

Continuity and Differentiability

1. Prove that if the function is differentiable at a point c , then it is also continuous at that point.
(MQP 2)
2. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.
3. Find $\frac{dy}{dx}$, if $x^3 + x^2y + xy^2 + y^3 = 81$.
4. If $\sin^2 y + \cos xy = k$, find $\frac{dy}{dx}$.

5. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$. Find $\frac{dy}{dx}$.
6. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1 < x < 1$, show that $\frac{dy}{dx} = -\frac{2}{1+x^2}$.
7. If $y = (\log x)^x$, then find $\frac{dy}{dx}$.
8. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$ with respect to x . **(MQP 1)**
9. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x . **(MQP 3)**
10. Find the derivative of $\cos x \cdot \cos 2x \cdot \cos 3x$ with respect to x .
11. If $xy = e^{x-y}$, then find $\frac{dy}{dx}$. **(S 20)**
12. If $y^x = x^y$, find $\frac{dy}{dx}$.
13. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.
14. If $y^x + x^y = a^b$, find $\frac{dy}{dx}$. **(MQP 5)**
15. Find $\frac{dy}{dx}$, if $x = 2at^2$ and $y = at^4$. **(M 20)**
16. If $x = \sin t$, $y = \cos 2t$, then prove that $\frac{dy}{dx} = -4 \sin t$. **(M 18)**
17. If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$, then prove that $\frac{dy}{dx} = \tan \frac{\theta}{2}$. **(A 21)(J 19)(M 19)(J 14)**
18. If $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$, then prove that $\frac{dy}{dx} = -\cot \frac{\theta}{2}$. **(J 18)(J 16)**
19. Find $\frac{dy}{dx}$, if $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$. **(M 14)**
20. Find $\frac{dy}{dx}$, if $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$. **(J 17)(J 15)**
21. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, then prove that $\frac{dy}{dx} = -\frac{y}{x}$. **(M 15)(MQP 4)**
22. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, prove that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$. **(M 16)**

23. If $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, find $\frac{dy}{dx}$. **(MQP 1)**

24. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$. **(M 17)**

25. If $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$, then prove that $\frac{dy}{dx} = \frac{1}{2}$. **(MQP 3)(MQP 4)**

26. For a positive constant a find $\frac{dy}{dx}$, where $y = a^{\left(t+\frac{1}{t}\right)}$ and $\left(t+\frac{1}{t}\right)^a$.

27. If $\cos y = x \cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. **(A 21)**

28. If $y = x^3 \cdot \log x$, find $\frac{d^2y}{dx^2}$.

29. Find second order derivative of $e^x \cdot \sin 5x$.