Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The HCF of the smallest 2-digit number and the smallest composite number is **[1]**

   a) 4 b) 10

   c) 20 d) 2

2. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is **[1]**

   a) 1600 b) 8000

   c) 1605 d) 320

3. If p = -7 and q = 12 and $x^2 + px + q = 0$, Then the value of x is **[1]**

   a) 3 and 4 b) 3 and -4

   c) -3 and -4 d) -3 and 4

4. The pair of equations ax + 2y = 9 and 3x + by = 18 represent parallel lines, where a, b are integers, if: **[1]**

   a) a = b b) 2a = 3b

   c) 3a = 2b d) ab = 6

5. For what values of k, the equation $kx^2 - 6x - 2 = 0$ has real roots? **[1]**

   a) $k \geq \frac{-9}{2}$ b) $k \leq -5$

   c) $k \leq -2$ d) $k \leq \frac{-9}{2}$

6. In what ratio does x-axis divide the line segment joining the points A(2, -3) and B(5, 6)? **[1]**

a) 1 : 2

b) 3 : 5

c) 2 : 1

d) 2 : 3

7. In $\triangle$LMN and $\triangle PQR, \angle L = \angle P, \angle N = \angle R$ and MN = 2QR. Then the two triangles are **[1]**

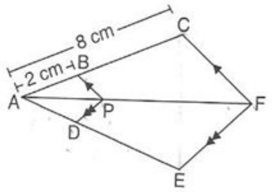
a) Similar but not congruent

b) Congruent but not similar

c) Congruent as well as similar

d) neither congruent nor similar

8. In the given figure if $BP || CF$, $DP || EF$,then AD : DE is equal to **[1]**



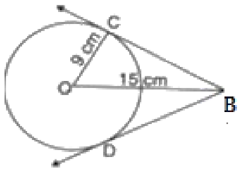
a) 1 : 3

b) 1 : 4

c) 3 : 4

d) 2 : 3

9. In the adjoining figure, If OC = 9 cm and OB = 15 cm, then BC + BD is equal to **[1]**



a) 24 cm

b) 18 cm

c) 12 cm

d) 36 cm

10. If 8 tan x = 15, then sin x - cos x is equal to **[1]**

a) $\frac{17}{7}$

b) $\frac{8}{17}$

c) $\frac{7}{17}$

d) $\frac{1}{17}$

11. A pole casts a shadow of length $2\sqrt{3}$ m on the ground when the sun's elevation is 60°. The height of the pole is **[1]**

a) 12 m

b) 6 m

c) $4\sqrt{3}$ m

d) 3 m

12. If cosecθ - sinθ = l and secθ - cosθ = m, then $l^2m^2(l^2 + m^2 + 3)$ = ________. **[1]**

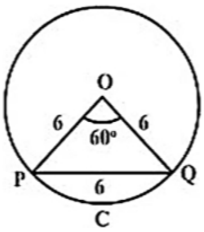
a) 1

b) 2 sinθ

c) 2

d) sinθ cosθ

13. PQ is a chord of a circle with centre O and radius 6 cm. PQ is of length 6 cm and divides the circle into two segments. The area of the minor segment is **[1]**



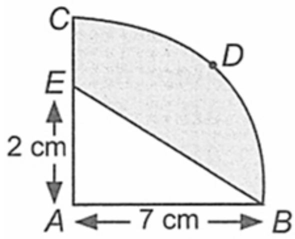
a) $(6\pi - 9\sqrt{3})\text{cm}^2$

b) $(6\pi + \sqrt{3})\text{cm}^2$

c) $(\pi - 3)\text{cm}^2$

d) $\left(\frac{8\pi}{3} + \sqrt{3}\right)\text{cm}^2$

14. In the figure, ABDCA represents a quadrant of a circle of radius 7 cm a with centre A. Find the area of the shaded portion. **[1]**



a) 14 cm^2
b) 31.5 cm^2
c) 24.5 cm^2
d) 38.5 cm^2

15. In a survey, it is found that every fifh person has a vehicle. The probability of a person <u>NOT</u> having a vehicle, is **[1]**

a) $\frac{1}{5}$
b) $\frac{4}{5}$
c) 5%
d) 95%

16. In the formula $\bar{X}$ = a + h$\left(\frac{1}{N}\sum f_i u_i\right)$ for finding the mean of grouped frequency distribution u_i = **[1]**

a) $\frac{x_i+a}{2h}$
b) h(x_i - a)
c) $\frac{x_i-a}{h}$
d) $\frac{x_i+a}{h}$

17. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged then the water level rises by **[1]**

a) 4 cm
b) 5 cm
c) 3 cm
d) 6 cm

18. The median of first 8 prime numbers is **[1]**

a) 9
b) 11
c) 13
d) 7

19. **Assertion (A):** Image of point (5, –16) under x-axis is (–5, 16). **[1]**

Reason (R): To find image of point (x, y) under x-axis change the sign of y and to find image under y-axis change sign of x.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162 **[1]**

Reason: If a, b are two positive integers, then H.C.F. × L.C.M. = a × b

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Solve the following pair of equations by substitution method: **[2]**

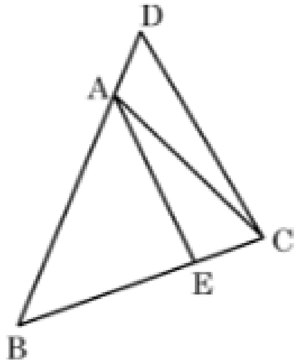
7x – 15y = 2 ...(1)

x + 2y = 3 ...(2)

22. In a $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D. If AD = 5.6 cm, BC = 6 cm and BD = 3.2 cm, find AC. **[2]**

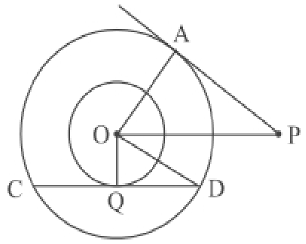
OR

In the given figure, ∠ABC = ∠ACB and $\frac{BC}{BE} = \frac{BD}{AC}$.



Show that ∠ABE ~ ∠DBC and AE || DC.

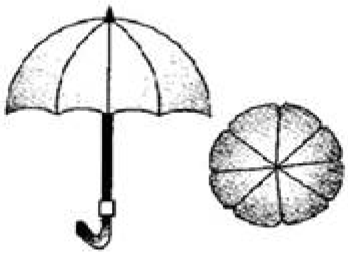
23. In two concentric circles, the radii are OA = r cm and OQ = 6 cm, as shown in the figure. Chord CD of larger circle is a tangent to smaller circle at Q. PA is tangent to larger circle. If PA = 16 cm and OP = 20 cm, find the length CD. **[2]**



24. Prove the trigonometric identity: **[2]**

$\frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A} = \frac{2}{\sin^2 A-\cos^2 A} = \frac{2}{2\sin^2 A-1} = \frac{2}{1-2\cos^2 A}$

25. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, Find the area between the two consecutive ribs of the umbrella. **[2]**

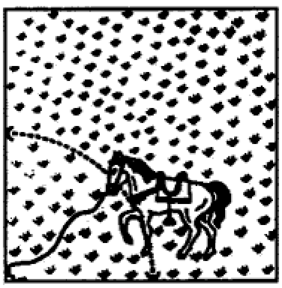


OR

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find

i. the area of that part of the field in which the horse can graze.

ii. the increase in the grazing area if the rope were 10 m long instead of 5 m (Use π = 3.14)



Section C

26. Prove that $7\sqrt{5}$ is irrational. **[3]**

27. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and other zero. **[3]**

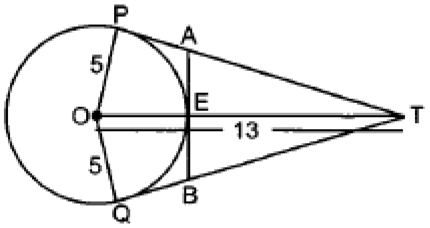
28. Check graphically whether the pair of equations x + 3y = 6 and 2x – 3y = 12 is consistent. If so, solve them graphically. **[3]**

OR

Use elimination method to find all possible solutions of the following pair of linear equations

ax + by - a + b = 0 and bx - ay - a - b = 0

29. In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB. where TP and TQ are two tangents to the circle. **[3]**

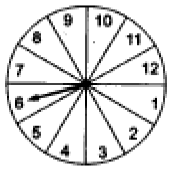


30. Prove that $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \frac{1}{\sec\theta-\tan\theta}$, using identity $sec^2\theta = 1 + tan^2\theta$. **[3]**

OR

If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that $q(p^2 - 1) = 2p$

31. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1,2,3,..., 12 as shown in the figure. What is the probability that it will point to **[3]**



i. 6
ii. an even number?
iii. a prime number?
iv. a number which is a multiple of 5?

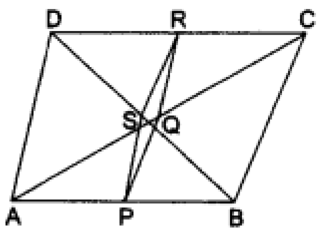
Section D

32. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately. **[5]**

OR

If the equation $\left(1+m^2\right)x^2 + 2mcx + \left(c^2 - a^2\right) = 0$ has equal roots, prove that $c^2 = a^2\left(1+m^2\right)$

33. ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the midpoints of AB, AC, CD and BD respectively, show that PQRS is a rhombus. **[5]**



34. Two cubes each of volume 125 cm^3 are joined end to end. Find the volume and the surface area of the resulting cuboid. **[5]**

OR

An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if one cubic cm of iron weighs 7.8 grams.

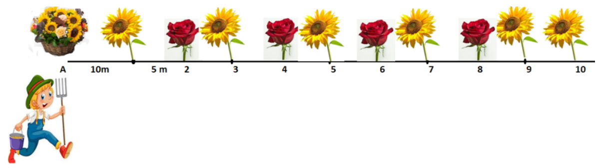
35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. **[5]**

Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

i. Write the above information in the progression and find first term and common difference. (1)
ii. Find the distance covered by Dinesh to plant the first 5 plants and return to basket. (1)
iii. Find the distance covered by Dinesh to plant all 10 plants and return to basket. (2)

OR

If the speed of Dinesh is 10 m/min and he takes 15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

The Chief Minister of Delhi launched the, 'Switch Delhi', an electric vehicle mass awareness campaign in the National Capital. The government has also issued tenders for setting up 100 charging stations across the city. Each station will have five charging points. For demo charging station is set up along a straight line and has charging points at $A\left(\frac{-7}{3},0\right)$, $B\left(0,\frac{7}{4}\right)$, C(3, 4), D(7, 7) and E(x, y). Also, the distance between C and E is 10 units.

i. What is the distance DE? (1)

ii. What is the value of x + y? (1)

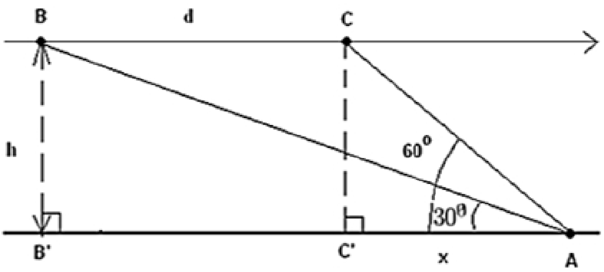
iii. Points C, D, E are collinear or not? (2)

OR

What is the ratio in which B divides AC? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Mr. Vinod is a pilot in Air India. During the Covid-19 pandemic, many Indian passengers were stuck at Dubai Airport. The government of India sent special aircraft to take them. Mr. Vinod was leading this operation. He is flying from Dubai to New Delhi with these passengers. His airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is 30^{o} and at 10:01 am, it is 60^{o}.



i. What is the distance **d** is covered by the airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour? (1)

ii. What is the altitude **h** of the airplane? (round answer to 2 decimal places) (1)

iii. Find the distance between passenger and airplane when the angle of elevation is 30^{o}. (2)

OR

Find the distance between passenger and airplane when the angle of elevation is 60^{o}. (2)

Solution

Section A

1.
   (d) 2
   Explanation: Smallest two digit number is 10 and smallest composite number is 4 .
   Clearly, 2 is the greatest factor of 4 and 10, so their H.C.F. is 2.

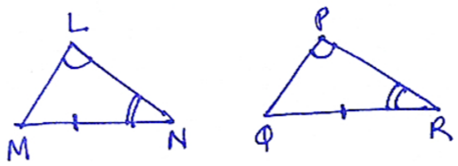
2.
   (d) 320
   Explanation: Let the two numbers be x and y.
   It is given that: x $\times$ y = 1600
   HCF = 5
   We know, HCF $\times$ LCM = x $\times$ y
   $\Rightarrow$ 5 $\times$ LCM = 1600
   $\therefore$ LCM = $\frac{1600}{5}$ = 320

3. **(a)** 3 and 4
   Explanation: Putting the values of p and q in given equation, we get
   x^2 + (-7)x + 12 = 0
   $\Rightarrow$ x^2 - 7x + 12 = 0
   $\Rightarrow$ x^2 - 4x - 3x + 12 = 0
   $\Rightarrow$ x(x - 4) - 3(x - 4) = 0
   $\Rightarrow$ (x - 3)(x - 4) = 0
   $\Rightarrow$ x - 3 =0 and x - 4 = 0
   $\Rightarrow$ x = 3 and x = 4

4.
   (d) ab = 6
   Explanation: for Parallel lines,
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
   $\frac{a}{3} = \frac{2}{b} \neq \frac{-9}{-18}$
   ab = 6

5. **(a)** $k \geq \frac{-9}{2}$
   Explanation: For real roots, we must have, $b^2 - 4ac \geq 0$.
   $(-6)^2 - 4 \times k \times (-2) \geq 0 \Rightarrow 36 + 8k \geq 0$
   $\Rightarrow 8k \geq -36 \Rightarrow k \geq \frac{-9}{2}$.

6. **(a)** 1 : 2
   Explanation: Let the x axis cut AB at P(x, 0) in the ratio K : 1
   Then $\frac{6k-3}{k+1} = 0 \Rightarrow 6k - 3 - 0 \Rightarrow 6k = 3 \Rightarrow k = \frac{1}{2}$
   required ratio = $\left(\frac{1}{2} : 1\right) = 1 : 2$

7. **(a)** Similar but not congruent
   Explanation: 

   $\because$ $\angle$L = $\angle$P (given)
   $\angle$N = $\angle$R (given)
   $\Rightarrow \triangle LMN \sim \triangle PQR$ (by AA Sim. rule)

But Not Congruent because

given MN = 2QR i.e. Sides are proportional Not equal.

8. **(a)** 1 : 3

Explanation: Since BP$\|$CF,

Then, $\frac{AP}{PF} = \frac{AB}{BC}$ [Using Thales Theorem]

$\Rightarrow \frac{AP}{PF} = \frac{2}{6} = \frac{1}{3}$

Again, since DP$\|$ EF,

Then, $\frac{AP}{PF} = \frac{AD}{DE}$ [Using Thales Theorem]

$\Rightarrow \frac{AD}{DE} = \frac{1}{3}$

$\Rightarrow$ AD : DE = 1 : 3

9. **(a)** 24 cm

Explanation: Here $\angle$C = 90° [Angle between tangent and radius through the point of contact]

Now, in right angled triangle OBC,

$OB^2 = OC^2 + BC^2$

$\Rightarrow (9)^2 = (15)^2 + BC^2$

$\Rightarrow BC^2 = 225 - 81 = 144$

$\Rightarrow$BC = 12 cm

But BC = BD [Tangents from one point to a circle are equal]

Therefore, BD = 12 cm

Then BC + BD = 12 + 12 = 24 cm

10.

(c) $\frac{7}{17}$

Explanation: 8 tan x = 15 $\Rightarrow \tan x = \frac{15}{8} = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem,

$(\text{Hyp.})^2 = (\text{Base}\}^2 + (\text{Perp.})^2$

$= (8)^2 + (15)^2$

$= 64 + 225 = 289 = (17)^2$

$\therefore$ Hyp. = 17 units

$\therefore \sin x = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$

$\cos x = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{8}{17}$

$\sin x - \cos x = \frac{15}{17} - \frac{8}{17} = \frac{15-8}{17}$

$= \frac{7}{17}$

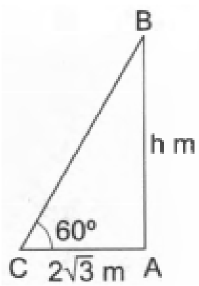
11.

(b) 6 m

Explanation: Let the height of the pole be h metres.

Then, $\frac{h}{2\sqrt{3}} = \tan 60^o = \sqrt{3}$

$\Rightarrow h = (2\sqrt{3} \times \sqrt{3}) = 6.$



12. **(a)** 1

Explanation: We have, $l^2m^2(l^2 + m^2 + 3)$

$= (\text{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2 \{(\text{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2 \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 \left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$

= $\frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta} \left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right\}$

= $\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta \times 1$

= $\{(\cos^2\theta)^3 + (\sin^2\theta)^3 + 3\cos^2\theta\sin^2\theta\,(\sin^2\theta + \cos^2\theta)\}$

= $(\cos^2\theta + \sin^2\theta)^3 = 1$

13. **(a)** $(6\pi - 9\sqrt{3})\text{cm}^2$

Explanation: Area of the minor segment = Area of sector OPCQ - area of $\triangle$OPQ

Area of the minor segment $= \left\{\frac{\theta}{360^\circ} \times \pi r^2 - \frac{\sqrt{3}}{4} \times r^2\right\}\text{cm}^2$

$= \left\{\frac{60^\circ}{360^\circ} \times \pi \times (6)^2 - \frac{\sqrt{3}}{4}(6)^2\right\}$ ..($\theta = 60^o$, r = 6 cm)

$= \left\{\frac{1}{6} \times \pi \times 36 - \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 36\right\} = (6\pi - 9\sqrt{3})\text{cm}^2$

Hence, the area of minor segment = $(6\pi - 9\sqrt{3})\text{cm}^2$

14.

(b) 31.5 cm^2

Explanation: Area of quadrant = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2}$ cm^2 = 38.5 cm^2

Area of $\triangle$BAE = $\frac{1}{2}\times$ base $\times$ height

= $\frac{1}{2} \times$ AB $\times$ AE = $\frac{1}{2} \times 7 \times 2$ = 7 cm^2

Hence, area of the shaded portion = Area of the quadrant ABDCA - Area of $\triangle$BAE

= (38.5 - 7) cm^2 = 31.5 cm^2

15.

(b) $\frac{4}{5}$

Explanation: Out of 5 persons , 1 person possess a vehicle

P (possessing vehicle) = $\frac{1}{5}$

Using Probability of the Complement

P (not A) = 1 - P(A)

P (not possessing vehicle) = 1 - P(possessing vehicle)

P (not possessing vehicle) = 1 - $\frac{1}{5}$

$\Rightarrow$ P (not possessing vehicle) = $\frac{4}{5}$

16.

(c) $\frac{x_i - a}{h}$

Explanation: Given $\overline{\text{x}}$ = a + h $\left(\frac{1}{\text{N}}\Sigma f_i u_i\right)$

Above formula is a step deviation formula, where

$\text{u}_\text{i} = \frac{x_i - a}{h}$

17.

(c) 3 cm

Explanation: Increase in volume of water = volume of the sphere

$\Rightarrow \pi \times 18 \times 18 \times h = \frac{4}{3}\pi \times 9 \times 9 \times 9$

$\Rightarrow$ h = $\left(\frac{4}{3} \times \frac{9\times9\times9}{18\times18}\right)$cm = 3 cm

18. **(a)** 9

Explanation: First 8 prime numbers are follows:

2, 3, 5, 7, 11, 13, 17, 19

N = 8 (even)

$\therefore \text{ Median } = \frac{\left(\frac{8}{2}\right)^{\text{th}} \text{ value} + \left(\frac{8}{2}+1\right)^{\text{th}} \text{ value}}{2}$

$= \frac{4^{\text{th}} \text{ value} + 5^{\text{th}} \text{ value}}{2}$

$= \frac{7+11}{2}$

$= \frac{18}{2}$

= 9

19.

(d) A is false but R is true.

Explanation: Rule: Image of (x, y) under x-axis is given by (x, –y) and under y-axis given by (–x, y).

20.

(d) A is false but R is true.

Explanation: $\frac{3072}{16}$ = 192 $\neq$ 162

Section B

21. **Step 1:** By substitution method, we pick either of the equations and write one variable in terms of the other.

7x – 15y = 2 ...(1)

and x + 2y = 3 ...(2)

Let us consider the Equation (2):

x + 2y = 3

and write it as x = 3 – 2y ...(3)

Step 2: Now substitute the value of x in Equation (1)

We get 7(3 – 2y) – 15y = 2

i.e., 21 – 14y – 15y = 2

i.e., – 29y = –19

Therefore y $= \frac{19}{29}$

Step 3: Substituting this value of y in Equation (3), we get

$\mathrm{x} = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$

Therefore, the solution is x = $\frac{49}{29}$, y $= \frac{19}{29}$

22. If is is given that AB = 5.6 cm, BC = 6 cm and BD = 3.2 cm

In $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5.6\text{cm}}{AC} = \frac{3.2\text{cm}}{2.8\text{cm}}$ [DC = BC - BD]

$AC = \frac{5.6 \times 2.8}{3.2}$ cm = 4.9

OR

It is given that $\frac{BC}{BE} = \frac{BD}{AC}$

$\Rightarrow \frac{BE}{BC} = \frac{AB}{DB}$ $(\because \angle\mathrm{ABC} = \angle\mathrm{ACB} \Rightarrow \mathrm{AC} = \mathrm{AB})$

Also $\angle$B is common

$\therefore \triangle$ABE $\sim \triangle$DBC (SAS similarity)

$\Rightarrow \angle$BAE = $\angle$BDC

But these are corresponding angles $\therefore$ AE || DC.

23. Since $PA \perp OA$ therefore $OA^2 = 20^2 - 16^2 = 144$

$\Rightarrow$ OA = r = 12 cm

In $\triangle OQD, QD^2 = 12^2 - 6^2 = 108$

$\Rightarrow QD = 6\sqrt{3}$ cm

Now OQ bisects CD

$\Rightarrow \mathrm{CD} = 2 \times 6\sqrt{3} = 12\sqrt{3}\ \mathrm{cm}$

24. We have,

$\mathrm{L.H.S} = \frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{(\sin A-\cos A)(\sin A+\cos A)}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{(\sin^2 A+\cos^2 A+2\sin A\cos A)+(\sin^2 A+\cos^2 A-2\sin A\cos A)}{\sin^2 A-\cos^2 A}$ $[\because (a \pm b)^2 = a^2 \pm 2ab + b^2]$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{(1+2\sin A\cos A)+(1-2\sin A\cos A)}{\sin^2 A-\cos^2 A}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{2}{\sin^2 A-\cos^2 A}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{2}{\sin^2 A-\cos^2 A} = \frac{2}{\sin^2 A-(1-\sin^2 A)}$ [$\because \sin^2 A + \cos^2 A = 1$]

$\Rightarrow \quad \mathrm{L.H.S} = \frac{2}{2\sin^2 A-1} = \frac{2}{2(1-\cos^2 A)-1} = \frac{2}{1-2\cos^2 A}$ = R.H.S $[\because \sin^2 A = 1 - \cos^2 A \ \&\ \cos^2 A = 1 - \sin^2 A]$

Hence proved.

25. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}$ cm ^{2}

OR

i. The area of that part of the field in which the horse can graze if the length of the rope is 5cm

$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times (5)^2 = \frac{1}{4} \times 78.5 = 19.625\text{m}^2$

ii. The area of that part of the field in which the horse can graze if the length of the rope is 10 m

$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times (10)^2 = 78.5\text{m}^2$

$\therefore$ The increase in the grazing area

= 78.5 - 19.625 = 58.875 m^2

Section C

26. We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers *a* and *b* ($b \neq 0$)

such that

$7\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ....(1)

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our assumption is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

27. Let $P(x) = 2x^2 + 3x + \lambda$

Its one zero is $\frac{1}{2}$ so $P(\frac{1}{2}) = 0$

$P(\frac{1}{2}) = 2 \times (\frac{1}{2})^2 + 3(\frac{1}{2}) + \lambda = 0$

$\Rightarrow 2 \times \frac{1}{4} + 3/2 + \lambda = 0$

$\Rightarrow \frac{1}{2} + \frac{3}{2} + \lambda = 0$

$\Rightarrow \frac{4}{2} + \lambda = 0$

$\Rightarrow 2 + \lambda = 0$

$\Rightarrow \lambda = -2$

Let the other zero be α

Then $\alpha + \frac{1}{2} = -\frac{3}{2}$

$\Rightarrow \alpha = -\frac{3}{2} - \frac{1}{2} = -\frac{4}{2} = -2$

28. The solution of pair of linear equations:

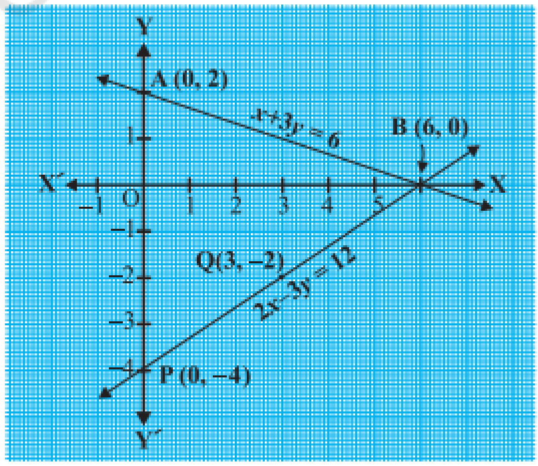
x + 3y = 6 and 2x – 3y = 12

x	0	6
$y = \frac{6-x}{3}$	2	0

and

x	0	3
$y = \frac{2x-12}{3}$	-4	-2

Plot the points A(0, 2), B(6, 0), P(0, – 4) and Q(3, – 2) on graph paper, and join the points to form the lines AB and PQ



We observe that there is a point B (6, 0) common to both the lines AB and PQ. So, the solution of the pair of linear equations is x = 6 and y = 0, i.e., the given pair of equations is consistent.

OR

Given pair of linear equation is $ax + by - a + b = 0$ .....(i)

and $bx - ay - a - b = 0$ .... (ii)

Multiplying $ax + by - a + b = 0$ by a and $bx - ay - a - b = 0$ by b, and adding them, we get

$a^2x + aby - a^2 + ab = 0$ and $b^2x - aby - ab - b^2 = 0$

$(a^2x + aby - a^2 + ab) + (b^2x - aby - ab - b^2) = 0$

$a^2x + aby - a^2 + ab + b^2x - aby - ab - b^2 = 0$

$a^2x + b^2x - a^2 - b^2 = 0$

$\Rightarrow (a^2 + b^2)x = (a^2 + b^2)$

$\Rightarrow x = \frac{(a^2+b^2)}{(a^2+b^2)} = 1$

On putting x =1 in first equation, we get

$ax + by - a + b = 0$

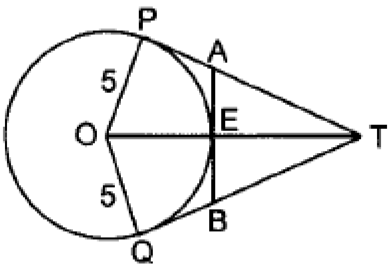
$a + by = a - b$

$\Rightarrow y = -\frac{b}{b} = -1$

Hence, x=1 and y=-1, which is the required unique solution.

29. According to the question,

O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E.



∵ OP ⊥ TP [Radius from point of contact of the tangent]

∴ ∠OPT = 90°

In right △OPT *

$OT^2 = OP^2 + PT^2$

$\Rightarrow (13)^2 = (5)^2 + PT^2 \Rightarrow PT = 12$ cm

Let AP = x cm AE = AP ⇒ AE = x cm

and AT = (12 - x)cm

TE = OT - OE = 13 - 5 = 8 cm

∵OE ⊥ AB [Radius from the point of contact]

∴ ∠AEO = 90° ⇒ ∠AET = 90°

In right △AET,

$AT^2 = AE^2 + ET^2$
$(12 - x)^2 = x^2 + 8^2$
$\Rightarrow 144 + x^2 - 24x = x^2 + 64$
$\Rightarrow \quad 24x = 80 \Rightarrow x = \frac{80}{24} = \frac{10}{3}$ cm
Also BE = AE = $\frac{10}{3}$ cm
$\Rightarrow \quad AB = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ cm

30. We have to prove that, $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using identity $sec^2\theta = 1 + tan^2\theta$

LHS = $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ [dividing the numerator and denominator by $\cos\theta$.]
$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} = \frac{\{(\tan\theta + \sec\theta) - 1\}(tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$ [Multiplying and dividing by $(\tan\theta - \sec\theta)$]
$= \frac{(\tan^2\theta - \sec^2\theta) - (tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$ [$\because (a-b)(a+b) = a^2 - b^2$]
$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$ [$\because \tan^2\theta - \sec^2\theta = -1$]
$= \frac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \frac{-1}{\tan\theta - \sec\theta}$
$= \frac{1}{\sec\theta - \tan\theta}$ =**RHS**

Hence Proved.

OR

Given,
$\sin\theta + \cos\theta = p$ ......(1)
And , $\sec\theta + \mathrm{cosec}\,\theta = q$....(2)
Now, L.H.S
$= q(p^2 - 1)$
$= (\sec\theta + \mathrm{cosec}\,\theta)[(\sin\theta + \cos\theta)^2 - 1]$ 【 from (1) & (2) 】
$= \left[\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right][\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta - 1]$
$= \left[\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right][1 + 2\cos\theta\sin\theta - 1]$ ($\because \sin^2\theta + \cos^2\theta = 1$)
$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\cos\theta\sin\theta$
$= 2(\sin\theta + \cos\theta)$
$= 2p$ ($\because \sin\theta + \cos\theta = p$)
=R.H.S
Hence, proved.

31. The possible outcomes are 1, 2, 3, 4, 5 ……….. 12.
Number of all possible outcomes = 12

i. Let E_1 be the event that the pointer rests on 6.
Then, number of favorable outcomes = 1
Probability = $\frac{Number\ of\ favourable\ outcome}{Total\ Number\ of\ outcomes}$
Therefore, P(arrow pointing at 6) = $P(E_1) = \frac{1}{12}$

ii. Out of the given numbers, the even numbers are
2, 4, 6, 8, 10 and 12
Let E_2 be the event of getting an even number.
Then, number of favorable outcomes = 6
Probability = $\frac{Number\ of\ favourable\ outcome}{Total\ Number\ of\ outcomes}$
Therefore, P(arrow pointing at an even number) = $P(E_2) = \frac{6}{12} = \frac{1}{2}$

iii. Out of the given numbers, the prime numbers are 2, 3, 5, 7 and 11.
Let E_3 be the event of the arrow pointing at a prime number.
Then, number of favorable outcomes = 5
Probability = $\frac{Number\ of\ favourable\ outcome}{Total\ Number\ of\ outcomes}$
Therefore, P(arrow pointing at a prime number) = $P(E_3) = \frac{5}{12}$

iv. Out of the given numbers, the numbers that are multiple of 5 are 5 and 10 only.
Let E_4 be the event of the arrow pointing at a multiple of 5.

Then, number of favorable outcomes = 2

Probability = $\frac{Number\,of\,favourable\,outcome}{Total\,Number\,of\,outcomes}$

Therefore, P(arrow pointing at a number that is a multiple of 5) = P(E_4) $= \frac{2}{12} = \frac{1}{6}$

Section D

32. Let time taken by pipe A be x minutes, and time taken by pipe B be x + 5 minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank

In one minute pipe B will fill $\frac{1}{x+5}$ tank

pipes A + B will fill in one minute = $\frac{1}{x} + \frac{1}{x+5}$ tank

Now according to the question.

$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$

or, $\frac{x+5+x}{x(x+5)} = \frac{9}{100}$

or, 100(2x + 5) = 9x(x + 5)

or, $200x + 500 = 9x^2 + 45x$

or, $9x^2 - 155x - 500 = 0$

or, $9x^2 - 180x + 25x - 500 = 0$

or,9x(x - 20) + 25(x - 20) = 0

or, (x-20)(9x + 25) = 0

or, $x = 20, \frac{-25}{9}$

rejecting negative value, x = 20 minutes

and x + 5 = 25 minutes

Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

OR

Here roots are equal,

$\therefore D = B^2 - 4AC = 0$

Here, $A = 1 + m^2, B = 2mc, C = \left(c^2 - a^2\right)$

$\therefore (2mc)^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right) = 0$

or, $4m^2c^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right) = 0$

or, $m^2c^2 - \left(c^2 - a^2 + m^2c^2 - m^2n^2\right) = 0$

or, $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$

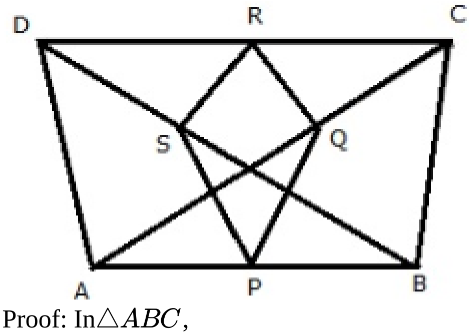
or, $-c^2 + a^2 + m^2a^2 = 0$

or, $c^2 = a^2\left(1 + m^2\right)$

Hence Proved.

33. Given: ABCD is a quadrilateral in which AD = BC. P, Q, R, S are the midpoints of AB, AC, CD and BD.

To prove: PQRS is a rhombus



Proof: In$\triangle ABC$,

Since P and Q are mid points of AB and AC

Therefore, $PQ \| BC$,PQ=$\frac{1}{2}$BC ....(1) (Mid-point theorem)

Similarly,

In $\triangle CDA$,

Since R and Q are mid points of CD and AC

Therefore, $RQ \| DA$,RQ=$\frac{1}{2}$DA=$\frac{1}{2}$BC ....(2)

In $\triangle BDA$,

Since S and P mid points of BD and AB

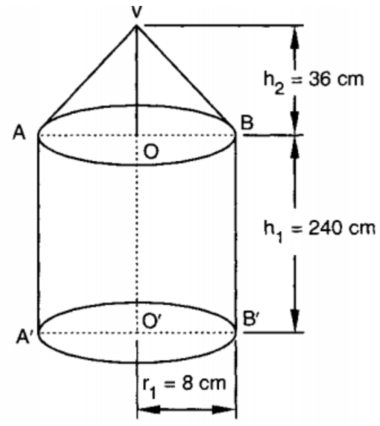
Therefore, $SP \| DA$,,SP=$\frac{1}{2}$DA=$\frac{1}{2}$BC ....(3)

In $\triangle CDB$,
Since S and R are mid points of BD and CD
Therefore, $SR || BC$,SR=$\frac{1}{2}$BC .....(4)
From (1) (2),(3)and (4) PQ || SR and (3) RQ || SP
PQ=RQ=SP=SR
So the opposite sided of PQRS are parallel and all sides are equal
Hence, PQRS is a rhombus.

34. Volume of one cube = 125 cm^3
$\therefore$ side of the cube = 5 cm
Volume of the resulting cuboid = volume of 2 cubes = 250 cm^3
$\therefore$ Length of new cuboid 5 + 5 = 10 cm
Breadth of new cuboid = 5 cm
Height of new cuboid = 5 cm
Surface area of the resulting cuboid = 2(lb + bh + hl)
= 2(10 $\times$ 5 + 5 $\times$ 5 + 5 $\times$ 10)
= 250 cm^2

OR

Let us suppose that r_1 cm and r_2 cm denote the radii of the base of the cylinder and cone respectively. Then,
$r_1 = r_2$ = 8 cm
Let us suppose that h_1 and h_2 cm be the heights of the cylinder and the cone respectively. Then,



h_1 = 240 cm and h_2 = 36 cm
$\therefore$ Volume of the cylinder = $\pi r_1^2 h_1 \text{cm}^3$
$= (\pi \times 8 \times 8 \times 240)\text{cm}^3$
$= (\pi \times 64 \times 240)\text{cm}^3$
Now, Volume of the cone = $\frac{1}{3}\pi r_2^2 h_2 \text{cm}^3$
$= \left(\frac{1}{3}\pi \times 8 \times 8 \times 36\right) \text{cm}^3$
$= \left(\frac{1}{3}\pi \times 64 \times 36\right) \text{cm}^3$
$\therefore$ Total volume of the iron = Volume of the cylinder + Volume of the cone
$= \left(\pi \times 64 \times 240 + \frac{1}{3}\pi \times 64 \times 36\right) \text{cm}^3$
$= \pi \times 64 \times (240 + 12)\text{cm}^3$
$= \frac{22}{7} \times 64 \times 252\text{cm}^3 = 22 \times 64 \times 36\text{cm}^3$
Total weight of the pillar =Volume$\times$Weight per cm^3
$= (22 \times 64 \times 36) \times 7.8\text{gms}$
= 395366.4 gms = 395.3664 kg

35.

Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7
200-300	x	7 + x

300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,

N = Σf_i = 100

$\Rightarrow$ 76 + x + y = 100

$\Rightarrow$ x + y = 24

It is given that the median is 525. Clearly, it lies in the class 500 - 600

$\therefore$ l = 500, h = 100, f = 20, F = 36 + x and N = 100

Now, Median = l + $\frac{\frac{N}{2}-F}{f}\times$ h

$\Rightarrow$ 525 = 500 + $\frac{50-(36+x)}{20}\times$ 100

$\Rightarrow$ 525 - 500 = (14 - x)5

$\Rightarrow$ 25 = 70 - 5x

$\Rightarrow$ 5x = 45

$\Rightarrow$ x = 9

Putting x = 9 in x + y = 24, we get y = 15

Hence, x = 9 and y = 15

Section E

36. i. The distance covered by Dinesh to pick up the first flower plant and the second flower plant,

= 2 $\times$ 10 + 2 $\times$ (10 + 5) = 20 + 30

therefore, the distance covered for planting the first 5 plants

= 20 + 30 + 40 + ... 5 terms

This is in AP where the first term a = 20

and common difference d = 30 - 20 = 10

ii. We know that a = 20, d = 10 and number of terms = n = 5 so,

$S_n = \frac{n}{2}[2a + (n-1)d]$

So, the sum of 5 terms

$S_5 = \frac{5}{2}[2\times 20 + 4\times 10] = \frac{5}{2}\times$ 80 = 200 m

Hence, Dinesh will cover 200 m to plant the first 5 plants.

iii. As a = 20, d = 10 and here n = 10

So, S_{10} = $\frac{10}{2}[2\times 20 + 9\times 10] = 5\times 130$ = 650 m

So, hence Ramesh will cover 650 m to plant all 10 plants.

OR

Total distance covered by Ramesh 650 m

Time = $\frac{\text{distance}}{\text{speed}} = \frac{650}{10}$ = 65 minutes

Time taken to plant all 10 plants = 15 $\times$ 10 = 150 minutes

Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

37. i. Here, CD = $\sqrt{(7-3)^2 + (7-4)^2}$

= $\sqrt{4^2 + 3^2}$ = 5 units

Also, it is given that CE = 10 units

Thus, DE = CE - CD = 10 - 5 = 5 units ($\because$ A, B, C, E are a line)

ii. Since, CD = DE = 5 units

$\therefore$ D is the midpoint of CE.

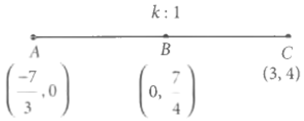
$\therefore \frac{x+3}{2} = 7$ and $\frac{y+4}{2} = 7$

$\Rightarrow$ x = 11 and y = 10 $\Rightarrow$ x + y = 21

iii. The points C, D and E are collinear.

OR

Let B divides AC in the ratio k : 1, then



$\frac{7}{4} = \frac{4k+0}{k+1}$

$\Rightarrow$ 7k + 7 = 16 k

$\Rightarrow$ 7 = 9k

$\Rightarrow$ k = $\frac{7}{9}$

Thus, the required ratio is 7 : 9.

38. i. Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

Given: Speed = 600 miles/hour

Thus, distance d = 600 $\times$ $\frac{1}{60}$ = 10 miles

ii. Now, tan 30^{o} = $\frac{BB'}{B'A} = \frac{h}{10+x}$ ...(i)

And tan 60^{o} = $\frac{CC'}{C'A} = \frac{BB'}{C'A} = \frac{h}{x}$

x = $\frac{h}{\tan 60^o} = \frac{h}{\sqrt{3}}$

Putting the value of x in eq(1), we get,

tan 30^{o} = $\frac{h}{10+\frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

$\tan 30^0 = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

$\Rightarrow 3h = 10\sqrt{3} + h$

$\Rightarrow 2h = 10\sqrt{3}$

$\Rightarrow h = 5\sqrt{3}$ = 8.66 miles

Thus, the altitude 'h' of the airplane is 8.66 miles.

iii. The distance between passenger and airplane when the angle of elevation is 30^{o}.

In $\triangle$ABB'

sin 30^{o} = $\frac{BB'}{AB}$

$\Rightarrow \frac{1}{2} = \frac{8.66}{AB}$

$\Rightarrow$ AB = 17.32 miles

OR

The distance between passenger and airplane when the angle of elevation is 60^{o}.

In $\triangle$ACC'

sin 60^{o} = $\frac{CC'}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{AC}$

$\Rightarrow$ AC = 10 miles