[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.7 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Q.1 If $f(x) = \min.\{|x-1|, |x-2|, |x-3|\}$, then the value of $I = \int_0^3 f(x)dx$ equals

(A) 1 (B) 2 (C) 3 (D) 4

Q.2 Let $f(x) = \frac{x}{x-1}$ and $g(x) = (\underbrace{f \text{ o } f \text{ o} \ldots\ldots \text{o } f}_{f \text{ occurs } 2013 \text{ times}})(x)$, then $\int_{1/6}^{1/3} g(x)d(x)$ equals

(A) $ln\left(e^{\frac{1}{2}} \cdot \frac{4}{5}\right)$ (B) $ln\left(e^{\frac{1}{6}} \cdot \frac{4}{5}\right)$ (C) $ln\left(e \cdot \frac{4}{5}\right)$ (D) $ln\frac{4}{5}$

Q.3 Let $f(x) = \frac{4x^2 - 3x + 1}{\int_0^{2\pi x} \sin^4 t \, dt}$. If $f'\left(\frac{1}{2}\right) = \frac{p}{q\pi}$, p, q $\in$ N, then $(p-q)$ equal

(A) 6 (B) 5 (C) 4 (D) 3

Q.4 If $\int e^{1-\cos x}(1 + x\sin x)dx = f(x) \cdot e^{1-g(x)} + c$, then number of solutions of the equation $|f(x)| = g(x)$ equals (c is arbitrary constant)

(A) 0 (B) 1 (C) 2 (D) 3

Q.5 If $A = \{a_1, a_2, \ldots a_n\}$ be set of all possible natural factors of 1000 and $P = \int_{-2}^{2} x\, ln(a_1^x + a_2^x + \ldots\ldots + a_n^x)dx$,

then value of $\frac{2P}{n}$ equals

(A) ln 10 (B) ln 2 (C) $\frac{ln\ 2}{16}$ (D) 8 ln 10

Q.6 Consider a real valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t f(t))\, dt$.

If maximum value of the function is equal to $k(\pi + 1)$, $k \in N$, then the value of k is

(A) 1 (B) 2 (C) 3 (D) 4

Q.7 $\int \frac{x - \sin x \cos x}{x^2 \cos^2 x} dx = F(x)$ such that $F(1) = 1$, then $\lim_{x \to 0} F(x)$ equals

(A) 1 (B) $1 - \tan 1$ (C) $2 - \tan 1$ (D) $\tan 1$

[PARAGRAPH TYPE]

Q.8 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Paragraph for question nos. 8 to 10

Let a continuous function f satisfies the relation

$$(1 + x^2)\, f(x) \cdot f\left(\frac{2}{x}\right) + x = x\, f(x) + (1 + x^2)\, f\left(\frac{2}{x}\right) \quad \forall\, x \in R - \{0\} \text{ and } f(0) \neq 1$$

Q.8 If $\lim_{x \to 0} \left(\tan^{-1}(f(x)) + a\right)^{\frac{4}{x}} = e^p$, then p is equal to

(A) 1 (B) 2 (C) 4 (D) 8

Q.9 If $\int_0^{\infty} \frac{\ln(2f(x))}{4 + x^2} dx = \frac{-\pi \ln 2}{\lambda}$, then λ is equal to

(A) 1 (B) 2 (C) 4 (D) 8

Q.10 If $g : [0, 2] \to \left[0, \frac{1}{2}\right]$ and $g(x) = f(x)$, then $\int_0^{1/2} g^{-1}(x)\, dx$ is equal to

(A) $1 + \ln 2$ (B) $2 - \ln 2$ (C) $1 + 2\ln 2$ (D) $1 - \ln 2$

[MULTIPLE CORRECT CHOICE TYPE]

Q.11 to Q.14 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct.

Q.11 If $I_1 = \int_0^{\pi} \frac{\cos x}{(x + 2)^2} dx$ and $I_2 = \int_0^{\pi/2} \frac{\cos x \cdot \sin x}{(x + 1)} dx$ and $\lambda I_2 = \frac{2}{\mu + \pi} + k - \gamma I_1$, then

(A) $\lambda = 4$ (B) $\mu = 2$ (C) $k = 1$ (D) $\gamma = 2$

Q.12 If $f(x) = \int_0^{x} (f(t))^{-1} dt$ and $\int_0^{1} (f(t))^{-1} dt = \sqrt{2}$, then

(A) $f(2) = 2$ (B) $f(8) = 4$ (C) $f(8) = 9$ (D) $f(128) = 8$

Q.13 If $f(x)=\sqrt{1-\frac{1}{x}}$ and $g(x)=\sec^{-1}x$ then identify which of the following statement(s) is(are) correct

(A) Range of $g(f(x))$ is $\left(0, \frac{\pi}{4}\right]$

(B) Domain of $g(f(x))$ is $(-\infty, 0) \cup [1, \infty)$

(C) $\int_{-4}^{-1} \cot(g(f(x)))dx = \frac{14}{3}$

(D) $\lim_{x\to-\infty}\left(\frac{\cot(g(f(x)))}{\sqrt{1-x}+\sqrt{2-x}}\right) = \frac{1}{2}$

Q.14 A function f satisfies $2+\int_2^x f(t)\,dt = \frac{x^2}{2}+\int_x^2 t^2 f(t)\,dt$, then number of real solutions of the equation $x^2-2x+2(1-f(x))=0$ is n, then n is a factor of

(A) 4 (B) 6 (C) 3 (D) 5

[MATRIX TYPE]

Q.15 has **four** statements (A, B, C, D) given in **Column-I** and **five** statements (P, Q, R, S, T) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II**.

Q.15 Let $\int x^2 \frac{d}{dx}\left(\frac{3x^2+1}{x^7+2x^5+2x^4+x^3+2x^2+5x}\right)dx = f(x)-\tan^{-1}(g(x))+c$,

where c is contant of integration and $f(0)=0$, $g(1)=\frac{3}{2}$. Also, $h(g(x))=x\ \forall\ x \in R$

Column-I		Column-II	
(A)	If the value of $\tan^{-1}(g(0))+\tan^{-1}\left(\frac{1}{2g(1)}\right)=\tan^{-1}(2g(p))$, then p is	(P)	0
(B)	If the number of solutions(s) of the equation $g(x)=h(x)$ is equal to $\frac{q}{\pi}\sin^{-1}(g(0))$, then q is	(Q)	1
		(R)	3
(C)	If $h'\left(\frac{31}{2}\right)=\frac{g(0)}{k}$, then k is	(S)	6
(D)	If $\int_{-3}^{3} g(x)\,dx = 2g(n)$, then n is	(T)	7

[INTEGER TYPE]

Q.16 to Q.19 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**)

Q.16 If $A = \int_{0}^{101} |[x]\sin \pi x|\,dx$, then find the value of $\frac{A\pi}{2}$.

[**Note:** [y] denotes greatest integer function less than or equal to y.]

Q.17 Let f be a twice differentiable function such that $f''(t) = \int_{12t}^{t^3+6t^2} ln\left|\frac{x+8}{(t+2)^3 - x}\right| dx \ \forall t \in R$

and $f(1) = 2f(2) = 2$, then find the value of $\int_{-1}^{1} f(x)\,dx$.

Q.18 If $\int_{0}^{1/2} \frac{\pi\,dx}{2(\cos^{-1}x)^2\sqrt{1-x^2}} = \frac{p}{q}$, where p and q are relatively prime then find $(p+q)$.

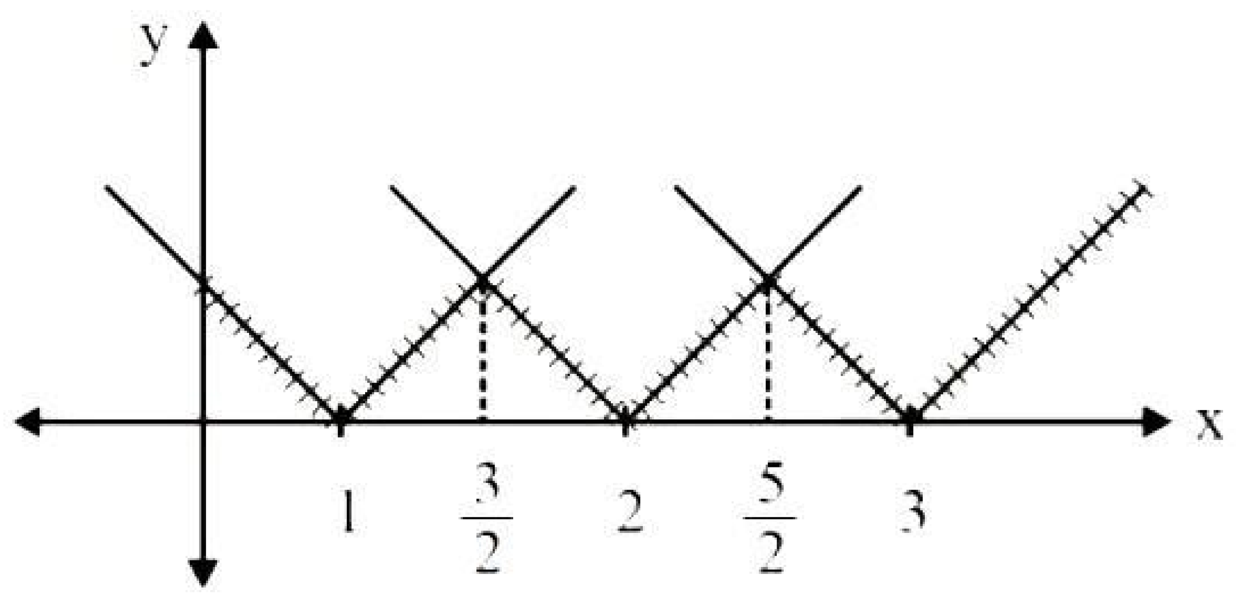
Q.19 If $\int e^x \left(\frac{1-x^n}{1-x}\right) dx = e^x \cdot P(x) + C$ where $n \in N$ and 'C' is constant of integration and $P(0) = 620$ then find the value of n.

ANSWER KEY

Q.1	A	Q.2	B	Q.3	B	Q.4	C	Q.5	A
Q.6	C	Q.7	C	Q.8	B	Q.9	C	Q.10	D
Q.11	ABCD	Q.12	AB	Q.13	CD	Q.14	ABCD		
Q.15	(A) P; (B) S; (C) T; (D) Q			Q.16	5050	Q.17	6	Q.18	0003
Q.19	7								

Q.1 (A)

$$\int_0^3 f(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 + \frac{1}{2}\cdot 1\cdot \frac{1}{2} + \frac{1}{2}\cdot 1\cdot \frac{1}{2} = \frac{1}{2}+\frac{1}{4}+\frac{1}{4} = 1$$



Q.2 (B)

$$f(x) = \frac{x}{x-1} \quad\Rightarrow\quad f(f(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-x+1} = x$$

$\Rightarrow$ $f(f(f\ldots\ldots x))$ when f is odd number of times $= f(x)$

$$\int_{1/6}^{1/3} \frac{x}{x-1}dx = \int_{1/6}^{1/3} \frac{x-1+1}{x-1} = x + \log(x-1)\Big|_{1/6}^{1/3}$$

$$= \left(\frac{1}{3}-\frac{1}{6}\right) + \log\frac{\frac{1}{3}-1}{\frac{1}{6}-1} = \frac{1}{6} + \log\frac{4}{5} = \log e^{1/6}\frac{4}{5}$$ **Ans.**

Q.3 (B)

$$f(x) = \frac{4x^2-4x+1+x}{\int_0^{2\pi x}\sin^2 t\,dt} = \frac{(2x-1)^2}{\int_0^{2\pi x}\sin^4 t\,dt} + \frac{x}{\int_0^{2\pi x}\sin^4 t\,dt}$$

$$f'(x) = g'(x) + \frac{\int_0^{2\pi x}\sin^4 t\,dt\ \cdot 1 - x\sin^4(2\pi x)\cdot 2\pi}{\left(\int_0^{2\pi x}\sin^4 t\,dt\right)^2}$$

$$f'\left(\frac{1}{2}\right) = g'\left(\frac{1}{2}\right) + \frac{1}{\int_0^{\pi} \sin^4 t\, dt} = 0 + \frac{1}{2\cdot\frac{3\pi}{16}} = \frac{8}{3\pi} \quad \Rightarrow p - q = 5$$

Q.4 (C)

$$\int e^{1-\cos x}(1 + x\sin x)dx = e\int e^{-\cos x}(1 + x\sin x)dx$$

$$e\left[\int e^{-\cos x}dx + \int x\, e^{-\cos x}\cdot \sin x\, dx\right]$$

Now integrating second integral using integration by parts.

$$e\left[\int e^{-\cos x}dx + x\left(e^{-\cos x}\right) - \int 1\cdot e^{-\cos x}dx\right] = e\cdot x\cdot e^{-\cos x} + c = x\cdot e^{1-\cos x} + c$$

$f(x) = x$ and $g(x) = \cos x$ number of solution is 2.

Q.5 (A)

$1000 = 5^3\cdot 2^3$

Number of factors $(3 + 1)(3 + 1) = 16$

Applying king, $P = -\int_{-2}^{2} x\, ln(a_1^{-x} + a_2^{-x} + ... + a_{16}^{-x})\, dx = -\int_{-2}^{2} x\, ln\left(\frac{1}{a_1^x} + \frac{1}{a_2^x} + ... + \frac{1}{a_{16}^x}\right)dx$

$$= -\int_{-2}^{2} x\, ln\left(\frac{(a_1^x + a_2^x + ... + a_{16}^x)}{1000^x}\right)dx = -\int_{-2}^{2} x\, ln(a_1^x + a_2^x + ... + a_{16}^x)dx + \int_{-2}^{2} x^2\, ln(1000)dx$$

$$2P = \frac{16}{3} ln\, 1000$$

$P = 8\, ln\ 10 \Rightarrow \frac{2P}{n} = \frac{16l}{16} = ln\ 10$ **Ans.**

Q.6 (C)

We have $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t f(t))\, dt = \sin x + \pi\sin x + \int_{-\pi/2}^{\pi/2} t f(t)\, dt$

$\therefore \quad f(x) = (\pi + 1)\sin x + A \quad .....(1)$

Now $A = \int_{-\pi/2}^{\pi/2} t\left((\pi+1)\sin t + A\right) dt = 2(\pi + 1)\int_0^{\pi/2} \underset{(I)}{t}\ \underset{(II)}{\sin t}\ dt$ (Bypart)

$\Rightarrow \quad A = 2(\pi + 1)$

Hence $f(x) = (\pi + 1)\sin x + 2(\pi + 1)$

$\therefore \quad f_{max.} = 3(\pi + 1) = M$

and $f_{min.} = (\pi + 1) = m \Rightarrow \frac{M}{m} = 3$

Q.7 $\int \frac{x - \sin x \cos x}{x^2 \cos^2 x} dx = F(x)$ such that $F(1) = 1$, then $\lim_{x \to 0} F(x)$ equals

(A) 1 (B) 1 – tan 1 (C*) 2 – tan 1 (D) tan 1

$$\int \frac{x - \sin x \cos x}{x^2 \cos^2 x} dx = \int \frac{dx}{x \cos^2 x} - \int \frac{\tan x}{x^2} dx$$

$$\frac{1}{x} \tan x + \int \frac{1}{x^2} \tan x \, dx - \int \frac{\tan x}{x^2} dx$$

$$F(x) = \frac{1}{x} \tan x + c; \; F(1) = 1 \quad \Rightarrow \quad c = 1 - \tan 1$$

$$\therefore \; F(x) = \frac{\tan x}{x} + 1 - \tan 1 \textbf{ Ans.}$$

Q.8 (B)
Q.9 (C)
Q.10 (D)

$$(1 + x^2) f(x) \cdot f\left(\frac{2}{x}\right) + x = x f(x) + (1 + x^2) f\left(\frac{2}{x}\right)$$

$$(1 + x^2) f\left(\frac{2}{x}\right) (f(x) - 1) = x(f(x) - 1)$$

$$(f(x) - 1) \cdot \left((1 + x^2) f\left(\frac{2}{x}\right) - x \right) = 0$$

$$f(x) = 1 \text{ (rejected) and } f\left(\frac{2}{x}\right) = \frac{x}{1 + x^2}$$

$$f\left(\frac{2}{x}\right) = \frac{x}{1 + x^2}, \; x \neq 0$$

$$f(t) = \frac{2t}{4 + t^2}, \; f(0) = 0 \qquad \{\because f \text{ is continuous}\}$$

(i) $$\lim_{x \to 0} \left(\tan^{-1}\left(\frac{2x}{4 + x^2}\right) + a \right)^{\frac{4}{x}} = e^p$$

a must be 1.

$$\Rightarrow e^{\lim_{x \to 0} \left(\tan^{-1}\left(\frac{2x}{4 + x^2}\right) \right) \cdot \frac{4}{x}} = e^p$$

$$\Rightarrow p = \lim_{x \to 0} \frac{\tan^{-1}\left(\frac{2x}{4 + x^2}\right)}{x \cdot \frac{2}{4 + x^2}} \cdot \frac{4 \cdot 2}{4 + x^2} = 2 \textbf{ Ans.}$$

(ii) $$I = \int_0^{\infty} \frac{ln(2f(x))}{4+x^2}dx = \int_0^{\infty} \frac{ln\left(\frac{2\cdot 2x}{4+x^2}\right)}{4+x^2}dx$$

Put $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta \cdot d\theta$

$$= \int_0^{\pi/2} \frac{\ln\left(\frac{4\cdot 2\tan\theta}{4+4\tan^2\theta}\right)}{4+4\tan^2\theta} \cdot 2\sec^2\theta \cdot d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} ln\,(\sin 2\theta)\,d\theta = \frac{1}{2}\left(\frac{-\pi}{2}\,ln\,2\right) = \frac{-\pi}{4}\,ln\,2 \textbf{ Ans.}$$

(iii) $$\int_0^2 g(x)\,dx + \int_0^{1/2} g^{-1}(x)\,dx = 1$$

$$\Rightarrow \int_0^{1/2} g^{-1}(x)\,dx = 1 - \int_0^2 \frac{2x}{4+x^2}dx = 1 - \left(ln(4+x^2)\right)_0^2 = 1 - (ln\,8 - ln\,4) = 1 - ln\,2 \textbf{ Ans.}$$

Q.11 (ABCD)

$$I_1 = \int_0^{\pi} \frac{\cos x}{(x+2)^2}dx, \quad x = 2t$$

$$I_1 = \int_0^{\pi/2} \frac{\cos 2t}{2(t+1)^2}dt = \left.\frac{-\cos 2t}{2(t+1)}\right|_0^{\pi/2} + \int_0^{\pi/2} \frac{-2\sin 2t}{2(t+1)}dt$$

$$I_1 = \frac{1}{2+\pi} + \frac{1}{2} - \int_0^{\pi/2} \frac{4\sin t\cos t}{2(t+1)}dt$$

$$2I_1 = \frac{2}{2+\pi} + 1 - 4I_2$$

$$4I_2 = \frac{2}{2+\pi} + 1 - 2I_1 \textbf{ Ans.}$$

Q.12 (AB)

$$f(x) = \int_0^x (f(t))^{-1}dt \Rightarrow f'(x) = \frac{1}{f(x)}\cdot 1$$

$$f'(x)\cdot f(x) = 1$$

$$\frac{(f(x))^2}{2} = x + c \Rightarrow f(x) = \sqrt{2(x+c)}$$

and $f(1) = \sqrt{2} \Rightarrow c = 0$ i.e. $f(x) = \sqrt{2x}$ **Ans.**

Q.13 (CD)

$$gof(x) = \sec^{-1}\sqrt{1-\frac{1}{x}}$$

Domain : $\sqrt{1-\frac{1}{x}} \ge 1 \Rightarrow x < 0 \Rightarrow x \in (-\infty, 0)$

Range : $\left(0, \frac{\pi}{2}\right)$

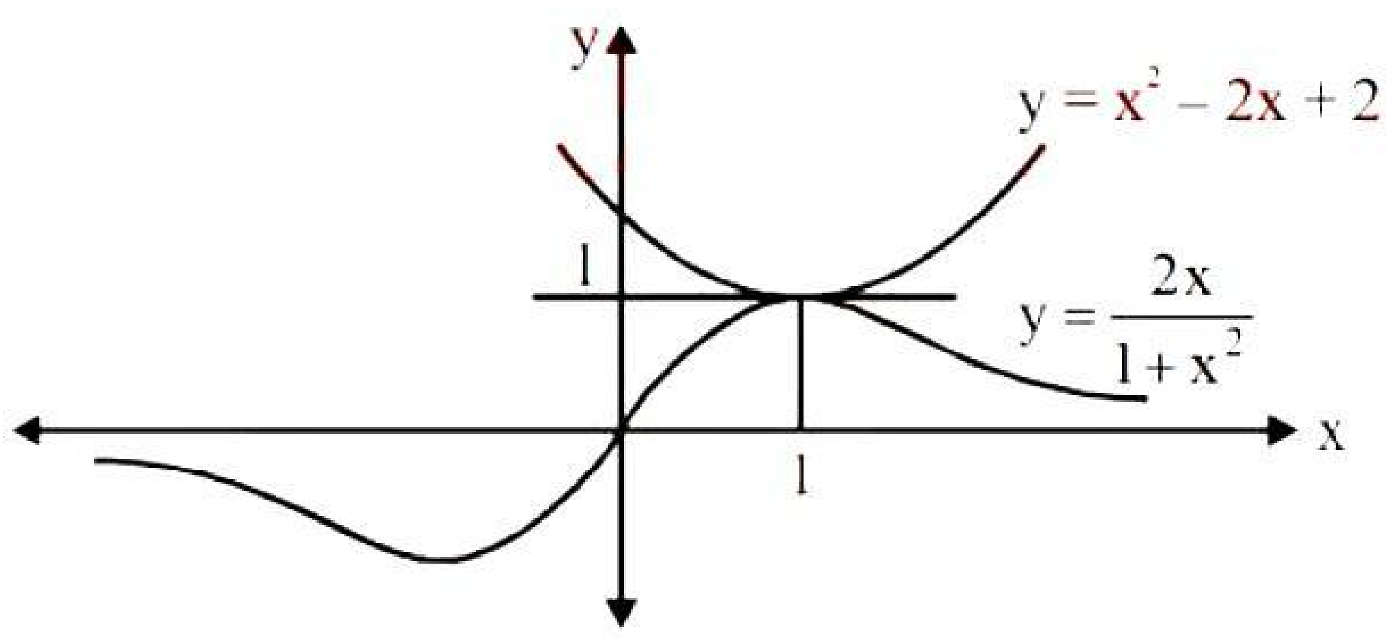
$$\cot(g(f(x))) = \sqrt{-x} \Rightarrow \int_{-4}^{-1}\sqrt{-x}\,dx = \frac{14}{3}$$

$$\lim_{x\to-\infty}\frac{\sqrt{-x}}{\sqrt{1-x}+\sqrt{2-x}} = \frac{1}{2}$$

Q.14 (ABCD)
Differentiating given relation

$$f(x) = x - x^2 f(x) \Rightarrow f(x) = \frac{x}{x^2+1}$$

$\therefore$ Given equation becomes $x^2 - 2x + 2 = \dfrac{2x}{1+x^2}$



The given equation is $x^2 - 2x + 2 = 2f(x)$
$\Rightarrow$ only one possible solution
Ans. (A), (B), (C), (D)

Q.15 (A) P; (B) S; (C) T; (D) Q

$$\int \underbrace{x^2}_{I} \cdot \underbrace{\frac{d}{dx}\left(\frac{3x^2+1}{x^7+2x^5+2x^4+x^3+2x^2+5x}\right)}_{II} dx$$

$$x^2 \cdot \frac{3x^2+1}{x^7+2x^5+2x^4+x^3+2x^2+5x} - \int \frac{2x(3x^2+1)\,dx}{x(x^6+2x^4+2x^3+x^2+2x+5)}$$

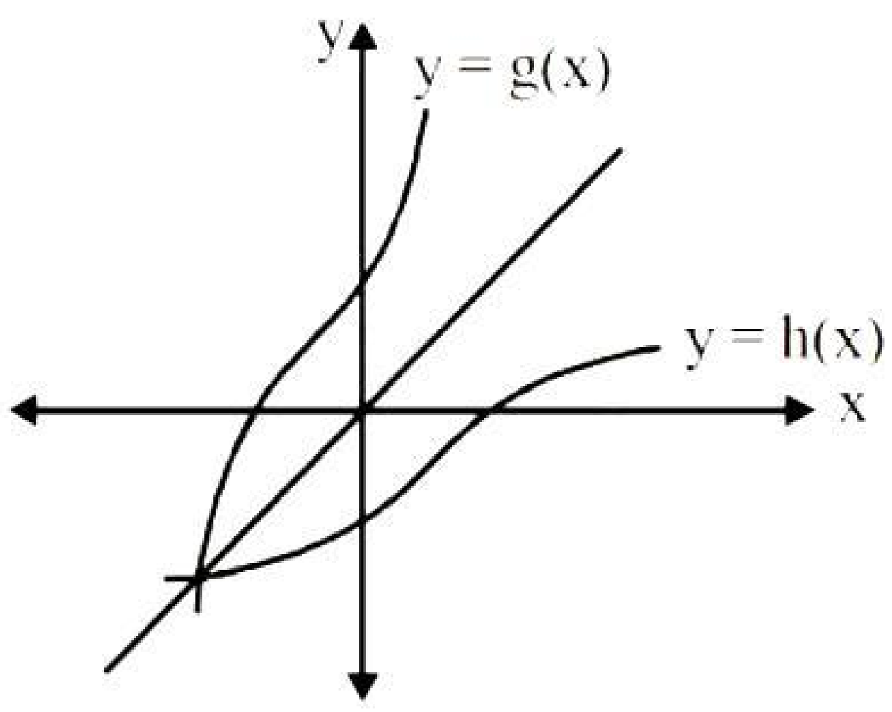
$$\frac{x(3x^2+1)}{x^6+2x^4+2x^3+x^2+2x+5} - \int \frac{2(3x^2+1)\,dx}{(x^3)^2+2x^3(x+1)+(x+1)^2+4}$$

$$\frac{x(3x^2+1)}{x^6+2x^4+2x^3+x^2+2x+5}-2\int\frac{(3x^2+1)dx}{\left(x^3+(x+1)\right)^2+4}$$

$$\frac{x(3x^2+1)}{x^6+2x^4+2x^3+x^2+2x+5}-\tan^{-1}\left(\frac{x^3+x+1}{2}\right)+c$$

$$g(x)=\frac{x^3+x+1}{2}$$

(A) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}(1)$ $\left\{\because\ g(0)=\frac{1}{2}\right\}$



(B) $g(x)=h(x)$ has exactly one solution

$\frac{q}{\pi}\sin^{-1}\left(\frac{1}{2}\right)=1\ \Rightarrow\ q=6$

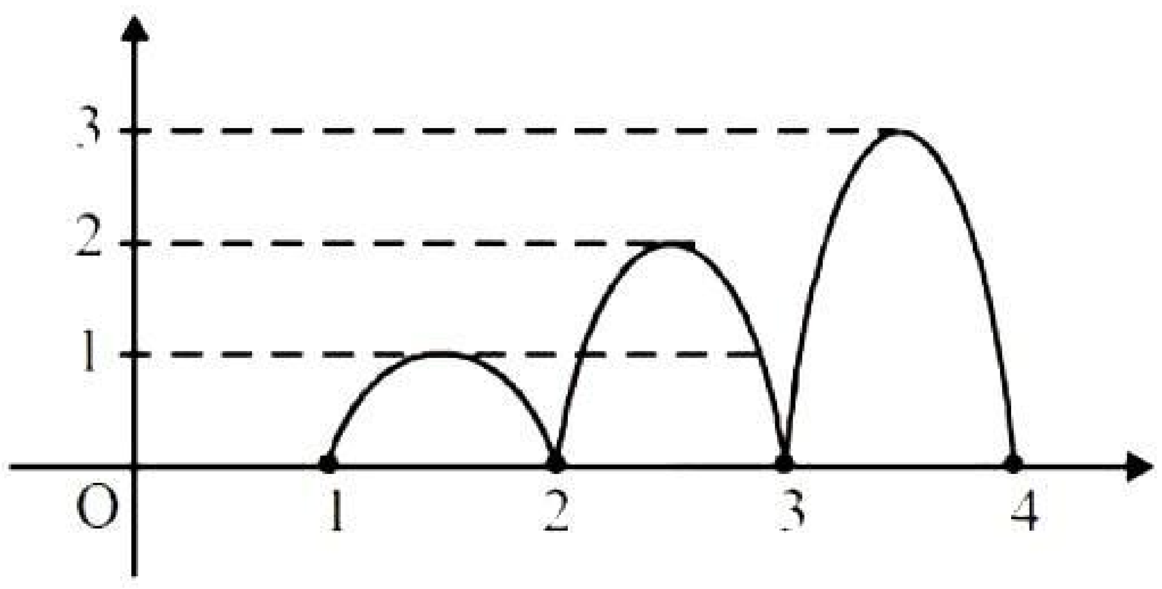
(C) $h'\left(\frac{31}{2}\right)=\frac{1}{g'(3)}=\frac{1}{14}=\frac{g(0)}{k}\ \Rightarrow\ k=7$

(D) $\int_{-3}^{3}g(x)\,dx=\int_{-3}^{3}\left(\frac{x^3+x+1}{2}\right)dx=\frac{1}{2}\cdot2\int_{0}^{3}dx=3=2g(1)$

$\Rightarrow\ n=1$ **Ans.**

Q.16 [5050]

Let $y=f(x)=\left|[x]\sin\pi x\right|$



$$A=\int_0^1 0\,dx+\int_1^2-\sin\pi x\,dx+\int_2^3 2\sin\pi x\,dx+\ldots\ldots$$

$$A=\frac{2}{\pi}+\frac{4}{\pi}+\frac{6}{\pi}+\ldots\ldots+\frac{200}{\pi}$$

$$A=\frac{2}{\pi}(1+2+\ldots\ldots+100)\ \Rightarrow\ \frac{A\pi}{2}=5050$$ **Ans.**

Q.17 [6]

$$f''(t)=\int_{12t}^{t^3+6t^2}ln\left|\frac{x+8}{(t+2)^3-x}\right|dx \qquad \ldots\ldots(i)$$

Applying king

$$f''(t)=\int_{12t}^{t^3+6t^2}ln\left|\frac{t^3+6t^2+12t-x+8}{(t+2)^3-(t^3+6t^2+12t-x)}\right|dx$$

$$f''(t)=\int_{12t}^{t^3+6t^2}ln\left|\frac{(t+2)^3-x}{8+x}\right|dx \qquad \ldots\ldots(ii)$$

(i) + (ii)

$$2f''(t) = 0$$

$\Rightarrow \quad f''(t) = 0$

$\Rightarrow \quad f'(t) = \lambda$

$$f(t) = \lambda t + \mu$$

$$\left.\begin{array}{l} f(1) = 2 \Rightarrow \lambda + \mu = 2 \\ f(2) = 1 \Rightarrow 2\lambda + \mu = 1 \end{array}\right\} \Rightarrow \lambda = -1,\ \mu = 3$$

$$\therefore \quad \int_{-1}^{1} (-x+3)\,dx = 2\int_{0}^{1} 3 \cdot dx = 6 \ \textbf{Ans.}$$

Q.18 [0003]

$$\frac{\pi}{2}\int \frac{dx}{(\cos^{-1}x)^2\sqrt{1-x^2}}$$

Let $\cos^{-1}x = t \Rightarrow \dfrac{dx}{\sqrt{1-x^2}} = -dt$

$$\frac{-\pi}{2}\int \frac{dt}{t^2} = \left(\frac{\pi}{2}\right)\cdot\frac{1}{t} + c = \frac{\sin^{-1}x + \cos^{-1}x}{\cos^{-1}x} + c = \left(\frac{\sin^{-1}x}{\cos^{-1}x}\right)_0^{1/2} = \left(\frac{\pi/6}{\pi/3}\right) = \frac{3}{6} = \frac{1}{2} \ \textbf{Ans.}$$

Q.19 [7]

$$\int e^x\left(\frac{1-x^n}{1-x}\right)dx = e^x P(x) + C$$

Let $P(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{n-1}x^{n-1}$

$= \int e^x(1 + x + x^2 + \ldots\ldots + x^{n-1})dx = e^x(a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{n-1}x^{n-1}) + c$

$P(0) = a_0 = 620$

Differentiating both sides

$e^x(1 + x + x^2 + \ldots\ldots x^{n-1}) = e^x\left(a_1 + 2a_2x + 3a_3x^2 + \ldots\ldots + (n-1)_{n-1}x^{n-2}\right)$
$+ (a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{n-1}x^{n-1})e^x$

Comparing coefficient of same power of x

$a_{n-1} = 1$

$a_0 = 620$

$a_1 + a_0 = 1 \Rightarrow a_1 = -619$

$a_1 + 2a_2 = 1 \Rightarrow a_2 = 310$

$a_2 + 3a_3 = 1 \Rightarrow a_3 = -103$

$a_3 + 4a_4 = 1 \Rightarrow a_4 = +26$

$a_4 + 5a_5 = 1 \Rightarrow a_5 = -5$

$a_5 + 6a_6 = 1 \Rightarrow a_6 = 1$

$\therefore\ n - 1 = 6 \Rightarrow n = 7.$