Playing with Numbers

Finding Factors and Multiples of Numbers

Can we exactly divide 26 by 2?

Yes, we can.

Can we exactly divide 26 by 3?

No.

Now, can we say that 2 is a factor of 26 while 3 is not?

Let us find out. Let us first understand the meaning of the term “factor” of a number.

The term “factor” can be defined as follows:

An exact divisor of a number is called a factor of that number.

In the above example, since 2 exactly divides 26 i.e., 2 is an exact divisor of 26, we can say that 2 is a factor of 26. On the other hand, since 3 is not an exact divisor of 26, we can say that 3 is not a factor of 26.

Till now, we were talking about the exact divisors of the numbers (i.e., factors). Let us now look at this from a different point of view.

Let us consider the relation, $24 = 4 \times 6$

We already know that 4 and 6 are the factors of 24. Apart from this, we can also say that 24 is a multiple of both 4 and 6.

Let us now look at some examples to understand the method of finding factors and multiples better.

Example 1:

Write all the factors of the following numbers.

(a) 8 (b) 18 (c) 64

Solution:

(a) We can write 8 as

1 × 8 = 8

2 × 4 = 8

Therefore, the factors of 8 are 1, 2, 4, and 8.

(b) We can write 18 as

1 × 18 = 18

2 × 9 = 18

3 × 6 = 18

Therefore, the factors of 18 are 1, 2, 3, 6, 9, and 18.

(c) We can write 64 as

1 × 64 = 64

2 × 32 = 64

4 × 16 = 64

8 × 8 = 64

Therefore, the factors of 64 are 1, 2, 4, 8, 16, 32, and 64.

Example 2:

Write the first four multiples of the following numbers.

(a) 6 (b) 9

Solution:

(a) The first four multiples of 6 can be calculated as

$6 \times 1 = 6$

$6 \times 2 = 12$

$6 \times 3 = 18$

$6 \times 4 = 24$

Therefore, the first four multiples of 6 are 6, 12, 18, and 24.

(b) The first four multiples of 9 can be calculated as

$9 \times 1 = 9$

$9 \times 2 = 18$

$9 \times 3 = 27$

$9 \times 4 = 36$

Therefore, the first four multiples of 9 are 9, 18, 27, and 36.

Properties of Factors and Multiples

Look at the following relations.

$2 = 2 \times 1$, $10 = 10 \times 1$, $37 = 37 \times 1$, $145 = 145 \times 1$

Can we make any observation from these relations?

Let us study the properties of numbers with respect to their factors or divisors with the help of these relations.

Let us now look at a few examples to have a better understanding of the concept.

Example 1:

How many multiples of 8 are there in total? Find all the multiples of 8 up to 100.

Solution:

By the property of multiples of a number, we can say that there are infinite multiples of 8.

The multiples of 8 can be calculated as:

$8 \times 1 = 8$

$8 \times 2 = 16$

$8 \times 3 = 24$

$8 \times 4 = 32$

$8 \times 5 = 40$

$8 \times 6 = 48$

$8 \times 7 = 56$

$8 \times 8 = 64$

$8 \times 9 = 72$

$8 \times 10 = 80$

$8 \times 11 = 88$

$8 \times 12 = 96$

$8 \times 13 = 104$

Therefore, the multiples of 8 upto 100 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, and 96.

Example 2:

Write the divisor which is common to the numbers 20, 10, 7, and 5.

Solution:

Since 1 is a divisor of all the natural numbers, the common divisor of 20, 10, 7, and 5 is 1.

Example 3:

Write the greatest and the least factor of 42.

Solution:

By the properties of factors of a number, 1 is the least factor and 42 is the greatest factor of 42.

Perfect Numbers

What are the factors of the number 28?

We know that the **factor** of a number is an exact divisor of that number. Let us now list the factors of the number 28.

They are 1, 2, 4, 7, 14, and 28. Now, let us add all these factors.

$1 + 2 + 4 + 7 + 14 + 28 = 56$

Can you relate this sum with the original number i.e., 28?

Yes, the obtained sum, i.e. 56, which is twice the original number 28.

Such a number is called a **perfect number.**

Note: 6 is the smallest perfect number.

Now, are all the numbers perfect numbers?

Let us take an example of 42.

The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

Now, sum of its factors = $1 + 2 + 3 + 6 + 7 + 14 + 21 + 42 = 96$

However, $96 \neq 2 \times 42$

Thus, we can say that 42 is not a perfect number.

From this example, we can conclude that **all numbers are not perfect numbers**.

Let us now look at a few more examples.

Example 1:

Find the perfect numbers from the following numbers.

(i) 35 (ii) 76 (iii) 496 (iv) 48

Solution:

1. Factors of 35 are 1, 5, 7, and 35.

   Now, $1 + 5 + 7 + 35 = 48 \neq 2 \times 35$

   Therefore, 35 is a not a perfect number.

2. Factors of 76 are 1, 2, 4, 19, 38, and 76.

   Now, $1 + 2 + 4 + 19 + 38 + 76 = 140 \neq 2 \times 76$

   Therefore, 76 is not a perfect number.

3. Factors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, 248, and 496.

   Now, $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 + 496 = 992 = 2 \times 496$

   Therefore, 496 is a perfect number.

4. Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

   Now, $1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 + 48 = 124 \neq 2 \times 48$

   Therefore, 48 is not a perfect number.

Example 2:

Find the perfect numbers between 1 and 30.

Solution:

There are only two perfect numbers between 1 and 30. They are 6 and 28.

The factors of 6 are 1, 2, 3, and 6.

And $1 + 2 + 3 + 6 = 12 = 2 \times 6$

The factors of 28 are 1, 2, 4, 7, 14, and 28.

And $1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28$

Prime and Composite Numbers

Consider the following set of numbers.

2, 3, 5, 7, 11, 13, 17, 19…

Is there any similarity between these numbers?

Each number in the above set of numbers is divisible by 1 and the number itself, or we can say that each number in the above set has only two divisors i.e., 1 and the number itself. Such numbers are called **prime numbers.**

Let us learn more about prime numbers.

A prime number can be written as a product of only two factors. These factors are called **prime factors.**

For example, $23 = 1 \times 23$

Hence, 1 and 23 are the prime factors of 23.

There are two kinds of prime numbers.

(1) Even prime numbers

(2) Odd prime numbers

Prime numbers which are divisible by 2 are known as **even prime numbers** and those which are not divisible by 2 are known as **odd prime numbers.**

Examples of odd prime numbers are 3, 5, 7 etc .

It is to be **noted** that 2 is the only even prime number.

Now, consider the numbers other than 1 and prime numbers as follows:

4, 6, 8, 9, 10, 12, 14, 15, 16, 18…

All these numbers have more than two divisors. Such numbers are called **composite numbers**.

Like prime numbers, composite numbers are also of two types.

(1) Even composite numbers

(2) Odd composite numbers

Composite numbers which are divisible by 2 are **even composite numbers** and those which are not divisible by 2 are **odd composite numbers.**

For example, 6, 24, 36, 42, etc. are even composite numbers while 9, 25, 39, etc. are odd composite numbers.

Let us go through the following video to have a better understanding about prime and composite numbers.

Note that 3 is the smallest odd prime number.

Twin primes:

If a pair of prime numbers is such that the difference between those numbers is 2 then such prime numbers are known as **twin primes** and the pair is called **twin prime pair**.

For example, the numbers 3 and 5, both are prime numbers and the difference between them is 2, so these are twin primes and make a twin prime pair.

Similarly, few more twin primes are: 5 and 7; 11 and 13; 17 and 19; 29 and 31; 41 and 43; 101 and 103.

There are many more such twin primes.

Sieve of Eratosthenes:

This is a method used for finding the prime and composite numbers from 1 to 100 which was given by the Greek Mathematician **Eratosthenes**, in the third century B.C.
Follow the steps given below:

Step 1: Prepare a table of natural numbers from 1 to 100 by taking ten numbers in each row.

Step 2: Cross out 1 because it is not a prime number.

Step 3: Encircle 2 as it is prime number and cross out every multiple of 2.

Step 4: Encircle another prime number which is 3 and cross out every multiple of 3. Crossed numbers are not required to be marked again.

Step 5: Encircle 5 as a prime number and cross out every multiple of 5. Crossed numbers are not required to be marked again.

Step 6: Continue this process till all the numbers from 1 to 100 are encircled or crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~22~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

All the encircled numbers are prime numbers. All the crossed out numbers, other than 1 are composite numbers.

This method is called the **Sieve of Eratosthenes**.

Let us now have a look at some examples to understand the concept better.

Example 1:

Find the prime numbers among the following numbers.

(i) 2 (ii) 15 (iii) 17 (iv) 27

Solution:

(i) The divisors of 2 are 1 and 2 i.e., 1 and the number 2 itself. Therefore, 2 is a prime number.

(ii) The divisors of 15 are 1, 3, 5, and 15 i.e., 15 has more than two factors. Therefore, 15 is not a prime number.

(iii) The divisors of 17 are 1 and 17. Therefore, 17 is a prime number.

(iv) The divisors of 27 are 1, 3, 9, and 27. Therefore, 27 is not a prime number.

Example 2:

Find an even prime number which is

(i) greater than 1

(ii) less than 100

Solution:

We know that 2 is the only even prime number. Therefore,

(i) A prime number greater than 1 is 2.

(ii) A prime number less than 100 is 2.

Example 3:

Find odd prime numbers less than 50.

Solution:

The odd prime numbers less than 50 are 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

Example 4:

Find even and odd composite numbers greater than 20 and less than 40.

Solution:

The even composite numbers greater than 20 and less than 40 are 22, 24, 26, 28, 30, 32, 34, 36, and 38.

The odd composite numbers greater than 20 and less than 40 are 21, 25, 27, 33, 35, and 39.

Example 5:

Express 27 and 34 as the sum of three prime numbers.

Solution:

27 can be expressed as follows:

27 = 3 + 11 + 13

And, 34 can be expressed as follows:

34 = 2 + 13 + 19

Example 6:

Write all twin primes lying between 100 and 200.

Solution: Twin primes lying between 100 and 200 are as follows:

101 and 103; 107 and 109; 137 and 139: 149 and 151; 179 and 181; 191 and 193; 197 and 199.

Common Factors and Common Multiples

Each number has factors (or divisors) and multiples. However, for any two or more numbers, these factors and multiples might overlap and we can have common factors (or common divisors) and common multiples of the given numbers. Let us look at the following video to understand what we mean by common factors and multiples.

Now, we are aware about common factors and multiples. Let us take the concept further to HCF and LCM.

HCF stands for **highest common factor** which is also termed as **GCD** which stands for **greatest common divisor**. It is clear from the name that the **greatest among all common factors of two or more numbers is called HCF of those numbers**.

For example, common factors of 14 and 28 = 1, 2, 7, 14

Among these, 14 is greatest, so it is the HCF of 14 and 28.

LCM stands for **least common multiple**. The **least among all common multiples of two or more numbers is called LCM of those numbers**.

For example, common multiples of 2 and 5 = 10, 20, 30, 40...

Among these, 10 is least or smallest, so it is the LCM of 2 and 5.

This method of finding LCM and HCF of numbers using their factors is known as **factor method**.

Let us now look at some more examples to understand the concept better.

Example 1:

Find the common factors of the following triplet of numbers. Also, find their HCF.

1. **77, 49, 63**
2. **50, 20, 100**

Solution:

1. Factors of 77 = 1, 7, 11, and 77

   Factors of 49 = 1, 7, and 49

   Factors of 63 = 1, 3, 7, 9, 21, and 63

   Therefore, the common factors of 77, 49, and 63 are 1 and 7.

   Highest factor among these is 7.

   So, HCF of 77, 49 and 63 is 7.

2. Factors of 50 = 1, 2, 5, 10, 25, and 50

   Factors of 20 = 1, 2, 4, 5, 10, and 20

   Factors of 100 = 1, 2, 4, 5, 10, 20, 25, 50, and 100

   Therefore, the common factors of 50, 20, and 100 are 1, 2, 5, and 10.

   Highest factor among these is 10.

   So, HCF of 50, 20, and 100 is 10.

Example 2:

Find the common multiples of the following pair of numbers. Also, find their LCM.

1. **80 and 30**
2. **2 and 3**

Solution:

1. Multiples of 80 = 80, 160, 240 …

   Multiples of 30 = 30, 60, 90, 120, 150, 180, 210, 240 …

   Therefore, the common multiples of 80 and 30 are 240, 480 …

   Least multiple among these is 240.

So, LCM of 80 and 30 is 240.

2. Multiples of 2 = 2, 4, 6, 8, 10, 12 …

   Multiples of 3 = 3, 6, 9, 12, 15 …

   Therefore, the common multiples of 2 and 3 are 6, 12 …

   Least multiple among these is 6.

   So, LCM of 2 and 3 is 6.

Example 3:

Find three numbers which are divisible by both 2 and 5.

Solution:

The numbers which are divisible by both 2 and 5 are the common multiples of 2 and 5.

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40 …

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40 …

Therefore, the first three common multiples of 2 and 5 are 10, 20, and 30.

Thus, the three numbers which are divisible by both 2 and 5 are 10, 20, and 30.

Co-prime Numbers

Consider the numbers 5 and 26.

Is there any common factor between them?

Let us find it.

An exact **divisor** of a number is called a **factor** of that number.

Therefore, the factors of 5 are 1 and 5 and the factors of 26 are 1, 2, 13, and 26.

The numbers which are common to the factors of both 5 and 26 are the common factors of 5 and 26.

Therefore, the common factor of 5 and 26 is 1 only. Such pairs of numbers are called **co-prime numbers.**

Co-prime numbers can be defined as follows.

Two numbers having only 1 as a common factor are called co-prime numbers.

In other words, if 1 is the greatest common factor between the two numbers, then the numbers are called co-prime numbers. Co-prime numbers are also known as **relative primes**.

For example, 3 and 14, 47 and 48, 87 and 91, etc. are pairs of co-prime numbers.

Important points about co-prime numbers:

1. Any two consecutive numbers can form a pair of co-primes.
2. A number, divisible by co-primes, is divisible by their product as well. For example, 2520 is divisible by co-primes 7 and 8. Hence, it is divisible by 7 × 8 = 56 also.

Let us now look at some examples to understand this concept better.

Example 1:

Find the co-prime numbers among the following numbers.

(i) 9 and 14

(ii) 12 and 27

(iii) 50 and 70

(iv) 77 and 97

Solution:

(i) The factors of 9 are 1, 3, and 9.

The factors of 14 are 1, 2, 7, and 14.

Therefore, the common factor of 9 and 14 is 1 only.

Thus, 9 and 14 are co-prime numbers.

(ii) The factors of 12 are 1, 2, 3, 4, 6, and 12.

The factors of 27 are 1, 3, 9, and 27.

Therefore, the common factors of 12 and 27 are 1 and 3.

Thus, 12 and 27 are not co-prime numbers.

(iii) The factors of 50 are 1, 2, 5, 10, 25, and 50.

The factors of 70 are 1, 2, 5, 7, 10, 14, 35, and 70.

Therefore, the common factors of 50 and 70 are 1, 2, 5, and 10.

Thus, 50 and 70 are not co-prime numbers.

(iv) The factors of 77 are 1, 7, 11, and 77.

The factors of 97 are 1 and 97.

Therefore, the factor common to 77 and 97 is 1.

Thus, 77 and 97 are co-prime numbers.

Example 2:

Is every pair of odd numbers co-prime?

Solution:

No, every pair of odd numbers is not co-prime.

For example, 3 and 9 is a pair of odd numbers but they are not co-prime numbers. Similarly, 55 and 77 is a pair of odd numbers which are not co-prime, etc.

Divisibility of a Number by 5 and 10

Hotel Grand Mansion has 432 rooms and 10 floors. Surbhi, who went to the hotel for the first time with her sister Shweta, was quite amazed by the numbers. She asked Shweta if she could tell whether each of the floors had the same number of rooms. Shweta replied that this is not possible as 432 is not exactly divisible by 10.

Surbhi wanted to check if what Shweta said was true. She quickly divided the numbers as

```
     43
10)432
   40
   --
    32
    30
    --
     2
```

She found that since the remainder is 2, 432 is not exactly divisible by 10.

Shweta had also said that the number 432 is not divisible by 10.

Do you want to know which trick Shweta used to give such a quick answer?

There is a rule to check if a number is divisible by 10. Similarly, we have a rule to check if a number is divisible by 5 or not.We can save the time used in division by applying this rule.

Let us consider a few examples to understand the concept better.

Example 1:

Use divisibility test to find which of the following numbers are divisible by 10.

(i) 11557468 (ii) 98746590 (iii) 95460 (iv) 74684255

Solution:

(i) 11557468 has 8 at its ones place. Therefore, it is not divisible by 10.

(ii) 98746590 has 0 at its ones place. Therefore, it is divisible by 10.

(iii) 95460 has 0 at its ones place. Therefore, it is divisible by 10.

(iv) 74684255 has 5 at its ones place. Therefore, it is not divisible by 10.

Example 2:

Which of the following numbers are divisible by 5 and 10 both?

(i) 8974 (ii) 5540 (iii) 58790 (iv) 57875

Solution:

(i) 8974 has 4 at its ones place. Therefore, it is neither divisible by 5 nor by 10.

(ii) 5540 has 0 at its ones place. Therefore, it is divisible by 5 and 10 both.

(iii) 58790 has 0 at its ones place. Therefore, it is divisible by 5 and 10 both.

(iv) 57875 has 5 at its ones place. Therefore, it is divisible by 5 but not by 10.

Example 3:

Find the value of x and y such that $98x2y$ is divisible by 10.

Solution:

The given number is $98x2y$.

We know that a number is divisible by 10 if the digit at its units place is 0.
Hence, $y = 0$.

According to the rule of divisibility by 10, the digit at the units place of a number must be 0, while the rest of the digits can take any value from 0 to 9.
Hence, x may take any value from 0 to 9.

Example 4:

Find the least number which, when

(i) subtracted from 624, gives a number divisible by 10

(ii) added to 624, gives a number divisible by 10

Solution:

(i) The given number is 624.

The number less than 624, which is divisible by 10 (which has a zero at its units place), is 620.

We can see that 624 − 620 = 4.

Hence, 4 is subtracted from 624 to get a number that is divisible by 10.

(ii) Similarly, the number larger than 624, which is divisible by 10 (which has a zero at its units place), is 630.

We can see that 630 − 624 = 6.

Hence, we need to add 6 to 624 to get a number that is divisible by 10.

Example 5:

How many numbers from 201 to 300 are divisible by 5?
Solution:
A number is divisible by 5 if and only if it ends with 0 or 5.
Now, the numbers from 201 to 300 which ends with 0 or 5 are 205, 210, 215, 220, 225, 230, 235, 240, 245, 250, 255, 260, 265, 270, 275, 280, 285, 290, 295 and 300.
Thus, there are 20 such numbers.

Divisibility of a Number by 2, 4 and 8

Ashok was standing with his friend Nikhil at a bus stop. The first bus that came was of route 432. Ashok looked at it and said to Nikhil that the route number of the bus was divisible by 2. Nikhil wanted to verify what Ashok had said and started calculating in his notebook. This is what he did:

```
   216
2)432
  4
  -
   3
   2
  --
   12
   12
  ---
    ×
  ---
```

Although he verified what his friend had said, Nikhil was amazed at Ashok's quick calculation. He asked Ashok how he did it.

Ashok said he used a trick to check the divisibility of 432 by 2. He also told Nikhil that he knows divisibility rules for 4 and 8 also.

Let us go through the following video to know these divisibility rules.

Note that **a number is also divisible by 4 if the number formed by its last two digits is 00.**

A number is also divisible by 8, if the number formed by its last three digits is 000.

For example, in the number 200, the number formed by the last two digits is 00. Therefore, 200 is divisible by 4. In the number 2000, the number formed by its last three digits is 000. Therefore, 2000 is divisible by 8.

Now, we do not have to divide a number by 2, 4, and 8 to check whether it is divisible by them or not. We can use the above stated divisibility rules.

Let us now look at some more examples to understand this concept better.

Example 1:

Which of the following numbers are divisible by 2?

(i) 48 (ii) 97 (iii) 345 (iv) 8460

Solution:

(i) The number 48 has 8 at its ones place, which is even. Therefore, 48 is divisible by 2.

(ii) The number 97 has 7 at its ones place, which is odd. Therefore, 97 is not divisible by 2.

(iii) The number 345 has 5 at its ones place, which is odd. Therefore, 345 is not divisible by 2.

(iv) The number 8460 has 0 at its ones place. Therefore, 8460 is divisible by 2.

Example 2:

Which of the following numbers are divisible by 4?

(i) 2348 (ii) 326

Solution:

(i) The given number is 2348.

The number formed by its last two digits is 48, which is divisible by 4.

Thus, the number 2348 is divisible by 4.

(ii) The given number is 326.

The number formed by its last two digits is 26, which is not divisible by 4.

Thus, the number 326 is not divisible by 4.

Example 3:

Find whether 56112 is divisible by 8 or not.

Solution:

The given number is 56112.

The number formed by its last three digits is 112, which is divisible by 8.

Thus, the given number is divisible by 8.

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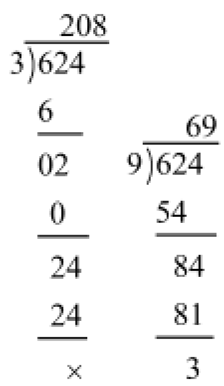
Divisibility of a Number by 3 and 9

Imagine if you had to check if numbers such as 27, 18, or 33 were divisible by 3 and 9. You will obviously check it mentally and give the answer without performing any long division. But what if you are asked to check if the number 123456789 is divisible by 3 and 9? You will take a lot of time calculating the divisibility of the number with both 3 and 9 before you give the answer.

A teacher asked the students in a class to check whether 624 is divisible by 3 and 9. Mayank said that 624 is divisible by 3 but not divisible by 9 by just looking at the number. Shikhar, another student, started performing the divisions.

This is what Shikhar did.



Although Shikhar and Mayank gave the same answer, Mayank took very little time to give the answer, while Shikhar had to perform the division. Do you know which trick Mayank used to give such a quick answer?

Mayank used two rules of divisibility to get the answer.

Let us go through the following video to learn the rules.

It can be observed that a number which is divisible by 9 is also divisible by 3.

For example, consider the number 252. The sum of its digits is 9, which is divisible by 3 and 9 both.

Let us discuss a few more examples to understand the concept better.

Example 1:

Which of the following numbers are divisible by 3?

(i) 163 (ii) 276

Solution:

(i) Sum of the digits of the number 163 = 1 + 6 + 3 = 10

Here, the sum of the digits (i.e., 10) is not divisible by 3. Therefore, 163 is not divisible by 3.

(ii) Sum of the digits of the number 276 = 2 + 7 + 6 = 15

Here, the sum of the digits (i.e., 15) is divisible by 3. Therefore, 276 is divisible by 3.

Example 2:

Find whether 477918 is divisible by 9 or not.

Solution:

The given number is 477918.

The sum of its digits is 4 + 7 + 7 + 9 + 1 + 8 = 36, which is divisible by 9.

Therefore, the given number is divisible by 9.

Example 3:

Fill in the blank space in the number 6587_41 so that it is divisible by 3.

Solution:

The given number is 6587_41.

The sum of the given digits is 6 + 5 + 8 + 7 + 4 + 1 = 31

We know that a number is divisible by 3 if the sum of its digits is also divisible by 3.

Therefore, the unknown digit can be 2 or 5 or 8 as then, the sum of the digits will be 33 or 36 or 39, and each of these sums is divisible by 3.

Example 4:

Check the divisibility of 388 by 3 and 9.

Solution:

Sum of the digits of the number 388 = 3 + 8 + 8 = 19

Here, the sum of the digits is not divisible by either 3 or 9.

Hence, 388 is not divisible by 3 or 9.

Example 5:

If *x* is a digit, then what are its possible values if the number 3*x*38 is divisible by 3?

Solution:

We know that x is a digit (i.e., x is a number between 0 and 9).

We also know that the number 3x38 is divisible by 3.

Thus, the sum of its digits will also be divisible by 3. [Divisibility rule of 3]

Hence, $3 + x + 3 + 8 = 14 + x$ is also divisible by 3.

The numbers larger than 14 and divisible by 3 are

15, 18, 21, and 24

To find the least value of x,

$14 + x = 15$

$x = 15 - 14$

$\therefore x = 1$

Similarly, $14 + x = 18$

$x = 18 - 14$

$\therefore x = 4$

Similarly, $14 + x = 21$

$x = 21 - 14$

$\therefore x = 7$

Similarly, $14 + x = 24$

$x = 24 - 14$

$\therefore x = 10$.

This is a two-digit number.

Hence, x cannot take this value.

Hence, the values of x are 1, 4, and 7 such that $3x38$ is divisible by 3.

Example 6:

The number 10*x*8 is divisible by 9. If *x* is a digit, then find its possible values.

Solution:

$10x8$ is divisible by 9.

Hence, the sum of its digits is also divisible by 9.

∴ $1 + 0 + x + 8 = 9 + x$ is divisible by 9

∴ $9 + x = 9$ or 18 or 27 ....

If we take

$9 + x = 9$

Then, $x = 0$

And, if we take

$9 + x = 18$

Then, $x = 9$

Here, we cannot take $9 + x = 27$ because it gives $x = 18$, which is a two-digit number.

Hence, the values of x are 0 or 9.

Example 7:

How many numbers from 130 to 200 are divisible by 5 but not by 3?
Solution:
A number is divisible by 3 if and only if the sum of its digits is divisible by 3 whereas a number is divisible by 5 if and only if it ends with 0 or 5.
The numbers from 130 to 200 which are divisible by 5 are 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195 and 200. Now, the sum of digits of these numbers is 4, 9, 5, 10, 6, 11, 7, 12, 8, 13, 9, 14, 10, 15 and 2 respectively. Among these sum of digits, only 6, 9, 12 and 15 are divisible by 3.
Thus, the numbers which are divisible by 5 but not by 3 are 130, 140, 145, 155, 160, 170, 175, 185, 190 and 200.
Hence, there are 10 such number.

Divisibility of a Number by 6

Consider the number 12.

Is it divisible by 6?

Yes, it is divisible by 6. Also note that the number 12 is divisible by both 2 and 3.

Consider some more numbers.

18, 42, 54, 72, 96, 264

Are these numbers divisible by 6?

On dividing by 6, all these numbers give remainder 0. Therefore, all the above numbers are divisible by 6.

Also note that the digit at ones place in each of the above numbers is even. Therefore, each of the above numbers is divisible by 2.

Also the sum of the digits in each of the above numbers is divisible by 3. Therefore, each of the above numbers is divisible by 3.

Now, can you think of any number which is divisible by 6 but not divisible by 2 or 3?

If you think of any number which is divisible by 6, then it will definitely be divisible by both 2 and 3. The reason behind it is that 6 is a multiple of both 2 and 3.

Thus, we can say that any number which is divisible by 6 is also divisible by both 2 and 3.

Using the above fact, we can check the divisibility of a number by 6. Let us state it as a rule to check the divisibility of a number by 6 as follows.

"**A number is divisible by 6, if it is divisible by both 2 and 3".**

For example, 756, 78, 696, 858, etc. are divisible by 6.

In the number 756, the digit at ones place is 6, which is even. Therefore, the number 756 is divisible by 2. Also the sum of the digits of 756 is 18, which is divisible by 3. Therefore, the number 756 is divisible by 3. Using the above divisibility rule, we can say that 756 is divisible by 6.

In the same way, 78, 696, 858 are divisible by 6.

Let us consider some more examples to understand the concept better.

Example 1:

Which of the following numbers are divisible by 6?

(i) 162 (ii) 286

Solution:

(i) The given number is 162.

We know that a number is divisible by 6 if it is divisible by both 2 and 3.

The digit at ones place is 2, which is even. Therefore, the number 162 is divisible

by 2.

Now, the sum of the digits of the number 162 is 1 + 6 + 2 = 9, which is divisible by 3. Therefore, 162 is divisible by 3.

Thus, the number 162 is divisible by 6.

(ii) The given number is 286.

We know that a number is divisible by 6 if it is divisible by both 2 and 3.

It has 6 at its ones place, which is even. Therefore, 286 is divisible by 2.

And the sum of its digits is 16, which is not divisible by 3. Therefore, 286 is not divisible by 3.

Thus, the number is not divisible by both 2 and 3 and hence, not divisible by 6.

Example 2:

Is the number 174 divisible by both 3 and 6?

Solution:

The given number is 174.

The sum of its digits is 1 + 7 + 4 = 12, which is divisible by 3.

Thus, the number is divisible by 3.

The number has digit 4 at ones place, which is even.

Thus, the number is divisible by 2 as well.

Since the number is divisible by 2 and 3 both, it is divisible by 6.

Thus, the number 174 is divisible by both 3 and 6.

Divisibility of a Number by 11

Consider the number 55. Is it divisible by 11?

Yes it is.

Now, consider the number 121. Is it divisible by 11?

Yes it is.

Is the number 868857 divisible by 11?

Now, if we check it by division method, then it will take a long time. Therefore, here we shall learn an easy rule to check whether a number is divisible by 11 or not.

Let us know this rule first and then we shall check the divisibility of this number by 11.

Divisibility of a number by 11 can be checked as follows:

If the difference between the sum of the digits at odd places from the right and the sum of the digits at even places from the right of a number is either 0 or a multiple of 11, then the number is divisible by 11.

Thus, to check the divisibility of a number by 11, we are not required to divide the number by 11; instead, we can use the above mentioned rule.

Let us now have a look at this video and find out whether the number 868857 is divisible by 11 or not.

Let's consider the number 656. If we reverse the digits of this number, then we again get back the same number. Such numbers are palindrome numbers. So, a palindrome number is a number which reads the same from left to right or right to left.

For example, 121 is a 3-digit palindrome number; 2332 is 4-digit palindrome number.

A four digit palindrome number is always divisible by 11.

Let us now look at a few more examples.

Example 1:

Which of the following numbers is divisible by 11?

(a) 286935

(b) 897562

Solution:

(a) The given number is 286935.

Counting from right, the digits occupying odd places in the number are 5, 9, and 8.

Sum of these digits = 5 + 9 + 8 = 22

Similarly, counting from right, the digits occupying even places in the number are 3, 6, and 2.

Sum of these digits = 3 + 6 + 2 = 11

Thus, difference between the two sums = 22 – 11 = 11

This difference is a multiple of 11. Therefore, the given number is divisible by 11.

(b) The given number is 897562.

Counting from right, the digits occupying odd places in the number are 2, 5, and 9.

Sum of these digits = 2 + 5 + 9 = 16

Similarly, counting from right, the digits occupying even places in the number are 6, 7, and 8.

Sum of these digits = 6 + 7 + 8 = 21

Thus, difference between the two sums = 21 – 16 = 5

This difference is not a multiple of 11. Therefore, the given number is not divisible by 11.

Example 2:

Is 829653 divisible by 11?

Solution:

The given number is 829653.

Counting from right, the digits occupying odd places in the number are 3, 6, and 2.

Sum of these digits = 3 + 6 + 2 = 11

Similarly, counting from right, the digits occupying even places in the number are 5, 9, and 8.

Sum of these digits = 5 + 9 + 8 = 22

Thus, difference between the two sums = 22 – 11 = 11

This difference is a multiple of 11. Therefore, the given number is divisible by 11.

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Special Cases of Divisibility Rules

Let us begin with an example.

Suppose Samay buys 75 sweets and distributes them equally among 25 students of his class. His friend, Sumit, buys 125 sweets and distributes them equally among the same 25 students. **Instead of this, if they both had bought 200 sweets in total, would it have been possible for them to distribute the sweets equally among those 25 students?**

Here, Samay distributes 75 sweets equally among 25 students, which means 75 is exactly divisible by 25. His friend Sumit distributes 125 sweets among the same 25 students, which means 125 is exactly divisible by 25. Now instead of this, if they both had bought 200 sweets in total, then it would be possible to distribute 200 sweets among 25 students if 200 is exactly divisible by 25. Can we find it without carrying out division?

Yes, we can solve it without division method. We can solve the above problem using a divisibility rule. We know the divisibility rules of 2, 3, 4, 5, 6, 8, 9 and 11. We do not know the divisibility rule of 25. So, let us learn some more divisibility rules and then we shall try to solve the above problem.

Let us now try to solve the above discussed problem. It is given in the problem that 75 and 125 both are divisible by 25 and we have to find if 200 is divisible by 25 or not.

Here, 200 is the sum of 75 and 125 and we know that if two numbers are divisible by a number, then their sum and difference are also divisible by that number. Thus, 200 is divisible by 25.

Thus, we can say that if they both had bought 200 sweets in total, then it would be possible for them to distribute the sweets among 25 students.

The fundamental principle on divisibility states that, if a and b are integers which are divisible by an integer $m \neq 0$,then m divides $a + b$, a −- b and ab.

Let us now see some more examples based on the above mentioned divisibility rules.

Example 1:

Two numbers 105 and 175 are divisible by 35. Check whether 280 and 70 are divisible by 35 or not.

Solution:
Here, it is given that 105 and 175 are divisible by 35.
Sum of 105 and 175 = 105 + 175 = 280
Difference of 105 and 175 = 175 −- 105 = 70

We know that if two numbers are divisible by a number, then their sum and difference are also divisible by that number.
Hence, 280 and 70 are divisible by 35.

Example 2:

252 is divisible by 18. Check the divisibility of 252 by 2, 3, 6, 9, and 18.

Solution:
Here, 2, 3, 6, 9, and 18 all are the factors of 18 and we know that if a number is divisible by a number, then it is also divisible by each of the factors of that number.
Hence, 252 is divisible by 2, 3, 6, 9, and 18.

Example 3:

Determine if the number 565128 is divisible by 24.

Solution:
The given number is 565128.
Now, 3 and 8 are co-prime numbers, since 1 is the only common factor of 3 and 8.
We know that if the given number is divisible by both 3 and 8, then it will be divisible by the product of 3 and 8, i.e., 24.
So, we have to check the divisibility of the given number by 3 and 8.
Now, the sum of the digits of the given number is 5 + 6 + 5 + 1 + 2 + 8 = 27, which is divisible by 3. So, the given number is divisible by 3.
The number formed by last three digits of 565128 is 128, which is divisible by 8.
So, the given number is divisible by 8 also.
Hence, the number 565128 is divisible by 24.

Prime Factorisation of Numbers

Consider the relation, 56 = 7 × 8

We can write 56 as a product of 7 and 8, and also note that 7 and 8 are the factors of 56.

When we write a number as the product of its factors, it is called **factorization.**

Thus, factorization of a number can be defined as follows:

The process of expressing a number as a product of its factors is called factorization.

The above way of writing the number 56 is not the only way.

We can also factorize 56 as 56 = 2 × 4 × 7 or 56 = 2 × 2 × 2 × 7

These processes are also called factorization.

Thus, prime factorization of a number can be defined as follows:

If a number is expressed as a product of its factors, all of which are prime numbers, then such factorization is known as prime factorization.

For example: Let us write the prime factorization of the number 420.

420 = 10 × 42

= (2 × 5) × (6 × 7)

= 2 × 5 × (2 × 3) × 7

= 2 × 2 × 3 × 5 × 7

Here, 2, 3, 5, and 7 are prime numbers and we cannot further factorize this number.

Thus, the prime factorization of 420 is 2 × 2 × 3 × 5 × 7.

There are two ways of expressing a number as a product of its prime factors – one is by drawing a **factor tree** and the other is by using the method of **successive division.**

Let us write the prime factorization of the number 56 with the help of a factor tree.

Let us now write the prime factorization of 56 using successive division method. In this method, the number is successively divided by prime numbers, till we obtain 1 as the result. In this method, it is best to start with the smallest prime number i.e., 2. This process can be shown as follows:

2	56
2	28
2	14
7	7
	1

Thus, by the method of successive division also, we obtain the prime factorization of 56 as 2 × 2 × 2 × 7.

Let us now look at some examples to understand this concept better.

Example 1:

Write the prime factorization of the number 1050 by successive division method.

Solution:

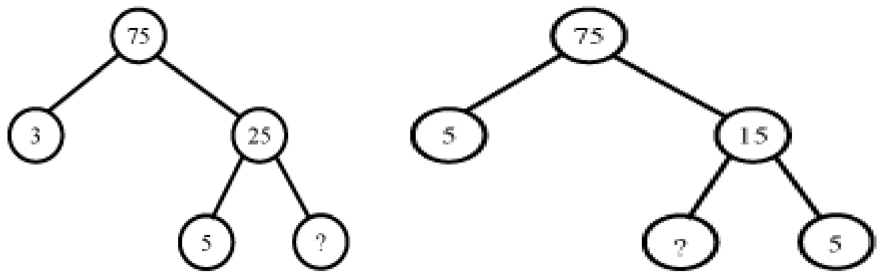
The prime factorization of the number 1050 by successive division method is shown below:

2	1050
3	525
5	175
5	35
7	7
	1

Thus, the prime factorization of 1050 is 2 × 3 × 5 × 5 × 7.

Example 2:

Find the missing values in the following factor trees.



(i) **(ii)**

Solution:

(i) 25 can be factorized as 5 × 5. Thus, the missing number is 5.

(ii) 15 can be factorized as 3 × 5. Thus, the missing number is 3.

Example 3:

Find all the prime factors of the smallest 4-digit number.

Solution:

The smallest 4-digit number is 1000.

The prime factorization of the number 1000 by successive division method is given as follows:

2	1000
2	500
2	250
5	125

5	25
5	5
	1

Thus, the prime factorization of 1000 is 2 × 2 × 2 × 5 × 5 × 5.

Example 4:

Find the smallest number having three different prime factors.

Solution:

We have to find the smallest number having three different prime factors.

The three smallest prime numbers are 2, 3, and 5.

Thus, the smallest number having three different prime factors is the product of 2, 3, and 5.

Thus, the required number is 2 × 3 × 5 i.e., 30.

Finding HCF and LCM of Numbers

The concept of highest common factor is derived from common factors and involves finding of the highest number among the common factors of given two or more numbers. Highest common factor (HCF) is also known as greatest common divisor (GCD).

We can find the HCF of given numbers by any of the two methods- **prime factorisation method or common division method.**

We have discussed about highest common factor. In the same way, can we find the highest common multiple of the given numbers?

We know that any given numbers can have infinite common multiples, so we cannot find the highest common multiple but we can find the lowest common multiple (LCM) of the given numbers.

There are two methods of finding the LCM of given numbers also. They are prime factorization method and common division method

Properties of HCF and LCM

(i) HCF of two or more co-prime numbers is 1.
(ii) LCM of co-prime numbers is equal to the product of the co-primes.
(iii) LCM is a multiple of HCF.
(iv) HCF of two or more prime numbers is 1.

Relationship between HCF and LCM

The relation between the HCF and LCM of two given numbers is given by:

> Product of LCM and HCF of two numbers = Product of the two numbers

Let us now look at some more examples to understand the concept of HCF and LCM better.

Example 1:

Find the HCF of 90, 108, and 180 using prime factorization method.

Solution:

2	90
3	45
3	15
5	5
	1
2	108
2	54

3	27
3	9
3	3
	1

2	180
2	90
3	45
3	15
5	5
	1

Therefore, 90 = 2 × 3 × 3 × 5, 108 = 2× 2 × 3 × 3 × 3, 180 = 2 × 2 × 3 × 3 × 5

Now, the common factors of the given numbers are 2 (occurring once) and 3 (occurring twice).

Therefore, the HCF of the given numbers is 2 × 3 × 3 = 18

Example 2:

Find the LCM of 9, 15, and 27.

Solution:

3	9, 15, 27
3	3, 5, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

Therefore, the LCM of 9, 15, and 27 is 3 × 3 × 3 × 5 = 135

Example 3:

Isha has 32 apples and 48 mangoes. She wants to keep these fruits in separate baskets such that there will be equal number of fruits in each basket and the two types of fruits are not mixed in any basket. Find the greatest number of fruits to be kept in each basket.

Solution:

The number of fruits to be kept in each basket must be an exact divisor of both the numbers 32 and 48.

Also, the number of fruits in each basket should be the greatest. Therefore, the number of fruits to be kept in each basket is the HCF of 32 and 48.

The prime factorization of 32 and 48 is

$32 = \underline{2 \times 2 \times 2 \times 2} \times 2$

$48 = \underline{2 \times 2 \times 2 \times 2} \times 3$

Now, the common factor of 32 and 48 is 2 (occurring four times).

Therefore, the HCF of 32 and 48 is $2 \times 2 \times 2 \times 2 = 16$

Thus, the greatest number of fruits to be kept in each basket is 16.

Example 4:

Samay and Sumit start running together around a circular park from the same point at the same time. If Samay completes one round of the park in 6 minutes and

Sumit completes one round of the park in 9 minutes, then after what time

will Samay and Sumit meet at the starting point?

Solution:

The required time would be the common multiple of 6 minutes and 9 minutes. Also, the time should be the least time. Therefore, the required time is the LCM of 6 minutes and 9 minutes.

2	6, 9
3	3, 9
3	1, 3
	1, 1

The LCM of 6 and 9 is 2 × 3 × 3 = 18

Thus, Samay and Sumit will meet at the starting point after 18 minutes.

Example 5:

The HCF and LCM of two numbers are 6 and 900 respectively. One of the two numbers is 150. Find the other number.

Solution:

HCF = 6

LCM = 900

We know that,

HCF × LCM = product of numbers

6 × 900 = 150 × required number

Required number = $\frac{5400}{150} = 36$