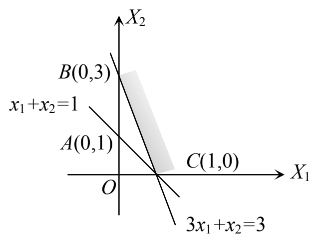
From $3x - y \geq 3, \frac{x}{1} + \frac{y}{-3} = 1$

From $4x - y \geq 4, \frac{x}{1} + \frac{y}{-4} = 1$

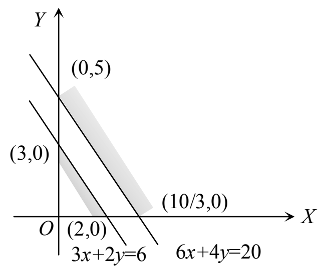
Clearly the common region of both the in-equations is true for positive value of (*x, y*). It is also true for positive values of *x* and negative values of *y*.

46. **(b)** Origin is not present in given shaded area. So $4x - 2y \leq -3$ satisfy this condition.

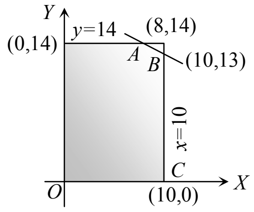
47. **(c)** Clearly from graph there is no feasible region.



48. **(a)** The equations, corresponding to inequalities $3x+2y\leq 6$ and $6x + 4y \geq 20$, are $3x + 2y = 6$ and $6x + 4y = 20$. So the lines represented by these equations are parallel. Hence the graphs are disjoint.

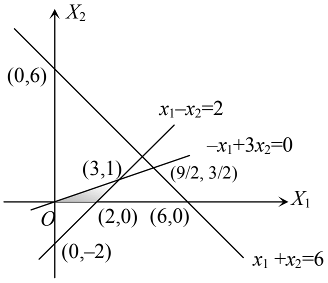


49. **(c)** Hence required feasible region is given by $ABCD$, and vertices are (8, 14), (10, 13), (10, 0) and (0, 14)



50. **(c)** $Max\, z = 3(10) + 5(13) = 95$.

51. **(a)** $(3,1), (2,0)$ are vertices of $Min\, z$ for (2, 0)



Hence $x_1 = 2$.

52. **(b)** After drawing a graph we get the vertices of feasible region are (1, 0), (10, 0), (2, 4), (0, 4) and (0, 1).
Thus minimum value of objective function is at (0, 1)
Hence $z = 0 \times 2 + 1 \times 1 = 1$

53. **(d)** Min z $= 2(0) + 2(40) = 80$.

54. **(c)** Mean $\bar{x} = \dfrac{1+2+3+4+5}{5} = 3$

$$\text{S.D.} = \sigma = \sqrt{\frac{1}{n}\sum x_i^2 - (\bar{x})^2}$$

$$= \sqrt{\frac{1}{5}(1+4+9+16+25) - 9} = \sqrt{11-9} = \sqrt{2}.$$

55. **(b)** A.M. $= \dfrac{3+4+5+6+7}{5} = 5$

$\therefore$ Mean deviation $= \dfrac{\Sigma|x_i - \bar{x}|}{n}$

$$= \frac{|3-5| + |4-5| + |5-5| + |6-5| + |7-5|}{5}$$

$$= \frac{2+1+0+1+2}{5} = \frac{6}{5} = 1.2.$$

56. **(a)** Variance $= (\text{S.D.})^2 = \dfrac{1}{n}\Sigma x^2 - \left(\dfrac{\Sigma x}{n}\right)^2$, $\left(\because \bar{x} = \dfrac{\Sigma x}{n}\right)$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2 = \frac{n^2-1}{12}.$$

57. **(a)** We know that, S.D. $= \dfrac{3}{2}$ Q.D.

$\therefore$ S.D. $= \dfrac{3}{2} \times 16 = 24$.

58. **(b)** Range $= X_{\max} - X_{\min} = 9 - 2 = 7$.

59. **(d)** Coefficient of variation $= \dfrac{\text{S.D.}}{\text{Mean}} \times 100 = \dfrac{19.76}{35.16} \times 100$.

60. **(b)** Mean $= \dfrac{-1+0+4}{3} = 1$.

Hence M.D. (about mean) $= \dfrac{|-1-1| + |0-1| + |4-1|}{3} = 2$.

61. **(a)** If each item of a data is increased or decreased by the same constant, the standard deviation of the data remains unchanged.

62. **(d)** $N = (\Sigma f) = 20$

$Q_1 = \dfrac{(N+1)}{4} th$ observation $= \left(\dfrac{21}{4}\right)^{th}$ observation=3

Similarly, $Q_3 = 3\left(\dfrac{N+1}{4}\right) th$ observation

$= \left(\dfrac{63}{4}\right) th$ observation = 5

Now Q.D. $= \dfrac{1}{2}(Q_3 - Q_1) = \dfrac{1}{2}(Q_3 - Q_1) = \dfrac{1}{2}(5-3) = 1$.

63. **(c)** Let the two unknown items be x and y, then

Mean $= 4 \Rightarrow \dfrac{1+2+6+x+y}{5} = 4 \Rightarrow x + y = 11$ . . . (i)

and variance = 5. 2

$\Rightarrow \dfrac{1^2 + 2^2 + 6^2 + x^2 + y^2}{5} - (\text{mean})^2 = 5.2$

$41 + x^2 + y^2 = 5[5.2 + (4)^2]$

$41 + x^2 + y^2 = 106 \quad x^2 + y^2 = 65$ . . . (ii)

Solving (i) and (ii) for x and y, we get

$x = 4, y = 7$ or $x = 7, y = 4$.

64. **(b)** On arranging the given observations in ascending order, we get; All negative terms $\underbrace{O}_{(n+1)^{th}\text{term}}$ All positive terms

The median of given observations $= (n+1)^{th}$ term = 0

$\therefore$ S. D. > M .D.

65. **(d)** When each item of a data is multiplied by λ, variance is multiplied by λ^2. Hence, new variance $= 5^2 \times 9 = 225$.

66. **(c)**

Class	f_i	y_i	$d = y_i - A$, $A = 25$	$f_i d_i$	$f_i d_i^2$
0-10	1	5	– 20	– 20	400
10-20	3	15	– 10	– 30	300
20-30	4	25	0	0	0
30-40	2	35	10	20	200
Total	10			– 30	900

$$\sigma^2 = \frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2 = \frac{900}{10} - \left(\frac{-30}{10}\right)^2$$

$\sigma^2 = 90 - 9 = 81 \Rightarrow \sigma = 9.$

67. **(d)** The given data in ascending order of magnitude is 7, 10, 12, 15, 17, 19, 25

Here Q_1 = size of $\left(\frac{n+1}{4}\right)^{th}$ item = size of 2nd item = 10

Q_3 = size of $\left(\frac{3(n+1)}{4}\right)^{th}$ item = size of 6th item = 19

Then Q.D. $= \frac{Q_3 - Q_1}{2} = \frac{19-10}{2} = 4.5$.

68. **(b)** We know that $S_k = \frac{M - M_o}{\sigma}$,

where M = Mean, M_o = Mode, σ = S.D.

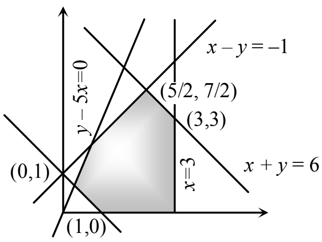
i.e., $0.32 = \frac{39.6 - M_o}{6.5}$

$\Rightarrow$ $M_o = 37.52$ and also know that, M_o = 3median – 2mean

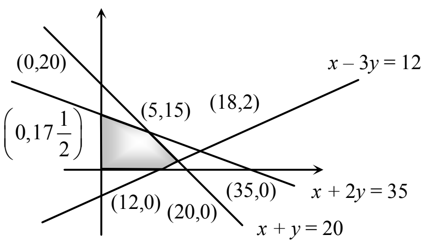
$37.52 =$ 3(Median) – 2(39.6)

Median = 38.81 (approx.).

69. **(d)** The shaded region represents the bounded region (3,3) satisfies, so $x = 3, y = 3$ and $z = 15$.

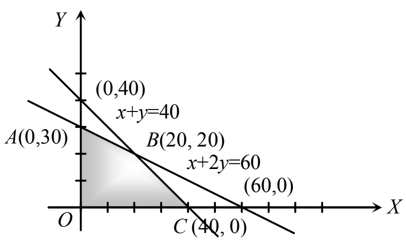


70. **(b)** Obviously, *max.* $4x + 5y = 95$. It is at (5, 15).



71. **(d)** Obviously *Max* $\mu = 3x + 4y$ at $(20, 20)$

$\mu = 60 + 80 = 140.$



NCERT Exemplar Problems

More than One Answer

72. **(b,d)** Finite minimum and An unbounded maximum solution

73. **(a,b)** (a) Both A and C are negative.

Now $Cov(AX + B, CY + D) = AC\ Cov.(X,Y)$ $\sigma_{AX+B} = |A|\sigma_x$

And $\sigma_{CY+D} = |C|\sigma_y$

Hence $\rho(AX + B, CY + D)$

$= \frac{AC.Cov(X,Y)}{(|A|\sigma_x)(|C|\sigma_y)} = \frac{AC}{|AC|}\rho(X,Y)$

$= \rho(X,Y) = r$, $(\because AC > 0)$

(b) $\rho(AX + B, CY + D) = \frac{AC}{|AC|}\rho(X,Y)$, $(\because AC < 0)$

$= \frac{AC}{-AC}\rho(X,Y) = -\rho(X,Y) = -r.$

74. **(b)** We know that $S_k = \frac{M - M_0}{\sigma}$,

where M = Mean, M_o = Mode, σ = S.D.

i.e., $0.32 = \frac{39.6 - M_o}{6.5}$

$\Rightarrow$ $M_o = 37.52$ and also know that, M_0 = 3 median– 2 mean

$37.52 =$ 3 (Median) – 2(39.6) Median = 38.81 (approx.).

75. **(a)** We have $r = \max_{i \neq j} |x_i - x_j|$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Now, $(x_i - \bar{x})^2 = \left(x_i - \frac{x_1 + x_2 + ... + x_n}{n}\right)^2$

$= \frac{1}{n^2}[(x_i - x_1) + (x_i - x_2) + ... + (x_i - x_i - 1)$

$+ (x_i - x_i + 1) + ... + (x_i - x_n)] \le \frac{1}{n^2}[(n-1)r]^2$, $[\because |x_i - x_j| \le r]$

$\Rightarrow$ $(x_i - \bar{x})^2 \le r^2 \Rightarrow \sum_{i=1}^{n}(x_i - \bar{x})^2 \le nr^2$

$\Rightarrow$ $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \le \frac{nr^2}{(n-1)} \Rightarrow S^2 \le \frac{nr^2}{(n-1)} \Rightarrow S \le r\sqrt{\frac{n}{n-1}}.$

76. **(c)** Let a, a, ...n times and – a, – a, – a, – a, ...n times

i.e., mean = 0 and S.D. $= \sqrt{\frac{n(a-0)^2 + n(-a-0)^2}{2n}}$

$3 = \sqrt{\frac{na^2 + na^2}{2n}} = \sqrt{a^2} = \pm a$

Hence $|a| = 3$.

77. **(b)** Let $y=\frac{ax+b}{c}$ *i.e.,* $y=\frac{a}{c}x+\frac{b}{c}$

i.e., $y=Ax+B$, where $A=\frac{a}{c},\ B=\frac{b}{c}$

$\therefore\quad \overline{y}=A\overline{x}+B$

$\therefore\quad y-\overline{y}=A(x-\overline{x})\Rightarrow (y-\overline{y})^2=A^2(x-\overline{x})^2$

$\Rightarrow\quad \Sigma(y-\overline{y})^2=A^2\,\Sigma(x-\overline{x})^2$

$\Rightarrow\quad n.\sigma_y^2=A^2.n\sigma_x^2\Rightarrow \sigma_y^2=A^2\sigma_x^2$

$\Rightarrow\quad \sigma_y=|A|\,\sigma_x\Rightarrow \sigma_y=\left|\frac{a}{c}\right|\sigma_x$. Thus, new S.D. $=\left|\frac{a}{c}\right|\sigma$.

78. **(c)** $r=\sqrt{b_{yx}.b_{xy}}=-\sqrt{1.5\times 0.5}=-0.87$

Answer is negative, because both of the regression coefficients are negative.

79. **(c)** Equation of regression line of y on x is,

$y-\overline{y}=r\frac{\sigma_y}{\sigma_x}(x-\overline{x})$ Or $y-\overline{y}=b_{yx}(x-\overline{x})$

m_1 = Slope of regression line of y on $x=b_{yx}$

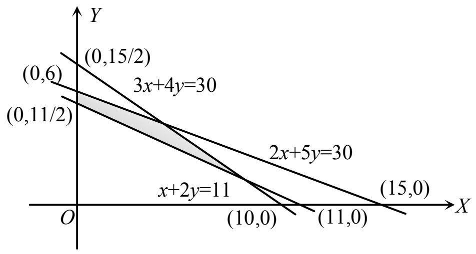
Now, equation of regression line of x on y is, $x-\overline{x}=b_{xy}(y-\overline{y})$

m_2 = slope of regression line of x on $y=\frac{1}{b_{xy}}$

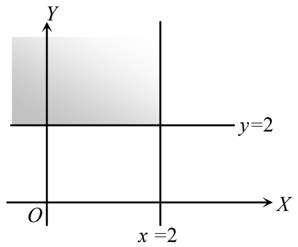
If angle between them is θ, then

$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}=\frac{b_{yx}-\frac{1}{b_{xy}}}{1+\frac{b_{yx}}{b_{xy}}}\text{ or }\tan\theta=\left(\frac{b_{xy}-\frac{1}{b_{yx}}}{1+\frac{b_{xy}}{b_{yx}}}\right).$$

80. **(c)** Obviously, solution set of constraints includes the point (3, 4).



81. **(a)**



Assertion and Reason

82. **(b)** Assertion: Sum of n even natural number $=n(n+1)$

Mean $(\overline{x})=\frac{n(n+1)}{n}=n+1$

Variance $=\left[\frac{1}{n}\sum(x_1)^2\right]-(\overline{x})^2$

$=\frac{1}{n}[2^2+4^2+\ldots+(2n)^2]-(n+1)^2$

$=\frac{1}{n}2^2(1^2+2^2+\ldots+n^2)-(n+1)^2$

$=\frac{4}{n}\frac{n(n+1)(2n+1)}{6}-(n+1)^2$

$=\frac{(n+1)[2(2n+1)-3(n+1)]}{3}$

$=\frac{(n+1)[4n+2-3n-3]}{3}=\frac{(n+1)(n-1)}{3}=\frac{n^2-1}{3}$

83. **(c)** A.M. of $2x_1,2x_2,\ldots,2x_n$ is $\frac{2x_1,2x_2+\ldots+2x_n}{n}$

$=2\left(\frac{x_1+x_2+\ldots+x_n}{n}\right)=2\overline{x}$

So, Assertion: is false. Variance $(2x_i)=2^2$ variance $(x_i)=4\sigma^2$

Reason: is true.

Comprehension Based

84. **(c)** Boys in XI $D=40$, Girls in XI $D=\left(\frac{25}{100}\times 40\right)=10$.

Boys in XII $D=40$, Ratio of boys and girls in XII $D=5:1$.

Let the number of girls in XII D be x.

Then, $\frac{40}{x}=\frac{5}{1}$ or $x=8$.

Girls in XII $D=8$

Girls failed in XI $D=(10-8)=2$.

85. **(b)** Boys playing Chess & Badminton $=(32+52)=84$

Girls playing Hockey & Football $=\left(\frac{25}{100}\times 84\right)=21$

$\therefore$ Required ratio $=84:21=4:1$.

86. **(a)** Boys in XI $A=44$, Girls in XI $B=\left(\frac{25}{100}\times 45\right)=12$

Girls in XI $C=\left(\frac{25}{100}\times 48\right)=12$

Let the total number of students be x. Then,

$\frac{25}{100}\times x=(44+12+12)$

Or $x=272$.

87. **(d)** 4 (Girls in XI *B* & XI *C*) $= 4(12+12) = 96$

But, none of the pairs of class given through (1) to (4) has this as the number of boys.

88. **(c)** Boys of XI *E* Playing Chess = 4.

Girls of XI *B* playing Table Tennis $= \left(\frac{25}{200}\times 16\right) = 4$

Girls of XI *C* playing Hockey $= \left(\frac{25}{100}\times 8\right) = 2$

Number selected at the college of sports

$= (4 = 4 = 2) = 10$

Total number of students $= \left(228 + \frac{25}{100}\times 228\right) = 285$

Let *x* % of 285 = 10. Then, $x = \left(\frac{10\times 100}{285}\right) = 3.51.$

89. **(b)** Girls = 25% of boys

∴ 25% of the boys can participate in Social work.

∴ 75% of the boys cannot participate in Social work.

90. **(d)** Part of body made of neither bones nor skin

$= 1 - \left(\frac{1}{6} + \frac{1}{10}\right) = \frac{11}{15}.$

91. **(c)** Required ratio $= \frac{1}{3} : \frac{1}{6} = 2:1.$

92. **(b)** Quantity of water in the body of a person weighing 50 kg. = 70% of 50 kg $= \left(\frac{70}{100}\times 50\right)$ kg = 35 kg.

93. **(b)** Weight of skin $= \frac{1}{10}$ part of 16% Proteins.

$= \left(\frac{1}{10}\times\frac{16}{100}\times 100\right)\%$ of Proteins

$= 1.6\%$ of Proteins.

94. **(c)** Percentage of Proteins & other dry elements = 30%

∴ Required angel $= \left(\frac{30}{100}\times 360\right) = 108°.$

Match the Column

95. **(a)** **(A)** Percentage of first divisioners in 2013

$= \left(\frac{20}{170}\times 100\right) = 11.76\%$

(B) Pass percentage in 2013 $= \left(\frac{140}{170}\times 100\right) = 82.3\%$

(C) Percentage of students in 2015 over 2013

$= \left(\frac{200}{170}\times 100\right) = 117.6\%$

(D) Number of students who passed $(140+150+160) = 450$

Number of students who appeared $= (170+190+200) = 560$

∴ Aggregate pass percentage $= \left(\frac{450}{560}\times 100\right)\% = 80.3\%.$

Integer

96. **(8)** We know that, Mean $= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

i.e $\quad 2.6 = \frac{1\times 4 + 2\times 5 + 3\times y + 4\times 1 + 5\times 2}{4+5+y+1+2}$

Or $\quad 31.2 + 2.6y = 28 + 3y$ or $0.4y = 3.2 \Rightarrow y = 8$

97. **(65)** Let the average marks of the girls students be *x*, then

$72 = \frac{70\times 75 + 30\times x}{100}$ (Number of girls = 100 – 70 = 30)

i.e., $\quad \frac{7200 - 5250}{30} = x,$

∴ $\quad x = 65.$

98. **(6)** Since frequency is maximum for 6

∴ Mode = 6

99. **(9)**

Class	Frequency	y_i	$u_i = \frac{y_i - A}{10}$, $A = 25$	$f_i u_i$	$f_i u_i^2$
0-10	1	5	– 2	– 2	4
10-20	3	15	– 1	– 3	3
20-30	4	25	0	0	0
30-40	2	35	1	2	2
	10			– 3	9

$\sigma^2 = c^2\left[\frac{\Sigma f_i u_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i u_i^2}{\Sigma f_i}\right)^2\right] = 10^2\left[\frac{9}{10} - \left(\frac{-3}{10}\right)^2\right]$

$= 90 - 9 = 81$

$\Rightarrow \quad \sigma = 9$

100. **(24)** We know that, $S.D. = \frac{3}{2} Q.D.$

∴ $\quad S.D. = \frac{3}{2}\times 16 = 24$